

Comparison of the learning outcomes in online and in-class environments in the divisibility lessons

Dina Kamber Hamzić^{1*} , Daniela Zubović¹ , Lamija Šćeta² 

¹ Faculty of Science, University of Sarajevo, Sarajevo, BOSNIA AND HERZEGOVINA

² School of Economics and Business, University of Sarajevo, Sarajevo, BOSNIA AND HERZEGOVINA

*Corresponding Author: dinakamber@pmf.unsa.ba

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ABSTRACT

In this paper, the effects of the online learning of the mathematical area of divisibility are studied, by comparing the achievements of students who have learned this mathematical topic in the online and in-class environments.

Data for this study were collected in seven schools at the beginning of the seventh grade of elementary education, with 383 participants aged 12 to 13. The test with four questions was designed according to the standards and levels set by APOSO (Agency for Pre-Primary, Primary, and Secondary Education in Bosnia and Herzegovina). Data were analyzed using a two-sample t-test and a Chi-square test.

The results highlighted that there was no statistical difference in the total scores between the students who learned divisibility in the in-class environment and those who learned it in the online environment. When comparing students' achievement in each question separately, statistical difference appeared only in the question of the highest level according to APOSO. The mistakes that students made when solving divisibility problems were also part of this research.

Keywords: divisibility, in-class learning, online learning

INTRODUCTION

The importance of an early education has been long recognized. The quality of knowledge and skills that students acquire at an early age significantly affects their later acquisition of knowledge. Claessens and Engel (2013) stress out that early mathematical knowledge and skills are good predictions for future mathematical, science, and reading achievements.

COVID-19 pandemic switched the regular, in-class teaching, and learning into an online model. Due to the more "relaxed" approach to the whole education process, an important question is what levels of knowledge students have achieved during this pandemic period (Owusu-Fordjour et al., 2020; Schult et al., 2022).

In this paper, the focus is on divisibility, an important topic in mathematics, which is taught in Bosnia and Herzegovina to students at the age of 11-12 (Agency for Pre-Primary, Primary, and Secondary Education, 2015). This study aimed to find out if learning about divisibility in an online environment had had a significant impact on the students' knowledge and understanding. The results of the study will show the quality of the online lessons in comparison to the in-class lessons in Bosnia and Herzegovina, but also the possible lacks in students' knowledge of divisibility, since one of the goals is also to discover common mistakes, students make when dealing with divisibility. This is important for Bosnian educators (teachers and administration) to know, so that the future divisibility lessons can be improved.

Divisibility

Findings of Siegler et al. (2012) show that high-school students' mathematics achievement is better determined by their knowledge of fractions and whole-number division in elementary school than by some other factors, like working memory, family education, verbal and non-verbal IQ, their knowledge in addition, subtraction, and multiplication.

One of the reasons for a strong relationship between the division in elementary school and mathematics achievement in high school according to Siegler et al. (2011) is that it reflects higher-order processing which is important to mathematical reasoning (see also Ellis et al., 2018).

Among other difficulties in understanding divisibility, Campbell and Zazkis (2002) mention the linguistic gap between informal natural language and formal mathematical language. It is common to confuse the concept of divisibility with the process of division, and the word multiple with the activity of multiplication, due to the linguistics itself, since the words 'divisibility' and

Table 1. Standards of student achievements in mathematics at the end of sixth grade related to the divisibility of natural numbers

Low level	Medium level	High level
Student knows the divisibility rules of 2	Student correctly selects, among several given choices, the product, which represents prime factorization of two-digit number	Student applies divisibility rules of 10, 100, and 1,000 to given numbers
Student knows the divisibility rules of 4	Student correctly selects the least common multiple of given numbers, among several given choices	Student determines odd numbers divisible by 3 in a given sequence of numbers
	Student determines the smallest natural number divisible by two given numbers	Student applies divisibility rule of 9 with the additional complex condition
	Student knows the concept of multiple and connects it to divisibility	Student understands the concept of a prime number and lists them according to the request "greater than/less than"
	Student identifies the number which is a factor of every number in a given sequence	Student applies the rules of product and sum divisibility
	Student calculates the greatest common divisor of two two-digit even numbers	Student determines the least common multiple of given numbers
		Student calculates the value of the product of fractions, applying proper simplifying
		Student sets up and solves the expression in the set of natural numbers, which corresponds to a real-life situation

'division', 'multiple' and 'multiplication' sound similar. Confusing the terms 'multiple', 'multiply', and 'multiplied' was common among the participants aged 10-12 in the study by Hurst et al. (2021). When it comes to the Bosnian language, there is no existing research about errors of linguistic type in the divisibility learning, but it is possible that the similar confusion will arise between words 'divisibility' (in Bosnian, 'djeljivost') and 'division' (in Bosnian, 'dijeljenje'). Due to the frequent use of the same expression for 'number' and 'digit' in informal Bosnian language, one expected mistake is for students not to distinguish between a number and a digit.

Brown et al. (2002) suggested that in divisibility issues, teachers should be more focused on the multiplication than the division role and they should strengthen the students' understanding of division as a reversal of the process of multiplication. Roscoe and Feldman (2016) noticed the advantage of "prime factorization as a conceptually rich tool for understanding divisibility" (p. 13). Their study supports the idea that abilities to solve divisibility problems rise across a variety of factor subtypes.

Young-Loveridge and Mills (2012) investigated how the students' understanding of divisibility rules affected their deeper understanding of multiplication and division with whole numbers.

There are numerous ways of approaching the concept of divisibility. Following the Piaget's stages of cognitive development (see Sternberg, 2003), the concept of divisibility could be built through the preoperational stage using examples such as to color every third square, and through the concrete stage using a hands-on approach described in West (2014) or dealing with tasks such as "find the value of 12:2". West (2014) observed how students developed their understanding of divisibility rules by starting from a hands-on approach, and encouraging them to rediscover divisibility rules for two, three, four, five, eight, nine, and ten.

Structure of the Bosnia and Herzegovina Education System

Bosnia and Herzegovina has a very complex administrative structure of the education system. There is one education department within the Government of Brčko District, two entities (Federation of BiH and Republika Srpska) have their ministries of education, there are also 10 cantonal education ministries (in the entity Federation of BiH) and the Ministry of Civil Affairs on the state level, which makes 14 ministries responsible for education at various levels of government. Curricula in all subjects, including mathematics, are not uniform in BiH, while there are also several different approved textbooks used in different areas of BiH. In such an environment, it is not surprising there are unequal pedagogical standards.

This study was conducted in the Sarajevo Canton, which is the second most populated canton in the Federation of BiH (Institute for Statistics of FBiH, 2021). However, even in this one canton, there is more than one mathematics textbook that has been selected in different schools to study mathematics. For example, currently, sixth-grade elementary school students from the Sarajevo Canton use four different textbooks (Catholic School Centre "St. Josip", 2022; Federal Ministry of Education and Science, 2020). In the previous years, the number was even greater (Ministry of Upbringing and Education of Sarajevo Canton, 2016). Considering that after finishing elementary school, students from this canton should take the uniform external exam, this diversity in curricula, as well as in approved textbooks, seems like a disorder.

In this disorder, the best attempt so far to unify curricula has been made by the Agency for Pre-Primary, Primary, and Secondary Education (APOS0). APOS0 is responsible for establishing standards of knowledge and evaluation of achieved results, development of a common core of curricula in pre-school, primary, and secondary education, and other activities in the field of standards of knowledge and quality assessment determined by special laws and other regulations (Agency for Pre-Primary, Primary, and Secondary Education, n. d.). APOS0 is also responsible for conducting international testing in BiH such as TIMSS and PISA.

Among others, APOS0 formed a common core curriculum for mathematics area defined on the learning outcomes defined for the end of third, sixth, and ninth grade of elementary school and the end of secondary school. For appropriate learning outcomes, APOS0 classifies standards of student achievements in mathematics (SSAM) as low, medium, and high level. Extracted student achievements related to the divisibility of natural numbers till the end of sixth grade are given in **Table 1**.

In-Class and Online Lessons

In the school year 2019/20, as well as in 2020/21, due to the COVID-19 pandemic, the learning that was held in classrooms up to that point moved to a home atmosphere for most students and teachers. Teachers and students were thus given the opportunity to compare the advantages and disadvantages of in-class and online teaching.

Comparing the benefits of in-class and online teaching and learning, Wright (2017) states the speed and convenience of learning as the advantages of online teaching, as well as flexibility in choosing the time and place for learning. As the benefits of working in the classroom, Wright (2017) talks about better motivation to work, better interaction between students and teachers, and between students, as well as the contribution of teachers.

Teachers' belief that the effectiveness of online teaching itself depends a lot on students' self-discipline (Cao et al., 2021) leads to the opinion that it will not have the same effect as classroom work. This leads to the conclusion that it is necessary to work more intensively on encouraging self-learning skills.

According to Zuo et al. (2021), teachers who have adapted their teaching strategies to the online environment in order to encourage learning motivation and greater student engagement, could see an improved teacher-student relationship as a benefit of online learning.

Khattar et al. (2020) reported that students felt that online teaching could be a supplement for classroom teaching, but that it was not a proper substitute for the experience of learning in the classroom environment.

Research Goals

The main goal of this research is to see whether there are differences between students' level of responses, depending on having studied divisibility in the in-class or online environment. Jiang and Xiong (2021) found that mathematically gifted students had a positive learning experience from online learning about divisibility in elementary number theory—however, these were gifted students and they studied divisibility at a higher level than it is learned in regular mathematics classes in school.

Common mistakes that students made when solving divisibility problems were also part of this research. Some mistakes might be of linguistic nature, but majority of them will probably happen due to the lack of mathematical knowledge and understanding.

Therefore, the research questions are, as follows:

1. Is there a significant difference in total scores between the students who studied divisibility in the in-class environment and students who studied divisibility in the online environment?
2. Is there a significant difference in answers to specific questions between the students who studied divisibility in the in-class environment and students who studied divisibility in the online environment?
3. What mistakes do students make when solving divisibility problems and are some mistakes more common than others?

MATERIALS, PARTICIPANTS, AND PROCEDURES

Tools

The research tool included a 12-task questionnaire related to divisibility of natural numbers. From each level (low, medium, and high) students were given four questions. The four questions that were a focus of this study and the reasons why they were selected follow below.

1. **Question 1.** Of the given numbers, one number does not belong to this group. Circle the letter in front of correct answer. Explain the answer.
 - a. 3
 - b. 11
 - c. 4
 - d. 7
2. **Question 2.** From the offered answers, choose a product that represents prime factorization of number 24. Circle the letter in front of correct answer. Explain the answer.
 - a. 2×12
 - b. 4×6
 - c. $2 \times 2 \times 2 \times 3$
 - d. 8×3
3. **Question 3.** By not calculating products, combine the relevant statements:
 - a. $29 \times 51 \times 47$ divisible by 2
 - b. $37 \times 101 \times 62$ divisible by 3
 - c. $35 \times 31 \times 124$ divisible by 4
4. **Question 4.** Write down all the digits that can replace * in the number $41*13$ so that this number is divisible by 3.

The aim of the question 1 was to examine whether students knew how to distinguish primes from composite numbers when there were some given numbers. In cases where students used the rule of divisibility by 2, without noticing that 4 is a composite number, this was also accepted as a correct answer. According to SSAM, this question satisfies low-level standard of student achievements at the end of sixth grade of elementary school.

Question 2 was given to check students' understanding of prime factorization, i.e., to examine whether students knew how to choose from offered answers a product that represented division of a two-digit number into prime factors. This question requests medium-level of student achievements by the standards given in SSAM.

The third question introduced here requests high level of student achievement. This question was introduced with the aim to investigate students' ability to correctly apply the rule of divisibility if the number is given as a product.

Question 4 was given in order to examine whether students understood divisibility by 3 and were able to easily apply rules for divisibility by 3. This question is also related to the students' ability to distinguish digit from number. In answering this question, students should be able to see that there is more than one correct digit as a solution. According to SSAM, this question satisfies high-level standard of student achievements at the end of sixth grade of elementary school.

After choosing the questions for the questionnaire, the panel of three experts (two university professors, one expert in number theory and other in geometry and mathematical education, and one school mathematics teacher, who has worked for several years as an advisor for one government educational agency) reviewed chosen questions for the content validity. They suggested some minor changes that were implemented in the questionnaire, before it was given to the participants.

Participants

The participants in this study were 383 students aged 12 to 13, from seven urban public elementary schools in the Sarajevo Canton. Among them, 196 were girls, while 187 were boys. The participants represented two generations of students who attended seventh grade in two school years (2019/20 and 2021/22), in September (first month of the school year). By curriculum, the participating students had studied the divisibility of natural numbers, divisibility rules, prime and composite numbers, prime factorization, and related topics at the end of the sixth grade. 128 of these students attended seventh grade in school year 2019/20, in school environment. Of the 128 students, 67 were girls and 61 were boys.

In the 2021/22 school year, there were 255 of participating students in the seventh grade in the observed schools. Of these 255 students, 129 were girls and 126 were boys. These students had the previous school year in an online environment (due to the COVID-19 pandemic), as well as mathematics classes and classes on divisibility of natural number. Lectures were held on the Microsoft Teams platform, and teachers in most cases used presentations and whiteboards to teach new lessons.

In the rest of the paper the first group of students will be named the 'in-class' group, while the second group of students will be named the 'online' group.

Procedures

At the beginning of the mathematics class, questionnaires were distributed to the participants in their classroom in presence of their mathematics teacher and the second author. Students were given 45 minutes to complete the 12-task questionnaire. The protocol of this study was approved by the Ministry of Upbringing and Education of Sarajevo Canton.

The answers were classified as correct, wrong, or incomplete. The student's answer was acknowledged as a correct one if it was completely correct and with a full explanation. The answer was considered to be wrong if the student did not answer it at all or if there were no elements of correct solution in it. The answer was considered to be incomplete, if there were some elements of correct solution in it, but not all (for example, if the student gave answer to the question 4, listing some correct digits, but not all). Only correct answers were given one point.

The statistical results in this paper are obtained using Stata/SE 12.0 software.

DATA ANALYSIS AND DISCUSSION

Analysis of the Total Scores for the 'In-Class' Group and the 'Online' Group

The total scores were calculated as the total number of correct responses on four selected questions (so, the maximum possible score was four points). Average score for the 'in-class' group on this test was 1.57. Average score for the 'online' group was 1.498.

A deeper analysis of the results in the 'in-class' group showed that only four participants (3.13%) gave all correct answers. Although 37.5% of the participants scored only one point, more than half of participants (51%) gave two or more correct answers.

In the 'online' group, none of the participants gave all correct answers. In this group less than half of participants (48%) gave two or more correct answers. The percentage of participants who scored only one point (32.2% of them) is similar to the same percentage in the 'in-class' group.

Figure 1 shows distribution (in percentages) of total scores for each of the two groups ('in-class' and 'online'). Comparison of the total scores of the two groups of students shows that in the 'in-class' group there are, in percentages, less zero answers and more one-correct, two-correct, and full score answers, while in the 'online' group of students, there are more three-correct answers.



Figure 1. Distribution (in percentages) of total scores for both groups

Table 2. Percentages of correct answers to each question

Questions	'In-class' group	'Online' group
Question 1 (low level)	55.47%	55.29%
Question 2 (medium level)	43.75%	52.16%
Question 3 (high level)	46.9%	40.78%
Question 4 (high level)	10.94%	1.57%

In order to answer the first research question, a two-sample t-test on 383 students was conducted to determine if online mode of learning led to a difference in mean of total scores of the four-task questionnaire. Results of the two-sample t-test had a p-value of 0.2787. Hence, there was no sufficient evidence to say that the mean of total score was different between the two groups of students, i.e., results showed that the mean of total scores was not significantly different between the two groups of students. This came as a surprise, since almost everyone in our education system (teachers, administration, parents, and students) claimed that online lessons were not as good as in-class lessons.

Analysis of Each Question Score for the 'In-Class' Group and the 'Online' Group

Percentages of correct answers to each question for the 'in-class' and 'online' groups are given in **Table 2**. The majority of students answered correctly to question 1, while the least correct answers were given to the question 4—this was expected, as the question 1 was a low-level (“easy”) question, and question 4 was a high-level (“difficult”) question.

To test whether there is difference in answers to questions 1 to 4 between the 'online' and the 'in-class' group, Chi-square test was conducted.

Regarding question 4, Chi-square test turns out to be 85.0417, and associated p-value is 0.000. These results provide sufficient evidence to conclude that there is a statistically significant association between the learning mode of students and their answer to the question 4. The 'in-class' group had better results on this question. Also, there were more students in the 'online' group who did not answer this question at all than in the 'in-class' group (37% and 28%, respectively). Meanwhile, 29.7% of the 'in-class' group gave incomplete answers (i.e., answers that had some elements of correct solution, but not all), unlike students from the 'online' group, where only 2.4% of them gave incomplete answers.

Regarding questions 1, 2, and 3, the associated p-value of Chi-square test turns out to be 0.678, 0.175, and 0.523, respectively. There is no sufficient evidence to conclude that there is a statistically significant association between the learning mode of students and their answers to questions 1, 2, and 3.

These results, together with the comparison of the total scores, imply that students have achieved basic levels of knowledge and understanding of the divisibility during the online lessons. However, the research results also imply that online lessons might have had an effect on students' understanding of some more complex and difficult concepts. Also, for one period of online lessons, classes were shortened to 30 minutes (instead of the regular 45 minutes), so teachers probably did not have enough time to work on more complex topics as they were committed to covering the basics. This needs to be improved in the future learning of the divisibility: more time needs to be dedicated to more complex topics and problems, so students could have the opportunity to achieve the highest level of SSAM, and not only low and medium levels.

Analysis of the Mistakes Students Made

Most mistakes were made on question 4. First, there were many “empty” answers, i.e., a lot of students left this question unanswered: 28% of the answers in the 'in-class' group were empty answers. Meanwhile, more than 37% of the 'online' group students left this question unanswered.

The second most common mistake for this question in both groups was not distinguishing the digit from the number, as well as not noticing all digits in the set of solutions. Some students did not list digit '0' as a solution to question 4, which was not surprising. Students know about '0' as a digit since first grade, but they learn about '0' as a number at the beginning of seventh

grade (when they learn about the set of integers), and since a lot of them do not distinguish a digit from a number, it could be that they just ‘forgot’ that zero exists (as a number and as a digit).

Considering the same question, the mistake that occurred only in the ‘online’ group of students is the use of signs like +, -, <, and >, instead of digits, which was an unexpected mistake.

In question 1, a great number of students in both observed groups selected incorrect answers without further explanation. In the ‘in-class’ group there were a few students who justified their choice as (b) 11, by explaining that it is a two-digit number, regardless of the oral remark before the test that this is a questionnaire that examines the divisibility of natural numbers.

The most common mistake that occurs in both groups of students for the question 2 is that they circle all the offered options with comments like “All these are equal to 24, so every option is true”. These comments from students who offered wrong answers show that they did not understand the concept of prime and composite numbers, nor did they understand the concept of prime factorization.

While answering question 3, a lot of students in both groups did not know the divisibility rule of 3. Also, students in both groups often made a mistake of connecting both expressions, $37 \times 101 \times 62$ and $35 \times 31 \times 124$ with the statement ‘divisible by 2’. Both expressions are divisible by 2, but they were supposed to differentiate which one of these two expressions was divisible by 4, which shows some of them did not understand what was asked of them to do.

Analysis of students’ mistakes shows that a greater emphasis must be put on differentiating between the concept of number and the concept of digit. Also, more attention needs to be given to the prime factorization of a number, since this was a major cause of wrong answers to question 2. Students had problems recognizing numbers divisible by 4, and not only by 2 (as in question 3), so divisibility by exponential numbers (i.e., the numbers of the form a^k) is another topic that needs more attention. One way to achieve this is to dedicate extra time to this topic during the pre-service mathematics teachers’ education. Another way is to add more problems involving these topics in textbooks.

CONCLUSION

Early mathematics knowledge, especially knowledge about divisibility, is an important predictor of later mathematics achievement (Claessens & Engel, 2013; Siegler et al., 2012). Transition to online classes due to the COVID-19 pandemic, as well as the pandemic itself, caused an increased amount of stress on students (Khattar et al., 2020) and obviously had effect on students’ learning. This research aimed to examine the effects of online learning on the area of divisibility, through a comparison of students’ achievements, depending on whether they learned about it in the classroom or in the online environment. Another goal was to see how well students understood some basic divisibility concepts and ideas.

The first research question was whether there was a significant difference in total scores between the students who studied divisibility in the in-class environment and students who studied divisibility in the online environment. Statistical test showed there was no significant difference. This was somewhat surprising, since the general attitude of everyone involved in the teaching process (teachers and students, parents, and administration) was that online teaching was noticeably poorer than in-class teaching. However, this result is in accordance with studies that showed no difference in learning outcomes between online and in-class courses (for example, Kirtman, 2009; Nichols et al., 2003). Of course, the focus of this research was on one area of mathematics, and it would be interesting to compare results for other mathematical areas and other age groups of students.

The following research question was about differences in specific questions. There was no statistically significant difference in answers to the first three questions, which were low, medium, and high level according to APOSO standards. The only significant difference was present in the answers to question 4 (high level), where the ‘in-class’ group of students had better results. One of the future research projects could focus on investigating whether students achieved higher levels of knowledge and understanding within different areas of mathematics during online lessons, and if they did not, what is the main reason for this: shortened lessons, pandemic stress, or online teaching itself.

Finally, the third research question was about mistakes students made when solving divisibility problems, especially the more common ones. Majority of the mistakes were expected: not knowing the divisibility rules of 3 and 4, lack of distinction between the concept of digit and concept of number, not noticing that question 4 had more than one correct solution.

This study demonstrates that online learning did not leave significantly bad consequences when it comes to learning outcomes in the area of divisibility (and hopefully the same holds for other mathematical areas). The only significant difference was noticed in the most difficult question, and now there is a difficult task up to everyone within the educational process to catch up on more complex topics that were probably missed during the pandemic.

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