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Peer interactions and their role in early mathematical learning in kindergarten discourses

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ABSTRACT

Since the 2000s, early co-constructive mathematical learning in kindergarten has focused on political discussions and (mathematical-didactic) research. This is because kindergarten is the first place for subject-specific learning, next to socialization in the family. Research on this first institutional learning in the kindergarten often focuses on the interaction between children and elementary school teachers, which is presented as a key variable for these learning processes. However, it is peer interaction that takes up a large part of the day in kindergarten. In these interactions, children negotiate a variety of issues that are relevant to them. The following contribution will present research that focuses on these peer interactions in the context of block play situations. There is a general analysis using methods of (qualitative) thematic analysis as well as a detailed analysis using methods of interpretative classroom research. These analyses will help answer the question as to what role these peer interactions play in early mathematical learning processes and to what extent conditions can emerge in such interactions that enable mathematical learning in the sense of Miller (1987).

Keywords: early mathematical learning, kindergarten, co-construction, peer interaction

INTRODUCTION

Several studies from the fields of (mathematics) education, social sciences and psychology have shown that peer interaction in primary and secondary school can have a particularly beneficial effect on children’s learning processes under certain conditions, such as the selection of suitable task formats (Cekaite et al., 2014; Cooper et al., 1984; Webb, 1989; Webb & Farivar, 1999).

In accordance with Vygotsky (1978), it can be assumed that interactions between peers can be particularly beneficial for the learning process, as these are characterized by a high degree of interactional responsiveness (Robertson et al., 2016), which in turn is characterized by joint (successful) support in the interactional learning process. This high responsiveness is due to the fact that peer interactions are characterized by developmental psychological similarities among said peers, resulting in high compatibility concerning the interpretation and contextualization of content. Coupled with what can be assumed to be different levels of experiential knowledge, new knowledge can thus emerge to a greater extent because more competent interaction partners of the same age presumably operate in the “zone of proximate development” (Vygotsky, 1978; e.g., Brandt & Höck, 2012).

These peer interactions can also play a key role in the field of mathematical learning for the reasons listed above (Forman, 1989; Saxe et al., 1993). In the following contribution they are understood as interactions based on symmetrical relationships in groups of children (Youniss, 1994, p. 154). For example, Brandt and Höck (2012) and Jung (2019), among others, demonstrate many patterns of interaction that are conducive to learning within these peer interactions. Krummheuer and Brandt (2001) point out that in children’s teamwork, even an additional teacher can stand in the way of a learning-promoting interaction process if children try to adapt their actions to those of the teachers, as children consider them to be more competent interaction partners. All these cited authors make arguments for more partner and group activities in primary school lessons in this context.

While the positive effects of various interaction patterns in the context of peer interaction in primary school are already taken into account by various studies, as described above, the (co-)constructive interaction of peers in the context of (mathematics-related) (play) situations in kindergarten and its significance for early mathematical learning has only been taken up in a few research papers thus far: Early childhood research primarily focuses on adults, such as legal guardians and parents and elementary education professionals. As role models in the sense of “more competent others in the matter” (Chaiklin, 2003, p. 42), they offer content and interaction patterns that children can adopt in their interactions (cf. Stude, 2014).

There are exceptions to this rule, as there is international work that focuses on adult interaction partners in contexts that promote learning in the parental home and kindergarten; this work is from the field of social, educational, and linguistic sciences published by Arendt (2015, 2019), Corsaro and Rizzo (1990), Eisenberg (1987), and Mercer et al. (1999), who describe peer interaction as a discursive acquisition context. As far as early mathematical learning is concerned, there has been rather little research that addresses the interaction between peers (cf. Flottorp, 2011; Zippert et al., 2019).

This contribution tries to help close this research gap and focuses on the interaction processes of peers in the context of block play activities in kindergarten. With the help of a (qualitative) thematic analysis as well as an interaction and argumentation analysis, a qualitative investigation with a reconstructive approach has been designed to study the mathematically meaningful ideas that emerge in these peer interactions. With this interconnecting analysis, it will be also considered in which situations children develop new mathematical knowledge through a process of mutual negotiation. Based on the assumption that (collective) argumentation is constitutive of the emergence of "new meanings" in discursive interaction (cf. Brunner, 2014; Krummheuer, 2007 in accordance with Miller, 1987), special attention will be paid to the children's collective argumentation processes in the following.

THEORETICAL FRAMEWORK

Early Mathematical Learning in Interactions

The fact that mathematical learning already takes place before starting school has shown in many ways in mathematics didactic research (cf. Melhuish et al., 2010). Researchers point out that entry into school cannot be seen as "zero hour" in mathematical learning. Rather, mathematical learning and mathematical socialization already take place in the family (cf. Acar Bayraktar, 2018; Civil et al., 2005; Tiedemann & Brandt, 2010) and in kindergarten before entry into primary school. In this context, Acar Bayraktar (2018) shows that, in addition to parents and elementary educational professionals' siblings and peers who interact in the role of the "more competent other" (Chaiklin, 2003, p. 41) are also of great importance for this initial learning. Children "grasp" first mathematical meaning in their everyday lives, both in the home environment and at day care, and therefore already begin approaching the process of mathematising in these early childhood settings.

The NCTM (2000) and various educational plans for early childhood education (in Germany, among other countries) (cf. KMK, 2005) state that this initial learning takes place primarily in a co-constructive way and those children actively discover mathematical content in interactions (with adults and other children). Key variables in this early co-constructivist, mathematical learning are the situational processes of negotiation of mathematical meaning (Vogler, 2019). Therefore, interactions will be considered a research object in this contribution. Before the importance and mathematical content of negotiation processes among peers are presented in the following, it should first be clarified how learning can be located under the (co-)constructivist perspective.

Conditions for enabling early mathematical learning in interactions

From the (co-)constructivist perspective, learning is to be seen as a process of becoming increasingly autonomous in interactions of discourse (cf. Lave & Wenger, 1991; Sfard, 2008). In line with this, "conditions for enabling learning" (Krummheuer & Brandt, 2001, p. 56; Miller, 1987) are created by offering children actively productive participation in the negotiation processes of the discourse, thus opening up so-called "participatory spaces" (Brandt, 2004) for them. Over the course of this participation, restructuring processes of the individual knowledge stocks of the interactants can occur. According to Vygotsky (1978), these processes start with situationally emergent negotiation processes in discourse at the intermental level of interaction. Such negotiation processes should be able to be described as "mathematically rich" so that new mathematical meanings can be developed that are relationally rich and outlast the situational context. Such interactions can on take very different forms.

(Co-)construction of "new meaning" through argumentatively structured negotiation processes in interactions

According to Miller (1987), collective argumentations in interactive negotiation processes are important for the development of "new meanings." They are essential for the participation in mathematically rich discourses and facilitating increasing autonomy. This is generally justified mainly by the fact that the possibilities of one's constructions or the framework of individual contextualizations are systematically exceeded in productive or receptive participation in collective argumentation processes (Miller, 1987, p. 29). The construction of new knowledge through collective argumentation occurs especially when the argumentation is comprehensible and convincing for the individual.

For mathematical learning, it is of great importance that collective argumentations not only have a generally beneficial effect on the development of "new meaning" (Miller, 1987), but also condition the "thematic shaping" (Bauersfeld et al., 1985) of the (new) individual constructions or contextualizations. In this context, Krummheuer (2007, 2009) speaks of the convergence function of collective argumentations. During the collective mathematical argumentation processes, individual interpretations and attributions of meaning are consequently "balanced," negotiated, extended, or rejected in the discursive interplay. Collective argumentation can therefore be seen as a constitutive element for a new, or reformation of already existing, mathematical attributions of meaning of the individual interactants. If, over the course of such collective argumentation, there is a routinization and a reconstructible increase in the autonomy of the interactants, Krummheuer (2007, 2009), in accordance with Bruner (1990), speaks of an "argumentation format." Over the course of such argumentation formats, "interpretations" (of the context) that are considered shared can also emerge situationally between the interactants. These interpretations represent the common starting point for the further (argumentative) negotiation process as a working consensus, or as a rather fragile, temporary basis for cooperation and work ("working interim", Cobb et al., 1992 in accordance with Krummheuer, 1983) and create the conditions for the interaction to continue.

The collective and argumentatively structured process of negotiation of meaning consequently conditions and influences the nature of the learning process.

More competent others as an interaction authority

Brandes (2008, p. 130), Vygotsky (1978), Youniss (1994), and many others attribute a special role to more competent others in co-constructive processes of negotiation. In many publications from the field of early (mathematical) learning, these more competent others are primarily considered to be guardians, parents, and elementary education professionals (cf. Acar Bayraktar, 2018; Vogler, 2020, 2021; Cooke & Bruns, 2018; Tiedemann & Brandt, 2010).

On the one hand, these adults can open opportunities for participation for the learning children by establishing specific patterns of interaction, such as the argumentation format presented earlier (Krummheuer, 2007, 2009), and support the children in their participation through the specific design of the interactions (Bruner, 1990).

On the other hand, Brandes (2008), van Oers (2019), and Youniss (1994) also attribute to the more competent persons that they interactively provide contexts that are both culturally and for the learning children relevant and are characterized by a certain "closeness" to their world of experience. This results in the claim that more competent persons realize interactional contributions in terms of linguistic discourse and content, both of which are located in the child's "zone of next development" (Vygotsky, 1978). The child can participate more fully in this zone of next development through scaffolding in interaction (Bruner, 1990). In this sense, such interactions are characterized by a high degree of responsiveness (Beck & Vogler, 2021; Robertson et al., 2016; Vogler & Beck, 2020).

Peer Interaction's Key Role in Early Co-constructive Mathematical Learning

While in research on language acquisition, for example, peers are considered important authorities of language socialization in interactions in the kindergarten context (Arendt, 2019), there have been few studies in mathematics didactics that focus on the important role of peer interactions in early mathematical learning in the kindergarten context. However, Henschen (2020), among others, can show that diverse mathematical content emerges in peer interactions (cf. Flottorp, 2011). Exceptions here are older works by Forman (1989), Saxe et al. (1993), and more psychologically oriented works by Zippert et al. (2019).

This "research gap" is particularly surprising because it can be assumed that learning in childhood takes place to a large extent through interaction in groups of children. In these peer interactions, the role of the teacher and the learner can constantly change. For example, the developmental psychologist Youniss (1994) coined the term "co-construction," which primarily describes learning with and from each other among peers. Co-construction in that case means that through the negotiation processes of peers, meanings that are shared, or taken as shared, are produced (Völkel, 2002, 2018) and, as a result, knowledge is constructed together. It is of central importance that the interaction partners are at the same level. Thus, a symmetrical relationship is assumed, but one in which the children negotiate their thoughts and plans through different knowledge constructions.

In comparison, in interactions between adults and children, a disparity can often be reconstructed which manifests itself in the fact that the adult interaction partners in particular "assert" certain interpretative perspectives in interactions. Some of the children's interpretations are not adopted by adults during the interaction and "fizzle out." It can be assumed that children (have to) make an effort to follow the interpretations of adults to be able to find opportunities for participation. Their constructions may be limited by the fact that adults do not act very responsively and can override children's creative interpretations during the interaction in favor of asserting their interpretative perspectives (Vogler, 2020). As Vogler and Beck (2020) describe this lack of supportive or responsive action by adults in interactions can lead to the successive exclusion of children from early mathematical discourse.

Research Desiderata and Research Question

In the previous chapters, it was pointed out that peer interactions in the kindergarten context can have key functions in early mathematical learning since they account for a large part of the "interaction time" and, from a research-logical perspective, it can be assumed that a particularly high degree of responsiveness emerges in interactions between peers. It can also be assumed by Henschen (2020) that in peer interactions, for example when building constructions, diverse mathematical contents are negotiated by the children. Nevertheless, there is a research gap in the field of empirical reconstructive research on peer interactions in kindergartens and their influence on early mathematical learning. Based on the finding of Henschen (2020), above and against the background of the research logical considerations on the facilitation of learning through negotiation processes in interactions between peers, the question arises: to what extent can be potential for participation in mathematical negotiation processes be reconstructed in peer interactions that can facilitate early mathematical learning? This question will be explored in the following sections of this contribution using the example of block play situations in kindergarten.

METHODOLOGY AND METHODS

Empirical Background

The empirical background for this contribution consists of video data of 30 to 90-minute free play situations with building materials from everyday life in kindergartens. This ethnographic data has been examined in the context of Henschen's work on block play concerning the emergence of various mathematical content (Henschen, 2020). In this contribution, the peer

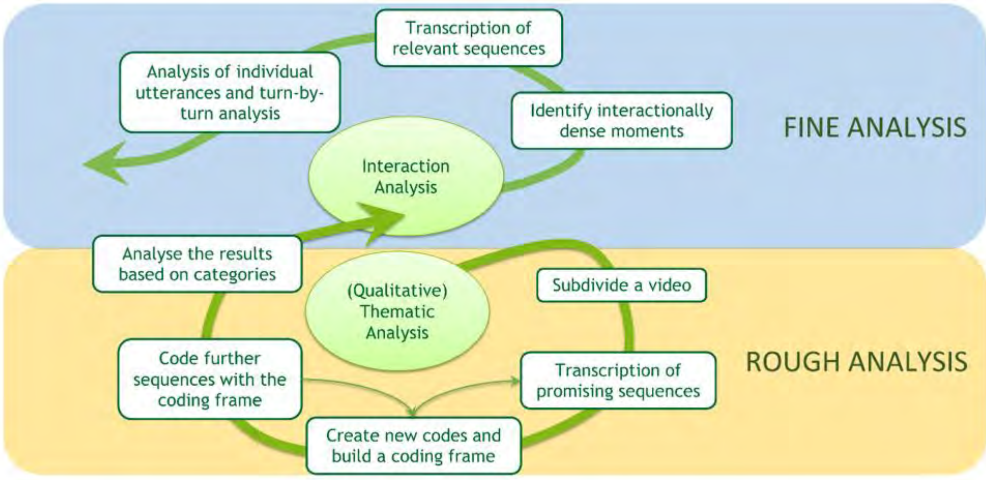


Figure 1. Combination and integration of methods

interactions in these play situations are focused on as a complement to and extension of Henschen (2020) accounts and made the subject of the research presented here. To be able to examine the wealth of ethnographic data with a focus on peer interaction, some methodological considerations are necessary for the research project, which are explained below.

Methodological (Pre-)Considerations

Block play activities are part of everyday mathematics practice in kindergarten (cf. Bruce et al., 1992a; Ertle et al., 2008). Making everyday activities the subject of mathematics didactic research goes along with analysing everyday practices. In line with Bishop, in this context Buchholtz (2019, 2021, p. 232) speaks of the research tradition of the "empirical scientist." This research tradition is characterised by adapting established research methods from other fields, such as social science, psychology, and educational research. If the question is to be answered as to the extent to which potentials for participation in mathematical negotiation processes emerge in play situations between peers, it is an obvious choice in this context to use qualitative methods from the field of ethnographic research that analyse everyday mathematical activities between children and enable a "controlled reconstruction" (Schütz & Luckmann, 1979). Therefore, it makes sense to rely on methods of qualitative-interpretative methodology (Miller, 1986; Schütz & Luckmann, 1979; in mathematics didactics research, e.g., Brandt & Tiedemann, 2019; Cobb & Bauersfeld, 1995), which is well established in mathematics didactics research (cf. Buchholtz, 2019, 2021, p. 233).

Because the peer interactions presented here are part of everyday life, they are necessarily characterized by the complexity of this everyday life, which is reflected in the (mathematical) process of negotiation. Making this complexity accessible for research requires being aware of the different layers of the research object and, if necessary, investigating them with different research methods (cf. Buchholtz, 2019, 2021, p. 236). The following will use sub-questions to illustrate which layers can be identified in the peer interactions as research object:

1. Which themes occur in children's interactions during their block play activities and which content-related mathematical aspects can be reconstructed and described in them?
2. How (through which interactional and argumentative structures) do children arrive at new mathematical interpretations in their mutual processes of negotiation?

When considering the first of the two questions, it becomes clear that it is first necessary to develop descriptive dimensions or categories that can be used to identify and describe mathematics in play situations. In a further step, we can reconstruct when and to what extent mathematical topics are introduced by the children during a play situation. Both were realized by Henschen (2020) in her work. This part of the methodological procedure can be described as a "rough analysis" and examined with methods of qualitative text analysis (Kuckartz, 2014b). These topics are summarized below and are in accordance with the previous research done by Henschen (2020).

The second research question requires a combination and integration of a further method through which a deepening of the insights gained through the rough analysis is possible; at the same time, the insights of the rough analysis support the development of the further analysis (cf. Buchholtz 2019, 2021; p. 227). This can be described as a "fine analysis" and draws on methods and analytical steps of interpretative research, as shown below. The following diagram (cf. **Figure 1**) illustrates the combination and integration of methods used in our research design.

Data Collection

The data we reused for this contribution were collected as part of a PhD project (Henschen, 2020) in a small kindergarten in a village with approximately 3,500 inhabitants in south-western Germany. A student known to the children from an internship during her studies conducted the video recordings with the consent of the kindergarten and the children's parents. From these internships, both the student and the children were already familiar with video graphing play situations. The videotaped situation analyzed in this text took place in the context of a so-called "free play phase," which is a common part of daily routines in German kindergartens as well as in other countries (Pyle et al., 2017). In this phase, the children are largely free to choose the game,

Table 1. Explanation of the transcription characters

Sign	Description
Italics	Actions and gestures of the actors
CAPITAL LETTERS	Emphasis on the word
–	Voice remains in the abeyance
#	Utterances follow one another immediately
<, >	Actions take place simultaneously or overlapping

material, place, and play partner without the direct influence of an adult. The data used here come specifically from a free play phase that regularly takes place in the morning after arrival in the kindergarten, which is influenced insofar as the children are expected to stick to their work with a material or their play with the chosen play partners for a longer period (at least half an hour). The observed children are from a group of essentially four children (two girls 6;0 and 5;3 years old and two boys 4;10 and 4;8 years old) who engaged with one of the materials available in the block play area and were filmed doing so over the period of four weeks during some of these free play phases. The children involved spoke to each other in German during their play. Apart from age, no other data was collected about the children. The names of the children were anonymized. The person filming focused on capturing the children's play in the block play area as well as possible, which is why a static camera was not chosen. The camera was tracked by hand and in some cases zoomed in. Because of the camera's perspective, there is not always an overall view of the situation. The ethnographic material was recorded according to the interpretative research paradigm (König, 1991; Wilson, 1973). During the recordings, the persons filming faced the challenge of generating data material while finding a balance between optimal recording quality and trying to influence the children's interactions the least they possibly could (Krummheuer, 2013). As far as was practically possible, the video recording starts as soon as the children have entered the block play area and ends when the children leave.

Methods of the Rough Analysis

Using (qualitative) thematic analysis (Kuckartz, 2014b), Henschen (2020) has developed two coding frames. The first coding frame addresses the connection between mathematical content and informal mathematics in children's block play. The second coding frame differentiates the ways children work in their building games. These ways of working in turn reveal connections to mathematical processes (cf. Henschen, 2020, p. 414). Examples of categories at the content level are "wrong way-right way," "small-large," "slanted-straight," "open-closed," "fixed-unfixed" and "equal-unequal." The categories can be understood as kinds of "natural categories" (Kuckartz 2014a, p. 44) because the children's spontaneous language is also considered in the category designations (Henschen, 2020). The categories at the process level reflect that for block play and "technical" learning opportunities, certain steps in problem-solving processes need to be described. Here Henschen (2020) ties in with Bruce and colleagues (1992b, p. 124), who identify the categories "making and monitoring," "planning" and "evaluation" as components of children's problem-solving processes. In addition, she also includes categories introduced by Siraj-Blatchford and MacLeod-Brudenell (1999, p. 68). Here the categories are labelled as "make or modify," "evaluate" and "design or adapt." These literature-based categories are expanded by Henschen (2020) with the category "making/monitoring," which considers the observation that children in their block play are either completely involved in their play or they communicate with others in their play. Furthermore, Henschen (2020) has developed the categories "constructing/building," "evaluating/titling," and "designing/adapting." The coding of the respective video sequences with the categories is done in MAXQDA. The resulting "codelines" are presented in the analysis section (Henschen, 2020).

Methods of the Fine Analysis

Following the rough analysis presented above, a detailed analysis of the interactive negotiation process has been realized in which the development of the mathematical theme is reconstructed with the help of interaction analysis. The basis for the fine analysis is the transcription of the actions. The transcription legend (**Table 1**) can be found in the notes at the end of the contribution. For details of the analysis, please refer to Krummheuer (2013) and Krummheuer and Fetzer (2005). The rough analysis corresponds to the division of the situation into interaction units, as required in the first step of the interaction analysis. Since the qualitative text analysis and it's structuring already facilitated the identification of interactionally dense moments with a particular accumulation of different mathematical themes that can be reconstructed, these are briefly described and then analyzed in step 2 of the interactional analysis as they pertain to their location in the situation and the situational kindergarten discourse. The analysis steps "analysis of individual utterances" (3) and "turn-by-turn analysis" are interlocked here and are presented in a summarizing analysis. The interaction analysis is supplemented by argumentation-analytical considerations that are based on functional argumentation analysis (Toulmin, 1969), which looks at the argumentation process as well as the individual argumentative links that are realized and negotiated by the peers involved in the interaction. For this purpose, the Toulmin's schema is used in its mathematics didactic adaptation according to Krummheuer (2007, 2009). Here, it is shown how an argumentative conclusion is secured based on data based on so-called warrants and backings by the naming of rules in the support. This functional argumentation analysis helps us understand the extent to which argumentation processes (which are described as constitutive moments in (mathematical) learning processes) emerge in peer interactions; it also tells us how mathematically rich and diverse the validation of conclusions through warrants and backings can be in these interactions.

Figure 2. "*SONOS*"

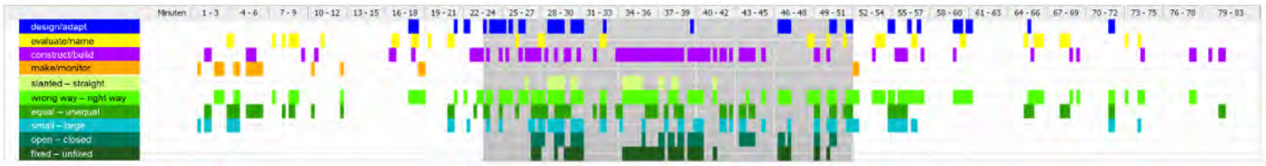


Figure 3. Codeline of play situation with "*SONOS*"(export from *MAXQDA*)

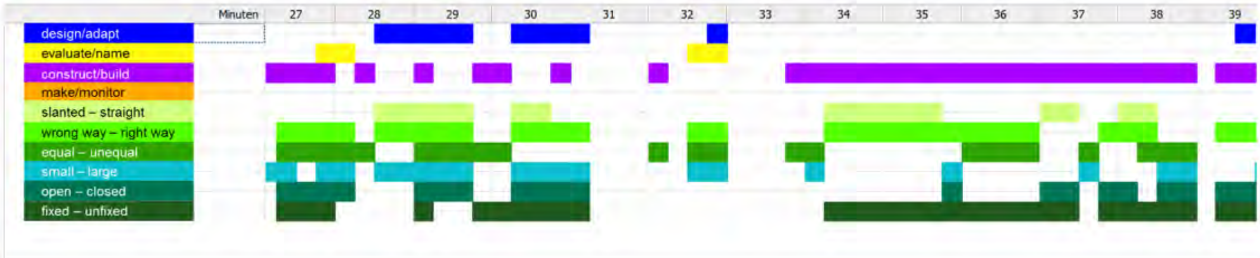


Figure 4. Extract of the codeline 27:00-39:45 (export from *MAXQDA*)

ANALYSIS OF A PARADIGMATIC EXAMPLE OF A PLAY SITUATION–"WE HAVE TO PUT THE LADDER RIGHT IN THERE! INSTALL IT CORRECTLY!"

The play situation considered in the analysis took place on the third day the four children were filmed during block play. On this day, the children were working with the material SONOS (**Figure 2**). For about the first 30 minutes, the four children (of the group described above) are in the block play area, then the two girls leave the building corner and the two boys, we will call them Max (4;10 years) and Ron (4;8 years), stay there for another 50 minutes. Max describes what they are doing as building and distinguishes it linguistically from playing: "Now we have had to build so much [...] Now we will play for a while and then we can build again."

Rough Analysis–Selection of the Paradigmatic Example "The Ladder"

The result of applying the two category systems (cf. chapter 3.3) to the recording of this play situation with SONOS can be displayed in MAXQDA, for example, with a codeline (**Figure 3**). The codeline shows the occurrence of the categories during the entire filmed play situation. Even though the overview does not show how the categories were assigned in detail, it is clear that there is a phase (highlighted in grey in **Figure 3**) in which the category density is higher. Here, the play and the exchange among the children seem to be much more intensive than in other phases of the videotaped play. Part of the intensive phase in **Figure 3** is shown again in a larger format in **Figure 4**.

From the 28th minute until around the 40th minute, the category fixed-unfixed (dark green) emerges very clearly. This shows a theme that seems to be particularly relevant for the children's building in this phase. At first, this category rather points to a technical discussion about stability and building techniques. From a mathematics-didactic perspective, however, the category is also quite interesting because a connection between form and stability can be assumed. The fact that this connection is also relevant for the children or is addressed by them is clearly shown in the codeline. In this video section, the category fixed-unfixed could be assigned together with the category slanted-straight (light green) or with the category wrong way-right way (neon green). This means that in the assigned situations there is a simultaneous examination of geometric concepts and the orientation of objects in space (cf. Henschen, 2020). In this way, the rough analysis shows a central theme that has a technical and mathematical reference, as well as variously pronounced references to all other categories at the content level.

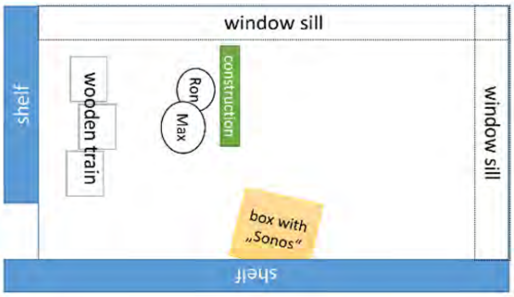


Figure 5. Diagram of the building corner

Figure 6. Illustration of the analyzed situation

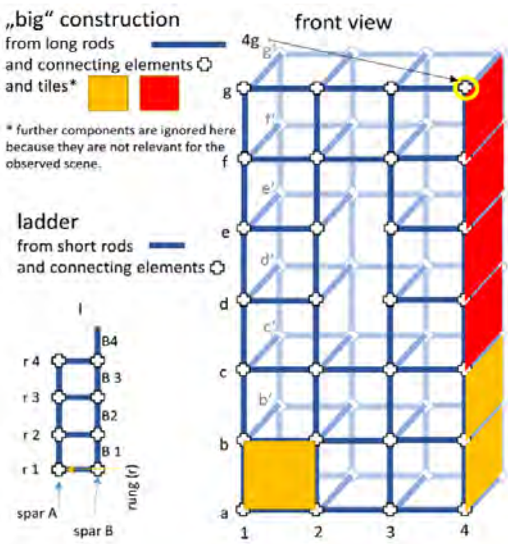


Figure 7. Sketch of the structure of the building

The points mentioned above suggest that this is a case of "interactional compression" (Krummheuer, 2002), it therefore seems worthwhile to subject this particular segment of material to further fine analysis. Furthermore, it would be interesting to do a fine analysis of the children's argumentation processes and, if necessary, of the differences between the working method of designing/adapting (blue), which is particularly evident between the 29th and 31st minutes, and the working method of constructing/building (purple), which is always present, but particularly so from the 33rd minute onwards.

Fine Analysis–Reconstruction of the Process of Negotiation of Mathematical Meaning in the "Ladder Example"

Description of the situation

Before the situation presented in the analysis began, the boys had already been working with a construction (see **Figure 5**) for some time; it had already been built before the children went to the building corner that morning. Max calls the structure an "aeroplane." During a preoccupation with "drivable" constructions to which wheels are attached, a common game (interest) of Ron and Max can be interpreted from minute 19 onwards. In minute 26, after Max has attached a construction with wheels (the children also refer to it as a "forklift truck") to the top of the structure (see **Figure 6**), Ron asks: "Why put it all the way on the top? They can't get it down that way." Max replies that "they" (imaginary people) have a "ladder." Ron then suggests building such a ladder. After the two have made a short piece of the ladder (four ladder rungs), they turn to the construction with this piece. The situation selected and considered in the following detailed analysis begins immediately afterwards. The sketch shown (**Figure 7**) helps to understand the material actions described in the transcript and the analysis.

Table 2. Scene I–"I have to fix it."

001	0:27:34	Max	I have to fix (incomprehensible).	
002		<	Holds the "ladder" (I) against the structure so that the end of "B4" lies on the point "4f", turns the ladder a little back and forth.	
003		< Ron	Looks at Max.	
004		Ron	And then how do they get up there?	

Table 3. Scene II–"Because otherwise they wouldn't get up there at all."

026	0:28:34	Max	No, we stack it to the floor!	
027			Places the ladder between 1d and 2d through the structure.	
028			We will stack it that way\.	
029		Ron	Makes a stacking gesture with his hand.	
030			Takes out the ladder.	
031			No, we stack it that high!	
032			Places the ladder diagonally into the "floor" so that Sp1 rests on the strut between 1d and 2d and A3 and B3 rest on the strut between 1e' and 2e'.	
033			Because otherwise they wouldn't get up there at all.	

Table 4. Scene III–"They will connect the ladders."

052		Ron	But otherwise, they can't get up there at all.
053	0:29:40	Max	But they will connect the ladders, you fool–.
054			Looks at Ron.
055		Ron	All ladders?
056		Max	Yes.

Scene I–"I have to fix it."

In the scene depicted in **Table 2**, it can be reconstructed that the fixing of the ladder is Max's focus. The boy's action in line 002, as well as his utterance in line 001, can be interpreted in this way. Obviously, from the boy's point of view, the previous interaction in which the children are looking for a solution to the problem of reaching the "forklift" on the building "forces" this action. The utterance can be interpreted here as a (form of) self-instruction. What is remarkable is the obvious reference to the everyday world in Ron's utterance in line 004, in which Ron raises the question of how "they get it up there." In this context, the indexical word "they" can be interpreted as the figures, the imaginary people. Both boys seem to agree on the "existence" of these people. In addition, the question can be understood as a problem outline that requires argumentative discussion (for further details see Vogler et al., in press). It is possible that at this point Ron does not agree with Max's proposal for fastening or with the ladder and therefore asks for clarification, but this does not directly follow. In various subsequent scenes, however, it can be seen that the topic continues to be the subject of negotiation between the children. From line 026 onwards, an argumentative structure emerges in this context.

Scene II–"Because otherwise they wouldn't get up there at all."

Max's statements in line 026 and line 028 can be understood as a counterproposal to attaching to 4f (**Table 3**). He suggests an integration of the ladder into the building with high flexibility and possible usability for the shared play context. In this context, Max uses the term "stacking" (line 028), as does Ron here. In the scene, Max identifies the ladder that has already been built as a representative of several ladders that are to be stacked.

Ron varies Max's idea from line 030 onwards by combining it with his idea of using the ladder diagonally at each storey, which he had already introduced 15 lines earlier without success. Ron takes up Max's expression "stacking" in the sense of positioning one on top of the other within the structure and varies the position of the ladder in that it is placed diagonally in the storey. This experimentation with different solutions can be seen as a propaedeutic of the heuristic "systematic searching" (Liljedahl et al., 2016); the children rule out rejected solutions. Max's use of the same term "stacking" can be interpreted as a sign of responsiveness on his part, but at this point in play no interpretation has emerged that can be taken as shared. The negotiation continues in the following, but it does not lead to an agreement.

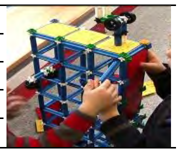
Scene III–"They will connect the ladders."

In line 52, Ron repeats the argument he already mentioned in line 033 (**Table 4**). This argument already served as the basis for the negotiation process in line 004 in the question, "And how do they get up there?". Max ties in with this initial question in line 053 as well. It can be assumed that while Ron sees the solution as a building task, Max shifts the solution to the narrative play action. A non-hierarchical variety of solutions is evident here, which is also evident in the further interaction. Between line 057 and line 077, there is a shift to the "construction task," there is a connection between the two ladders or ladder parts that have been created in the meantime, and this ladder is then extended by several components. Subsequently, the connection of the ladder and further supports of the warrants are integrated into the argumentation process, such as the necessity of using or not using certain components (lines 089 and 094) or to pay attention to the design of the construction (line 097).

Table 5. Scene IV–”No, you don’t have to attach it at all.”

117	0:35:10		Ron	I HAVE TO ATTACH IT!	
118			Max	Removes the part attached by Ron, in addition to the connecting element, the rod “A8” also comes off, pushing Ron’s hand away and upwards in the process.	
119				No, you don’t have to attach it at all.	
120				Detaches the bar from the connecting element and tries to reattach it to the ladder.	
121				Only this one must be quite simple.	
122			Ron	Holds a white connector in his hand and looks at Max.	
123				But otherwise, it won’t stay up there\.	
124		<	Max	It lifts completely#	
125		<	Ron	Kneeling next to Max.	
126				Please (.) and besides, I had the idea.	
127			Max	Look, it doesn’t move.	

Table 6. Scene V–”They build the ladder together.”

140	0:36:12	Ron	But you can attach it like that and that’s good.	
141			Then it will hold better.	
142		Max	Observes Ron. But first we need another long one.	
143			Turns to the components lying on the floor, he picks up a piece off the floor.	
144			I’ll get one of my long ones here–.	
145			Returns to the building with the piece and holds it up against the upper right end of the ladder.	
146		Ron	Yes, and then you can put it there. Wait, I’ll give it to you.	
147			Attaches a connecting element to spar A8.	
148		Max	No, wrong, this has to go in here.	
149			Grabs Ron’s hand and leads it to bar B, holds the fastener to B8 together with him and then takes his hand away.	
150		Ron	Fastens the connecting element.	
151		Max	Holds the long rod from his hand with one end at the upper corner of the structure (4g) the other pointing towards the ladder (B8).	
152			And attached here.	
153			Attaches the bar perpendicular to spar B to the connecting element at B8, causing the structure to sway.	
154		Ron	Uh oh!	
155		Max	Uh oh!	
156			Tries to connect the rod he has attached to the ladder with the corner “4g” of the structure.	
157		Ron	Wait, we have to#	
158			Together with Max he pulls the ladder (V) upwards above 3d-4d out of the building.	
159		Max	#Remove it#	
160			Tries again to attach the rod to the corner 4g.	
161		Ron	Together with Max, he grabs the ladder.	
162			We have to really get the ladder in there#	
163		Max	#Install correctly.	
164			Together they connect the ladder to the corner 4g of the structure, the ladder now hangs parallel.	

Scene IV–”No, you don’t have to attach it at all.”

The negotiation subsequently intensifies (line 116), while Ron continues to argue that the ladder should be attach and makes attempts to implement it, Max vehemently blocks this idea and Ron’s attempts to attach it (lines 108 and 114) (**Table 5**). The argumentative culmination is reached in the following section.

After Ron again insists loudly on attaching the ladder, Max deconstructs the component that was supposed to be used to attach it and–through his physical intervention and pushing Ron’s hand away–prevents further attempts to attach the ladder. He comments on this in line 119 by repeating his earlier statement that the ladder does not need to be attached. On the one hand, this could be interpreted as a support of the warrant “flexibility” from lines 026 and 053. On the other hand, this can be interpreted as a further warrant of a conclusion that emerges in line 083 and which can be interpreted as a shared assertion from line 123 onwards. In line 123, the children agree that a ladder on which the imaginary people can reach the upper floor must be stable: On the one hand, the children’s statements in lines 119, 124, and 127 can be interpreted as a generally valid rule that even leaning ladders have this stability and can still be flexibly accommodated in the building’s structure. On the other hand, they can also be interpreted as a justification for the above-mentioned conclusion.

Line 126 can also be interpreted as supporting the warrant “anchoring.” Social persuasive power can be attributed to this statement by Ron. In “Please,” (line 126), the rule can be recognised that actions also receive their validity in the block play if they are demanded by a game partner. The statement “And besides, I had the idea” can also be interpreted Ron’s reference to his authority as the initiator of the game idea “ladder,” as well as the initiator of the problem-solution “attaching.”

Scene V–”They build the ladder together.”

Starting at line 140, a turn in the interaction can be reconstructed (**Table 6**). After the children had previously struggled over the relevance of their different solutions, Ron’s argumentation now seems to catch on with Max.

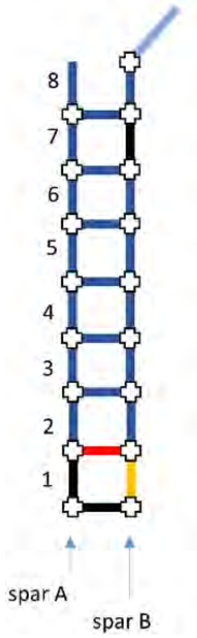


Figure 8. Ladder (V)

Figure 9. Building with ladder

This could result from the fact that Ron uses gestures to illustrate the distance or gap between the end of the ladder and the building in line 141, while the ladder is stuck diagonally through the "upper floors" of the building. Through this, Max develops the idea of not only connecting the ladder and the building with a white connecting piece, but of using another long stick as an intermediate piece (line 142).

In this context, it is particularly noteworthy that the distance between the end of the ladder and the building corresponds to the length of the longer rod chosen. It can be assumed that Ron's illustration of the distance using gestures initiates an estimation process that leads to Max choosing a suitable rod that also enables the attachment. In this situation, the diagonal placement of the ladder in the building creates an angle between the rod attached to the ladder and the building (lines 149-153), which does not allow it to be attached to the building (lines 156 to 157). Therefore, the children pull out the ladder and attach it to the corner of the building (lines 158 to 164) (**Figure 8** and **Figure 9**). After the upper point of the ladder has been cleared, the ladder is extended to the ground. The heuristic strategy of working backwards can be found here (Schoenfeld, 1985).

EMPIRICAL FINDINGS

From a mathematics didactics perspective, it can be particularly emphasized in the situation presented that different mathematical content areas emerge in the overall view of the argumentatively structured process of negotiation of meaning. Thus, the attaching or bridging gestured made by Ron can be assigned to the topology. Subsequently, Max estimates the length of the rod needed for bridging. Furthermore, it can be assumed that the children become aware of the angles between the building and the ladder since an attachment is only possible at right angles. Concerning these angles, it can be assumed that the children (mentally) rotate the ladder (**Figure 8**) until they can connect it to the edge [4g] of the construction (**Figure 7**). A fourth mathematical content reference can be seen in the structure of the building and ladder. The material enforces building in parallels and with right angles as well as the alternating use of rods and connecting elements, the limited use of which the children must conform to. It is precisely these structures that cannot be consistently found in the children's interactive building activity. Only through the argumentatively rich and ongoing process of negotiation of meaning as well as the experiences gained through actions with the material do they arrive at an "attaching" solution through the interpretation of the structures. Both children are consistently involved in interpretations and actions from these different content areas. They participate actively and productively and clarify the topics in detail in peer interaction.

Something that is particularly remarkable about these peer interactions is that the negotiation process between the children does not stop even when terms to describe these structures and actions are presumably missing from the material. This becomes obvious in lines 150-160, in which the children communicate through their actions while attaching the ladder. The fact that a strong interactive proximity between the two children emerges here is made clear by the fact that, among other things, they make utterances that can be identified as an echo or completion of the previous utterance. This could be attributed to the fact that the children realize actions in the zone of each other's next development (Vygotsky, 1978).

Epistemologically, it can be concluded that the controversial negotiation and the resulting collective argumentation processes presumably result in new interpretative perspectives for the two children, which can be interpreted as mathematically substantial insofar as experiences are gained here in the field of topology, sizes, and geometric relations (cf. Henschen, 2020; Vogler et al., in press). In the analysis of the negotiation process, it also becomes clear that, in addition to mathematical content, processes that are particularly relevant to mathematics also emerge. These include the process of problem-solving realized by the children in the context of which heuristic strategies such as systematic trial and error and working backwards can be reconstructed (cf. Helenius et al., 2016).

CONCLUSION & OUTLOOK

On a theoretical level, this contribution initially argued that participation in argumentative processes is essential for the condition of the possibility of learning. Through collective argumentation, children can gain new knowledge. Using this as a reference, the aim of the presented study was to find out to what extent potentials for participation in argumentatively structured processes of negotiating mathematical meaning can be reconstructed in peer interactions in kindergarten. For this purpose, block play situations were first described and systematized in the rough analyses concerning their informal mathematical content. This made it possible to identify sequences in which the negotiation processes between children intensify. These sequences can be interpreted as interactional condensation (Krummheuer, 2007) in which "conditions for enabling mathematical learning" (Miller, 1986) emerge through the mutual negotiation process. With a view of the emergence of these learning processes, the subsequent detailed analysis allowed us to reconstruct how the mathematical topics are negotiated and the extent to which opportunities for participation arose here for the corresponding children, as well as how the topics were used by them. Thus, it is precisely in the theoretical and methodological connection of the two analytical perspectives that a further gain in knowledge can be seen.

The analyses also clearly showed that in peer interactions in block play, diverse mathematical content (cf. Henschen, 2020) emerges that can be interpreted in a predominantly cross-domain manner in the children's building activities as described above. This seems to be relevant for the (re-)construction of mathematical learning because networking of mathematical areas of experience can be assumed here, which contributes to the development of networked mathematical knowledge (cf. Krummheuer & Schütte, 2014). According to Bauersfeld (1988), it can be assumed that networked mathematical knowledge is particularly sustainable.

Such a potential for sustainable and networked development of mathematical knowledge is also made possible through collective argumentation. In addition to the emergence of cross-cutting mathematical themes, such collective argumentations were reconstructed in the analyses of the play situations in kindergarten. From the perspective of argumentation theory, it could be shown in this context that so-called "deep" argumentations emerge presumably through the equal negotiation of ideas (play ideas, mathematical ideas), which are characterized by a richness of variants as well as by their long-chain nature. The multifaceted and ongoing negotiations can be constitutive for mental restructuring processes, as redundancies emerge alongside different argumentative elements. These redundancies, which are presumably realized in this form primarily in peer interactions in discourses of equal authority, can have an important function for the "intramental" reorganization process (the re-appropriation or reorganization of subjective areas of experience): This is due to the fact that these redundancies in the process of negotiation of mathematical meaning are what enable the individual participants in the interaction to enter the process of interpretation at different points in time. Therefore, a small step "evolutionary" development of knowledge is made possible by following pre-existing areas of experience. Play situations can thus have the potential for sustainable networked knowledge (cf. Vogler, 2020, 2021).

Concerning the formulated research question of identifying potentials for participation in mathematical discourse in peer interactions in block play in kindergarten, the contribution was also able to highlight that the assertion of Helenius et al. (2016) that playing is not only mathematical when explicit mathematical content areas are negotiated, but also when heuristics of problem-solving emerge are highly relevant. These can also be attributed to mathematical discourse and are thus part of a mathematical learning process (Helenius et al., 2016). In line with this, the empirical data presented here also show that heuristics emerge in peer interactions in building games that can be attributed to mathematical problem-solving (Vogler et al., 2022, submitted). It can therefore be concluded that play situations can facilitate learning processes that relate to mathematical content as well as those that relate to mathematical processes.

Subsequently, some interesting topics and questions can be identified studied based on further analyses. For example, it should be investigated how the collective argumentations of equal peers, which are considered particularly conducive to the emergence of new (mathematical) knowledge, differ from such interactions in which pedagogical professionals are directly involved. Furthermore, it has to be asked to what extent learning support, for example by competent pedagogical professionals, could enrich play situations by mediating between the children's informal mathematics and the formal mathematical language, which is seen as a requirement for success in "mathematization processes". This is also accompanied by the question of how mathematics teachers in primary schools can succeed in specifically addressing children's subjective areas of experience and thus do justice to the idea of sustainable networked knowledge.

From the perspective of argumentation theory, it seems worthwhile to show in detail which support, and warrants emerge in different play situations. In the situations analyzed so far, interesting aspects have already been identified:

(1) If the interaction comes to a halt due to a technical or linguistic "overload" of the children, for example, the children switch to narrative argumentation processes in which the solution is brought about through role play and
(2) likewise, interpretations that are taken as shared are produced by convincing the play partner on a social level. Social backings emerge here (Vogler, in press).

Furthermore, play situations between peers could be examined from a structural point of view. It would be worthwhile to look at the rhythmization of actively productive phases and so-called "co-construction pauses" (Brandt & Höck, 2012) if one assumes that the "co-construction pauses" are possibly constitutive for mental restructuring processes because they enable the interactants to intramentally summarize interpretations that were previously realized intermentally (Bruner, 1990). A comparison with situations with polyadic structures would also be interesting for further research because the game situation analyzed above has a continuous dyadic structure.

The questions answered in this contribution and those posed beyond it contribute to research on the conditions for success in early mathematics education. Finally, it can be pointed out that early mathematical learning processes are not only driven by "others who are more competent in the matter," but that peer interaction seems to play a key role because they create opportunities for participation that are also attuned to the discursive competences of the children due to their discursive proximity. The potential of these interactions is shown not least by the intensity, but also by the richness of variation with which mathematical topics are negotiated by the children in the example presented here. A further aspect has also become clear: Such negotiations need time and a calm setting so that children have the opportunity and space to engage with the accessible materials in a multifaceted way.

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REFERENCES

Acar Bayraktar, E. (2018). How can a father be supportive for the mathematics learning process of a child?–The relationship between scaffolding and the interactional niche in the development of mathematical learning in the familial context. In C. Benz, A. S. Steinweg, H. Gasteiger, P. Schöner, H. Vollmuth, & J. Zöllner (Eds.), *Mathematics education in the early years: Results from the POEM3 conference, 2016* (pp. 255-280). Springer. https://doi.org/10.1007/978-3-319-78220-1_13

Arendt, B. (2015). Kindergartenkinder argumentieren–Peer-Gespräche als Erwerbskontext [Kindergarten children argue–Peer talks as an acquisition context]. *Mitteilungen des Deutschen Germanistenverbandes* [*Communications from the German Association of German Studies*], *62*(1), 21-33. https://doi.org/10.14220/mdge.2015.62.1.21

Arendt, B. (2019). Discourse acquisition in peer talk–The case of argumentation among kindergartners. *Learning, Culture and Social Interaction, 23*, 100342. https://doi.org/10.1016/j.lcsi.2019.100342

Bauersfeld, H. (1988). Interaction, construction, and knowledge: Alternative perspectives for mathematics education. In D. A. Grouws, & T. J. Cooney (Eds.), *Perspectives on research on effective mathematics teaching* (pp. 27-46). Lawrence Erlbaum.

Bauersfeld, H., Krummheuer, G., & Voigt, J. (1985). Interactional theory of learning and teaching mathematics and related microethnographical studies. In H. G. Steiner, & H. Vermandel (Eds.), *Foundations and methodology of the discipline mathematics education (didactics of mathematics)*. University of Antwerp.

Beck, M., & Vogler, A.-M. (2021). Teachers' responsiveness as a key element in supporting learners' participation in arithmetic discourses in primary classroom. In J. Novotná & H. Moraová (Hrsg.), *Proceedings of the International Symposium Elementary Mathematics Teaching (SEMT) – Broadening experiences in elementary school mathematics* (pp. 72-82). Prague.

Brandes, H. (2008). *Selbstbildung in Kindergruppen. Die Konstruktion sozialer Beziehungen* [*Self-education in children's groups. The construction of social relationships*]. Ernst Reinhard Verlag.

Brandt, B. (2004). *Kinder als Lernende. Partizipationsspielräume und -profile im Klassenzimmer* [*Children as learners. Participation scopes and profiles in the classroom*]. Peter Lang.

Brandt, B., & Höck, G. (2012). Mathematical joint construction at elementary grade–A reconstruction of collaborative problem solving in dyads. In *Proceedings of CERME 7*. Rzeszów, Polen.

Brandt, B., & Tiedemann, K. (Eds.). (2019). *Mathematiklernen aus interpretativer Perspektive* [*Learning mathematics from an interpretative perspective*]. Waxmann.

Bruce, T., Gura, P. & Froebel Blockplay Research Group. (1992b). Children being scientific and solving problems. In P. Gura, & T. Bruce (Eds.), *Exploring learning. Young children and blockplay* (pp. 107-131). P. Chapman.

Bruce, T., Gura, P., & Froebel Blockplay Research Group. (1992a). Being mathematical. In P. Gura, & T. Bruce (Eds.), *Exploring learning. Young children and blockplay* (pp. 75-91). P. Chapman.

Bruner, J. (1990). *Acts of meaning*. Harvard University Press.

Brunner, E. (2014). *Mathematisches Argumentieren, Begründen und Beweisen. Grundlagen, Befunde und Konzepte* [*Mathematical reasoning, reasoning and proof. Basics, findings and concepts*]. Springer. https://doi.org/10.1007/978-3-642-41864-8

Buchholtz, N. (2019). Planning and conducting mixed methods studies in mathematics educational research. In G. Kaiser, & N. Presmeg (Eds.), *Compendium for early career researchers in mathematics education* (S. 131-152). Springer. https://doi.org/10.1007/978-3-030-15636-7_6

Buchholtz, N. (2021). Voraussetzungen und Qualitätskriterien von Mixed-Methods-Studien in der mathematikdidaktischen Forschung [Prerequisites and quality criteria of mixed-methods studies in mathematics didactic research]. *Journal für Mathematik-Didaktik* [*Journal for Mathematics Didactics*], *42*(1), 219-242. https://doi.org/10.1007/s13138-020-00173-0

Cekaite, A., Blum-Kulka, S., Grøver, V., & Teubal, E. (Eds.). (2014). *Children's peer talk: Learning from each other*. Cambridge University Press. https://doi.org/10.1017/CBO9781139084536

Chaiklin, S. (2003). The zone of proximal development in Vygotsky's analysis of learning and Iinstruction. In A. Kozulin, B. Gindis, V. S. Ageyev, & S. M. Miller (Eds.), *Vygotsky's educational theory in cultural context* (pp. 39-64). Cambridge University Press. https://doi.org/10.1017/CBO9780511840975.004

Civil, M., Planas, N., & Quintos, B. (2005). Immigrant parents' perspectives on their children's mathematics education. *ZDM, 37*(2), 81-89. https://doi.org/10.1007/BF02655717

Cobb, P., & Bauersfeld, H. (Eds.). (1995). *The emergence of mathematical meaning: Interaction in classroom cultures*. Lawrence Erlbaum.

Cobb, P., Yackel, E., & Wood, T. (1992). Interaction and learning in mathematics classroom situations. *Educational Studies in Mathematics, 23*(1), 99-122. https://doi.org/10.1007/BF00302315

Cooke, A., & Bruns, J. (2018). Early childhood educators' issues and perspectives in mathematics education. In I. Elia, J. Mulligan, A. Anderson, A. Baccaglini-Frank, & C. Benz (Eds.), *Contemporary research and perspectives on early childhood mathematics education* (pp. 267-289). Springer. https://doi.org/10.1007/978-3-319-73432-3_14

Cooper, J., Prescott, S., Cook, I., Smith, L., Mueck, R., & Cuseo, J. (1984). *Cooperative learning and college instruction. Effective use of student learning teams*. Fundation Publikation.

Corsaro, W., & Rizzo, T. A. (1990). Disputes in the peer culture of American and Italian nursery-school children. *Conflict Talk*, 21-66.

Eisenberg, A. R. (1987). Learning to argue with parents and peers. *Argumentation, 1*(2), 113-125. https://doi.org/10.1007/BF00182256

Ertle, B. B., Ginsburg, H. P., Cordero, M. I., Curran, T. M., Manlapig, L., & Morgenlander, M. (2008). The essence of early childhood mathematics education and the professional development needed to support it. In A. Dowker (Eds.), *Mathematical difficulties. Psychology and intervention* (pp. 59-83). Academic Press. https://doi.org/10.1016/B978-012373629-1.50006-X

Flottorp, V. (2011). How and why do children classify objects in free play? A case study. In M. Pytlak, T. Rowland, & E. Swoboda (Eds.), *Proceedings from the 7th Congress of the European Society for Research in Mathematics Education, Rzeszów, Poland* (pp. 1852-1862). European Society for Research in Mathematics.

Forman, E. (1989). The role of peer interaction in the social construction of mathematical knowledge. *International Journal of Educational Research, 13*(1), 55-70. https://doi.org/10.1016/0883-0355(89)90016-5

Helenius, O., Johansson, M. L., Lange, T., Meaney, T., Riesbeck, E., & Wernberg, A. (2016). When is young children's play mathematical? In T. Meaney, O. Helenius, M. L. Johansson, T. Lange, & A. Wernberg (Eds.), *Mathematics education in the early years. Results from the POEM2 Conference*, 2014 (pp. 139-156). https://doi.org/10.1007/978-3-319-23935-4_8

Henschen, E. (2020). *In Bauspielen Mathematik entdecken. Aktivitäten von Kindern mathematikdidaktisch analysieren und verstehen* [Discover mathematics in construction games. Analyzing and understanding the activities of children in mathematics education]. Springer VS.

Jung, J. (2019). Möglichkeiten des gemeinsamen Lernens im inklusiven Mathematikunterricht. Eine interaktionistische Perspektive [Possibilities of joint learning in inclusive mathematics lessons. An interactionist perspective]. In B. Brandt, & K. Tiedemann (Eds.), *Mathematiklernen aus interpretativer Perspektive* [*Learning mathematics from an interpretative perspective*] (pp. 103-126). Waxmann.

KMK. (2005). *Beschlüsse der Kultusministerkonferenz* [*Decisions of the Conference of Ministers of Education*]. Luchterhand–Wolters Kluwer.

Krummheuer, G. (1983). Das Arbeitsinterim im Mathematikunterricht [The work interim in mathematics lessons]. In H. Bauersfeld, B. H., G. Krummheuer, J. H. Lorenz, & J. Voigt (Eds.), *Lernen und Lehren von Mathematik. Analysen zum Unterrichtshandeln II* [*Learning and teaching mathematics. Analysis of classroom action II*] (S. 57–104). Köln: Aulis.

Krummheuer, G. (2007). Argumentation and participation in the primary mathematics classroom. Two episodes and related theoretical abductions. *Journal of Mathematical Behavior, 26*(1), 60-82. https://doi.org/10.1016/j.jmathb.2007.02.001

Krummheuer, G. (2009). Inscription, narration and diagrammatically based argumentation. The narrative accounting practices in the primary school mathematics lesson. In W.-M. Roth (Ed.), *Mathematical representation at the interface of the body and culture* (pp. 219-243). Information Age Publishing.

Krummheuer, G. (2013). The relationship between diagrammatic argumentation and narrative argumentation in the context of the development of mathematical thinking in the early years. *Educational Studies in Mathematics, 84*(2), 249-265. https://doi.org/10.1007/s10649-013-9471-9

Krummheuer, G., & Brandt, B. (2001). *Paraphrase und Traduktion. Partizipationstheoretische Elemente einer Interaktionstheorie des Mathematiklernens in der Grundschule* [*Paraphrase and traduction. Participation-theoretical elements of an interaction theory of mathematics learning in elementary school*]. Beltz.

Krummheuer, G., & Schütte, M. (2014). Das Wechseln zwischen mathematischen Inhaltsbereichen–eine Kompetenz, die nicht in den Bildungsstandards steht [Switching between math content areas–a skill that is not in the educational standards]. *Zeitschrift für Grundschuldidaktik* [*Journal for Elementary School Didactics*], *7*(1), 112-124.

Kuckartz, U. (2014a). *Qualitative Inhaltsanalyse. Methoden, Praxis, Computerunterstützung* [*Qualitative content analysis. Methods, practice, computer support*]. Beltz.

Kuckartz, U. (2014b). *Qualitative text analysis. A guide to methods, practice & using software*. SAGE. https://doi.org/10.4135/9781446288719

Lave, W., & Wenger, E. (1991). *Situated learning. Legitimate peripheral participation*. Cambridge University Press. https://doi.org/10.1017/CBO9780511815355

Liljedahl, P., Santos-Trigo, M., Malaspina, U., & Bruder, R. (2016). Problem solving in mathematics education. In *Problem solving in mathematics education. ICME-13 topical surveys* (pp. 1-39). Springer. https://doi.org/10.1007/978-3-319-40730-2_1

Mercer, N., Wegerif, R., & Dawes, L. (1999). Children's talk and the development of reasoning in the classroom. *British Educational Research Journal, 25*(1), 95-111. https://doi.org/10.1080/0141192990250107

Miller, M. (1987). Argumentation and cognition. In M. Hickmann (Ed.), *Social and functional approaches to language and thought*. Academic.

NCTM. (2000). Principles and standards for school mathematics*. National Council of Teachers of Mathematics*. https://www.nctm.org/standards/

Pyle, A., DeLuca, C., & Danniels, E. (2017). A scoping review of research on play-based pedagogies in kindergarten education. *Review of Education, 5*(3), 311-351. https://doi.org/10.1002/rev3.3097

Robertson, A. D., Scherr, R., & Hammer, D. (Eds.). (2016). *Responsive teaching in science and mathematics*. Taylor & Francis. https://doi.org/10.4324/9781315689302

Saxe, G. B., Gearhart, M., Note, M., & Paduano, P. (1993). Peer interactions and the development of mathematical understandings: A new framework for research and educational practice. In H. Daniels (Ed.), *Charting the agenda: Educational activity after Vygotsky*. Routledge.

Schoenfeld, A. (1985). *Mathematical problem solving.* Academic Press.

Schütz, A., & Luckmann, T. (1979). *Strukturen der Lebenswelt* [*Structures of the living environment*]. Suhrkamp.

Sfard, A. (2008). *Thinking as communicating. Human development, the growth of discourses, and mathematizing*. Cambridge University Press. https://doi.org/10.1017/CBO9780511499944

Siraj-Blatchford, J., & MacLeod-Brudenell, I. (1999). *Supporting science, design and technology in the early years*. Open University Press.

Stude, J. (2014). The acquisition of discourse competence: Evidence from preschoolers' peer talk. *Learning, Culture and Social Interaction, 3*(2), 111-120. https://doi.org/10.1016/j.lcsi.2014.02.006

Tiedemann, K., & Brandt, B. (2010). Parents' support in mathematical discourses. In U. Gellert, E. Jablonka, & C. Morgan (Eds.), *Proceedings of the 6th International Conference on Mathematics Education and Society* (pp. 457-468).

Toulmin, S. E. (1969). *The uses of argument*. Cambridge University Press.

van Oers, B. (2019). Dynamics of enculturation: Results and consequences of engagement in ECE practices. *European Early Childhood Education Research Journal, 27*(4), 433-435. https://doi.org/10.1080/1350293X.2019.1634231

Vogler, A.-M. (2019). *Chances and obstacles of 'indirect' learning processes in situations with preschool teachers* [Paper presentation]. Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education (CERME11, February 6 – 10, 2019), Utrecht, Netherlands.

Vogler, A.-M. (2020). *Mathematiklernen im Kindergarten. Eine (mehrperspektivische) Untersuchung zu Chancen und Hürden beim frühen mathematischen Lernen in Erzieher*innen-Situationen* [Learning mathematics in kindergarten. A (multi-perspective) study of opportunities and obstacles in early mathematical learning in early childhood education settings]. Waxmann.

Vogler, A.-M. (2021). Indirect learning processes as key variable in early mathematics leaning In M. Inprasitha, N. Changsri & N. Boonsena (Eds.), *Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education* (pp. 639-647). PME.

Vogler, A.-M., & Beck, M. (2020). Förderung responsiven Handelns durch den Einsatz mathematischer Situationspattern [Promotion of responsive action through the use of mathematical situation patterns]. In M. Beck, L. Billion, M. Fetzer, M. Huth, V. Möller, & A.-M. Vogler (Hrsg.), *Ein multiperspektivischer Blick auf Lehr-Lernprozesse – Konzeptionelle, multimodale und digitale Analysen im elementaren und hochschuldidaktischen Kontext* [A multi-perspective view of teaching-learning processes - conceptual, multimodal and digital analyzes in the elementary and university didactic context] (pp. 193-214). Waxmann.

Vogler, A.-M., Henschen, E., & Teschner, M. (2022). *Collective problem-solving in peer interactions in block play situations in kindergarten*. Manuscript submitted for publication.

Vogler, A.-M., Heschen, E., & Teschner, M. (in press). Facettenreichtum kollektiver Argumentationen in Peerinteraktionen in Bauspielsituatio-nen [Diversity of collective arguments in peer interactions in construction game situations]. In B. Brandt & K. Tiedemann (Eds.), *Mathematiklernen aus interpretativer Perspektive II. u.a* [Learning mathematics from an interpretive perspective II]. Waxmann.

Völkel, P. (2002). Geteilte Bedeutung–Soziale Konstruktion [Shared meaning–social construction]. In H.-J. Laewen, & B. Andres (Eds.), *Bildung und Erziehung in der frühen Kindheit* [*Education and upbringing in early childhood*] (pp. 159-207). Beltz.

Völkel, P. (2018). Konstruktion der Wirklichkeit. Die Bedeutung der Kindergruppe für die frühkindliche Bildung [construction of reality. The importance of the children's group for early childhood education]. In *TPS 7/2018 Kinder unter Kindern* [*TPS 7/2018 Children among children*] (pp. 4-7). BETA.

Vygotsky, L. S. (1978). *Mind in society. The development of higher psychological processes*. Harvard University Press.

Webb, N. M. (1989). Peer interaction and learning in small groups. *International Journal of Educational Research, 13*(1), 21-39. https://doi.org/10.1016/0883-0355(89)90014-1

Webb, N. M., & Farivar, S. (1999). Developing productive group interaction in middle school mathematics. In A. M. O'Donnell, & A. King (Eds.), *The Rutgers Invitational Symposium on Education Series. Cognitive perspectives on peer learning* (pp. 117-149). Lawrence Erlbaum.

Youniss, J. (1994). *Soziale Konstruktion und psychische Entwicklung* [*Social construction and psychological development*]. Suhrkamp.

Zippert, E. L., Eason, S. H., Marshall, S., & Ramani, G. B. (2019). Preschool children's math exploration during play with peers. *Journal of Applied Developmental Psychology, 65*, 101072. https://doi.org/10.1016/j.appdev.2019.101072