International Electronic Journal of Mathematics Education
2022, 17(4), em0699
e-ISSN: 1306-3030
https://www.iejme.com

MODESTUM

Research Article

OPEN ACCESS

Relational equity: Adapting an elementary mathematics teaching methods course to online contexts

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Citation: Ruef, J. L., & Shepard, R. (2022). Relational equity: Adapting an elementary mathematics teaching methods course to online contexts. *International Electronic Journal of Mathematics Education, 17*(4), em0699. https://doi.org/10.29333/iejme/12224

ARTICLE INFO

Received: 22 Feb. 2022

Accepted: 29 Jun. 2022

ABSTRACT

Background. This article describes the redesign, implementation, and students' experiences with an elementary mathematics teaching methods course in the Pacific Northwest of the United States that was forced online due to COVID-19 restrictions. We share course design principles, changes to reflect online teaching, and students' experiences of *relational equity,* "the care and consideration students and teachers must take with each other to establish and maintain a culture of equitable learning."

Methods: This study took place online. 50 out of a cohort of 54 pre-service elementary teacher candidates participated. Our methods included analysis of course documents, analytic memoing, and analysis of students' responses to online open response surveys. We qualitatively analyzed the responses from the surveys, coding for a priori and emergent themes (Charmaz, 1995; Emerson et al., 2011; Miles et al., 2019).

Findings: The results share the ways the instructors modified the course for online instruction and indicate that our participants experienced successes or challenges to enacting relational equity in the following: (i) curriculum and assessments, (ii) instructors, (iii) generic reference to the course, (iv) technology limits, and (v) future teaching.

Contribution: This work sheds light on how two mathematics teacher educators redesigned their course for online instruction while centering and modeling relational equity teaching practices.

Keywords: equity, online instruction, elementary mathematics teaching methods

INTRODUCTION-COVID-19 CHALLENGES: MAINTAINING RELATIONAL EQUITY FOR AN ONLINE ELEMENTARY MATHEMATICS TEACHING METHODS COURSE

2020 was a challenging year for all humans. Those tasked with preparing pre-service teachers (PTs) met unprecedented challenges as education lurched online in the Spring of 2020. This research report shares the intentional planning, ongoing revision, and students' responses to the implementation of an elementary mathematics teaching methods course taught in Fall, 2020 for a university located in the Pacific Northwest of the United States of America.

A common problem of practice in mathematics teacher education is enculturing new teachers to ways of learning, knowing, and doing mathematics that differ from their prior experience as students. Often, PTs arrive in methods classes believing that to teach mathematics is to show students how to calculate answers. Learning to teach in ways that center student-inquiry and exploration of mathematical concepts and problem-solving techniques is thus a transformative experience for many PTs.

Mathematics teacher educators (MTEs) are responsible for designing teaching methods courses that allow PTs to learn and engage with the theory and practice of student-centered teaching. To immerse themselves in these transformative experiences, PTs may enact multiple roles, including student, teacher, and colleague. In our teaching, we refer to this practice of enactment and perspective-taking as wearing "hats." For example, depending on the activity, we might ask the PTs to wear student, teacher, or colleague "hats," and we refer to moments of our own explicit sharing of pedagogical choices as "teacher hat moments." Because so much of our work depends upon empathetic perspective taking and interpersonal communication, our methods courses are founded on the principle of *relational equity* which we operationalize as the ways in which students are seen, heard, and valued by their peers within a learning community (Boaler & Staples, 2008; Lo & Ruef, 2020; Sun et al., 2022). Relational equity is both a theoretical and pedagogical stance of recognizing and honoring the humanity of all members of a learning community. This work requires care and consideration as the instructors and students must create effective and caring communities of learning. Building effective learning contexts online is all the more challenging because of the ways subtle interpersonal communications, such as body language, vocal inflection, and eye contact are all limited or distorted.

This is an unusual research report, befitting an unusual time. Our analysis first considers the changes in planning and instruction required to center relational equity as we moved our course from in-person to online in Fall 2020. Second, we analyze the students' experience across the term of instruction. Thus, we frame our work as both anticipating, during the time of planning, reflecting concurrently during instruction, and looking back as we analyzed the students' reflections at the end of the course. We position ourselves as researchers of our own practice (e.g., Ball, 1993), which is based on principles of mathematics teaching and teacher education delineated by the National Council of Teachers of Mathematics (NCTM, 2014, 2018) and the Association of Mathematics Teacher Educators (AMTE, 2017). To these ends, we describe the theoretical and pedagogical foundations of our course design and its conversion from brick-and-mortar to online contexts with the goals of maintaining the foundational ethos and content of the original course. We then share the students' experiences of the new format and curriculum, reported both during the term and in a summary survey.

LITERATURE

The literature review is organized in terms of related research on the impacts of COVID-19 on mathematics instruction and online instruction. These sections are followed by explanations of course design principles, including public sensemaking, leading to our conceptual framing of relational equity.

Impacts of COVID-19 on Mathematics Instruction

Our study joins a small, but growing, number of emergent studies on the impacts of COVID-19 on mathematics instruction. These include reports on the challenges of establishing online learning communities, which included willingness and opportunities for students to participate and access to stable and effective technological interfaces (Calder et al.2021; Jordaan & Havenga, 2021; Ruef et al., 2022; Sun et al., 2022; Uegatani et al., 2021).

In general, these studies report that there were many challenges to online instruction. From the students' perspectives, these included inequitable access to functional technological interfaces and equipment, the organization and implementation of online instruction, and inequitable distributions of resources and demands on time. For example, some students faced challenges due to unstable or absent internet access, shared devices with siblings, or competing demands for their time and energy as they worked or provided childcare to support their family during a time of duress (Jordaan & Havenga, 2021; Sun et al., 2022). Teachers and schools struggled to create effective online learning environments and provide special education or emergent bilingual supports (Ruef et al., 2022; Sun et al., 2022; Uegatani et al., 2021).

There were also online instructional successes, including the emergence of previously reticent students, the creation and adaptation of technical tools, and flattening of power structures that resulted in more democratic student participation (Ruef et al., 2022; Sun et al., 2022; Uegatani et al., 2021).

Considerations of Online Mathematics Instruction

Specific to teaching mathematics and mathematics teaching methods courses online, teachers and MTEs struggled to quickly create and find workarounds for effective online group work and the use of mathematics manipulatives, physical materials used to model mathematical problem solving (e.g., unifix cubes or base ten blocks) (Borba et al., 2021; Ruef et al., 2022; Sun et al., 2022).

Technology that supports online instruction has made great strides in recent years (e.g., Desmos, Padlet, interactive whiteboards) but remains inadequate for fully replicating in-person interactions (Borba et al., 2021). The challenges of fostering equitable participation in online contexts is not a new concern (Engelbrecht & Harding, 2005; Jones & Long, 2013; Pope & Mangram, 2015; Thanheiser et al., 2020). COVID-19 created new urgency for teachers and MTEs to solve these problems of practice in support of online instruction.

Course Design Principles

Our work as MTEs is grounded in the standards and principles set forth by the AMTE (2017) and the NCTM (2014, 2018). This collective vision of mathematics teacher education and mathematics teaching and instruction is built upon an integrated core: inclusive, agentic, and collaborative problem solving. In this work, teachers are positioned as experts in designing and facilitating inquiry-based and student-centered learning opportunities for students. Students are positioned as capable and collaborative sensemakers. If we think of the larger project of teaching people to teach math, we include several nested projects. These include students working together to complete mathematical tasks, teachers working together to solve problems of practice, and teacher educators working together to research and improve mathematics education.

If we consider mathematics education as comprising a community of problem solvers engaged with various problems of practice, we find a common purpose: shared work completed by problem-solving peers. And when people collaborate effectively, they are stronger together in solving problems (Horn, 2012). This work is informed by overarching considerations of teaching mathematics for equity, which we define as effectively including every student as a learner, knower, and doer of mathematics (e.g., Hand, 2010; Jones & Long, 2013; Langer-Osuna, 2011; Lubienski, 2002; Martin et al., 2010; Moschkovich, 2013; Ruef, 2021; Tate et al., 2018). As MTEs, we operationalize equity in terms of the culture we co-construct with our students. To that end, we consider core principles that support equitable instruction as we design our syllabi, write our lesson plans, choose activities, and model instruction for our students. The following sections describe two key principles: public sensemaking and relational equity.

Public sensemaking

Public sensemaking (Ruef, 2021; Ruef & Torres, 2020) classrooms position students as active agents in proposing, refining, and validating mathematical arguments. Co-creating a culture of public sensemaking is intentional work. Teachers and students work together to co-construct socioamathematical norms (Yackel & Cobb, 1996) and practices that support productive discourse (e.g., Chapin et al., 2009) and appreciations for mathematical contributions (Jilk, 2016).

Historically, to be good-at-math meant avoiding mistakes (Horn, 2012). But to learn mathematics in deep and robust ways, students must be comfortable with sharing incomplete and emergent understandings. They must be brave in sharing their thinking and this demands vulnerability. Publicly sharing ideas, conjectures, or questions implicitly invites critique by peers and experts, and may invite appreciation or censure depending on the culture and norms of the classroom. Students need support in developing a culture of academic risk-taking inherent in presenting and refining mathematical ideas (Beghetto, 2009; Jansen, 2020; Ruef, 2021; Ruef & Torres, 2020; Sun & Evans, 2016). Simply put, it is easier to be brave when one anticipates appreciation rather than censure.

Relational equity

Boaler and Staples (2008) describe schools as "places, where students learn ways of acting and being that they are likely to replicate in society, making respect for students from different circumstances an important goal." And while "it is not commonly thought that mathematics classrooms are places where students should learn about cultural respect," classrooms, even in online spaces, are inherently social spaces (p. 625).

To see, hear, and appreciate the lived experiences and intellectual contributions of one's colleagues is to manifest *relational equity* (Boaler & Staples, 2008; Ruef & Torres, 2020; Ruef et al., 2022).

This core principle is essential in establishing classroom practices such as *productive discourse*, which supports effective interpersonal communication between participants that is essential as students debate, refine, and validate commonly shared understandings (e.g., Hufferd-Ackles et al., 2004; Kazemi, 1998; Turner et al., 2013).

In practice, relational equity can be observed in the ways students debate mathematical ideas–when students show respect and appreciation for the ideas and questions of their colleagues, they contribute to a culture of equity (Jilk, 2016; Ruef, 2021). Additionally, relational equity accounts for participants' needs to be seen, heard, and appreciated in their efforts to learn. Our students are humans striving to succeed as new educators and must surmount unprecedented challenges in the process. The pandemic taxed educators and schools in new ways. In redesigning our course in response to these realities, we aimed to recreate the humanizing aspects of our teaching, the heart of our in-person classes, for online settings. To enact public sensemaking we needed to attend to opportunities to build relational equity. To know if we had succeeded in doing so, we needed to ask our students how they had experienced relational equity.

CONCEPTUAL FRAMING

Our study is framed by considerations of relational equity in planning for and assessing students' experiences of a course of study. As a theoretical and practical construct, relational equity requires that we recognize the humanity of our colleagues. In the practical context of a mathematical teaching methods class, this means creating and assessing opportunities for students to interact, share ideas, debate, refine, and present their thinking. The following questions framed our study:

1. How can we center relational equity in our course design as we transition to an online context?
2. How did our students experience relational equity in the online context of the course?

As a theoretical construct, relational equity informed the revisions of our curriculum and pedagogy for online instruction and shaped our assessment of the students' experience of the course. To that end, we considered aspects of social interaction for brick-and-mortar instruction and re-envisioned them for online setting. These aspects included interaction structures such as small groups of whole-class instruction, class activities, assessments of student learning, technology and tools, communication between students and instructors, and the use of feedback to assess and inform the effectiveness of the course both during and after the term of instruction. In practice, we mapped from brick-and-mortar to online by mapping what we had done in the past in person to what we planned to enact online. **Table 1** offers a visual model for this mapping.

To assess our students experiences with relational equity, we defined it for them as "the care and consideration students and teachers must take with each other to establish and maintain a culture of equitable learning" (Ruef et al., 2022, p. 2). This allowed us to ask them explicitly how they had experienced relational equity across the course.

METHODS

This context for this study was unusual. First, it was forced rather than chosen. The restrictions necessitated by COVID-19 created conditions for a natural experiment that should not pass any institutional review board. We would not have voluntarily pivoted from in-person, small group, inquiry-based, manipulative-rich instruction to weekly videoconference meetings. Such a pivot would have violated our principles of teaching for equity, but the urgency of creating online learning environments created new and different equity considerations: public and personal health. For these reasons, online education became the most

Table 1. Model for mapping from in-person to online contexts

	Brick-and-mortar context→	Online context
Interaction structures/ activities		
Assessments		
Technology/tools		
Communication		
Feedback		

equitable option under the circumstances. Second, the study was conducted entirely online, and our research usually situates us within brick-and-mortar classroom contexts.

Participants and Context

The study participants include the instructors and 50 members of a cohort of 54 elementary teacher candidate cohort in a masters-level teacher education program in the Pacific Northwest of the United States. The cohort self-identified as follows: Alaska Native; Asian; Black or African American; Hispanic or Latino; Native Hawaiian or Other Pacific Islander; two or more races, 17 students (31%); white, 37 students (69%).

As much as possible, we wanted the students to experience the course as just that: a course of study in mathematics teaching methods. For this reason, we withheld our request for participation in the study from the students until the end of the last class of the term. Because we knew we could utilize the students' coursework as data, pending their eventual consent, we were able to teach the course as planned with the knowledge that we might gain consents and collect data from a post-course survey after we had concluded instruction. This turned out to be true.

The researchers were a teaching team comprising a jigsaw of expertise. Jennifer Ruef, who identifies as white and female, is a MTE, researcher, and former secondary (grades 6-12) mathematics teacher in the United States. Reid Shepard, who identifies as white and male, is a mathematics staff development specialist with experience teaching at the elementary grade levels. Between us, we embodied decades of professional experience, deep pedagogical knowledge of mathematics teaching and learning, and understanding of mathematics education research. We co-constructed a significant revision of our institution's elementary mathematics methods course sequence in 2018, refined it in 2019, and adapted it for online instruction in 2020.

Data Sources and Analysis

Data collection took place during 2020 and analysis during 2020 and 2021. Data included the previous and revised course planning and instructional documents for the second of a two-course sequence in K-5 mathematics teaching methods and the students' survey responses to a prompt asking how they had experienced relational equity in the course.

Our course documents data includes notes from planning sessions, syllabi, lesson plans, class activities, weekly student feedback, and assessments from previous and current years. We analyzed this data through planning and lesson debriefing discussions and memoing (Emerson et al., 2011). Through an iterative process, we documented emergent themes, challenges, and successes toward the goal of effectively preparing new elementary teachers of mathematics (Charmaz, 1995). The **Appendix A** includes an example of a final version of a series of iterative memos documenting revisions to the key assessments for the course.

35 of the 50 participants completed a post-course survey. Our analysis of the post-course survey included iterative rounds of coding, code refinement, and validation via calculating inter-rater agreement (Miles et al., 2019). Ruef created the initial codebook and Shepard assessed the codes for completeness and face validity, by reading the participants' responses, noting themes across the data, and confirming those themes were present in the codebook. Both authors discussed the outcome of the code validity exercise and modified the codebook accordingly. Ruef recoded all participant responses, and Shepard coded 20% of the responses using the refined codebook. Ruef then calculated inter-rater agreement to be 90.48%, and Cohen's kappa for inter-rater agreement to be .78125. This statistic indicates substantial agreement on coding between the two raters.

FINDINGS

The findings are organized in terms of the research questions. To answer the first question, we analyzed the course re-design, adaptation, and implementation in an online context. We include descriptions of major assessments. To answer the second question, we analyzed the students' experiences with relational equity in the course.

RQ 1: How Can We Center Relational Equity in Our Course Design as We Transition to an Online Context?

We first present the recent history of the course design to inform the re-design for online instruction. Next, we describe the adaptation and implementation for online instruction. We then discuss both in-person and online organizations of class hours. We then share the role of new and additional technology and conclude with adaptations of major course assessments.

Initial course design principles and theory

The transition to online instruction was not our teaching team's first re-design challenge. Our teacher preparation program underwent a reorganization in 2018, and as a result, the mathematics teaching methods course sequence was restructured to ensure it continued to meet the needs of the students and supported all licensure requirements. The course we inherited, and its evolutionary redesign, were both built from a well-established body of mathematics education research and practice (AMTE, 2017;

Ball et al., 2005, 2008; NCTM, 2014, 2018). Additional emphasis on teaching for social justice, antiracist instruction, and decolonizing education articulate the program's core tenets of teaching for equity, social justice principles, and anti-racist education within our mathematics teaching methods courses (Gutiérrez, 2017; TODOS, 2020; Ruef et al., 2020).

These principles share commonalities with our operationalization of relational equity. The central theme of building respectful relationships, with a focus on making sense of mathematics, informed the re-design of the course sequence. For example, each class meeting included students working in small groups to solve mathematics problems, share and appreciate questions and solutions, and collectively validate the solutions. We discussed the ways in which mathematics has historically marginalized groups of people and what teachers can do to counter problematic framings about who is good-at-math. The instructors modeled the use of *status interventions* and *multiple abilities treatments,* tenets of *complex instruction,* a pedagogical theory and practice for teaching heterogenous groups of students (Cohen & Lotan, 2014). The instructors also modeled how to scaffold risk-taking, which helps students normalize and manage anxieties over sharing emergent mathematical thinking (Jilk, 2016; Ruef, 2021; Ruef & Torres, 2020; Sun & Evans, 2016).

The PTs conducted in-class *teaching rehearsals*, where groups of 3-4 teachers were responsible for planning and implementing a full elementary-level mathematics lesson following a launch-explore-summarize model (Jackson et al., 2012; Van de Walle et al., 2019). Mathematics has long been a gated profession, with barriers to people identified as female, Black, Indigenous, or of Color (BIPOC), emergent bilingual, disabled, or low socioeconomic status (Goins, 2017; Gutiérrez, 2017; Langer-Osuna, 2011; Martin et al., 2010; Ruef, 2021). Our courses were designed to engage PTs in discussions and rehearsals to support them in learning to better advocate for their students, often within imbalanced political power dynamics due to their status as new teachers working within established departmental hierarchies (Gutiérrez, 2016). In practice, these rehearsal structures created opportunities for students to acknowledge and engage in the political work of teaching and learning, and practice new-to-them strategies and procedures for enacting everything from assessing students' prior knowledge of mathematics to facilitating a discussion of mathematical ideas to advocating for their own students.

Many elementary PTs suffer from mathematics anxiety. Our course sequence was designed to help PTs who had struggled to develop a productive disposition toward mathematics in their own past (National Research Council, 2001; Ruef et al., 2020). We did so by modeling the humanizing of mathematics, which includes debunking pernicious myths about what it means to be good-at-math (Ruef, 2021; Sun, 2018). In practice, this included normalizing and celebrating the following: not-knowing as an important part of the process of building understanding of mathematics; the value of asking a good question; the importance of appreciating bravery in sharing a mathematical idea or quandary; and "rough draft" reasoning (Jansen, 2020). In sum, these themes were interwoven within the planning and implementation of the course.

Adaptation and implementation of course for online instruction

March 2020 brought the realization that we must learn to implement effective small group, inquiry-based mathematics instruction online. Additionally, we needed to prepare PTs to learn and teach mathematics both online and in person. It was unclear how long the pandemic would persist, how and where field placements might take place, and how candidates would be assessed for licensure. In the face of much uncertainty, we began our planning on an immutable foundation: whatever we built, it would rest on the principle of relational equity. In practice, this meant accounting for how students would be seen, heard, and valued for their contributions.

From this stance, we considered how students would interact online. Because our course is one component of a teacher preparation program that centers teaching for equity and social justice, we knew our students would begin our course with an established foundation in these principles. Specifically, our program prepares teachers to be critical analysts of content and pedagogy and to advocate for students.

By the time our students arrived in our classes, they were enculturated to using video break out rooms for small group interactions and were developing the theoretical lenses and practices necessary to reframing mathematics teaching and learning for equity. Our program used Zoom and Canvas as platforms for online interaction and delivery and receipt of course materials and assignments. As instructors, we met to discuss student-facing Canvas pages which resulted in general agreements about providing advance organizers for students. In the past, many student-facing documents were delivered on paper and in person. To support online document delivery, we renamed all electronic course files for clarity and consistency and shared them before class so that students could download and print them, if so desired. We included a lesson plan for synchronous sessions so students could follow the course of the lesson and organize student-facing documents accordingly.

All classes were held on the Zoom platform, which allowed the instructors to see and interact with students. Many students opted to keep their cameras off, reporting unstable internet connections made worse when their cameras were on or "Zoom fatigue," a sense of being tired or overwhelmed by being present for online video meetings.

Pre-COVID-19 course schedule, interaction structures, and activities for each class

Having considered how students would interact, we were confronted with twin imperatives: minimize Zoom fatigue and re-envision brick-and-mortar interactions. We managed both by reorganizing how the students spent the three hours and fifty minutes allotted for each of nine class sessions. In the past, we used the first hour for small group teacher rehearsals. In brick-and-mortar classes, for the first two weeks all groups met during that first hour to complete a planning guide for a full class mathematics rehearsal. The instructors offered feedback on the planning guide before the groups revised and finalized their plans. The following weeks, the groups took turns teaching their lesson during the first hour of class. The groups acted as a "hive mind" teacher; functioning as one collective teacher with each participant taking one of the following facilitation roles:

1. conducting a number talk (e.g., Parker & Humphreys, 2018),

Table 2. Allotment of hours of fall methods course for brick-and-mortar and online settings

Hours	Brick-and-mortar	Online
1 ~60 minutes	**Small groups:** "Hive Mind" teaching rehearsal with debrief	**Small groups in breakout rooms:** workshop time for TPA assignments; reading/video discussion time, recorder writes and uploads short group reflection
2-3 ~110 minutes	**Whole class instruction:** brief lectures on content, whole class teaching rehearsals **Small groups:** solving math problems, practice with manipulatives and modeling, reading discussion, doing math together, analyzing student work, rehearsals on advocating for students, 20-minute dinner break	**Whole class instruction:** brief lectures on content, whole class teaching rehearsals, complete feedback form **Small groups:** Solving math problems, practice with manipulatives and modeling, analyzing student work, rehearsals on advocating for students short break
4 ~60 minutes	**Small groups:** workshop time for TPA assignments	**Asynchronous (on your own):** engagement with classroom video records of brick-and-mortar teaching and learning or pre-recorded webinars

2. launching a task (Jackson et al., 2012), with the other hive mind teachers monitoring "students," a role enacted by non-teaching classmates, in completing the task,
3. orchestrating a whole class discussion (Smith & Stein, 2018), and
4. debriefing the rehearsal with the class.

The second part of those in-person classes was devoted to readings discussions and workshop time for Teaching Practice Assessments (TPAs), which included

1. a diagnostic interview, wherein a PT interviewed a student to assess their conceptual and procedural understanding of problems in the numbers base ten strand of the CCSS-M (National Governors Association Center for Best Practices & Council of Chief State School Officers [NGACBP & CCSSO], 2010),
2. facilitating a mini-problem, usually a number talk, with a small group of students, and
3. planning, implementing, and assessing student learning for an entire mathematics lesson.

The third part of class included instruction on the use of manipulatives in class, how to advocate for students, pedagogical theory, task selection and modification, assessment, and additional mathematical content practice. Our key mathematical foci included the numbers base ten, operations and algebraic thinking, and fractions strands of mathematical content.

For example, one lesson first asks the students to find two solutions for how "seven people [could] share four brownies." After solving the problem on their own, the instructors select students to share their solutions with the class (Smith & Stein, 2018). The PTs then analyze copies of student work on the same problem to assess mathematical understandings, both emergent and complete. After conferring, a group representative shares their analysis and supporting evidence with the whole class.

Adapted online course schedule, interaction structures, and activities for each class

To adapt to online instruction, we reapportioned class time as follows. The first 50 minutes included a brief orientation and directions for how to use small group time. The pre-assigned groups of 3-4 students were then released to Zoom break out rooms to discuss course readings and videos, write and upload a short response based on assigned prompts, and workshop TPA assignments. The instructors remained available in the main Zoom room and could be called in to join discussions and answer questions. The instructors reserved the right to "pop in" and visit groups. After a ten-minute break, the class reconvened for the second and third hours of class, which consisted of a concentrated version of the original whole class section. The instructors made use of various Zoom features, including break out rooms for small group work, polls to gather student feedback, and the chat bar to elicit questions and observations from students. This portion of class was allotted one hour and forty minutes with a brief two-minute break in the middle. This schedule was adopted based on the predominant wishes of the students to finish early enough for a seven pm dinner hour. What we referred to as the "fourth hour" was reallocated to online videos and webinars, and while it ostensibly took place from 7:00-7:50 pm, in practice that work could happen whenever it best fit the students' schedule and ability to interact with online video formats. In effect, this made the fourth hour asynchronous. **Table 2** shows an approximate distribution of the four instructional hours allotted to the course for both settings.

Adapted and additional technology

One success in our shift to online instruction was curating a set of high-quality video footage of brick-and-mortar instruction. This had long been a planning imperative, particularly given the unequal access the PTs had to observing and taking part in diverse communities of students and educators. Shifting to online instruction made that work essential. Ruef reached out to colleagues over social media and followed related posts of other mathematics teacher educators seeking to solve similar problems and compiled a list of video resources. The instructors then watched several videos, curated an annotated list of content mapped to course themes, and compared notes. The syllabus was then organized to pair video content with readings for each weekly class. Each week's reading list included one practitioner article, one chapter from the course text (Van de Walle et al., 2019), and one webinar or a set of brick-and-mortar elementary mathematics classroom videos. The syllabus included hotlinks to online content.

Canvas modules, which included the plan for weekly class meetings, assignment due dates, and links to content, were similarly organized to facilitate students' access to readings and videos. The addition of high-quality webinars and videos of mathematics instruction was an important and timely improvement to course content. To make Canvas more user friendly, the instructors

created naming conventions for all electronic files and unified the student-facing Canvas pages into modules that included "before class" pages with links to all files the students needed to download before whole class instruction.

Another success was the use of a document camera. Toggling between face-framing and web/document cameras, the instructors were able to model the use of manipulatives and scribe number talks. Relatedly, the PTs were required to purchase a personal manipulatives kit, which included base ten blocks, fractional pattern blocks, linking cubes, Cuisenaire rods, and an analog movable clock face. The PTs were able to practice using these manipulatives as they learned to conceptually model mathematical methods. We also made use of online interactive software such as padlets, jamboards, and virtual manipulatives.

One challenge was the uncertainty of field placements. It was a big ask, and a big give, for teachers to take on a mentorship role for the teacher preparation program. While all students were provided placement by the end of the term, our revisions and plans for implementing online versions of key assessments were solidified before placements were secured for all teacher candidates.

Teaching practice assignments

We named the major course assignments *teaching practice assignments* to highlight their centrality in preparing to teach and because they also prepare PTs for the educational teacher performance assessment (edTPA). Thus, our TPAs must meet two important criteria:

a. authentically support students in learning and demonstrating competence at key teaching practices, and

b. prepare them to pass a high stakes performance assessment.

Fortunately, these are compatible goals. All TPAs are assessed using rubrics with accompanying diagnostic feedback in the form of comments, both to model these assessment practices for students and to make feedback authentic and actionable. In brick-and-mortar placements, relational equity centers on our students' interactions with their own students.

To adapt these assignments to an online context, we had to consider how to center relational equity as our students assessed the teaching and learning opportunities they rehearsed with colleagues wearing "student hats" or records of classroom instruction. In practice, we asked students to highlight the brilliance of students' reasoning, including messy and incomplete reasoning, and compassion for the earnest efforts of students and educators. The appendix includes a detailed explanation of the revision of the TPAs.

Timely communication

We made efforts to communicate frequently and clearly, prioritized portions of class time towards organizing course tasks and assignments, and were responsive to student needs throughout the course. In practice, this meant attending to requests for help with quick responses and sending strategic course update messages. The announcements were written as summaries of course announcements, new opportunities related to teaching and learning mathematics, and advance organizers for upcoming classes to avoid overwhelming students with messages. In these ways, we intentionally reinforced our goals to create productive feedback loops and increase availability to meet with students both within and outside of designated class workshop times. While these were not new approaches, our adapted versions of timely responses were appreciated by our students in the relatively new online context.

Feedback from students

A key element in the ongoing adaptation of the course was weekly feedback from students. In practice, this took the form of short reflections and synopses of the reading/video discussions held in small groups. The responsibility for writing the reports rotated among group members. This collaborative assignment reflected the course goals of synthesizing pedagogical and mathematical knowledge. If a student was absent, they had the opportunity to complete the report as an individual. In practice, few students needed that accommodation as attendance was notably improved from previous years. There were only three absences the entire quarter, an attendance rate of .994 which may reflect students' ability to attend electronically and a decrease in communicable diseases due to COVID-19 restrictions.

Additionally, students submitted individual reflections on the whole class portion of class focusing on "stars and deltas," the activities that they felt were most impactful (stars) as well as how the instructors might improve facilitation of the course (deltas). The "deltas" feedback informed decisions about when and how long to break during synchronous instruction.

Summary of findings on centering relational equity in the course re-design

Beginning with the conceptual and practical frameworks for the previous in-person class, we were able to map to online versions of the interaction structures, activities, assessments, communication, and feedback structures and tools we had already developed. Learning to implement online versions of these course elements required savvier use of familiar technology and the implementation of new technological tools. Student familiarity with Zoom and Canvas interfaces were important factors in the overall success of the course (see **Appendix A**, TPA memos). The use of break out rooms for small group interactions and the chat feature on Zoom supported the goals of public sensemaking and productive discourse, markers of relational equity. We now turn to analysis of how the students experienced the course.

RQ 2: How Did Our Students Experience Relational Equity in the Online Context of the Course?

As we taught the course, we were mindful of opportunities for students to enact relational equity and used the weekly feedback reports as formative assessment for what was working and what we might modify in course design and implementation. At the

Table 3. Emergent themes from participants' experiences with relational equity (RE) in the course

Code ID	# / % (n=35)	Code	Source of relational equity experience
1	12 34%	Curriculum and assessments modeled RE	curriculum and assessment reflected equity, modeled pedagogy, reasonable workload, relevant, clear directions, clear criteria for success, choice in assignments,
1a	6 17%	Feedback surveys	feedback loops, student felt seen and heard in feedback surveys, response to feedback surveys
1b	3 9%	Flexibility	flexibility, flexible, deadlines, modifications
2	21 60%	Instructors/pedagogy modeled RE	collaborative, ability to share screens, student felt respected, Instructors built relationships, supportive instructors, responsive, communication, kind, patient
2a	10 29%	Group work/discussions	student mentions discussing, sharing, or working with groups/ peers/ breakout rooms
2b	7 20%	Student-centered	learning from students, sensemaking, students are experts, choice in assignments,
2c	18 51%	Relationships/culture	voices mattered, mutual respect, Cultural responsiveness/ Inclusion, collaborate, collaboration build relationships
3	10 29%	Course modeled RE	entire course as immersive experience in RE
4	8 23%	Technology limits	did the best possible, tech limited…
5	5 14%	Future teaching	future students, classrooms, teaching…

Note. Alpha-numeric code identifiers (e.g., 2a) indicate a parent-child code relationship

end of the course, we surveyed the students about their experiences with relational equity in the course. The 35 students who completed the after-course survey responded to this prompt:

Our final question relates to the theme of our current study of moving [our course] to an online context. We focused the course on the concept of "relational equity, the care and consideration students and teachers must take with each other to establish and maintain a culture of equitable learning." We would appreciate it if you could share your thoughts on how you did (or did not) experience this principle in our work together in [our course].

Table 3 reports the codes and frequencies of their application to the participants' responses. We noted five main sources of students' experiences with relational equity:

1. Curriculum and assessments,
2. Instructors,
3. Generic reference to the course,
4. Technology limits, and
5. Future teaching.

Curriculum and assessments

34% of the participants' responses mentioned the structure of the course, assessments, course readings, or in-class assignments. The students experienced the pedagogy they were reading about as enacted by the instructors, which was an intentional design feature of the course. One participant wrote: "I did experience [relational equity] through having choice in lessons, seeing changes after feedback was given, having appropriately leveled assignments." Of note, 17% of the responses mentioned the weekly requests for feedback or how the instructors responded to student feedback to modify the structure of the course. 9% appreciated the instructors' flexibility in modifying course assignments or due dates.

Instructors

60% of the responses referenced either interactions with the instructors or the pedagogy modeled by the instructors. These responses mentioned ways that the instructors facilitated collaborative interactions, such as permitting students to share screens during Zoom meetings, and ways that the students felt seen, heard, and respected by their instructors through class discussions, messages, or email communications. Again, this was an intentional feature of instruction, to model compassion in the face of the pandemic and for all challenges to learning. The instructors wanted the students to experience respect for their mathematical and intellectual work, compassion for their human needs, and respect for their humanity. One participant commented:

> I believe that [the instructors] do an excellent job of recognizing students as also being experts. Because this was idea of our knowledge and strategies also being valuable. I feel that they make this course equitable between instructors and students.

This work was intentionally modeled by the instructors and mentioned in the responses as experienced by the students in small and whole class mathematical discussions, the student-centered nature of the course, and the co-construction of a culturally relevant and inclusive culture of learning.

The course

29% of the responses referenced "the course" as a generic whole. These responses described the immersive experience in learning how to teach mathematics in a manner similar to how it was modeled throughout their experiences in our shared work. This included responses like the following:

> It was a great time to hear about how we could bring relational equity into a math setting. I think sometimes we only want to talk about equity in our literacy time, so by learning how to do it in a math setting we are able to become more equitable all around for our students.

This response connects the student's experiences with learning the specifics and nuances of teaching mathematics for equity to the broader goals of the teacher preparation program to do the same.

Technical limitations

23% of the responses referenced limitations on relational equity related to the online nature of the course. These included the ways video conferencing limited interactions and ability to share ideas. One participant expressed doubt that relational equity in an online setting was possible:

> [The course] brought to life the idea of relational equity, as much as it could. Equity isn't an option when school is online. However, in this class, I've never felt more respected for my ideas. I've never felt more respected for my ideas, even if they were wrong. I'm not sure why that was. I'm sure it was a combination of the professors giving us lots of space to think and discuss and my classmates being confident enough to do so.

Despite the challenges of learning online, this student indicated they felt seen, heard, and valued for their contributions. This was also evidence of students' willingness to take intellectual risks.

Future teaching

We note that 14% of the participants referenced themselves as future teachers of their future students. One participant shared:

> The course overall felt like it always went back to that principle. The coursework showed real ways that we can establish and maintain culture of equitable learning that we could use in our future classrooms.

This response, and others, described how the participants connected learning mathematics and pedagogy collaboratively in the course, and a collaborative and student-centered vision of future teaching.

Summary of themes

In their responses, the PTs shared the ways they were seen, heard, and valued in the course. They also indicated how these principles could apply to their own future teaching. Overall, we found evidence that our intentionality in constructing and modifying the course for online instruction resonated with the students' collective experiences with relational equity in taking the course.

DISCUSSION

The shift to online instruction challenged us to solve old and new problems of practice in an online setting. The following sections include a synopsis of the findings, followed by our contributions to theory and practice, concluding with limitations and next steps.

Synopsis of Findings

As we planned, implemented, revised, and reflected on our efforts to move our brick-and-mortar elementary mathematics methods course online, we encountered varying degrees of challenge and success, some anticipated outcomes, and some surprises. The following sections describe those successes, challenges, and surprises organized by analytical themes.

Embedded principles

Beginning and ending with our core principle of relational equity, we found ways to embed the principles and practices of facilitating inquiry-based, student-centered teaching and learning practices (AMTE, 2017; NCTM, 2014; 2018; Ruef, 2021). Our successes included the transition from table groups to Zoom break out rooms. We shifted from shared paper artifacts and manipulatives to shared screens, document cameras, and individual manipulatives sets. Our TPAs leveraged the ability of PTs to role-play as teachers and students, which helped them to envision and embody children's anticipated experiences in learning mathematics (Ball et al., 2005, 2008; Jackson et al., 2012; Parker & Humphreys, 2018; Smith & Stein, 2018). By necessity, we offered role-playing and online records of practice in place of field experiences. The latter was both a challenge, in that there is no real substitute for in person interactions with children, and a success, in that it forced the instructors to complete a longstanding goal of curating a set of high-quality free access online instructional resources. Based on their feedback, the students felt seen, heard, and supported. The feedback on assignments supported their revisions and completing assignments in a timely manner.

Doing more with less

To combat Zoom fatigue, we limited the amount of synchronous instruction. This forced us to distill content to essential elements. Relatedly, we redesigned the TPAs and weekly assignments to interweave as much as possible. Our goal was, as always, to prepare teachers to be effective early career teachers of mathematics. We were pressed to enact our common call to our students to "do more [learning] with less [content]." This motivated our careful selection of rich tasks and fewer assignments to braid, rather than layer, content and theory–in the days of COVID-19, we needed every activity to serve multiple goals more than ever.

Flexibility for the win

As was evident in the feedback from our students, flexibility was appreciated. We argue it was essential in providing meaningful instruction. To that end, we planned as much of the course as made sense before launching to provide a stable and predictable experience. The students knew when, where, and how to meet, access course materials, and complete and turn in assignments. They also understood what remained "under construction," notably, the guidelines for upcoming TPA assignments, and that these would be shared with them as the course unfolded. We modified meeting times after the second class meeting, based on student feedback. We learned that we could effectively plan for two major activities in addition to the introduction and closing of each whole class segment, so our initial planning included sorting our lesson plans in order of most to least essential content. As we learned to teach online, our planning better reflected what we could effectively enact (Sun et al., 2022).

Students rose to the occasion

Though we were intentional, resourced, and persistent in the design and implementation of this online version of the class, none of it would have been possible without the dedication, intention, and persistence of the students. We were regularly heartened by their engagement and appreciation. The weekly collection of feedback provided students a handy way to express their feelings, both negative and positive. Always respectful, frequently grateful, our students were our best teachers (Sun et al., 2022).

Contributions

The following sections share our contributions to pedagogical theory and practice for MTEs and researchers. We conclude with limitations and our next steps as researchers and educators.

Theory and practice

We took up the challenge of replicating, and where possible, improving upon our brick-and-mortar classroom in an online setting centered on relational equity (Boaler & Staples, 2008; Lo & Ruef, 2020; Sun et al., 2022). This guiding principle was important in making planning and pedagogical decisions in advance, in the moment, and in reflectively providing feedback to students (Lampert, 2010). We believe this ethic of care is reflected in the positive feedback from students, indicating they felt seen, heard, and supported. We believe it was instrumental in the students' willingness to unmute and share their thinking–the students were enculturated to intellectual risk-taking (Beghetto, 2009; Jansen, 2020; Lo & Ruef, 2020; Sun & Evans, 2016).

Through our adaptations of in class rehearsals of teaching and learning practices, we were able to embed essential theory and practice that center teaching for equity and social justice (Ball et al., 2005, 2008; Gutiérrez, 2016). Overall, our records offer a reflective roadmap of the considerations and problem solving required to teach this course in an online setting.

As we return to in-person settings, we anticipate utilizing some of the successes we found in online settings. Specifically, a four-hour in-person class can be reapportioned to three hours in-person with the "fourth hour" offered asynchronously. We plan to keep the weekly "stars and deltas." We will use and update the curated set of online videos of teaching and learning and webinars to provide a commonly shared set of observational opportunities. The streamlining of readings, content, and assignments will better integrate future lessons. Forced to walk our frequent mantra of "do more with less," we renew our commitment to that guiding principle (Sun et al., 2022).

Finally, there will always be a call for some forms of online learning (Calder et al., 2021; Jordaan & Havenga, 2021; Ruef et al., 2022; Uegatani et al., 2021). Because of our collective work in response to COVID-19 requirements, we are much better equipped to support long distance learning, which may be of benefit to students whose pre-service teaching placements situate them far from campus. Many PTs serve students in rural, urban, or reservation schools, which makes traveling back to campus for coursework challenging, particularly during inclement weather. Thus, our ability to provide a meaningful and effective educational experience in online settings will remain an important equity consideration, for our PTs, the students they teach, and the communities in which they live (Ruef et al., 2022).

Research

Our study of our own practice extends the research on mathematics teacher education curriculum and instruction (Lampert, 2010). This includes work on teaching rehearsals (Ball et al., 2005, 2008), noticing and wondering (Jilk, 2016; Van es et al., 2015), and teaching mathematics for equity (e.g., Ruef, 2021; TODOS, 2020). We extend this work by showing how we incorporated equitable teaching and learning practices and principles into an online context (e.g., Engelbrecht & Harding, 2005; Jones & Long, 2013, Pope & Mangram, 2015; Thanheiser et al., 2020).

Limitations

Study of one's own practice inherently positions the researcher in a thought experiment: the observation of one's own environment and interactions from the perspectives of within and an imagined without. We must attempt to understand from the imagined and observed perspective of others. To that end, we acknowledge our positionality as both instructors and researchers in this study and that this was both a strength and a limitation. Further, we would have preferred to measure the students' experiences of relational equity in the online version of the course with those of the cohort that took the class in person. For this study, we seized an unanticipated moment and opportunity with the data and participant consents available.

Next Steps

We will utilize what we have learned in revising our course for in-person learning with a refined understanding of how to support future cohorts of PTs. Teaching online, we found the chat bar an effective tool for sharing in whole class discussions. We wondered if it created a less threatening space to contribute to discussions, and if some students were thus more forthcoming. We do not have data to support this claim, because there is no way to compare with what might have been had the course taken place in-person. We believe there may be opportunities for researching how to recreate equitable online spaces for in-person settings (Sun et al., 2022).

CONCLUSION

The advent of COVID-19 presented nested challenges: teacher educators had to learn to teach online and support PTs in teaching and learning mathematics both online and in person. We are grateful to the colleagues, near and far, who supported our efforts to create and deliver the best quality online elementary mathematics methods course we could muster. We are especially grateful to our students, who more than rose to the occasion. In conversations with colleagues, we have heard the claim that anything that can be done in a brick-and-mortar classroom can be accomplished online. We found this to be mostly true as some things defy replication. It is hard, but not impossible, to give an online hug.

Author contributions: All authors have sufficiently contributed to the study, and agreed with the results and conclusions.

Funding: No funding source is reported for this study.

Acknowledgements: The authors would like to thank the participants in our study. As always, our students were our best teachers.

Declaration of interest: No conflict of interest is declared by authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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APPENDIX A

Appendix A includes data collection prompts and coded data. See **Table 3** in the body of the data for the coding scheme.

Survey Prompt for Data Presented in Table 4

> Our final question relates to the theme of our current study of moving [our course] to an online context. We focused the course on the concept of "relational equity, the care and consideration students and teachers must take with each other to establish and maintain a culture of equitable learning." We would appreciate it if you could share your thoughts on how you did (or did not) experience this principle in our work together in [our course].

Table 4. Data codes corresponding to coding scheme shared in **Table 3**. Yellow bands indicate data coded by Author 2 to calculate inter-rater agreement ratio. Author 1 codes=x, Author 2 codes=y

	1	1a	1b	2	2a	2b	2c	3	4	5
I did experience this through having choice in lessons, seeing changes after feedback was given, having appropriately leveled assignments.	x y	x y		x y		x y	y			
The class was very thoughtful of our (the students) time. We were treated like our thoughts and feelings mattered & our voices & opinions were heard.				x			x			
We were given a lot of grace and support during our course, encapsulating the very idea of relational equity. It was clear that everyone was going to be successful in the course, with accommodations and flexibility being integral to our class functioning.	x						x			
The group work & the checkins helped keep us all on the same page and allowed us to take care of each other. I would say the given the circumstances of it being a completely virtual class, it was still relationally equitable.		x			x		x		x	
I felt that this class focused on relational equity in all aspects. This course teaches the principle that math should be based on sense-making and caring about student thinking, not just about right answers. All students are given opportunities to solve math equations equally.						x				
I believe that [our instructors] do an excellent job of recognizing students as also being experts. Because this was idea of our knowledge and strategies also being valuable I feel that they make this course equitable between instructors and students.	y			y		x y	x y			
[our course] brought to life the idea of relational equity, as much as it could. Equity isn't a an option when school is online. However, in this class, I've never felt more respected for my ideas. I've never felt more respected for my ideas, even if they were wrong. I'm not sure why that was. I'm sure it was a combination of the professors giving us lots of space to think & discuss and my classmates being confident enough to do so.				x	x				x	
I think relational equity is important and experiencing this in the classroom is so important!								x		
I saw relational equity throughout the entirety of [our course] course. Both professors made sure to always take into consideration the mental health of their students. There was never a time that I felt like I couldn't easily go to [instructor 2] or [instructor 1]. Both were lovely & constantly checked in with us to evaluate where we were at & how they could support us further. We experienced relational equity in the form of curriculum. If an assignment didn't make sense for where we were at then they would adjust assignment to fit our needs.	x		x	x			x	x		
It was a great time to hear about how we could bring relational equity into a math setting. I think sometimes we only want to talk about equity in our literacy time, so by learning how to do it in a math setting we are able to become more equitable all around for our students.								x		x
The professors in this course took the time to get to know their students and be there for them. They did a good job of building relationships.				x y			x y	y		
I think the biggest way I saw relational equity not only presented, but actively used, was with the surveys after each class. I felt more listened to in your class than any other so far, because you made it safe to share our thoughts, and we often saw you follow through on what we wrote in the deltas, or continue doing the stars. I am positive that your commitment to student feedback and the way you normalized it will have an impact on my future role as classroom teacher. I already notice myself trying to get feedback from my students in my student teaching placement, though it is hard on zoom with young children. I valued readings about making math culturally responsive & accessible to all students by building background knowledge, having students work in groups, & emphasizing that all students deserve to have their voices heard in math class (i.e. there are no right answers, & discussions should always be happening about deeper understanding of mathematics).	x	x							x	x
I had a great experience with equitable learning. I think [our instructors] always put an emphasis on equity in all aspects of their teachings				x				x		
The lesson plans we created in this class allowed us to consider how to include equity in our planning and support students.	x									x
My experience in [our course] was a beneficial and fun one. I was able to discuss concepts discussed in class with other members of my cohort. My fellow pre-teachers were open to thoughts and ideas in the area of mathematics. They & instructors were helpful in clarifying any misunderstandings that may have occurred due to being a virtually distanced course. Instructors were considerate when some technical issue occurred.			x	x	x				x	
You did an amazing job! I miss this class a lot and one of the reasons is that you both made an effort to build a relationship and maintain an equality among all of the students and groups. I feel like it also worked really well because of both you and [instructor 2] as educator.				x y	x y		x y			

Table 4 (Continued). Data codes corresponding to coding scheme shared in **Table 3**. Yellow bands indicate data coded by Author 2 to calculate inter-rater agreement ratio. Author 1 codes=x, Author 2 codes=y

	1	1a	1b	2	2a	2b	2c	3	4	5
The course overall felt like it always went back to that principal. The coursework showed real ways that we can establish and maintain culture of equitable learning that we could use in our future classrooms.	x						x	x		x
I experienced this principal in the communication with peers and teachers as well as through the readings and assignments that we were assigned in the class.	x			x	x					
I believe that I did experience this principle. The focus of our learning was always in equity, and equity was always at the root of our discussions both with instructors and classmates.				x	x			x		
I experienced this principle through giving feedback about class and having that feedback taken into consideration and acted upon. I have experienced many times where myself and other students provided teachers feedback yet nothing was ever changed, mentioned, or discussed. It was apparent that [our instructors] really read and considered the feedback they got from my class and made changes that were beneficial to us and our learning.		x								
I experienced this principle in our work together in [our course] when noticing that the professors valued our participation and discussion				x y	x		x y			
I experienced it but it was hard to pretend to be a student even though students either 'get it' or 'don't get it' in my experience. My third graders I work with usually all use the equation provided, I haven't been able to encourage creative problem-solving.									x	
I experienced relational equity on many levels in [our course]. The work that [instructor 1] and her coteacher did to build relationships with students online during a pandemic was helpful to my learning and confidence in the subject. Additionally our class was given opportunities to work together to build our cohort community in a way that could positively affect us all.				x	x		x			
I felt a great sense of relational equity throughout this course. The professors continuously asked for feedback after each class with a student reflection that included "stars and deltas". This helped students have a voice in how the class was run and allowed them to get the most that they could out of the course. The workload of the course was beneficial to our learning and didn't feel like busy work. The professors made themselves available, offered help frequently, and were kind and patient with all students, which made it a productive and equitable course.	x	x		x						
I did, I felt accepted and understood throughout the entire course!				x						
I experienced relational equity by learning about the importance of number talks and communication between students and teachers to get a better sense of students critical thinking when it comes to mathematics. It is important as educators that we learn from our students. To get a sense and understanding of where our students are and their thinking about a concept. The readings and course assignments gave me more knowledge about these concepts.	x y					x y				
I certainly did not experience anything that went against this principal. I could tell that both instructors were operating from a place of respect with attitudes of inclusion. I appreciated the thoughtfulness and ongoing consideration of students' lives as the class was run online.				x					x	
I feel like I learned a ton about 'relational equity' throughout this course. Much of the learning/work was done collaboratively with peers and I always felt like my professors were trying to learn from and about me as the student as much as I was learning from them as educators.				x	x	x	x			
I experienced a culture of relational equity in [our course]. My instructors were deeply invested in my growth and success as a student teacher and really worked to build my confidence and meet with me to support my understanding of concepts. I felt like the class culture was extremely positive and affirmative. Directions were clearly and consistently communicated and I always felt comfortable communicating with my instructors.	x			x			x			
I thought the manner in which [our instructors] taught this course 100% captured the essences of the UOteach program. They focused on different learning styles and teaching styles and connects with students. They modeled these in how they taught class and flexibility and scaffolding of class and the assignments.	x		x	x						
I thought that the course did a good job to give the opportunity for students and teachers to connect.						y	x y	x y		
I do not remember relational equality being explicitly stated, but I do remember our focus on the cultural responsiveness of math and the ways that we can make math more accessible to more students.							x			
Throughout the term, there were many opportunities for supports for students. This was extremely helpful because there was so much going on so this extra support they provided was extremely beneficial. Every class it was clear that the professors made a space for students to ask their questions and provided ample time for students to process the information they were given. There was a big emphasis on doing things with multiple strategies and making sure that students were able to discuss their process. With class being on zoom, there were restrictions but also many possibilities to involve everyone. Everyone had the ability to screen share and talk things through as a group. Overall, the classroom was set up for success and a culture of equitable learning was present.		x		x	x		x	x	x	
I experienced relational equity in [our course] because there was a mutual respect between the teachers and students. Learning in this course was very collaborative.				x			x	x		
I experienced this principle in our work together in [our course] by connecting with the professors when I am needing extra help, time or further explanations. In this course for me it is more beneficial to learn in person as I am a hands on learner. But throughout extra guidance and help from the professors I was able to successfully not only pass the class but learn and find new skills to teach my future students when we do go in person. Alot of what I learned about car and consideration we need to obtain in order to sustain a equitable learning culture can be used not only online but also in person. Taking this class online showed me how students can thrive in one category online but not in another in person, and vice versa.				x		x	x		x	x
I did well.										
	12	6	3	21	10	7	18	10	8	5

Finalized Iteration of Memoing for Teaching Practice Assessments (TPAs) Course Assignments

TPA 1: Diagnostic Interview. In brick-and-mortar practicum placements, we ask PTs to conduct one-on-one interviews with a student. Ideally, the interview is audio-recorded and copies of students' mathematical work are retained for analysis. The first TPA is a diagnostic interview, an intentionally slowed-down, focused, and evidence-based consideration of what a student does and does not yet know about mathematical concepts and procedures specific to a chosen task. The diagnostic interview is an important gateway to building asset-based rapport with mathematical learners and the collection and analysis of evidence of learning. Because we did not know which PTs had access to actual students, we modified the diagnostic interview assignment to include a choice of interviewing either a child or discussion group colleague. For example, the "teacher" might request that a colleague enact the role of "fourth-grade student." The interviews with colleagues were conducted and debriefed in discussion groups during workshop time. Because this was the first online TPA assignment, we allowed extra time for its completion. All students achieved scores of meets or exceeds for all rubric categories. There were two important related results: 1) the reports from the diagnostic interviews were generally quite strong, with evidence-based claims differentiating conceptual and procedural knowledge, correctly identified CCSS-M content standards (NGACBP & CCSSO, 2010) and asset-framed claims about the students' learning, and 2) the extended schedule pushed the remaining three TPAs further into the term, which compressed due dates at the end of the term and increased stress for some students.

TPA 2: Facilitating a Number Talk or Warmup Problem. The next TPA shifted the focus from an interview with an individual student to facilitating a group of students in a small segment of a full mathematics lesson. We focused on Number Talks because of the research base on their effectiveness in supporting a culture of public sensemaking and nurturing students' number sense (Parker & Humphreys, 2018). Again, in a brick-and-mortar setting, the PTs would have taught and recorded their Number Talk with groups of students. In online settings, while at this point in the term more of our PTs did have access to groups of students, there was still variation in how much opportunity our students had to teach their own students. Again, we provided workshop time for the discussion groups to rotate facilitating and debriefing Number Talks with their colleagues acting as students. One colleague had the additional task of taking notes on the process to share with the teaching student during the debrief. Again, we were pleased with the results, which included articulation of presentation principles, effectiveness of questions and noticings of mathematical connections, and reflections on revised or additional questions (e.g., Jilk, 2016; van Es et al., 2015).

TPA 3: Designing a Lesson. In a brick-and-mortar setting, our PTs have opportunities to teach full mathematics lessons, assess students learning, reflect, and plan re-engagement opportunities. In that version of the assignment, the PTs chose their own content, completed an initial planning guide, and utilized feedback from the planning guide to refine planning and complete formal lesson plans. In online settings, though this TPA took place later in the term, there remained wide variation in the PTs' opportunities to teach online within their field placement. As noted earlier in the paper, every activity planned for this course was carefully considered for its importance, effectiveness, and centrality to equitable teaching. We leveraged the variation in placement experiences alongside a common challenge in mathematics instruction: the need to modify existing content. Specifically, we asked students to begin by revisiting a lesson analysis assignment they had completed during a summer methods course. For that assignment, the students chose from a first- or fourth-grade lesson chosen for its problematic pedagogical framing. Because our teacher education program emphasizes critical analysis and revision of curriculum, the PTs were learning these skills in multiple courses. Because the students started from the base of their prior critique, they were able to build on their earlier analysis as they planned a revised lesson based on the mathematical content of their chosen lesson. In short, they decided what was worth keeping, what to cut, and what to modify.

In practice, the planning guide included thinking through all mathematical tasks from the perspective of students to anticipate multiple solutions, methods, and possible misconceptions. Pre-planning also considered cultural and mathematical contexts of real-life contexts, the construction of conceptual, procedural, and academic language learning objectives, notes on what the teacher might say or do to facilitate instruction, and an assessment of the cognitive demands of the mathematical tasks (Smith & Stein, 2018). The instructors provided extensive feedback on the planning guide, after which the PTs wrote formal lesson plans, adding details such as the size of student groups (individual, pairs, small groups, whole class) and appendices of student-facing materials. Overall, we were pleased with the PTs' ability to critically analyze and revise a mathematics lesson, to think from the perspective of students, and enact a formal lesson plan that could be enacted as they envisioned it by a "well qualified substitute teacher." Five students, whose rubric scores did not meet or exceed proficiency on all measures, were invited to revise their work, and those students took advantage of the opportunity to improve their work based on instructor feedback. Final grades for cohort reflected scores of meets or exceeds proficiency on all rubric measures.

TPA 4: Observing and Assessing a Re-engagement Lesson. As with the previous assignments, we designed our final TPA under the assumption that access to elementary students would vary. We were challenged with assessing our students' emerging abilities to teach when we could not observe their records or analysis of their own instruction. We also knew that our program's mathematical edTPA focus was on re-engagement lessons: how to plan instruction based on what students still need to learn about mathematical concepts or procedures. In short, what do teachers do next when a lesson does not meet all the intended learning objectives? We returned to our core principle of relational equity and designed an observation protocol for our students to use while analyzing a series of videos from *Inside Mathematics* (Inside Mathematics, n.d.). There were four different grade level lesson sequences to choose from. Each lesson sequence totaled approximately an hour of video footage, including a pre-lesson planning debrief, several videos of teaching and learning, and an assessment of the lesson de-brief. In effect, we asked the PTs to take field notes and analyze the planning, instruction, and assessment of students. Additionally, we asked for evidence-based claims of student learning, teacher facilitation, and a *potential* status intervention. Status interventions must, by definition, raise the academic status of a student, which cannot be determined solely by observation. Thus, we could only ask for interactions that met the second criteria: a teacher positions a student by publicly praising their mathematical contribution (Cohen & Lotan, 2014). We concluded the observation protocol with a request for two noticings and one wondering. Again, we were pleased with the overall results, and in particular, the respect and appreciation the PTs showed for the mathematical contributions of students and facilitation work of teachers. Once again, all students rubric scores at the meets or exceeds level for all categories for this assessment.