

# Configuring the landscape of research on problem-solving in mathematics teacher education

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## ABSTRACT

The purpose of this study is to configure the landscape of empirical mathematics educational research on problem-solving in teacher education, and thereby disentangle how mathematical problem-solving is understood and used. The method consists of a configurative review of empirical mathematics education research on problem-solving in teacher education. A two-dimensional model is presented to illustrate how different aspects of problem-solving in teacher education are connected to and complement each other. Using the model, the configuration results in the proposition of four major categories of research on problem-solving in teacher education. The result indicates an almost equal distribution of research which views problem-solving as an aim for mathematics education versus research which views problem-solving as a means for learning mathematics. However, within the former, roughly three quarters of the articles focus on content knowledge, and only a quarter on pedagogical content knowledge. Implications for teacher education and future research are discussed.

**Keywords:** mathematics teacher education, pre-service teacher, problem-solving, teacher education, configurative review

## INTRODUCTION AND BACKGROUND

Although there is widespread agreement that problem-solving is an important part of mathematics education (see, however, de Ron, in press), what problem-solving is, what a problem is, and how to teach problem-solving is not commonly agreed upon (Lester, 1994; Stein et al., 2003; Xenofontos & Andrews, 2014). There is a significant number of texts from the past 50 years—e.g., research articles, conference reports, curriculum guides, and textbooks—discussing the role of problem-solving in mathematics education (for instance, Carlson & Bloom, 2005; Liljedahl et al., 2016; Ryve, 2007). When it comes to what this means to teacher education, the research is sparser, and positions not as clearly drawn. Problem-solving as an ambiguous phenomenon and how problem-solving is engaged in teacher education will be unpacked in this configurative review.

A useful distinction is made by Andrews and Xenofontos (2014), between the *problem-solving process* and *problem-solving teaching* (see also Ryve, 2007). A similar distinction is made by Cai and Nie (2007) between *problem-solving as the aim of instruction* and *problem-solving as a means of learning mathematics*. In both cases, the former aims to provide learners with opportunities to enhance flexible and independent thinking, reasoning skills and overall problem-solving competencies. The second perspective addresses problem-solving as an instructional approach that provides a context or means for students to learn mathematics. The authors stress that the distinction is neither exclusive nor to be seen as a hierarchy where one is more important than the other. Rather, they are complementary to each other and should both be emphasized in mathematics education (Andrews & Xenofontos, 2014; Cai & Nie, 2007) as well as in teacher education (Ryve, 2007). Hence, a mathematics teacher could be expected to engage with problem-solving in both respects.

Whether the aim is to develop learners' problem-solving competences or problem-solving as a means of teaching mathematics, the teacher's preparedness includes their own problem-solving competences and knowledge, as well as competences to facilitate the relevant learning. This is closely linked to the distinction between *content knowledge* (CK) and *pedagogical content knowledge* (PCK) (Shulman, 1986). Säfström and Lithner (2020) argue for investigating the links between teaching and the knowledge and behaviors required for problem-solving.

What does it mean to apply the distinction CK-PCK to problem-solving? As pre-service teachers (PSTs) come to teacher education with what is sometimes referred to as general CK (Ball, 1988), "unpacking" of mathematical content may play a role in developing specific CK directed at teaching (Ball et al., 2008). In relation to problem-solving, this specific CK may imply learning how to solve problems using mathematics. PCK on the other hand includes knowledge about learning and teaching problem-

solving, for instance, about how to facilitate learning about problem-solving, or how to teach mathematics through didactical situations (Brousseau & Gibel, 2005).

The distinction between PCK and CK has been widely used in mathematics education and to some extension also connected to problem-solving. Csíkos and Sztányi (2020) investigated pre-service and in-service teachers' PCK concerning the teaching of problem-solving, suggesting that it often is implicit in the sense that the respondents have not elaborated on different aspects of PCK before being asked about them. In their result, Csíkos and Sztányi (2020) focused on some specific aspects of PCK: The teaching of solution strategies or algorithms, assessment of students' solutions, and making drawings during the solution process, suggesting that knowledge in these aspects informed their favored teaching of the process of problems-solving. In contrast, a study by Leikin and Levav-Waynberg (2008) focuses mostly on CK and address the complexity of teachers' knowledge by connecting PCK and CK to another classification of teacher knowledge focusing on the sources of that knowledge as either through learning (systematic mode) or through teaching (craft mode). In their result they recommend that courses for mathematics teachers address knowledge of both PCK and CK as well as systematic and craft mode. Hence, mathematics teachers could be expected to have knowledge about problem-solving in all these respects.

The distinction CK-PCK is not the only way to section research in teacher education. Does the research engage student teachers or in-service teachers? Does it address changes in beliefs, CK, PCK, reflections, practice, awareness, or identity? From which theoretical perspective does it start? In reviews of research perspectives in mathematics teacher education, Lerman (2001) and Sánchez (2011) touch upon all of these. However, problem-solving is not addressed in either of these reviews, and is only mentioned once as a motivating factor for including creativity in mathematics education in the more focused review of Joklitschke et al. (2018). Indeed, Kontorovich and Liljedahl (2018) found the area under-researched and called for further research, both theoretical and empirical. This accentuates the need for this review.

Existing research overviews of mathematical problem solving (e.g., Lester, 1994; Schoenfeld, 2013; Törner et al., 2007) have emphasized problem-solving in schools as opposed to teacher education. Research about problem-solving in mathematics teacher education has certainly been conducted, but there are no existing overviews of which perspective on problem-solving is promoted in teacher education practices and research. This review, which is part of a larger study on problem-solving in teacher education, sets out to provide a first attempt to categorize existing research. The aim is to configure the landscape of empirical mathematics educational research on problem-solving in teacher education. In relation to the aim, we also elaborate on a model for this configuration which points to two issues. The first concerns which content which teacher education should include and what knowledge PSTs should have. The second is how the findings of the study may be of interest to teacher education and the development of courses including problem-solving. The review offers one possible answer to the following research question: "How is mathematical problem-solving described/understood in empirical research on problem-solving in mathematics teacher education?"

## MATERIALS AND METHOD

The aim of the study was to generate a configuration of how problem-solving is understood and researched in the mathematics education research field. Therefore, a review of empirical studies that explicitly addressed problem-solving in teacher education was conducted. As no existing categorizations were found in the selected literature (see below about the process of searching for relevant articles), and in accordance with the open and inductive approach, aiming at exploring and developing new insights, it was decided to do a configurative review (see Gough et al., 2017). In contrast to an aggregating review where findings from studies are synthesized, a configurative review aims to configure characteristics across and between studies, describing the landscape of the studied area. Configurative approaches are therefore apt to investigate and clarify variation and differences in research with a focus on generating arrangements that offer meaningful pictures of perspectives in studies (Gough et al., 2017). These approaches usually do not adopt an exhaustive search but a purposive search which aims at identifying sufficient amounts of studies to provide coherent and meaningful configurations that have the potential to widen the understanding of the phenomena. The resulting configurations are arguably useful for addressing the heterogeneity of educational research (Levinsson & Prøitz, 2017).

### Process of Selecting the Articles

Research literature was located through searches in the database ERIC via EBSCO and ProQuest Social Sciences, and in the databases Scopus and PsycInfo, which together provide comprehensive access to education-related literature.

Several inclusion criteria were used for the review. First, the contributions had to be published in peer-reviewed academic journals. This criterion was applied to guarantee research of a scholarly nature. Second, in order to respect the focus of this study, selected articles had to deal explicitly with problem-solving in the field of mathematics education. Combined with that the context had to be teacher education, not school. Third, only empirically based research articles were included—that is, studies utilizing data from engaging teacher educators' and/or teacher students in problem-solving activities or reflections thereon. Fourth, the chosen timespan was 1990 to 2021 (March). Fifth, limited by our language abilities, only articles written in English were included. This could of course be seen as limiting the representativity of the sample; a common problem with reviews that is unfortunately unavoidable. We hope that if a substantial body of literature exists in another language, other researchers will supplement our study.

Practically, these criteria framed the searches using words from three search blocks: problem-solving, mathematics education and teacher education. The search block *teacher education* included words that either were connected to teacher education (e.g.,

teacher education, pre-service teacher education, teacher education programs) or words describing PSTs (pre-service teacher, preservice teacher, student teacher). Mathematics education and mathematics instruction were words in the *mathematics education* search block, and problem solving and problem-solving in the *problem-solving* search block. Words were combined with Boolean operators OR within and AND between the blocks. In ERIC, abstracts were searched for the search terms, while abstract, keywords and title were searched in Scopus and PsycInfo. The rationale for this was that search in abstracts only in Scopus and PsycInfo resulted in very few hits. In total, the search resulted in 299 articles. This search was then combined with a previous search conducted in 2016, including the database ProQuest Social Sciences. That search resulted in 71 hits.

Since the purpose of the review was configurative, we opted not to search for so-called gray literature or dissertations, or to use snowballing or targeting specific journals. A potential shortcoming of using the aforementioned databases is that they are skewed in favor of research published in English, and perhaps even American research in particular.

The resulting 370 articles were screened twice. In the first screening, doublets were removed, and abstracts were read with a focus on the inclusion criteria described above. To ensure consistency and consensus between all three authors in the screening process some articles were first read together and discussed. Subsequently, screening results were compared between authors. In this screening 31 were doublets, and 185 did not meet the inclusion criteria; or the text was written in another language than English; resulting in 154 articles being included. The included articles were then screened a second time based on a reading of the text in full. The same screening process as described above was conducted, resulting in exclusion of additional 43 articles for the reasons described above. Consequently, 110 articles were included in the review (see **Appendix A**). **Appendix B** shows the continents and countries in which the studies were conducted.

### Analytic Questions and Guidelines for Categorization

The unit of analysis was one article, and each article was summarized in a tabular format under the following headings:

1. Research method(s) and research design,
2. The research questions, the rationale for the research question and/or the aim/objective of the study,
3. Rationale for including problem-solving in mathematics education when applicable, and
4. Findings.

In response to the research question, “(h)ow is problem-solving described in empirical research on problem-solving in mathematics teacher education?”, the articles were grouped according to the research questions and rationale for including problem-solving in mathematics education. The findings were not used in the grouping of the research articles, as the focus was on configuring the research according to how problem-solving was understood. However, we do briefly comment on the results in the generated groups, as will be clarified in the results section.

Utilizing this coding, we compared the articles in the dataset to identify similarities, differences, and patterns within and across articles and codes. Again, to ensure consistency and consensus between the authors, some articles were read together and coded separately. By comparing coding, ambiguities could be solved. Discussions about coding were recurrent during the process. The processes of analyzing and synthesizing the data in a configurative approach are mainly inductive and aim at opening up different understandings of complex phenomena (Gough et al., 2017). The analyses include reading and re-reading, coding, and recoding, development of themes, comparison, and iteration. In this case, one distinction that emerged was between studies that researched pre-student teachers' knowledge, affect, beliefs or competencies/skill regarding problem-solving versus studies researching pedagogy or PCK. A second emerging distinction was between studies that saw problem-solving as an aim versus studies which saw problem-solving as a means to learning mathematics. Unavoidably, these distinctions are influenced by our own experiences from teacher education, our acquaintance with the knowledge base of mathematics teachers, and familiarity with earlier research.

The review revealed merits as well as pitfalls. Certain decisions were made which likely reduced the total number of articles included in the review; for example, the limitation to only include articles in English. However, only eight articles were excluded on account of language issues. While this undeniably is a relatively small amount of the total it can be seen as a bias in the studies included in the review, mirroring the overrepresentation of articles published in English. The time span was more than 30 years and articles available through the three largest relevant databases were included, which increased the total number of articles. The intention to review a sample that was representative of the field always carries the risk that important articles are missed due to decisions of inclusion and exclusion in data selection. There is also a possible bias in how the grouping was constructed, where transparency regarding the iterative and explorative way in which a configurative approach is carried out is a challenge. The advantage of an iterative and explorative approach is that it can be adjusted continuously. In this study, the groupings were reorganized repeatedly during the analysis which on the other hand makes it difficult to describe the process accurately in a transparent way.

In the next section, we briefly summarize the characteristics of the studies so that readers may relate to the representability of the articles. Thereafter, we present an analysis of the articles from two perspectives.

## CHARACTERISTICS OF THE STUDIES

There were almost equal numbers of papers from Europe (43) and North America, with 42 papers from the USA—the dominant nation—and 16 from Turkey. There were 15 papers from Oceania, five from Asia, two from Africa—whereof one is a cross-national study, and none from South America. There has been a strong focus on problem-solving in mathematics education research in the

USA from the 1970's and forward (Schoenfeld, 2007). A possible explanation for the lack of Asian studies is fewer English-language research articles on problem-solving in Japan since the early 21st century (see also Hino, 2007).

There were no articles from before 1996. There is an increase over time in the number of papers which seems to reach its peak in the mid-2010s. There is only one article from 2021, which is in part because only articles from the first part of the year were included.

As the understanding of problem-solving reflected in research articles may vary with the level the prospective teachers will teach upon graduation, we categorized all the papers according to level. As some papers engaged prospective teachers at more than one level, we only state approximate numbers here. Prospective primary teachers were in the majority, represented in about 60% of the papers, while secondary PSTs were represented in just over a fifth of the papers. In roughly one of 10 papers, we could not determine the level, and in around the same number of papers, the PSTs were to be middle teachers. This skewness would make it difficult to detect any distinct differences in understanding depending on level. The levels were fairly equally distributed amongst the categories we introduce in the results section, which supported our decision not to include level as a variable.

## ANALYSIS

Below, we describe the distinctions that began to take shape as we grouped the articles, first according to their research questions (focus) and second according to the rationales for including problem-solving in mathematics education (rationale). These two distinctions concern different aspects which allowed us to develop a cross-product model. This model will be discussed after the introduction of the distinctions.

### Focus: Content versus Pedagogy

From the research questions or stated aims of the research studies it was evident that some of the 110 studies focused on the PSTs' own knowledge of or competencies in mathematical problem-solving—what we would categorize as CK in this context. A clear example is this aim:

“The purpose of this study is to investigate how pre-service teachers (PSTs) in a sequenced, methodological and process integrated course (a) solved a total integration levers simulator problem, (b) explained how they solved the problem ...” (Cormas, 2016, p. 2558).

Some studies did not concern knowledge and competencies of problem-solving as much as affective or belief issues related to mathematical problem-solving. However, they clearly focused the affect and beliefs related to problem-solving itself, not to the teaching thereof. We have therefore grouped these articles with the articles on knowledge and competencies. This included research questions such as the following:

1. Is there a significant difference between the mathematical problem-solving beliefs of teacher candidates with low and high problem-solving success?
2. Does the interaction of problem-solving success with gender create a significant difference in the mathematical problem-solving beliefs of teacher candidates?
3. Does the interaction of problem-solving success with the overall level of success create significant differences in the mathematical problem-solving beliefs of teacher candidates?
4. What are the opinions of teacher candidates with low and high problem-solving success about their mathematical problem-solving beliefs? (Bal, 2015, p. 1377).

In total, 62 of the 110 articles were classified as concerning knowledge, affect, beliefs or competencies/skill regarding mathematical problem solving.

In contrast, the research questions or aims in 44 of the studies foregrounded the teaching of problem-solving, related knowledge or competencies, and related affect and beliefs. PCK for problem-solving in mathematics may include knowledge of how to facilitate learners' awareness of their own work (the notion of “control” in Schoenfeld, 1985). Some examples of research questions with a PCK focus:

What factors support or inhibit secondary mathematics PSTs' implementation of problem-solving tasks during professional experience? (Little & Anderson, 2016, p. 508).

Question 2: In what ways does engagement in an instructional unit on problem posing and solving influence the types of problems designed for use with children in grades 1-4? (Leavy & Hourigan, 2020, p. 352).

Another aspect related to teaching for or through problem-solving concerns PSTs' belief in the appropriateness and applicability of including problem-solving in mathematics instruction, as for instance Gur (2017). Xenofontos and Kyriakou (2017) not only wanted to map PSTs' beliefs but also the effect of a specific course:

“What are prospective elementary teachers’ beliefs about [collaborative problem solving] and dialogue before and after a course designed to provide them with such experiences?” (p. 143).

A minority (five) of the studies addressed both content aspects and pedagogical aspects. This included focusing on PSTs’ knowledge or experiences of learning mathematics through problem-solving, followed by considerations of how this affected their teaching. For instance, Bjuland (2004) asked “Do the students reflect on their experience as learners of mathematics or as teachers of mathematics?”

### **Rationale: Problem-Solving as Aim versus as a Means for Learning Mathematics**

As mentioned in the introduction, Cai and Nie (2007) made a distinction between problem-solving as the aim of instruction and problem-solving as a means of learning mathematics. Often, one of these views was reflected in the rationales of the studies, or in the general discussion of the relevance of problem solving.

Amongst the 110 papers, 50 focused on problem-solving as an aim in itself, not on the mathematics learning it facilitates. For instance, one study was interested in “whether prospective mathematics teachers indeed develop alternative strategies when solving plane geometry problems” (Koyuncu et al., 2015, p. 439). Another example is a Turkish study, which focused on improving creative thinking: “The aim of this exploratory study is to note and discuss some of the diversified views from the perspectives of the prospective mathematics teachers on improving the creative thinking in problem solving” (Kandemir & Gur, 2007, p. 111).

In several other articles, a problem-solving approach was instead seen as a means to learn mathematics; often an explorative approach where cooperative learning, communication, and reflection are in focus, and frequently with an agenda of reforming mathematics teaching. Of the 110 articles reviewed, 52 were characterized thusly. This view is reflected in this statement from an introduction to an Australian study on factors influencing student teachers’ use of problem-solving during their practicum: “The challenge for teacher education programs is to develop PSTs’ understanding of the role of problem solving in learning mathematics” (Little & Anderson, 2016, p. 505). Ozgen and Alkan (2012) have a lengthy discussion of the relation between problem-solving and learning which they sum up as “Accordingly, problem solving is not a topic per se, but a learning tool for all stages of learning” (p. 1174), which we have also interpreted as reflecting a view of problem-solving as a means.

A few studies (eight) addressed problem-solving as both means and aim (in one article the view of the relevance of problem solving were not clear). This was very clear in an article on teaching Irish prospective mathematics teachers to pose “mathematically worthwhile problems”, where the authors explain the importance of problem-solving thus:

Problem posing is valuable as a goal in itself but also constitutes a means to accomplish a myriad of other mathematical goals such as developing confidence in mathematics, deepening mathematical understanding, advancing mathematical problem-solving skills, and developing mathematical aptitude and learning autonomy (Leavy & Hourigan, 2020, p. 343).

The result of this first reading hence led to two dimensions: focus and rationale, where focus was characterized as content or pedagogy, and rationale as aim or mean. Combining these two dimensions led to a two by two categorization of articles. This is presented in the results section below, followed by a discussion of the articles within each of the four resulting categories.

## **RESULTS**

### **The Resulting Model**

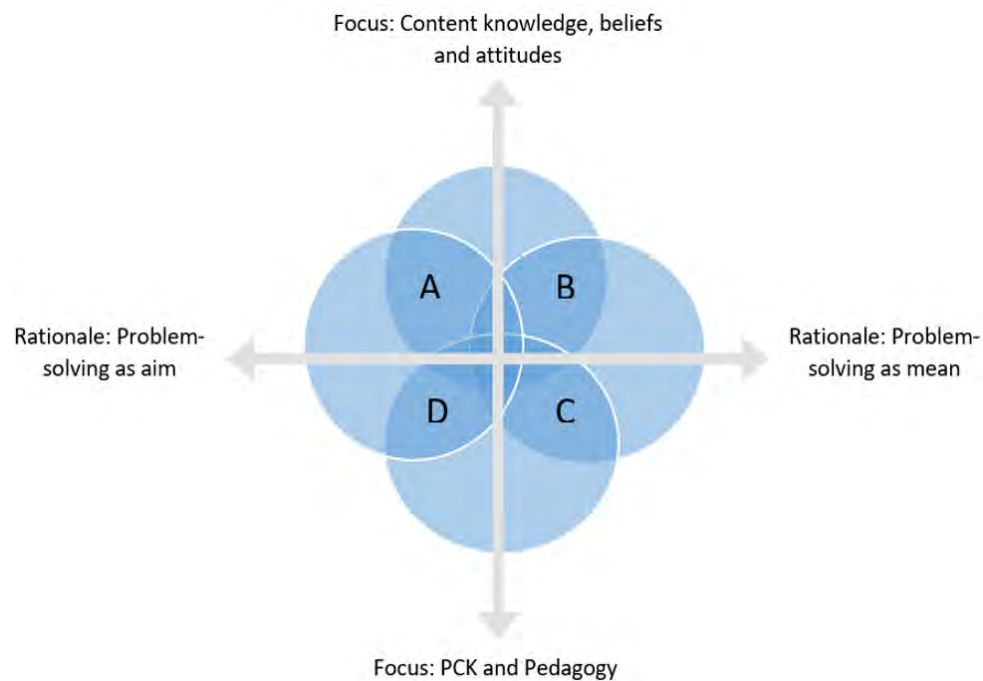
As mentioned above, combining the two dimensions of focus and rationale led to a two by two model, which is illustrated in **Figure 1**.

The distinction between content and PCK in the research focus of the articles is represented by the vertical axis, while the distinction in the articles between problem-solving as an aim in itself and problem-solving as a means of learning mathematics is represented by the horizontal axis. This gives rise to four categories:

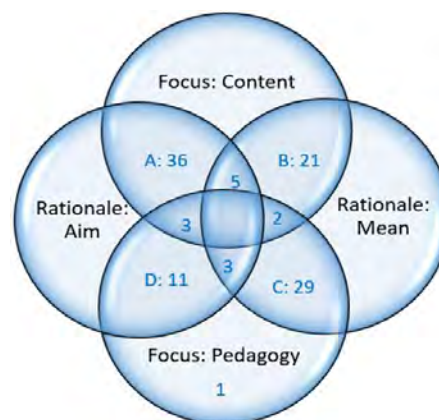
1. *Research focusing on CK, beliefs or attitudes, and seeing problem-solving as an aim in itself.* The typical article considered PSTs’ own mathematical problem-solving processes as basic skills and knowledge required for teaching (36 articles).
2. *Research focusing on CK, beliefs or attitudes and seeing problem-solving as a means of learning mathematics.* Articles here could describe studies of PSTs’ engagement with teaching mathematics through problem-solving based on their mathematical knowledge or on their own experiences with problem-solving as a way to learn mathematics (21 articles).
3. *Research focusing on pedagogy, seeing problem-solving as means of learning mathematics.* In this category, articles describe studies with PSTs engaging problem-solving as an instructional approach (29 articles).
4. *Research focusing on pedagogy, seeing problem-solving as an aim in itself.* For instance, articles looked at PSTs’ knowledge of different types of problems and problem-solving approaches and how to engage these in teaching (11 articles).

### **Categorization of Articles**

The categorization of articles led to 97 of the 110 articles fitting cleanly into one of the four categories, leaving 14 articles. The Venn diagram in **Figure 2** shows how many of these articles had either overlapping categorizations or could not be categorized in one dimension.



**Figure 1.** The model for characterizing research perspectives on problem-solving in mathematics teacher education



**Figure 2.** The result of categorizing articles according to focus and rationale

Using the categories, A-D, we now discuss the characteristics of the categories and provide examples that illustrate the range of articles within each category. We conclude the result section by touching on the articles that did not fall clearly in one of these four categories.

#### **Category A: Research focusing on CK, beliefs and attitudes, and seeing problem-solving as an aim in itself**

The underlying assumption of studies in this category—explicitly stated or implied—is that problem-solving skills are a key part of PSTs' CK; that to teach mathematical problem-solving, a teacher must be able to solve problems themselves (e.g., Kaya et al., 2014) and understand the nature of problem-solving (e.g., Ma et al., 2008). Therefore, PSTs need to be involved in a variety of problem-solving experiences, many of the articles state (e.g., Capraro et al., 2012). How problem-solving skills translate into the teaching of problem-solving or teaching of mathematics through problem-solving is not the focus of category A articles.

Within this group of articles, some focus on the processes, or strategies engaged by PSTs, whilst others focus on the performance, or the success of PSTs' problem-solving. A *strategy focus* is seen in the purpose statement of this Turkish study:

The primary purpose of the present study was to test this assumption about whether prospective mathematics teachers indeed develop alternative strategies when solving plane geometry problems in two different environments. In addition, we also sought to understand how they present evidence for the correctness of their solutions, i.e., their proving strategies (Koyuncu et al., 2015, p. 839).

The findings of these articles typically describe successful or unsuccessful strategies used. One example of this is an intervention by Lopez-Real and Lee (2006) where PSTs presenting computer-based solutions to problems are more inclined to engage in the process of solving problems than when working without computers.

In several of these studies, a variety of solutions, as in the example above, using different strategies, or using a specific heuristic strategy (as in Applebaum et al., 2011; Avcu & Avcu, 2010; Gurat, 2018) are taken as indications of development in the ability to solve problems.

The *performance* of the PSTs—their ability to solve problems—is another focus. Poor results from learners and/or the importance of problem-solving are the rationales for an interest in PSTs' abilities to solve problems themselves. An example of a performance focus is seen in the formulation in this Indonesian study: “the purpose of this study was to describe the mathematical problem-solving ability of undergraduate students of mathematics education in solving the well-structured problem” (Paradesa, 2018, p. 2).

The intervention in the study consists of giving PSTs opportunities to practice mathematical problems by modeling or using ICT, with the assumption that this would improve their problem-solving skills, which was then found to be the case (see also Bal & Doganay, 2014; Cekmez, 2020). Generally, studies found the interventions successful, a common trend in teacher education according to previous reviews (Österling & Christiansen, 2018).

### **Category B: Research focusing on CK, beliefs and attitudes, and seeing problem-solving as a means to learning mathematics**

As in category A, category B articles work from the assumption that a teacher's ability to solve problems, learn mathematics from problem-solving, engage in collaborative problem-solving or perceptions of mathematics and learning are essential parts of PSTs' CK, as prerequisites for teaching. In contrast to category A, the articles in category B foreground problem-solving as a productive means of learning mathematics.

These studies generally assume that PSTs' knowledge, beliefs, and attitudes influence their teaching and learners' learning outcomes. Nonetheless, few of these articles explore how PSTs' CK affect their mathematical teaching activities explicitly. While “subject-matter expertise is critical for effective teaching” (Nathan & Petrosino, 2003, p. 906), most of the category B studies appear to imply that by partaking in modelled desired practices—for instance through cooperative learning (Gil et al., 2019) - PSTs will learn mathematics and change their beliefs and/or practices at the same time. This can be seen in statements that PSTs need experiences as learners of mathematics (e.g., Caballero et al., 2011) and in the following statement by Namukasa et al. (2009): “... including warm mathematics in a teacher education program may facilitate consistency between what PSTs profess to be their teaching beliefs and what they practice in the classroom” (p. 60).

Of the studies in this category, nine report on the effect of specific interventions and three on the effect of teacher education as is. The effects on PSTs' deepening of mathematical knowledge and skills (Gil et al., 2019; Hickman, 2013), or on beliefs or perceptions of mathematics (e.g., Baki, 2004; Verschaffel et al., 1997) are generally positive. For instance, a focus on the emotional aspects of problem solving led to PSTs becoming more aware of the importance of considering the emotions of learners (Caballero et al., 2011). One distinct exception to this is a Samoan study which shows that students' attitudes to mathematics become more negative after the intervention (Afamasaga-Fuata'i & Sooaemalagi, 2014).

Within this category, several studies appear to have focused on PSTs' mathematical skills and knowledge separated from any links to teaching. For instance, links between PSTs' mathematical problem solving and their learning style (Ozgen & Alkan, 2012), critical orientation (Kurniati et al., 2019), cognitive style (Faradillah et al., 2018), or attention during group work (Cheng et al., 2011) were researched. Contreras and Martínez-Cruz (2011) explored the extent to which PSTs used their real-world knowledge in problem-solving, and Irfan et al. (2019) looked into incorrect schemes triggered for some PSTs engaged in problem-solving. An action research study by Gil et al. (2019) modeled problem-based teaching and studied “whether PSTs made more effective connections as the course progressed” (p. 10).

Some studies specifically focused on collaborative problem solving. This helped PSTs' feelings of difficulty while performing mathematical problems (Hurme et al., 2009), their transition from learner to teacher (Sharon, 2012), or it generated opportunities to learn (Baki, 2004; Cheng et al., 2011; Sharon, 2012; Tatsis & Koleza, 2006). An example of this can be seen in this research question from Sharon (2012): “What roles do prospective teachers assume while involved in cooperative problem-solving?” (p. 19). In this study, there is a focus on cooperative learning when problem-solving and the role PSTs take when verifying, correcting, or helping each other to better achievements.

As mentioned, only a few of the studies explicitly describe links between PSTs' own problem-solving and their teaching. Three studies compared aspects of PSTs' own solutions to problems to the ways in which they engaged with learners' solutions (Nathan & Petrosino, 2003; van Doreen et al., 2002; Verschaffel et al., 1997). Van Dooren et al. (2002) found a significant positive correlation between the number of times participants used a strategy and their favorable evaluations of that strategy. PSTs with advanced mathematics but lacking knowledge of how novices learn the subject, tend to assume that learning should follow the structure of the content rather than the learners' learning process, challenging the view that subject-matter should be the main consideration in licensure of new teachers (Nathan & Petrosino, 2003).

### **Category C: Research focusing on PCK and seeing problem-solving as means of learning mathematics**

The underlying assumption in category C is, as in category B, that problem-solving is a productive way of learning mathematics, but in contrast these articles focus more on how PSTs' PCK can affect teaching mathematics through problem-solving or on the development of such PCK.

PSTs' experiences with or analysis of teaching through problem-solving, are seen as productive ways of learning to teach mathematics. Inviting PSTs to experience and reflect on teaching through problem-solving are designed to challenge their ways of thinking about the subject and its teaching. As Bailey and Taylor (2015) ask: “Does experiencing and reflecting on a problem-solving approach in mathematics support novice teachers to learn about core high-leverage teaching practices?” (p. 113). Here, the PCK is knowledge about core high-leverage teaching practices developed through *experiencing* or reflecting on an

implemented *problem-solving approach* to teaching mathematics. Another example of this is the study by Osmanoğlu et al. (2013). They showed that by observing an expert teacher engaged in problem-solving as a teaching strategy, the PSTs developed a professional vision for reform teaching.

Behind the argument that by engaging in a problem-solving approach, PSTs will change their beliefs and knowledge about mathematics teaching and learning, lies a perception that PSTs need to change their views. Whether it is concerned with changed views on mathematics (Blanco, 2004), changed beliefs (Gur, 2013), an attempt to make PST's more reflective (Runesson, 1997), directing PSTs to think less "traditionally" about instruction (King et al., 2020), not avoid problem solving (Mallart et al., 2018), or to furthering PSTs' sensibility towards language and culture (Santistevan et al., 2009), the PSTs are expected to change.

Finally, in this group of studies, there is a strong coherence between the designed situations of interventions and the intended outcome or effect. For example, interventions where PSTs discussed experiences in groups facilitated their capability to reflect on teaching (Cavanagh & Garvey, 2012; Cavanagh & McMaster, 2015). In general, the findings show that the approach had the intended effect in the sense that PSTs reported that the activities provoked and added a different dimension to teaching mathematics. A problem-solving approach is, therefore, according to these studies, confirmed as a way of implementing the reform agenda for mathematics education.

#### **Category D: Research focusing on pedagogy and seeing problem-solving as an aim in itself**

Papers in this category assume that a teacher's problem-solving PCK is essential for teaching problem solving. This includes knowledge of how to engage learners in different types of problems, how to facilitate problem-solving strategies, creativity and courage, and which challenges to be prepared for. This is clearly reflected in the research questions or aims in the studies in this category. It is therefore not surprising that these studies report on researchers' own courses, interventions and/or student teachers' perceptions of (aspects of) problem solving.

For instance, one study researched Turkish PSTs' views on problem-solving techniques (Kandemir & Gur, 2007). Creative problem-solving techniques were introduced into the mathematics courses in the programme, over the course of which the PSTs were observed and interviewed. The findings suggest that the PSTs developed positive attitudes towards mathematics and became more open-minded about using different approaches when teaching problem-solving. Furthermore, PSTs also came to believe that creativity could be learned and improved by training and problem-solving (Kandemir & Gur, 2007). Other studies focused on transfer of skills (Cormas & Middlemiss, 2019), learning through reflective analysis (Karp 2010), group discussions (Peled & Hershkovitz, 2004), or the use of digital storytelling (Walters et al., 2016, 2018) in developing PSTs' ways of teaching (for) problem solving. A Spanish study illustrates that the skill of facilitating learners' problem solving can be learned. The study compared the discourses of in-service and pre-service mathematics teachers teaching problem solving (Rosales et al., 2008), finding that the in-service teachers acted more strategically so that essential elements of the task at hand were addressed.

The remaining studies in this category were surveys of PSTs' expected problems in facilitating problem solving (Keazer & Jung, 2020), PSTs' views of creativity in teaching problem solving (Yazgan-Sag & Emre-Akdogan, 2016), and how PSTs think about teaching problem solving/word problems in a medium of instruction different from the learners' mother tongues (Kasule & Mapolelo, 2013).

There was a limited number of studies in this category, even when including studies on PSTs' views on teaching problem solving. PCK about problem-solving, for example knowledge about creative problem-solving techniques applied to teaching or ways of discursively directing learners' attention, is relatively underrepresented in research and this gives rise to questions about whether this is sufficiently studied. We return to this point in the discussion.

#### **Studies Outside the Four Categories**

Of the fourteen articles which did not fall neatly into one of the four categories, eight reported on the outcomes of specific courses, and in half of these cases, the courses appeared to have been successful. The remaining 10 studies had more qualitative results. For instance, Bjuland (2004) explored how Norwegian students used their own problem-solving experiences to reflect on learners' situation, and a French study found that the problem task from the course was transformed when implemented in schools, mediated by the official syllabus (Kuzniak et al., 2013).

Amongst the 110 articles, a total of eight concerned the link between PSTs' own problem-solving experiences and their related PCK or teaching approaches. There were also eight articles that engaged the doubleness of problem solving as both an aim in itself and a means to learning mathematics.

## **DISCUSSION**

The attempt to generate an overview of how problem-solving is understood and researched through a configurative review has led to a characterization of existing research in mathematics teacher education. A configurative review was deemed particularly suitable for synthesizing the complex body of research on problem-solving in mathematics education. The aim of configurative reviews is to find sufficient studies to provide a meaningful configuration that has the potential to deepen the understanding of the phenomena (Gough et al., 2017). Although there is an increased interest for configurative reviews in the field of education (Levinsson & Prøitz, 2017), they are rarely used, which also seems to be the case in mathematics education generally and problem-solving in particular. There are systematic reviews that describe a methodology reminiscent of the explorative and iterative approach of configurative reviews, without calling them configurative (e.g., Depaepe et al., 2013; Simsek & Boz, 2016). We believe the results presented here illustrate the fruitfulness of undertaking the task.



The articles included in this review were first mapped according to the focus of the research as reflected in the research questions and second according to the rationales for including problem-solving in mathematics education. This led to a two-dimensional model for characterizing research on problem solving in mathematics teacher education. The presented model enables a disentangling of the different ways problem-solving is described/understood in mathematics teacher education, which is important for both teacher education and school education. The configuration reflects the research community's emphasis on problem-solving as a vast, ambiguous, and complex concept and provides one way of describing this complexity. On the other hand, the result also shows that descriptions of problem-solving vary. Previous arguments that the research field has not yet provided a common understanding of what problem-solving is (Andrews & Xenofontos, 2014; Lester, 1994; Stein et al., 2003) and how problem-solving relates to teacher education, therefore, seem still to apply.

All articles were categorized using the model. The results show that there was a predominance of studies focusing on CK, beliefs or attitudes (categories A and B, 62 articles) compared to focus on PCK (categories C and D, 44 articles). Hence, there seems to be a bigger focus on investigating PSTs' skills in or knowledge of solving problems than on PCK including knowledge of how to engage learners in different types of problems, how to facilitate problem-solving strategies, creativity and courage, and which challenges to be prepared for. Regarding the other axis in the model, between problem-solving as an aim in itself (categories A and D) or as means of learning mathematics (categories B and C) the distribution is relatively even (50 versus 52 articles). When considering the number of articles in each category (A:36, B: 21, C:29, D:11) there is an overweight on category A with a focus on PSTs' skills in solving problems and relatively few in category D with a focus on PSTs' PCK about problem-solving.

Only five studies addressed both content aspects and pedagogical aspects, while eighth studies addressed problem solving as both an aim in itself and as a means to learning mathematics. Perhaps it is in the nature of research articles as a genre to focus on delimited aspects of a phenomenon, but it is refreshing to see some try to make connections across complex issues.

Surprisingly enough, research investigating PCK about the teaching and learning of problem-solving, for example knowledge about creative problem-solving or ways of discursively directing learners' attention, is relatively underrepresented. Looking across the sub-themes within the four categories A-D, some patterns are worth noting. Firstly, there are very few studies explicitly engaging links between PSTs' own problem solving and their teaching—those there were fell within category B. Secondly, the papers that focused PCK more rarely engaged teaching PSTs specific aspects of PCK or of teaching problem solving; more prevalent was a focus on how PSTs experiencing problem solving as learners—or even observing it taught—changed their view of mathematics teaching. As previously mentioned, this reflects a perception that PSTs need to change their views to be good mathematics teachers; we will engage this in more detail in forthcoming work (de Ron & Skog, 2022; Österling & Christiansen, 2022). Indeed, this is where the insights generated by this configuration of the field truly come to light; while PSTs come to recognise that problem solving can be learned or that problem solving is one powerful way to teach mathematics, the field remains surprisingly tacit about how they are to translate this recognition into practices in the classroom. The results of this review raise questions about implications for research and practice.

### Implications for Research and Practice

One implication of the imbalance of research in the four categories of the model could be that *more research focusing on PCK* in relation to mathematical problem solving is needed. Such research could, for example, investigate PSTs' pedagogical knowledge about problem-solving, how PSTs disentangle the notion of problem-solving, how problem-solving is connected to other teaching practices in mathematics education, how the notion of problem-solving has been understood over time (de Ron, in press), or how knowledge of problem-solving is represented in teacher education, for example, in the literature (de Ron & Pansell, forthcoming). However, the types of studies within the existing research on PCK and problem solving may even suggest the need to home in on how teacher education can facilitate PSTs' learning of well-informed teaching strategies in mathematical problem solving—using the terminology from MfT (Ball et al., 2008), we could refer to this as KCT-PS.

It would be unreasonable to assume that the reviewed studies indicate anything about the practices in teacher education. However, should the lack of research on KCT-PS reflect the content of teacher education, it is worth reflecting on what the desirable balance may be between activities aiming to change PSTs' beliefs and activities facilitating their KCT-PS. Indeed, the issue of desired knowledge of PSTs and the related issue of selection of content for mathematics teacher education is evident, explicitly, or implicitly, in most of the articles across the four categories in the model. The four categories can be regarded as complementary, together giving a broader picture of problem-solving and contributing to the field of teaching and teacher education. The model for characterizing research perspectives could therefore also be utilized in informing selection of content, possibly making more explicit decisions about which of the four categories to include or emphasize.

We have briefly touched upon the issue of how PSTs are positioned as in need of changing beliefs and attitudes. The research field may want to engage such assumptions more critically, and reflect on the desired teacher projected in research, policy, and practice (see also Christiansen et al., 2021). This review also points to concerns regarding research design and trustworthiness. Several studies made claims about the impact of an intervention on student learning, yet these claims were seldom supported by comparing intervention groups to controls or by scrutinizing whether the results could likely be explained in other ways. Mathematics teacher education is a young field, and our readings of the studies in this review suggests that it would be beneficial for the field to develop stronger practices for reporting.

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APPENDIX A

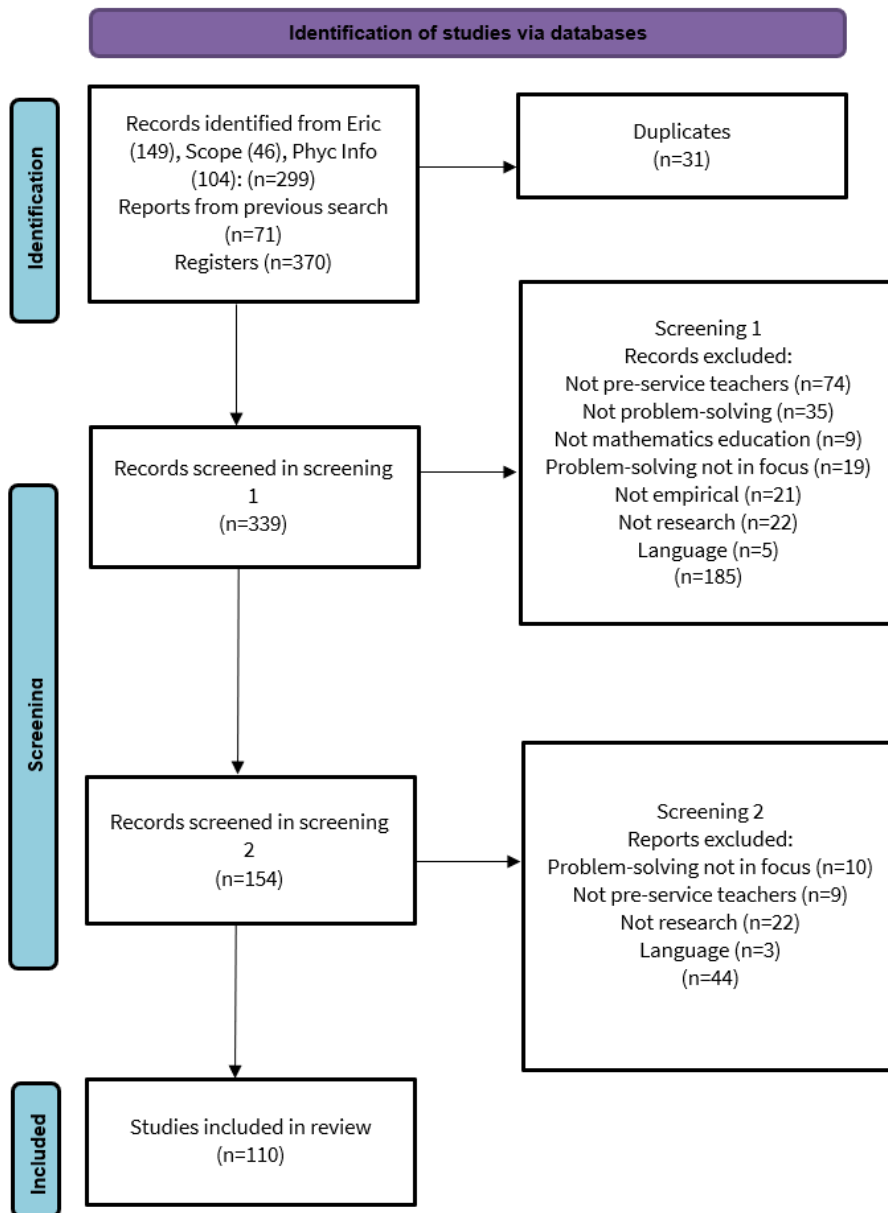


Figure A1. PRISMA diagram



## APPENDIX B

Table B1. Continents and countries in which the studies were conducted

Continent (n)	n	Country	n	Authors
Asia (5)	1	China	1	Chen et al. (2011)
	1	Hong Kong	1	Lopez-Real and Lee (2006)
	2	Korea		Kim and Cho (2016) & Taplin and Chan (2001)
	1	Singapore		Toh et al. (2014)
	3	Belgium	1	van Dooren et al. (2002, 2003) & Verschaffel et al. (1997)
Europe (43)	1	Cyprus	1	Xenofontos and Kyriakou (2017)
	2	Finland	1	Hurme et al. (2009) & Löfström & Pursiainen (2015)
	2	Greece		Tatsis and Koleza (2006, 2008)
	1	Ireland		Leavy and Hourigan (2020)
	5	Israel	1	Daher (2009), Even (1998), Guberman and Leikin (2013), Peled and Hershkovitz (2004), & Tabach and Schwarz (2018)
	2	Norway		Bjuland (2004, 2007)
	2	Slovenia	1	Istemic Starcic et al. (2016) & Kuzle (2013)
	5	Spain	1	Blanco (2004), Blanco et al. (2013), Caballero et al. (2011), Mallart et al. (2018), & Rosales et al. (2008)
	2	Sweden	2	Runesson (1997) & Ryve (2007)
	16	Turkey	4	Avcu and Avcu (2010), Baki (2004), Bal (2014, 2015), Bal and Doganay (2014), Bulbul et al. (2020), Cetinkaya et al. (2016), Cekmez (2020), Fadlelmula and Cakiroglu (2011), Gur (2013), Hidiroglu and Guzel (2013), Kandemir and Gur (2007), Kaya et al. (2014), Koyuncu et al. (2015), Yazgan-Sag and Emre-Akdogan (2016), & Ozgen and Alkan (2012)
	2	UK		Abidin and Hartley (1998) & Hickman (2013)
North America (41)	40	USA	1	Abramovich and Nabors (1998), Aguirre et al. (2013), Atkins (1997), Berk et al. (2009), Boote and Boote (2018), Brown et al. (2011), Capraro et al. (2012), Cheng et al. (2011), Contreras and Martinez-Cruz (2011), Cormas (2016), Cormas and Middlemiss (2019), Flores et al. (2014), Gil et al. (2019), Hallman-Thrasher (2017), Harkness (2009), Karp (2010), Keazer and Jung (2020), Kim et al. (1998), King et al. (2020), Kuzniak et al. (2013), Lee (2005), Lim and Morera (2011), Ma et al. (2008), Maher et al. (2014), Manouchehri (2011), Masingila et al. (2018), Nathan and Petrosino (2003), Novak and Tassel (2017), Osmanoglu et al. (2013), Rosli et al. (2020), Santistevan Matthews et al. (2009), Sharon (2012), Spungin (1996), Thanheiser et al. (2016), Timmerman (2004), Wachira et al. (2008), Wall et al. (2016), Walters et al. (2018), Walters et al. (2016), & Weidemann and Humphrey (2002)
	1	Canada	1	Namukasa et al. (2009)
Oceania (15)	5	Australia	3	Afamasaga-Fuata'i (2009), Afamasaga-Fuata'i and Sooaemaleagi (2014), Cavanagh and Garvey (2012), Cavanagh and McMaster (2015), & Little and Anderson (2016)
	6	Indonesia		Faradillah et al. (2018), Irfan et al. (2019), Kurniati et al. (2019), Paradesa (2018), Thalib et al. (2021), & Wahyudi et al. (2020)
	2	Malaysia		Abdullah et al. (2015) & Yildiz (2020)
	1	New Zealand		Bailey and Taylor (2015)
	1	Philippines		Gurat (2018)
Africa (1)	1	South Africa		Steyn and Adendorff (2020)
Cross National (5)		Indonesia		
		the Netherlands		
		Canada		Johar et al. (2017)
		Israel		Applebaum et al. (2011)
		Cyprus		Xenofontos and Andrews (2014)
		England		Kasule and Mapolelo (2013)
		Botswana		Clivaz and Miyakawa (2020)
		Swaziland		
	Switzerland			
	Japan			