# Approaches of Science Teacher Candidates to Errors in Regarding the Concepts of General Mathematics 

## OPEN ACCESS

Volume: 10

Special Issue: 1

Month: August

Year: 2022

E-ISSN: 2582-1334

Received: 21.06.2022

Accepted: 28.07.2022

Published: 18.08.2022

## Citation:

Altun, Solmaz Damla Gedik. "Approaches of Science Teacher Candidates to Errors in Regarding the Concepts of General Mathematics." Shanlax International Journal of Education, vol. 10, no. S1, 2022, pp. 21-30.

DOI:
https://doi.org/10.34293/
education.v10iS1-Aug. 4989


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#### Abstract

The teacher must, first of all, have a good knowledge of mathematics to detect the errors in the student's mathematical knowledge and to identify the general situation in the classroom. This depends on the mathematical knowledge gained in the teacher training schools. General mathematics topics occupying an important place in the departments of mathematics and science education form the basis for a better understanding and comprehension of subsequent mathematics and science topics (Gökçek \& Açıkyldız, 2015).In this study, approaches of teacher candidates to errors in the questions on limits, derivatives, integrals, and asymptotes were determined as the case of the research. Perspectives regarding this case were examined in detail in line with the candidates' answers to the questions "what, how, why" and presented to the reader (Yin, 1984). Participants of the study consisted of sixty first-year students studying in the Department of Science Education of a university in Central Anatolia. In the research, a knowledge test consisting of six open-ended questions was used as the data collection tool.


Keywords: General Mathematics, Mistake Approaches, Science Teacher Candidates.

## Introduction

There is no doubt that everybody makes errors. People instinctively consider errors as unpleasant experiences and avoid them. However, studies on brain analysis suggest that errors, though unpleasant, are necessary for effective learning (Moser et al., 2011; Boaler, 2015). Therefore, going further than avoiding errors, one should develop a positive perspective about them in order to render learning more meaningful. Boaler (2015) argues that establishing an environment where students feel comfortable dealing with errors is beneficial for learning mathematics. Establishing such an environment depends on how teachers deal with not only students' errors but also their own. This underlines not only the importance of detecting students' errors but also the role of errors in classrooms and the importance of teachers' managing these errors (Borasi, 1987; Heinze, 2005; Santagata, 2005; Steuer et al., 2013; Tainio \& Laine, 2015).

Teachers' awareness of how to manage errors is based on acquiring mathematical thinking skills and knowing that errors do not constitute an obstacle to mathematical understanding. A teacher with that awareness may make different inquiries before making any decision about the students' errors. Teacher's way of interpreting student's error is also important. As that error gives further information about the student's mathematical knowledge and skills. This determines how the teacher will respond to the errors pedagogically. Many reasons lay behind the error of a student, and each reason has different informative implications. For example, a student can answer a question on any subject incorrectly while giving a correct answer to another one. Students' mathematical knowledge of the concerned subject can be examined by checking and questioning the errors in
the solution of the questions on a specific topic (Ginsburg 1997). Thus, the teacher can ascertain students' understanding of the subject and determine the characteristics of the general situation.

The teacher must, first of all, have a good knowledge of mathematics to detect the errors in the student's mathematical knowledge and to identify the general situation in the classroom. This depends on the mathematical knowledge gained in the teacher training schools. General mathematics topics occupying an important place in the departments of mathematics and science education form the basis for a better understanding and comprehension of subsequent mathematics and science topics (Gökçek \& Açıkyıldız, 2015). Mathematics is a sequential and cumulative discipline, and any concept should not be given without introducing its prerequisite concepts. Accordingly, it is vital that the students have primarily learned the concepts of general mathematics, forming the basis for advanced mathematics topics and other courses.

Considering the science education courses in the teacher training programs, functions, limits, derivatives, applications of derivatives and integrals are among the topics of the general mathematics courses. As these topics form the basis for other courses, they play an important role for the university students studying in the science education departments.

Research addressing these topics, which form the basis for the departments of science education, has generally identified the students' errors and provided several suggestions to correct these errors. However, there is also research focusing on teachers' reactions to students' errors. Yet, there is a limited number of research on the role of errors in classrooms and teachers' management of these errors. Different types of errors and incorrect answers can offer different opportunities for learning (Stockero \& Van Zoest, 2013). Researches underline the importance of teachers' addressing incorrect questions in the classroom and interacting with students over the errors (Bray, 2014; Brodie, 2014; Santagata, 2005; Silver et al., 2008).

In this research, considering the importance of interacting with students over the errors, incorrect questions about functions, limits and derivatives were
given to science students in the last two weeks of the General Mathematics II course. Their approaches to these questions were examined in depth.

## Method

## Research Design

Creswell (2007) defined a case study as a qualitative research approach analyzing the bounded system(s) in-depth with certain data collection tools and defining the themes determined based on the emerging situation or situations following the analysis (Subaşı \& Okumuş, 2017). In this study, approaches of teacher candidates to errors in the questions on limits, derivatives, integrals, and asymptotes were determined as the case of the research. Perspectives regarding this case were examined in detail in line with the candidates' answers to the questions "what, how, why" and presented to the reader (Yin, 1984).

## Study Group

Participants of the study consisted of sixty first-year students studying in the Department of Science Education of a university in Central Anatolia. Criterion sampling was used in the study. Criterion sampling enables working with people, cases, or conditions with the qualifications determined concerning the problem in the relevant research (Yıldırım \& Şimşek, 2016). In the research, considering that the students having had the General Mathematics I-II courses might have a different perspective on the errors, this was considered a criterion, and individuals meeting this criterion were selected. Teacher candidates participating in the research were coded M1, M2,..., M43, and the data were presented with these codes.

## Data Collection Tool and Data Collection

In the research, a knowledge test consisting of six open-ended questions was used as the data collection tool. Two of the open-ended test questions in the knowledge test were on limits, one on derivatives, two on integrals, aimed at measuring the required information. The whole part of the open-ended questions consists of questions solved incorrectly and the questions intended to detect the error and examine its reasons. This study focuses on examining teacher candidates’ mathematical knowledge of limits,
derivatives and integrals within the framework of the main theme of error detection and approaches to existing errors. In the study, incorrect solutions and true-false questions in the test were formed by the researcher. A knowledge test, prepared as a data collection tool, was provided to the students in written form. No time limit was applied. Students were asked to detect the errors in the questions' solutions and explain the reasons for these errors together with the justifications. In addition, students assuming the solution was incorrect were also asked to provide their own correct solutions.

Questions in the knowledge test given to teacher candidates are as follows.

```
1. If \(A=2\) cosk-sinx, what are the minimum and maximum idegry values of \(A\) ?
    Seltime- \(1 \leq \sin \leq 1\) and also \(-1 \leq\) coser \(<1\)
        \(\left.\begin{array}{l}-2 \leq 2 \cos x \leq \\ -1 \leq-\sin x \leq 1 \\ +\end{array}\right]-\begin{aligned} & \text { anc the minimum integer value } \\ & \text { is }-3 \text { and the maximum integer } \\ & \text { vatue is }+3\end{aligned}\)
    \(-3 \leq 2\) coss-silix \(<3\)
2. \(\int \tan x d x=\) ?
    Selntima: \(\int\) tancix \(=\int \frac{\operatorname{sinx}}{\operatorname{cosx}} d x\)
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    \(\int \tan x d x=\int \frac{\sin _{\cos }^{\cos x}}{\cos x}=\frac{1}{\cos x}(-\cos x)-\int(-\cos x)(\tan x \cdot \operatorname{sen} x) d x\)
    \(\int \tan x d x=(-I)+\int \tan x d x \Rightarrow 0=1\)
    3. \(\int \frac{1}{x} d x=\) ?
    Selminta:
    Here \(\frac{1}{x}=y \Rightarrow \frac{-1}{x^{2}} d x=d u\) and \(d x=d v \Rightarrow x=v\)
    \(\int \frac{1}{x} d x=\frac{1}{x}-x-\int \frac{(-1)}{x^{2}} \cdot x d x \Rightarrow \int \frac{1}{x} d x=1+\int \frac{1}{x} d x \Rightarrow 0=1\)
    4. \(\lim _{x \rightarrow-\infty} \frac{2 x^{8}-4 x^{2}+2}{x^{2}+2 x+3}=\) ?
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    \(=\lim x^{2} .0=\lim 0=0\)
    5. \(\lim _{x \rightarrow 0} \frac{\max (2 x)}{x}=\) ?
    Selwinhe \(\lim _{x \rightarrow 1} \frac{a x(2 x)}{x}=\lim _{x \rightarrow 0} \frac{\left(x s(2 x)^{\prime}\right)}{(x)^{\prime}}=\lim _{x \rightarrow 0} \frac{\left(-2 \sin (2 x) y^{\prime}\right.}{1}=0\)
6. \(f(x)=\left\{\begin{array}{ll}\left\{x^{2},\right. & x \leq 1 \\ 3 x-2 & x>1\end{array}\right.\) What is the derivative value off this fimation \(x t x=1\) ?
Selmane. For \(f(x)=\left\{\begin{array}{rl}x^{2}, & x \leq 1 \\ 3 x-2, & x>1\end{array} \Rightarrow x=1\right.\) when \(x \leq 1, f(x)=x^{2}\) and tor \(f(x)=2 x\)
\(\Rightarrow f(1)=2\)
```


## Data Analysis

Descriptive analysis is presenting the research data to the reader with direct quotations by adhering to the authentic version without any changes (Miles \& Huberman, 1994). In the descriptive analysis, qualitative data are processed pursuant to a predetermined framework, findings are defined, and those defined findings are interpreted (Yıldırım \& Şimşek, 2016).

In this study, the data obtained based on the answers of the teacher candidates to the questions
in the knowledge test used as a data collection tool were coded according to students' ability to identify the errors in the solutions of the questions and to explain the reasons of these errors; and classified under predetermined categories in line with the purpose of the study. Analyzed data were presented and interpreted in tables. Answers in each category were supported with direct quotations. In accordance with the characteristics of the questions in the test applied to the mathematics teacher candidates, the test data were classified based on students' ability to find the errors in the solutions of the questions on the related topic and to explain their reasons correctly. Then, the frequencies of the data obtained from the answers of the teacher candidates were calculated, and their statements explaining the reasons were analyzed.

## Reliability and Validity

In order to ensure the reliability of this research, teacher candidates were first assured that their names would not be used in any way, a comfortable classroom environment was provided, and no time limit was applied to pave the way for detailed answers. In line with the research objective, the study was carried out with the teacher candidates at a convenient time which they required and by providing the necessary time. Thus, it was aimed to enable teacher candidates to provide more accurate answers.

## Findings

In this section, teacher candidates; Identifying incorrect questions and solutions given ability and from the analysis of their explanations for the reasons for these errors. findings are included. In this direction, it was obtained from the answers of the teacher candidates. The obtained data are grouped according to the predetermined frame and frequency values calculated and the values for each question are presented in the table below. Moreover preservice teachers' statements about the reasons for the mistakes in the questions and solutions. The answers are supported by direct quotations.

Table 1 Frequency Regarding the First Question

| Categories | Codes | Participants | f |
| :--- | :---: | :--- | :---: |
| 1. Failure to detect the error | 1-Y | M2, M4, M5, M6, M7, M8, M10, M11, M12, M13, M14, <br> M15, M19, M20, M24, M26, M30, M31, M33, M35, M36, <br> M38, M39, M40, M41, M43, M47, M49, M50, M51, M57, <br> M60 | 32 |
|  | 2-D | M18, M21, M22, M23, M28, M32, M34, M37, M44, M45, <br> M46, M52, M55, M58 | 14 |
|  | 2-Y | M9, M42, M16, M53 | 4 |
| 3. Accepting the error as partially <br> correct | 3-D | M1 | 1 |
| 4. Detecting the error correctly | 3-Y | M26, M56 | 2 |
|  | 4-D | M3, M17, M25, M29, M48, M54, M59 | 7 |

Upon examining Table 1, it is observed that many students did not understand this question, could not detect the error in the solution correctly or could not find the correct answer. Thirty-two of the teacher candidates reckoned the question and, thus, the solution as correct. However, one of the teacher candidates accepted the error as partially correct and found the correct solution, while 14 teacher candidates detected the error incorrectly but found the correct solution. It is observed that seven teacher candidates detected the error correctly and found the correct solution. Some answers of the students are given below.


Figure 1 M2's Thought on Question 1


Figure 2 M9's Thought on Question 1

Table 2 Frequency Regarding the Second Question

| Categories | Codes | Participants | f |
| :---: | :---: | :---: | :---: |
| 1. Failure to detect the error | 1-Y | $\mathrm{M}_{2}$ | 1 |
| 2. Detecting the error incorrectly | 2-D | $M_{3}, M_{5}, M_{6}, M_{7}, M_{9}, M_{10}, M_{12}, M_{13}, M_{14}, M_{15}, M_{16}, M_{11}$, $\mathrm{M}_{57}, \mathrm{M}_{23}, \mathrm{M}_{24}, \mathrm{M}_{25}, \mathrm{M}_{26}, \mathrm{M}_{27}, \mathrm{M}_{28}, \mathrm{M}_{29}, \mathrm{M}_{30}, \mathrm{M}_{31}, \mathrm{M}_{32}, \mathrm{M}_{33}$, $\mathrm{M}_{34}, \mathrm{M}_{35}, \mathrm{M}_{36}, \mathrm{M}_{37}, \mathrm{M}_{38}, \mathrm{M}_{39}, \mathrm{M}_{40}, \mathrm{M}_{41}, \mathrm{M}_{42}, \mathrm{M}_{43}, \mathrm{M}_{44}, \mathrm{M}_{45}$, $\mathrm{M}_{46}, \mathrm{M}_{47}, \mathrm{M}_{48}, \mathrm{M}_{49}, \mathrm{M}_{50}, \mathrm{M}_{51}, \mathrm{M}_{52}, \mathrm{M}_{53}, \mathrm{M}_{54}, \mathrm{M}_{55}, \mathrm{M}_{56}, \mathrm{M}_{58}$, $\mathrm{M}_{59}, \mathrm{M}_{60}$ | 56 |
| 3. Accepting the error as partially correct | 3-D | $\mathrm{M}_{1}, \mathrm{M}_{8}$ | 2 |
| 4. Detecting the error correctly | 4-D | $\mathrm{M}_{17}, \mathrm{M}_{18}, \mathrm{M}_{19}, \mathrm{M}_{20}, \mathrm{M}_{21}, \mathrm{M}_{22}$, |  |
|  | 4-Y | $\mathrm{M}_{4}$ | 1 |
| Total |  |  | 60 |



Figure 3 M13's Thought on Question 2

Upon looking at Table 2, it is observed that 56 students detected the error in this question incorrectly but gave a correct answer. Since M4 detected the error correctly but did not find any solution afterwards, they were considered one of those detecting the error
correctly but providing the incorrect solution. It is notable that other teacher candidates could detect the error partially correct, find the correct solution based on the formula, and not clearly explain the reason for the error. Some of the answers are given below.


Figure 4 M4's Thought on Question 2

Table 3 Frequency Regarding the Third Question

| Categories | Codes | Participants | f |
| :---: | :---: | :---: | :---: |
| 1. Failure to detect the error | 1-Y | $\mathrm{M}_{5}, \mathrm{M}_{10}, \mathrm{M}_{15}, \mathrm{M}_{26}, \mathrm{M}_{27}, \mathrm{M}_{30}, \mathrm{M}_{31}, \mathrm{M}_{35}, \mathrm{M}_{53}$ | 9 |
| 2. Detecting the error incorrectly | 2-D | $\begin{aligned} & M_{1}, M_{4}, M_{6}, M_{7}, M_{8}, M_{9}, M_{11}, M_{12}, M_{13}, M_{14}, M_{16}, M_{17}, M_{18}, M_{19}, \\ & M_{20}, M_{21}, M_{22}, M_{24}, M_{22}, M_{32}, M_{33}, M_{34}, M_{37}, M_{38}, M_{40}, M_{41}, M_{43}, \\ & M_{44}, M_{45}, M_{46}, M_{47}, M_{48}, M_{50}, M_{51}, M_{54}, M_{55}, M_{56}, M_{57}, M_{58}, M_{59}, \\ & M_{60} \end{aligned}$ | 41 |
|  | 2-Y | $\mathrm{M}_{2}, \mathrm{M}_{36}, \mathrm{M}_{49}, \mathrm{M}_{52}$ | 4 |
| 4. Detecting the error correctly | 4-D | $\mathrm{M}_{3}, \mathrm{M}_{23}, \mathrm{M}_{25}, \mathrm{M}_{29}, \mathrm{M}_{39}, \mathrm{M}_{42}$ | 6 |
| Total |  |  | 60 |



Figure 5 M33's Thought on Question 3
Upon examining Table 3, it is observed that 41 of the students detected the error incorrectly but found the correct solution. It is also noticed that these students benefited from the formula while finding the correct solution but did not provide an
adequate explanation. It is observed that nine teacher candidates could not detect the error and reckoned the solution as correct. Six teacher candidates explained the error in the question precisely and found the correct solution.


Figure 6 M23's Thought on Question 3

Table 4 Frequency Regarding the Fourth Question

| Categories | Codes | Participants | f |
| :---: | :---: | :---: | :---: |
| 1. Failure to detect the error | 1-Y | $\mathrm{M}_{2}, \mathrm{M}_{4}, \mathrm{M}_{33}, \mathrm{M}_{36}, \mathrm{M}_{43}, \mathrm{M}_{49}, \mathrm{M}_{50}$ | 7 |
| 2. Detecting the error incorrectly | 2-D | $\mathrm{M}_{11}$ | 1 |
|  | 2-Y | $\mathrm{M}_{8}, \mathrm{M}_{24}, \mathrm{M}_{30}, \mathrm{M}_{56}, \mathrm{M}_{57}, \mathrm{M}_{59}$ | 6 |
| 3. Accepting the error as partially correct | 3-D | $\mathrm{M}_{14}, \mathrm{M}_{18}, \mathrm{M}_{39}$ | 3 |
| 4. Detecting the error correctly | 4-D | $\begin{aligned} & M_{1}, M_{3}, M_{5}, M_{6}, M_{7}, M_{9}, M_{10}, M_{12}, M_{13}, M_{15}, M_{16}, \\ & M_{17}, M_{19}, M_{20}, M_{21}, M_{22}, M_{23}, M_{25}, M_{26}, M_{27}, M_{28}, \\ & M_{29}, M_{31}, M_{32}, M_{34}, M_{35}, M_{37}, M_{38}, M_{40}, M_{41}, M_{42}, \\ & M_{44}, M_{45}, M_{46}, M_{47}, M_{48}, M_{51}, M_{52}, M_{53}, M_{54}, M_{55}, \\ & M_{58}, M_{60} \end{aligned}$ | 43 |
| Total |  |  | 60 |



Figure 7 M30's Thought on Question 4
Upon examining Table 4, it is observed that seven students reckoned the question as correct and could not detect the error. Out of the students detecting the

Three students reckoned the error as partially correct and reached the correct result, while 43 students detected the error correctly and reached the correct result.


Figure 8 M60's Thought on Question 4 error incorrectly, six of them provided an incorrect solution, and only one found the correct answer.

Table 5 Frequency Regarding the Fifth Question

| Categories | Codes | Participants | f |
| :---: | :---: | :---: | :---: |
| 1. Failure to detect the error | 1-Y | $\begin{aligned} & M_{1}, M_{2}, M_{4}, M_{11}, M_{14}, M_{15}, M_{19}, M_{22}, M_{26}, M_{27}, M_{31}, \\ & M_{33}, M_{34}, M_{37}, M_{44}, M_{49}, M_{51}, M_{55}, M_{56}, M_{57} \end{aligned}$ | 20 |
| 2. Detecting the error incorrectly | 2-D | $\mathrm{M}_{39}$ | 1 |
|  | $2-\mathrm{Y}$ | $\begin{aligned} & M_{5}, M_{9}, M_{10}, M_{16}, M_{20}, M_{21}, M_{35}, M_{40}, M_{41}, M_{42}, \\ & M_{43}, M_{46}, M_{50}, M_{53}, M_{58}, M_{60} \end{aligned}$ | 16 |
| 3. Accepting the error as partially correct | 3-Y | $\mathrm{M}_{12}, \mathrm{M}_{24}$ | 2 |
| 4. Detecting the error correctly | 4-D | $\begin{aligned} & M_{3}, M_{7}, M_{8}, M_{13}, M_{17}, M_{23}, M_{25}, M_{28}, M_{29}, M_{32}, \\ & M_{38}, M_{45}, M_{47}, M_{48}, M_{52}, M_{54}, M_{59} \end{aligned}$ | 17 |
|  | 4-Y | $\mathrm{M}_{6}, \mathrm{M}_{18}, \mathrm{M}_{30}$ | 3 |
| Total |  |  | 59 |

(M36 left the answer blank.)


Figure 9 M46's Thought on Question 5

Upon looking at Table 5, it is seen that twenty students could not detect the error. Twenty-six students out of the ones who detected the error incorrectly reached incorrect solutions, while one student had the correct answer.

It is seen that two students reckoned the error as partially correct and provided incorrect answers, while 17 students detected the error correctly and reached the correct result. Three students detected the error correctly but left the solution blank and did not provide any solution. One teacher candidate left the entire answer blank.


Figure 10 M9's Thought on Question

Table 6 Frequency Regarding the Sixth Question

| Categories | Codes | Participants | f |
| :---: | :---: | :---: | :---: |
| 1. Failure to detect the error | 1-Y | $M_{2}, M_{4}, M_{5}, M_{11}, M_{12}, M_{15}, M_{27}, M_{47}, M_{50}, M_{57}$ | 10 |
| 2. Detecting the error incorrectly | $2-\mathrm{Y}$ | $\mathrm{M}_{42}$ | 1 |
| 4. Detecting the error correctly | 4-D | $\begin{aligned} & \mathrm{M}_{1}, \mathrm{M}_{3}, \mathrm{M}_{6}, \mathrm{M}_{7}, \mathrm{M}_{8}, \mathrm{M}_{9}, \mathrm{M}_{10}, \mathrm{M}_{13}, \mathrm{M}_{14}, \mathrm{M}_{16}, \mathrm{M}_{17}, \mathrm{M}_{18}, \mathrm{M}_{19}, \\ & \mathrm{M}_{20}, \mathrm{M}_{21}, \mathrm{M}_{22}, \mathrm{M}_{23}, \mathrm{M}_{24}, \mathrm{M}_{25}, \mathrm{M}_{28}, \mathrm{M}_{29}, \mathrm{M}_{30}, \mathrm{M}_{31}, \mathrm{M}_{32}, \mathrm{M}_{33}, \\ & \mathrm{M}_{34}, \mathrm{M}_{35}, \mathrm{M}_{36}, \mathrm{M}_{37}, \mathrm{M}_{38}, \mathrm{M}_{39}, \mathrm{M}_{40}, \mathrm{M}_{41}, \mathrm{M}_{43}, \mathrm{M}_{44}, \mathrm{M}_{45}, \mathrm{M}_{46}, \\ & \mathrm{M}_{48}, \mathrm{M}_{49}, \mathrm{M}_{51}, \mathrm{M}_{52}, \mathrm{M}_{53}, \mathrm{M}_{54}, \mathrm{M}_{55}, \mathrm{M}_{56}, \mathrm{M}_{58}, \mathrm{M}_{59}, \mathrm{M}_{60} \\ & \hline \end{aligned}$ | 48 |
|  | 4-Y | $\mathrm{M}_{26}$ | 1 |
| Total |  |  | 60 |



Figure 11 M54 's Thought on Question 6
Upon examining Table 6, it is observed that ten students could not detect the error. Only one student detected the error incorrectly and reached an incorrect result; similarly, only one detected the error correctly and got a wrong result. Forty-eight students detected the error correctly and also provided the correct solution.


Figure 12 M12 'sThought on Question 6

## Discussion and Conclusion

This study examined whether the science teacher candidates can detect the errors in the six-question knowledge test on topics in general mathematics, forming the basis for future science topics and explaining the reasons for these errors accurately. Researcher provided a solution for each question in the test. Solutions to the questions in the knowledge test were presented incorrectly. It was not stated whether there was an error in the solutions to these
questions or not. Teacher candidates evaluated the solutions to the given questions.

All the solutions to the questions in the knowledge test included an error. The first question was solved like any other standard inequality question. However, half-angle formulas should be used here to reach the correct solution. Upon examining the answers, it is observed that most candidates declared the solution correct. This displays that students do not exactly know where and how to use half-angle formulas in trigonometry. Moreover, the fact that students stated that the inequality solution used in the question was correct points out that they also do not exactly know the topic of inequality. Mathematical concepts are related to each other, and students make inferences by attributing different meanings to these concepts, and these cause students to make errors in the learning process (Türkdoğan, Güler, Bülbül, \& Danişman, 2015). Lack of knowledge of trigonometry also leads to errors in other topics requiring trigonometry knowledge (Kuzu, 2017).

Concerning the solutions to the second and third questions, candidates stated that the integration by parts was correct, there was an error in the next phase, but they could not find it. Students regarded the solution as definitely incorrect as the result of $0=1$ is not possible. Error in the solutions to these questions results from ignoring the number " $c$ " at the end of the integral calculation. Likewise, it is seen that the candidates also ignored the number " $c$ ". Teacher candidates detected the error incorrectly in the integral questions but reached a correct solution. They said that their solutions to the second and third questions were based on rules. Upon scanning the literature, integral has been emphasized as one of the most challenging topics to learn (Gürbüz, 2021; Zakaria \& Salleh, 2015). Therefore, teacher candidates should be provided with several solution methods regarding the integral. Moreover, they should also comprehend the importance of some concepts that are not highlighted much and their effect on the result.

It is observed that in the solution of the fourth question, the candidates detected the error correctly and reached the correct solution. It has been determined that the candidates who could not detect the error and reach the solution could not
fully comprehend how to solve the questions on the infinity at limits (e.g., $\infty / \infty$ ). It is observed that in the fifth question, there were candidates who did not realize that the limit was $1 / 0$ but thought it was $0 / 0$ instead and assumed that the solution was correct. On the other hand, the number of teacher candidates realizing that the limit was $1 / 0$ but stating that the right and left limits should be considered in order to reach the right solution is rather few compared to the whole number of the students in the class. The result shows that teacher candidates have difficulty in perceiving and distinguishing the concepts of infinity, undefined and indeterminate (Alkan \& Güven, 2018; Baştürk \& Dönmez, 2011).

The last question in the research is on derivatives, and to find the correct solution to this question, right and left derivatives must be equal. In the solution to the problem, only the left derivative was considered by ignoring the critical point. It is seen that the candidates generally detected the error in this question correctly and reached the correct result. However, it is also observed that some candidates assumed the given solution as correct or found the solution incorrect but also provided an incorrect solution. This displays that the candidates cannot fully understand the concepts of limit and continuity related to the derivatives, so they also made errors concerning the derivatives (Ulaş \& Biber, 2020).

It is observed that in examining the solutions to the given questions, the teacher candidates had difficulty detecting the errors. Some of the teacher candidates reckoned the incorrect solutions as correct. Moreover, it has been noticed that the teacher candidates could not provide adequate explanations regarding the reasons for incorrect solutions. Some studies have concluded that the teacher candidates should be able to evaluate a given solution, i.e., to explain the reasons for declaring the solution as correct or incorrect (Billi, Özkaya, Çiltaş, \& Konyalıoğlu, 2020).

Adequate knowledge of general mathematics does not mean applying the given rules directly to a specific question. To ensure a significant field knowledge, it is required to be able to detect incorrect questions and solutions and comprehend them (Türkdoğan \& Yıldız, 2021). Indeed, field knowledge is an important factor in detecting
students' errors and examining their reasons (Boz, 2004). It has been established in this study that the teacher candidates could not precisely detect the incorrect solutions in general mathematics and could not provide an adequate explanation of the reasons for the errors. When the research results are considered, it is thought that teacher candidates' field knowledge of general mathematics should be improved. Studies have also revealed that the practices aimed at detecting errors have positive effects on learners and teachers (Demirci, Özkaya \& Konyalığlu, 2017; Konyalıoğlu, 2013).

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