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8th Graders' Interpretation of Equal Sign in Scratch: Pan Balance Activities

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Abstract

The purpose of this study was to investigate how the 8th graders interpreted the equal sign in the pan balance activities they created on Scratch. Five eighth graders (4 girls and 1 boy) in Turkey participated in this study. Case study was employed as a qualitative research method. Data was gathered through the Scratch files created by the students and the think-aloud protocol interviews with them. The questions used in the activities were prepared in accordance with the test created by Matthews et al. (2012). For data analysis, content and descriptive analysis methods were employed. Findings showed that participants interpreted the operational questions as operational and relational questions as relational through using Scratch. Another significant finding emerged in Scratch activities was the fact that in order for the code block to provide an equation, the pan balance was supposed to balance. Employing Scratch in the class teachings may provide variety in learning environments for teaching of equal sign.

Keywords: Equal Sign, Scratch, Programming, Algebra Teaching

1. Introduction

In every stage of education of algebra and mathematics, the notion of *equation* is treated as a main concept, and the significance of *equal sign* is noted (Alibali et al., 2007). It is especially observed that pupils in early stages are unable to fully comprehend what *equal sign* means*,* and that they face serious difficulties in interpreting equations (Knuth et al., 2005; Knuth et al., 2006). *Equal sign* can roughly be defined as a symbol that signifies a result or a relation (Öksüz, 2007). In many studies, *equal sign* is handled as a concept which is operational and relational (Alibali et al., 2007; Knuth et al., 2005; Knuth et al., 2006; Stephens et al., 2013). For example, when asked to identify the □ in the equation 8+4=□+5, pupils expressed that □ can be defined with number 7 (after adding 8 to 4 as in the left side of the equation, and then extracting 5 from the result) (Yıldız & Atay, 2019). As seen in the example, *operational concept* defines that after the *equal sign* the result or the addition of the two numbers is expected (Alibali et al., 2007; Knuth et al., 2006). Relatively, Stephens et al. (2013) have proven that pupils have solved the equation 57+22=58+21 by applying *addition* to both sides of the equation.

Relational concept is based on acquiring equivalence on both sides of the equation (Carpenter et al., 2003). Van de Walle et al. (2010) defines *relational concept* as a process that emphasizes numerical relations rather than quantitative calculations of either side of the equation. Studies, on the other hand, indicate that *equal sign* isn't assumed as a symbol that signifies a requirement as to refer to the relations between the figures on both sides of the equation (Behr et al., 1980). Pupils tend to recognize *equal sign* as symbol that signifies result rather than a symbol that reflects relational concept (Kieran, 1981).

In formal education, it is aimed that pupils perceive the *equal sign* as a relational symbol rather than an operational one (Şimşek et al., 2019). It is generally recommended that *equal sign* to be emphasized as a relational symbol that signifies equivalence (Sert Çelik & Masal, 2018). It is important that the teachers describe the *equal sign* as a symbol that signifies the equivalence on both sides of the equation, but not as a symbol that signifies the result of a mathematical operation (Akyüz & Hangül, 2014). Baratta (2011) expressed that the notion of pan balance scale – which signifies the concept of balance – can be used to notify the relational concept of the *equal sign*. Similarly, Akkaya and Durmuş (2006) noted that a pan balance – which is a concrete material – can be employed to show the scale will still balance if same numbers were subtracted from both sides of an equation, and various computer software can also be used to concretize and generalize the subject. In conclusion, for the teaching of *equal sign*, the use of the notion of balance, and concrete materials similar to the pan balance and alike software are frequently mentioned.

Nowadays, various geometry software (i.e., Cabri Geometry, Geometer's Sketchpad, Geogebra, and Cinderella) and algebra software (i.e., Mathematica, Mathlab, and Maple) are employed in mathematics education. Also, software regarding programming exists in mathematics education. These include software like Logo, LogoTurk and Scratch that are based on programming. *Logo* -a software used in Computer Assisted Mathematics Education developed by Seymour Papert in Massachusetts Institute of Technology (MIT) Artificial Intelligence Laboratory- is an application that helps students to learn via programming. *Logo* uses basic programming logics and simple graphic commands in order to support analytical and visual thinking (Baki, 2000). Since it is based on constructivism, *Logo* is a very efficient tool at concretization of abstract concepts (Ar, 2012). Another software nowadays used in programming and gamification is *Scratch*. The coding program *Scratch,* is a visual programming environment developed by Lifelong Kindergarten group -a part of MIT Media Lab- designed for the use of individuals aged between 8 and 16 (Scratch, 2017). *Scratch* mainly aims to provide to the newcomers a nice and endearing introduction to the world of programming (Erol, 2015). Besides programming education, gamification can make the teaching of classes such as mathematics, science, foreign language, and social sciences more enjoyable (Çatlak et al., 2015). In this context, the purpose of this study was to investigate the eighth graders' interpretation of the equal sign in pan balance activities they created on Scratch; and the research question was: *How do eighth graders employ equal sign in their pan balance designs using Scratch?*

1.1 Theoretical Context

1.1.1 Equal sign

One of the important symbols in algebra is *equal sign* (Alibali et al., 2007). *Equal sign* is a relational symbol that signifies that both sides of an equation are equivalents and are exchangeable (Kieran, 1981). Many studies put forward that students pupils *equal sign* as an operational symbol not a relational one (Behr et al., 1980; Byrd et al., 2015; Falkner et al., 1999; Kieran, 1981; Knuth et al., 2005; Knuth et al., 2006; Yaman et al., 2003). For example, Kieran (1981) stated that elementary and high school and university students perceived *equal sign* as a symbol that divides the operation and the solution. Baiduri (2015) expressed that undergraduate students perceived *equal sign* not as a relational symbol but as operational. Behr et al. (1980) denoted that even though the ages of pupils' progress, the perception of *equal sign* remained the same. Despite that fact that pupils are expected to comprehend *equal sign* as a relational symbol signifying that expression on both side of an equation are equivalents, they perceive as an operational symbol.

The interpretation of the *equal sign* by pupils has an influence on their comprehension of mathematical equivalence and relation in early algebra education (Byrd et al., 2015). Generally, a mathematical equivalence represented by *equal sign* signifies the principle that both sides of an equation represent equivalent symbols (Rittle-Johnson et al.,

2011). Rittle-Johnson et al. (2011) expressed that the comprehension of a mathematical equivalence requires perceiving that both sides of an equation represent equivalent symbols. Similarly, Knuth et al. (2006) pointed out a meaningful relation regarding pupils' in solving equivalent equations, who perceived *equal sign* as a relational symbol. Baiduri (2015) stated that the number of pupils using *equal sign* in solving equivalent equations was low. It was observed that pupils who perceived *equal sign* as a relational symbol performed better with equivalent equations (Alibali et al., 2007; Knuth et al., 2005). Students, in defining equivalent equations, have not used the conversions that they referred to in solving equations (Steinberg et al., 1991). Temur and Sancak (2012) expressed that in order to encourage students to find out different methods and develop strategies to explore equal sign, asking questions (e.g., why, how, is there an alternative method for the solution, and is an easier method possible?) might support their learning process. To sum up, students who perceive *equal sign* as a relational symbol are observed to better comprehend mathematical equivalence expressions and to be more successful at solving equations that contain equivalent expressions.

Rittle-Johnson et al. (2011) aimed to prepare an assessment that will define systematically changes in the knowledge of equivalence. Mathematical equivalence signifies the principle that both sides of an equation represent equivalent values. They stated that equivalence assessments - in order to meet their individual needs - may help the educators with evaluating and diversifying education. Four different levels were referred to for the coding of the test. These levels have been also referred to in determining the students to take part in this study. Similarly, Matthews et al. (2012) conducted a study to define how much the perception of *equal sign* by children contributes to algebraic thinking. During the research, to assess the knowledge of equal sign, evaluation scales (Rittle-Johnson et al., 2011) -examined and rearranged by four different mathematic educators experienced in researching the algebraic thinking in children- was used. Evaluation clauses were mostly based upon previously published studies or queries included in previously published studies. Queries were handled in three different categories: (Equation-solving items), deciding if it right or wrong (Equation-structure items), and definition of *equal sign* (Equal-sign items). Study included advanced reasoning items such as conversions regarding the preservation of equivalence ($76 + 45 = 121$, $76 + 45 - 9 = 121 - 9$) and equation items that embodied variable letter ($n+n+n+2=17$). It is stated that students with advanced knowledge of *equal sign* were more likely to solve the algebraic items included in the test. This study was conducted with participant who had no former experience with variables or letters in equations. It was observed that participants faced more difficulties in solving equations that included variables (letters) than maintenance of equivalence that included only numeric. It was also concluded that the use of pen and paper test technique for this study has been a restricting factor and that students had abilities and knowledge that they were unable to put forward due to use of pen and paper test technique. It was also noted that the unification of interviews with the participants and the written evaluations would help build a better understanding of the performance of the participants.

Studies regarding *equal sign* are usually conducted with the participation of students from different classes (i.e., Knuth et al., 2005; Stephens et al., 2013; Yaman et al., 2003). Studies conducted with the participation of single class students also exist (Falkner et al., 1999; Yıldız & Atay, 2019). In addition, during the data collection stage of these studies, participants were given questions on paper (i.e., Alibali et al., 2007; Knuth et al., 2006; Matthews et al., 2012) and they were also interviewed (i.e., Yaman et al., 2003; Yıldız & Atay, 2019). Stephens et al. (2013) recorded that it is possible to collect data via individual interviews with the participants. Furthermore, the age 13 has been noted as critical for the relational comprehension of *equal sign* (Yaman et al., 2003). Therefore, it is considered to be fair to conclude if and when a research about *equal sign* will be conducted, the participants of the study may be of a single grade, and the ages of participants must be 7th grade minimum. Various studies have concluded the fact that pupils perceive *equal sign* as a figure that signifies result rather than, in fact, a figure that signifies equivalence, and that they face difficulties in comprehending the use of *variables* (Akkaya & Durmuş, 2006; Knuth et al., 2005). Providing a relational perception of equal sign helps pupils to be more successful in solving equivalent equations (Knuth et al., 2005). Studies have shown that even though *equal sign* is presented as a symbol that signifies a relational concept, it is perceived by the pupils as an operational symbol.

1.1.2 Scratch

Scratch is a simple programming software that enables users to code by using code blocks. It is a free programming language with which it is possible to create interactive stories, games, and animations (Saez-Lopez et al., 2016).

Scratch programming language offers programming blocks grouped in eight different categories: motion, looks, sound, events, control, sensing, operators, and variables. Pupils can freely choose code blocks required to make real life objects move like they would in the real world (Kakavas & Zacharos, 2019). It is possible to use Scratch software in teaching of many classes since, with the help of diverse multimedia items, it enables putting forward various projects (Çatlak et al., 2015).

Saez-Lopez et al. (2016) recorded that it is possible to make use of visual programming language software in order to understand logic, mathematics and content creation items. Scratch helps students develop and utilize their mathematical thinking abilities (Taylor et al., 2010). Calder (2010) noted that Scratch functions as an incentive for the students' learning mathematical concepts, and that it could be used during the problem solving process and developing mathematical thinking. Gökdağ et al. (2022) emphasized students developed aesthetic experiences through exploring tessellations with Scratch. Kilhamn and Brating (2019) stated that during the programming educations given at schools, students will encounter algebraic variables, structures, and patterns. Calao et al. (2015) emphasized that the use of Scratch in Mathematic classes creates a motivational teaching environment where students develop their problem solving, reasoning, and mathematical exercise performances. Lai and Yang (2011), after interviews with the students, reflected that they with low academic achievement are less interested in classes, and that employment of Scratch in lessons may provide an individual learning opportunity. Lewis and Shah (2012) expressed that Scratch supports the mathematical understanding, and that is especially efficient in comprehension of geometrical relations, and that the teaching of programming curriculum is highly dependent on knowledge of mathematical content. Programming applications provide the students the opportunity to consolidate and enrich mathematical content. To sum up, it is observed that middle school students enjoy coding activities using Scratch and they find the application to be easy to use and entertaining. It is also observed that the use of Scratch in Mathematics classes helped students to motivate to the lessons better. While programming with Scratch, students were observed to be efficient in presenting and utilizing their mathematical skills. Students with low mathematical skills were observed to get bored when using Scratch, and they mostly tend to dislike the application.

2. Method

Single case-holistic design -one of the qualitative research designs- was employed in this study to understand the question *why* to perform an in-depth examination of event that the researcher cannot control (Yıldırım & Şimşek, 2013). In this context, the process of 8th graders' creating pan balance activities using Scratch were thoroughly examined by researchers asking *why questions* to the participants.

Five 8th graders (4 girls and 1 boy) in Konya participated in this study. Participants were selected according to criterion sampling. The *criterion* was the participant to be familiar with Scratch and to score Level 4 in the test developed by Matthews et al. (2012). Participants, who scored Level 4 in the test, successfully solved and evaluated the equation by comparing the expressions on both sides of the equation through various strategies. While participants were creating pan balance activities on Scratch, they were asked to evaluate equations. Real names of the participants were kept hidden; instead, they were referred to with pseudonyms: Sude, Furkan, Rümeysa, Helin and Ayşe.

2.1 Data Collection

The data source of this study consisted of think-aloud protocol interviews with students and Scratch files created by the participants. The activities of participants using Scratch were screen recorded; and these code blocks and outputs were examined thoroughly.

 2.1.1 Think-Aloud Protocol Interviews

Patton (2018) defines the strategy of *think-aloud protocol interviews* as participants expressing their emotions and thoughts into words while they are engaged in an activity. Participants were asked *why questions* to reveal their thoughts while they were working on activity worksheets. Questions included in the activity worksheets were

prepared by the researchers and an expert with a PhD in mathematics education. Among 31 questions prepared by Matthews et al. (2012) four questions were selected and revised to be used in Scratch. The questions are as follows:

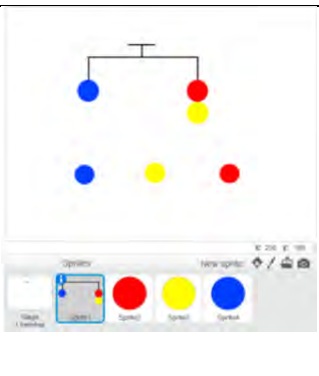> *Question (Q1) of the first activity*: Do you think the expression 7=3+4 is right or wrong? Open the 3rd file in Scratch and design your activity.
>
> *Question (Q2) of the second activity*: Do you think the expression 31+16=16+31 is right or wrong? Open the 5th file in Scratch and design your activity.
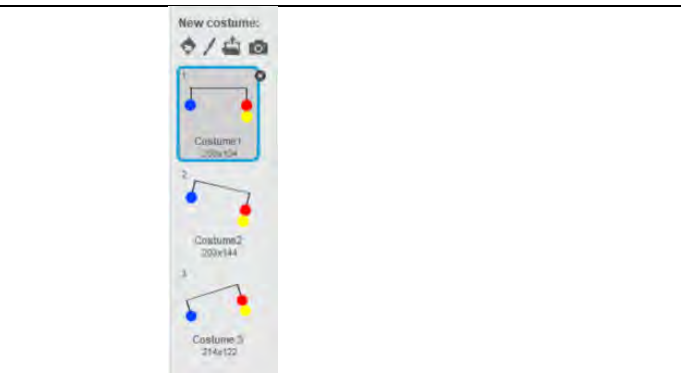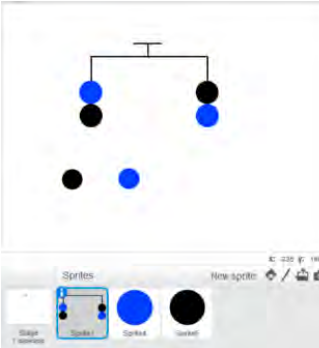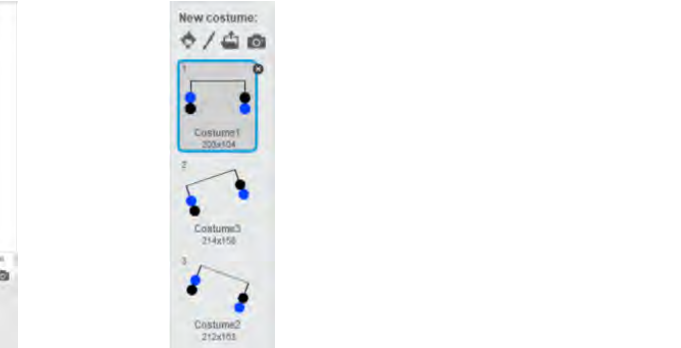>
> *Question (Q3) of the third activity*: Do you think the expression 56+85=141 is right or wrong? Can you tell if the expression 56+85-7=141-7 is right or wrong without extracting 7 from either side of the expression? Open the 8th file in Scratch and design your activity.
>
> *Question (Q4) of the fourth activity*: Can you tell if the expression 67 + 86 = 68 + 85 is right or wrong without performing the addition 67+86. Open the 9th file in Scratch and design your activity.

These questions contained the different levels of mathematical equivalence knowledge developed by Rittle-Johnson et al. (2011). Q1 was aimed to assess the students' solving, evaluating, and encoding atypical equation structures coherent with the operational aspect of equal sign (Level 2), whereas Q2 tested their solving, evaluating, and encoding the equivalence structures on both sides of the equation (Level 3) by using Scratch. Q3 and Q4 assessed the students' knowledge of solving and evaluating the equations by comparing the expressions on both sides of the equation including the employment of compensatory strategies and conversions providing the equivalence (Level 4) by using Scratch.

2.1.2 Student Scratch Files

Files and outputs that include Scratch code blocks used in the activities created by the students were used in the data analysis. Since the students were expected to create pan balance activities, they were given the scene and characters that they could use with Scratch. They were expected to write codes for these characters. Code blocks were analysed to determine what kind of strategies the students referred to while creating the activities. Scratch files given to the students are as in Figure 1.

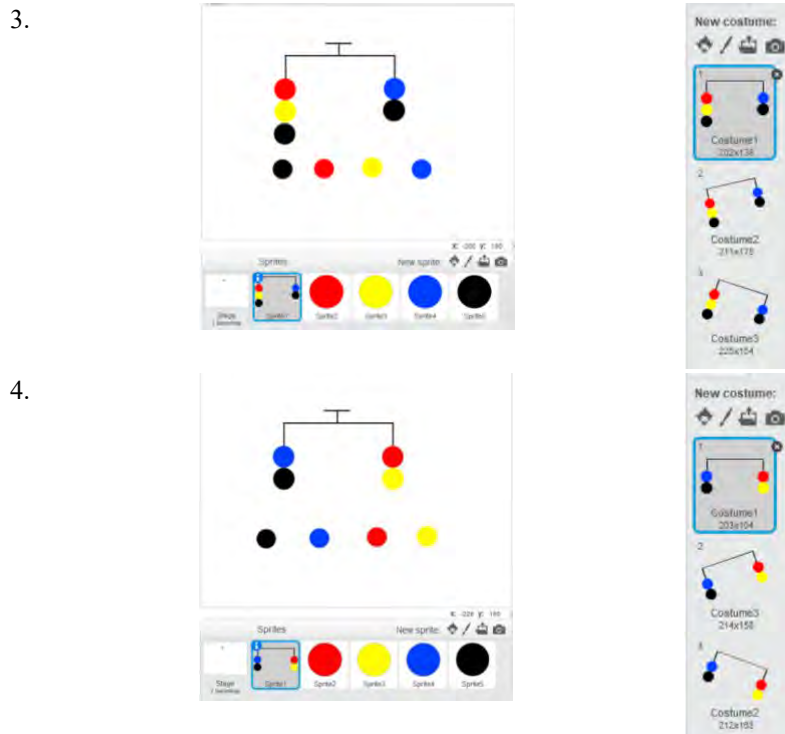| Question #	Code Blocks
1.	
2.	

3.

4.

Figure 1: Scratch files given to students

Images seen in Figure 1 were given to participants were instructed to create the required activities. For each activity, students were given *costumes* that provided the shapes to exhibit different movements. A *Costume* was the various animations of the characters included in the Scratch files. Costumes were created because in the Scratch file only Character-1 represented the movement of the scale. These costumes provided the shape to be able to lean to left and right. The images administered to the students were selected to be similar to those included in the activity worksheets. Students were instructed to create the activities making use of the images and the code blocks. Coloured characters represented the variables.

 2.2 Data Collection

Individual interviews with the students were performed. It was observed that students spent the most time in their first activity, and that they spent less and less time as the students performed more activities. The reason why the students spent the most time for the first activity was considered to be the fact that it was not very easy for the students to develop strategies for a pan balance activity using Scratch.

The students were asked to read the questions on the activity papers, and their answers were recorded. Then, they opened the Scratch files given to them. While they were in the process of creating the activities on Scratch, they were asked questions aiming to explore the reasons why they created their activities the way they did. Participants were allowed to have breaks between the activities.
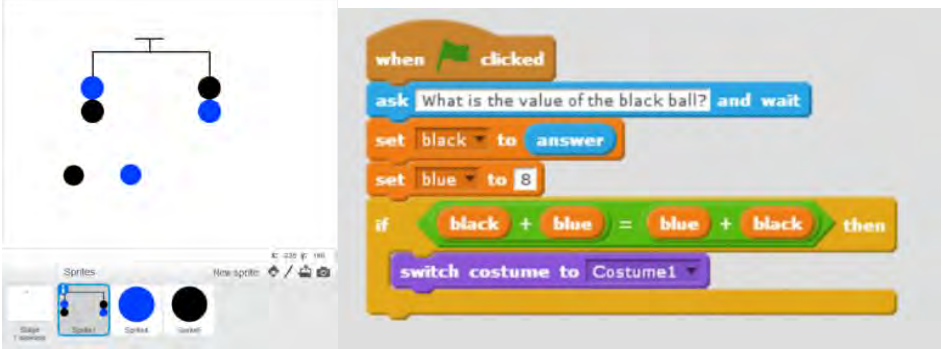
 2.3 Data Analysis

The data consisted of the recordings of think-aloud protocol interviews and the Scratch code blocks created by the participants for the four different activities. *Content analysis* and *descriptive analysis* methods were employed. Firstly, the recordings of think-aloud protocol interviews were transcribed to Microsoft Word files for each student. The transcripts were analysed based on the interpretation of equal sign as *operational* and *relational* (Knuth et al., 2005). The way how the participants interpreted the equal sign was examined. For instance, we interpreted the equal sign as *operational* when the participants checked if expressions on both sides were equal or not by making addition; and as *relational* when they performed comparisons of the expressions on both sides of the equation. Examples of the participants' interpretations were given in Table 1.
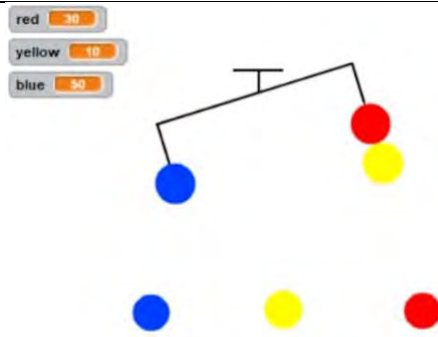
Table 1: Samples of participants' interpretations of equal sign

	Relational	Operational
1. Activity	We can express with different numbers. That is, instead of 7=3+4, here we can write 7=1+6. (Helin)	Yes. 7=3+4. When add 3 and 4, we get 7 that is 7 equals to 7. (Sude)
2. Activity	Replacing the numbers with each other doesn't change the sum. (Rümeysa)	
3. Activity	Figures on both sides of the equation were already equal to each other. When the same number is subtracted from both sides, the result will equal to each other on both sides. (Sude)	
4. Activity	Added 1 to one side and got 68, subtracted 1 and got 85. It is the same on both sides (Sude)	

Scratch code blocks were used to provide information about how the participants created code blocks providing balance or imbalance for the scale. Table 2 shows how they used the code blocks in their activities.

Table 2: Samples of participants' code blocks

Scratch code blocks	

Student's interpretation (Balance)	Both sides of the equation contain the same things. This time we do not have three options. Because, no matter what numbers are appointed, the sum of blue and black will always be equal to each other. Numbers appointed to blue and black can be changed, but it will not affect the equation (Rümeysa)
Student's interpretation (Imbalance)	If blue equals to red plus yellow, switch to Costume 1. If not, a different costume is needed. Red + yellow can be greater or smaller than blue. Therefore, I added two more costumes. If blue is smaller than red + yellow, switch to Costume 2; if greater switch to Costume 3. (Helin)

The first author carefully followed the activities during data collection, and aimed to obtain in depth knowledge by asking the students *why* and *how* questions. Activity worksheets and Scratch files administered to the participants were analysed by the second author, who had PhD degree in mathematics education and by the third author, who had PhD degree in computer and education technology. Moreover, all transcripts and code blocks were independently coded by three researchers. All disagreements were resolved through discussion. Coding continued until the researchers came to an agreement.

3. Results

Findings were analysed as *relational* and *operational* strategies used in the interpretation of equal sign. Findings were evaluated according the activities they were instructed to create. Table 3 contains the interpretations of the participants classified as relational and operational.

Table 3: The interpretations of the participants classified as relational and operational

	1st Activity		2nd Activity		3rd Activity		4th Activity	
	R	O	R	O	R	O	R	O

Sude	x	x	x	x
Furkan	x	x	x	x
Rümeysa	x	x	x	x
Helin	x		x	x	x
Ayşe	x	x	x	x

Note: R: Relational, O: Operational

While the first activity contained an operational question, the other three activities consisted of relational questions. As seen in Table 3, all students perceived the second, third and fourth activities as relational. It is also noted that Helin also interpreted the first activity as relational.

3.1 First Activity

In the first activity students were asked "Do you think the expression 7=3+4 is correct or not?" and were instructed to open the first file on Scratch and create the activity. All students have replied as *correct*. Table 4 contains the answers of the students.

Table 4: Students' interpretations of equation

Student	Statements
Sude	Yes, 7=3+4. When you add 3 to 4 you get 7, that is 7 is equal to 7.
Furkan	Seven is equal to three plus four. When we add three to four, we get seven. This proves that seven is equal to seven.
Rümeysa	When I add three to four, I get seven. That is, they are equals.
Helin	We can increase the expressions. That is, instead of 7=3+4, we can also write 7=1+6
Ayşe	Let's assume that you have four pencils. Someone comes and gives you three more pencils. Now, you can count and see you have seven pencils. Let's also assume another person who also has seven pencils. You ask him/her "is the number of pencils you and I have equal to each other?" Upon counting the pencils both you and the other person you get the number seven, so yes, equal.

Only Helin interpreted the equation as relational by comparing the figures without making an addition. The rest of the participants (Sude, Furkan, Rümeysa, and Ayşe) interpreted the equation as operational, by making addition. Figure 2 contains the code blocks created by the students.

Student	Code blocks
Sude	

Ayşe
(left column)

Furkan
(right column)

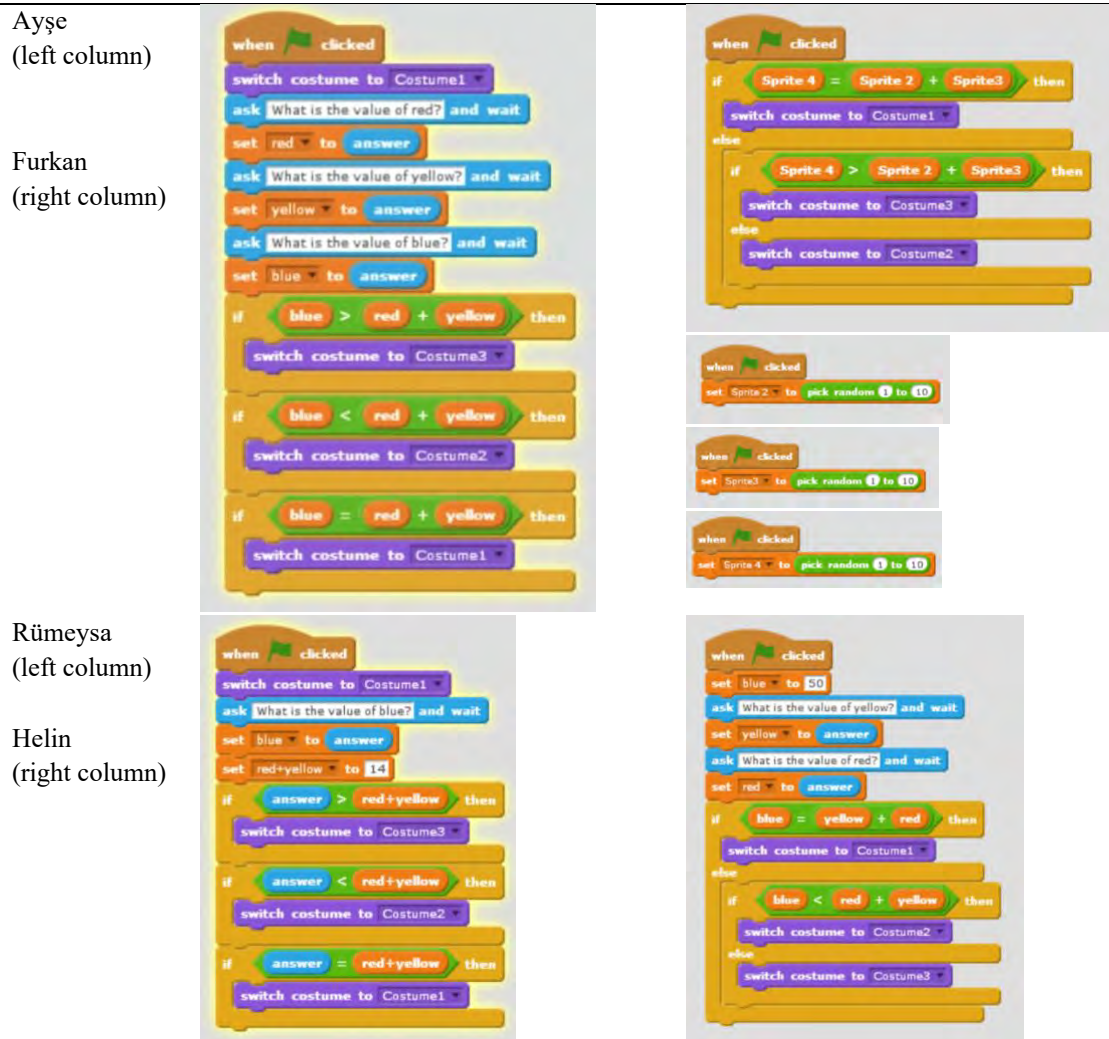Rümeysa
(left column)

Helin
(right column)

Figure 2: Code block samples created by the students in the first activity

Sude created an activity signifying the shape will never change. All other four students (Furkan, Rümeysa, Helin, and Ayşe) created additional activities where the shape unbalances and leans to right or left despite defining the equation as *correct*. Students, by assigning numbers to the round shaped sprites, made the shapes move. They thought the shape must unbalance and lean right or left when the numbers assigned to sprites were changed. Conversation with Sude: (P is Participant, R is Researcher)

      P: I will see if the equation is correct or not by creating the codes

      R: hmmm

      P: I will check this

      R: Let's think

      P: Now blue = yellow + red

      P: Then, it should also be replaced here

      R: Ok. Does it always shift?

      P: But, there is an equation here. No, it doesn't shift.

      P: When we add yellow, yellow and red…..

      R: Yes?

      P: We should show they equal to blue

      P: If they are equal, the scale must balance.

      R: Right. Why did you place the sum to the right instead of left?

      P: It matters not. I can place the sum on the left as well.

      R: Why did you prefer to place the 3+4 to the right instead of left?

      P: Well, because 7=3+4, that is red + yellow, on the right side

      R: Which side?

P: Right side

R: So that's why you placed red + yellow on the right.

P: Yes. If it is equal, it is ok.

R: How will the scale behave if it is equal?

P: What does it add to? Which costume is this? It must be equal, ok, costume 1 it is. Is it always equal? Switch to costume 1 constantly.

R: Ok. It will remain in costume1 constantly. How should the scale behave if yellow plus red is greater than blue? But, when I assign random numbers, I now notice that blue has a smaller number. How should the scale act in this case?

P: It should switch to another costume.

R: Switch? How?

P: If blue is smaller?

R: Yes.

P: Then the right side will be greater

R: What happens when the right side is greater? Does the scale lean towards right?

P: Yes.

R: What are you expected to provide?

P: Equality

P: That is blue= red + yellow

While designing the activity, the student codes as instructed blue= red + yellow; however, she referred the expression as 3+4=7 when defining it as *correct*. The same pattern was observed in other participants (i.e. Rümeysa, Ayşe, and Furkan) as well. Additionally, the students recorded that blue could be either greater or smaller than the sum of red and yellow, but since she was instructed to provide an equation, she coded accordingly. Similarly, Helin noted *Yellow plus red must be equal to blue*, but the rest of the participants (i.e. Rümeysa, Ayşe, and Furkan) expressed that there could occur three different results. There were some statements of students in the following.

> If blue equals to red plus yellow, switch to Costume 1. If not, a different costume is needed. Red + yellow can be greater or smaller than blue. Therefore, I added two more costumes. If blue is smaller than red + yellow, switch to Costume 2; if greater switch to Costume 3. (Helin)

> Well, let's see. If both sides are equal, the scale will balance. Red plus yellow equals to fourteen, then if blue is greater or red plus yellow is greater, the scale will lean towards the greater side. (Rümeysa)

> If blue is greater (than red + yellow), the scale should lean this way, which is Costume 3. If red plus yellow is greater (than blue), switch to Costume 2. I need one more costume for balance. There are only 3 possible results. Either left side of the equation is greater, or the right side, or both sides are equal to each other. (Ayşe)

> What happens if Sprite 2 plus Sprite 3 is greater? The scale should lean towards right, so switch to Costume 2. If Sprite 4 is greater, switch to Costume 3, if not to Costume 2. What happens if I use *if not*? Let me try. It didn't switch to Costume 1. (Furkan)

Participants coded the double pan scale to act according to the figures appointed to each colour. They ignored the fact that they were expected to pay attention to the point that blue must equal to the sum of red and yellow as instructed in the question.

3.2 Second Activity

In the second activity, the students were asked to identify the expression 31+16=16+31 as correct or wrong, and create an activity on Scratch using the second file. All participants identified the expression as *correct*. Sude claimed about her solution: "When making an addition, placement of the addends doesn't affect the sum. In other words, when making an addition, we can replace the addends with each other and still get the same sum". Furkan interpreted the equal sign in the same way: "This signifies to the fact that when making an addition, the placement of the addends does not affect the result. When making an addition, if we replace the addends with each other and the sum is equal to each other, it means that the expression is correct." Ayşe' explanation was also similar: "Both sides of the equation are the same. The left side is 31+16, and the right side is 16+31. Both sides consist of same

addends and placement of the figures does not affect the sum when making an addition." All students signified the aspect that even when addends are replaced with each other, both sides of the equation will remain equal to each other. It could be stated that they interpreted the question as *relational*. Figure 3 contains the activities created by the students.
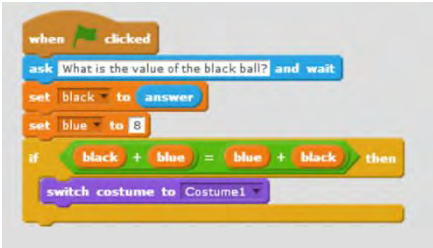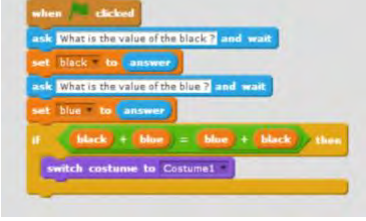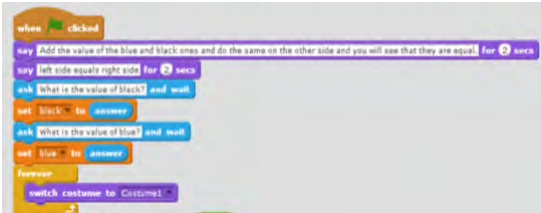
Student	Code Blocks
Sude (left column) Furkan (right column)	
Rümeysa (left column) Helin (right column)	
Ayşe	

Figure 3: Code block samples created by the students

Sude, Helin, and Rümeysa used only *equal* costume, Furkan provided two additional costumes where the scale will lean to right or left, and Ayşe used an explanatory code block. They created codes that only will provide balance for the scale. Two additional costumes provided by Furkan would not function. Even though all students pointed to the aspect that the placement of addends did not affect the sum, only Sude, Rümeysa, and Helin referred this feature in their codes. Interview with Ayşe:

> P: How can I conclude that blue plus black will be equal to black plus blue? Well, if it is equal, will switch to Costume 1 which is balance. Eight and one. The scale will go off balance.
> R: If eight and one are appointed, will it go off balance?
> P: Yes.
> R: Let's try. What number did we appoint to black?
> P: Wait a second. It will not go off balance.
> R: Why not?
> P: Because they are the same. Even though they placed differently, both addends contain the same numbers.
> R: So?
> P: So, if I appoint 1 for blue, and eight for black, both sides of the equation will sum to 9, and will not go off balance.
> R: Why did you switch to *if* from *if not?*
> P: There is no *if not*. The sum will always be the same. Because both sides of the equation consist of only blue and black. The places of blue and black are different, but the sum will always be equal to each other.

Ayşe, at first, mistook that the scale would balance only if blue was equal to black; then realized that even if she appoints different numbers to the colours both sides would remain equal to each other. Rümeysa similarly recorded:

> Both sides of the equation contain the same things. This time we do not have three options. Because, no matter what numbers are appointed, the sum of blue and black will always be equal to each other. Numbers appointed to blue and black can be changed, but it will not affect the equation" She also stated the reason for using *if* as *if not* command is not applicable.

Rümeysa, Helin, and Sude all used a single *if*. Furkan explained why he used two intertwined *if not* codes:

> How should it function if equal? It should switch to Costume 1. If not equal, we will need another button. I will code if Sprite 4 plus Sprite 5 is greater than Sprite 5 plus Sprite 4. What will happen if Sprite 4 plus Sprite 5 is greater than Sprite 5 plus Sprite 4? Since the sum is greater, left pan will go down and right pan will go up, which is Costume 2.

He also noted "I need to provide sprite 4 plus Sprite 5 equals to Sprite 5 plus Sprite 4." and provided the equation by the numbers he appointed to the sprites. During this activity, participants realized that when both sides of the equation contain same addends, the scale must balance. Accordingly, students created code blocks that will provide balance for the scale. Especially, Sude, Helin, and Rümeysa have shown in their codes that the placement of addends will not change the sum.

3.3 Third Activity

In this activity, participants were asked if the expression 56+85=141 is correct or not, and if they could tell if the expression 56+85-7=141-7 is correct or not without making a subtraction; then they were instructed to create code blocks Scratch using the third file. All students identified the expressions as *correct*. There were some examples of students' statements regarding their equal sign strategies:

> It is correct. 56 plus 85 sums to 141. Both sides were already equal to each other before adding minus 7 to both sides. Subtracting same number from both sides will not affect the equation. The result will remain equal to each other. (Sude)
>
> Correct. Erm, without making a subtraction. Well, subtracting seven from both sides does not affect the equation. Because we are doing the same operation on both sides, subtracting seven. Let's assume that we added seven to both sides, this wouldn't affect the equation either. Both sides would remain equal to each other. (Rümeysa)
>
> Yes, correct. There already is an equation. This means the scale will balance. If we put 5 kg apples to both pans, the scale will not go off balance. Because we added equal amounts to both sides. Same applies to the equation. Adding to or subtracting from the same numbers will not affect the equation, the result on both sides will remain equal to each other. Since the sum of 56+85 was already equal to 141, adding or subtracting any number will not affect the equation. The result will remain equal, and the scale will remain balanced. (Helin)

All participants pointed to the fact that subtracting same number from both sides would not affect the equation and that the scale would remain balanced. Figure 4 contains the activities created by the students.
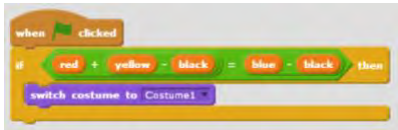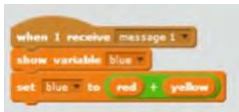
Student	Code blocks
Sude (left column) Furkan (right column)	

Rümeysa
(left column)

Helin
(right column)

Ayşe

Figure 4: Code block samples created by the students

All students except Furkan used a single *if* in their activities. While Sude, Rümeysa, Helin, and Ayşe created activities signifying the balance, the code created by Furkan included options that signified imbalance. Even though Furkan noted:

> If we subtract the same number from both sides of an equation, the result would remain equal after the subtraction. The scale would remain balanced. In order for the scale to remain balanced, the result on both sides of the equation must remain equal. In order for the scale to balance, we need to subtract same number from both sides.

He could not provide this in his code block he created for Sprite 1. Code blocks he created for Sprite 2, Sprite 3, Sprite 4, and Sprite 5 all result in imbalance; because all codes include random numbers, resulting in scale leaning right or left. Rümeysa stated that

> If equal it is balanced, if smaller leans to right, if greater leans to left. Let's appoint a number to blue. One second, if blue is red plus yellow, it is not possible to appoint a number to blue. Let me appoint red and yellow. Black will be appointed. Where is blue? Hold on. Blue doesn't exist. Well, blue will be calculated automatically. So, it (blue) cannot be smaller or greater.

Rümeysa at first considered she needed to create code blocks for imbalance, but upon realizing that the sprite (blue) is equal to the sum of two sprites (red and yellow), she created a code that result in balance only. Similarly, Sude said that "We need to provide that blue equals to yellow plus red. The sum of red and yellow equals to blue, and we subtract black from both sides, so both sides remain equal." Helin also stated that "We need to show yellow

plus red equals to blue. So, in this equation, we need to show that the result will remain equal if we subtracted or added the same number to both sides". Ayşe claimed that

> Blue equals to the sum of red and yellow. What need to do is to show that both sides are equal, and subtract the same number from both sides. That's why it will switch to costume1 (balance) when the same number (black) is subtracted from both sides in the *if* block.
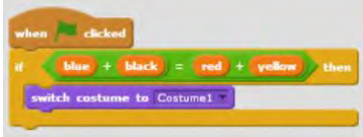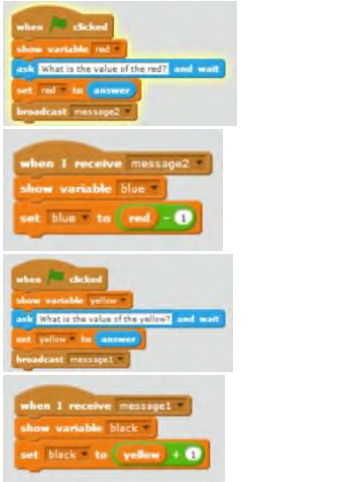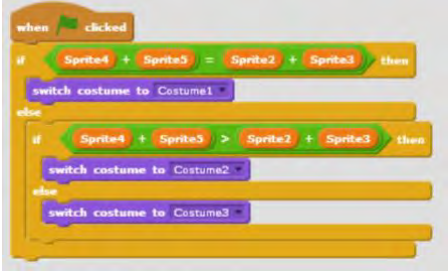
All students, except Furkan, created activities that signified balance.

3.4 Fourth Activity

In the fourth activity, the students were asked to identify the expression 67 + 86 = 68 + 85 as correct or wrong without performing the addition 67+86; and create an activity on Scratch using the fourth file. All participants identified the expression as *correct*. For instance, Sude stated that "Correct. Because, plus one is added resulting 68, and minus one subtracted resulting 85. So minus one and plus one equals to zero, and equation remains the same." Furkan explained it by exemplifying the sprites:

> I will have two sprites, and by performing addition with these sprites….. Actually, I need four sprites. By performing addition of the first two sprites…… If the sum of first two sprites is equal to the sum of other two sprites, the scale will balance; if the sums are not equal the scale may lean to right or left. 67 + 86 = 68 + 85, first cluster includes 67, and the second cluster 68, so second cluster is plus one. Also, first cluster contains 86, and the second 85, so the second cluster is minus one. Now it is always balanced. Where did I go wrong? Sprite2 (67) is one less than (68)… and (86) is one greater than (85); this will provide balance. (Furkan)

All students interpreted the equations as relational. They realized that one addend is added and the other is subtracted. Figure 5 contains the activities created by the students.

Student	Code Blocks
Sude (left column) Furkan (right column)	

Rümeysa
(left column)

Helin
(right column)

Ayşe

Figure 5: Code Block Samples

All students employed a single *if*, except Furkan. While all four students created activities signifying balance, Furkan created an activity signifying imbalance. Furkan simply managed to provide balance, by appointing numbers to the variables in his code block. All students identified the question as correct, and reflected this on their activities. Furkan explained how they managed the balance: "The scale will balance only if the sum of Sprite 2 and Sprite 3 is equal to the sum of Sprite 4. If not the scale may lean towards right or left". Furkan created code blocks accordingly. However, he was able to provide balance by appointing certain numbers to Sprite 2, Sprite 2, Sprite 4 and Sprite 5. He stated "Where did I go wrong? Sprite 2 (67) is one less than (68)… and (86) is one greater than (85); this will provide balance. Similarly, Helin signified the balance in this way:

> The scale will balance. If I appoint 67 for blue, and 86 for black, erm, and then I suppose red should be 67 plus one that is 68, and yellow should be 86 minus one that is 85. There are three costumes, but it will always switch to Costume 1. Because, both sides will always be equal to each other.

Rümeysa also said that "Black plus one equals to yellow. Blue minus one equals to red. Red and yellow will be appointed automatically. That is, the scale will always balance, which means Costume 1". It could be argued that while the students were designing the activities, they paid attention to the balance of the scales. For this purpose, that they wrote code blocks that would keep the scales in Costume 1 constantly. When the students wrote the code blocks in which the scales lean towards right or left, they kept the scales in balance with the code blocks they wrote to the variables they created.

4. Discussion

Of the four questions administered to participants, Q1 is operational and Q2, Q3, and Q4 are relational. In Q1, participants were asked 'Do you think the expression 7=3+4 is correct or wrong?' and almost all students made the statement 'three plus four equals to seven'. Behr et al. (1980) noted that when the left side of an equation consists of a single figure, students tend to rearrange the equation by replacing the figures on the right with the one on the left. Similarly, all participants were observed to replace the figure on the left with the figures on the right. Despite verbally describing the equation as 'three plus four equals to seven' (3+4=7), when coding the students were observed to refer to the equation in its original state (7=3+4). Helin, who provided a relational response, recorded 'We can increase the expressions. That is, instead of 7=3+4, we can also write 7=1+6'. Despite stating that both sides of the equation are equal to each other, only Sude was observed to take this fact into account in her Scratch coding. Sude was observed to employ a single *if* in her coding, aiming for the scale to balance.

The second question administered to students was 'Do you think the expression 31+16=16+31 is correct or wrong?' which is a relational question. All students pointed to the fact that when performing an addition if the addends were replaced with each other, the sum would remain the same. The fact that all students interpreted this question as relational coheres with the findings of Behr et al. (1980). In a research conducted by Behr et al. (1980) with participants aged six and seven, it was recorded that the participants were unable to interpret the equation 3+2=2+3, that both sides of the equation are equal to each other. It was also noted that in an operation that contains two different figures and addition, the participants stated that there should be a result. The inspection of the Scratch code blocks shows that while four of the participants created code blocks that provide balance, Furkan additionally employed greater than and less than symbols. Furkan managed to provide balance, by appointing numbers to the variables in his code block.

The third question administered to students was 'Do you think the expression 56+85=141 is correct or wrong? Can you tell if the equation 56+85-7=141-7 is correct without performing a subtraction?', which is a relational question. All participants stated that the results on both sides of the equation will remain equal to each other if the same number was subtracted from either side. Stephens et al. (2013) notes a higher level of reasoning is required for this kind of questions regarding the relations between both sides of an equation. In their research, 3rd, 4th and 5th graders were asked 'It is known that 15+8=23. Do you think the expression 15+8+12=23+12 is correct?' The participants were unable to interpret that to comprehend the equation they were expected to take into consideration the first expression (15+8=23). However, participants capable of relational interpretation of an equation stated that adding the same number to both sides of an equation will not break the equation. Very few of the 5th graders were observed to have this interpretation. That all participants displayed a relational interpretation of equations differs from the research conducted by Stephens et al. (2013); the reason for this may be that the participants of our research were all 8th graders. While, Sude, Rümeysa, Helin, and Ayşe created Scratch activities providing balance, the code created by Furkan included options that provided imbalance.

The fourth question administered to students was 'Can you tell if the equation $67 + 86 = 68 + 85$ is correct without performing an addition?' which is a level4 and relational question. Stephens et al. (2013) in his research conducted with the participation of 3rd, 4th and 5th graders concluded that very few of the students provided answers by focusing on the *structure* of the equation. The other participants recorded that they reached to the result by performing operations not by focusing on the structure of the equation. The inspection of the Scratch code blocks shows that while four of the participants (Sude, Ayşe, Rümeysa & Helin) created code blocks that provide balance, Furkan created code blocks that make the scale lean to right or left. Furkan managed to provide balance, by appointing numbers to the variables in his code block. All four students who created code blocks that provide balance were observed to employ a single *if* in their code blocks.

The inspection of the findings shows that participants mostly provided operational interpretations to operational questions, and relational interpretations to relational questions. Researches show that students tend to interpret equal sign as a symbol that signifies operation and result (Behr et al., 1980; Byrd et al., 2015; Kieran, 1981; Knuth et al., 2005; Knuth et al., 2006). Interpretations of the participants were observed to differ according to the type of the question they were given. Knuth et al. (2005), after conducting a research with the participation of 6th, 7th and 8th graders, states that 8th graders were more likely to provide a relational interpretation of equations compared to 6th and 7th graders. Similarly, Alibali et al. (2007) recorded that the relational interpretation of equations increases as the age of the participant increases, in a lengthwise research on 6th, 7th and 8th graders.

Another finding regarding the interpretation of equal sign is that the participants paid attention to providing balance in their Scratch activities. In the first question, only Sude, and in the second question all participants created code blocks that provided balance. Regarding third and fourth questions, all participants but Furkan created code blocks that provided balance. Even though creating similar code blocks for all questions, Furkan provided balance by creating extra codes for two questions (2nd and 4th). In this context, in accordance with the researches in the field, participants were observed to focus on balance in their double pan scale activities. For example, according to Akkan (2009), in Algebra the equal sign signifies balance. In conclusion, the participants created code blocks using the equal sign to provide the scale to balance.

All participants were Level 4 students, in accordance with the four different levels defined by Matthews et al. (2012). All participants were 8th graders. Further research may be conducted with participants from other grades. Students can be encouraged to use Scratch for various subjects (first-degree equations in one unknown, factorization, first degree inequalities in one variable).

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