What Was and Is Algebraic Thinking Skills at Different Education Levels?

Ufuk Töman[1,*] & Ömer Gökburun[2]

[1]Fatih Faculty of Education, Department of Primary Education, Trabzon University, Turkey

[2]Ministry of Education, Turkey

*Correspondence: Fatih Faculty of Education, Department of Primary Education, Trabzon University, Turkey. E-mail: ufuktoman@trabzon.edu.tr ORCID: https://orcid.org/ 0000-0003 3545-7097

Received: June 28, 2022 Accepted: July 25, 2022 Online Published: August 15, 2022

doi:10.5430/wje.v12n4p8 URL: https://doi.org/10.5430/wje.v12n4p8

Abstract

The aim of this study is to determine the algebraic thinking levels, misconceptions and understanding levels of algebraic expressions of 6^{th}, 7^{th} and 8^{th} grade students. In this research, developmental research methods, which is one of the descriptive studies, were used. This study was carried out with secondary school students (6^{th}, 7^{th} and 8^{th} grades) studying in a public school located in a province of the Eastern Anatolia Region in Turkey. The sample of the study consists of 82 students studying at secondary school in the second semester of the academic year 2020-2021. Algebraic thinking levels determination form (ATLDF), consisting of 8 questions and 4 levels, and interview were used as data collection tools. The test was administered to 33 students from 6^{th} grades, 21 students from 7^{th} grades and 28 students from 8^{th} grades, and then interviews were conducted. From the tests and interviews, it was seen that the students had 30 misconceptions about algebraic expressions, that their algebraic thinking levels were quite low, that there were only 3 students who could reach Level-4 and all of these students were female students. From the results, it was concluded that the algebraic thinking level of the students who have problems in the transition from arithmetic to algebra is low; and therefore, it was suggested that the connection between arithmetic and algebra should be established firmly, and studies should be done in this direction.

Keywords: Algebra, misconception, Algebraic thinking, Algebraic thinking levels

1. Introduction

The development of the concept of algebra also provides the development of algebraic thinking skills. Many definitions of algebraic thinking have been made up to now. It is the ability to analyze mathematical situations with relations by using various symbols (Kieran, 2004b). The reflexes of the students who can think algebraically in the problem situations they encounter in their daily lives are very different from the students who cannot think algebraically. The way in which students who can think algebraically make sense of life and interpret events are close to constructivist approaches, far from traditional approaches. In the constructivist approach, individuals make sense of information based on their own experiences and thoughts (Yarimkaya & Ünsal, 2019). It is necessary to increase the number of individuals who try to solve their own problems in line with their experiences and thoughts, reason about them, and use patterns and symbols (Töman & Çimer, 2014). For this, algebraic thinking is of great importance. Algebraic thinking not only provides the development of the concept of algebra and the abstract thinking, but also makes a great contribution to other disciplines (Türkoğlu, 2017).

It has a great contribution to the field of science (Ünsal, Kizilcik, & Yarimkaya, 2018), which has an important place in reaching the level of contemporary civilization. We solve the problems we encounter in our business life by making use of arithmetic and algebra (Usiskin, 1999).

In order for students to be successful in teaching algebra, they need to use and understand the basic concepts, symbols and expressions well (Kieran, 1992a). Good learning and understanding of algebraic concepts and symbols facilitates the development of algebraic thinking. In this way, both the prejudices of the students are broken down and the transition between disciplines becomes easier (Acar, 2019). Therefore, learning algebra has an important place in the development of society and in increasing academic success. It is necessary to give importance to the teaching of algebra. Algebraic thinking includes not only academic success in mathematics lessons, but also mental processes in overcoming difficulties in problem situations encountered in daily life (Kaya and Keşan, 2017).

In order to teach algebra, first of all, transition from arithmetic to algebra should be healthy, and misconceptions should be determined well and prevented in this process. In an algebra teaching process that is free from misconceptions, algebraic thinking will be built on solid foundations and algebraic thinking will be developed. The literature and practice show that algebraic thinking is of great importance in our country and should be developed and placed on solid foundations (Birgin and Demirören, 2020; Gülpek, 2006; Kaya and Keşan, 2017; Usta and Gökkurt Özdemir, 2018). However, when the literature is examined, it is seen that there are studies examining misconceptions about algebra, but there are not enough studies on algebraic thinking. Examining the levels of algebraic thinking in the 6^{th}, 7^{th} and 8^{th} grades in the second level of primary education reveals the reason and importance of the study from different aspects in terms of revealing and examining the difficulties experienced in the transition from arithmetic to algebra and the precautions to be taken against misconceptions. In this study, it is aimed to determine the algebraic thinking levels and learning status of the students in the 6^{th}, 7^{th} and 8^{th} grades of secondary school regarding the concept of algebra, and to reveal their misconceptions.

2. Method

In this part of the research, explanations are given about the research model, study group, data collection, data collection process and data analysis processes.

2.1 Research Model

This research, which aims to investigate and determine the algebraic thinking levels of secondary school students about algebra concepts according to their grade levels in line with the target outcomes in the mathematics curriculum, is a study with a latitudinal feature from the developmental research method, one of the descriptive research methods.

Descriptive research is the study of an existing situation without deteriorating the natural aspect of the researched environment, and as it is the defining feature of these studies, it seeks answers to the questions of "what was it, what is it now" (Çepni, 2007). Unlike discovering a new feature in a research, developmental research focuses on the monitoring and follow-up processes related to that situation and reveals a defining feature (Munn, Johnstone and Holligan, 1990).

This study, in which students' algebraic thinking levels, misconceptions and learning levels were investigated, was also carried out in order to reveal the changes between their education levels and to determine their learning status. However, in this study, since it was not possible to work on the same grade level in the time period of the research, different grade levels were studied in order to achieve the purpose of the study.

In the stage before the application stage, the opinions and information of the experts in the field of algebra and education regarding the concept of algebra and algebraic thinking were taken, studies were made on the levels of algebraic thinking in the light of this information, and then the knowledge of the experts in the field was used on how to use them in the education dimension. Then, the mathematics curriculum was examined and the outcomes and grade levels related to the algebra subject were determined. In this direction, a test that includes all of the target outcomes was researched and the Algebraic Thinking Level Determination Form (ATLDF) prepared by Usta and Gökkurt Özdemir (2018) was used. In terms of whether this test is suitable for students or not, it was presented to the opinion of a senior teacher in the field of mathematics and algebra and an expert lecturer. The test to be applied has been prepared in line with the feedback and corrections received.

2.2 Study Group

This research was conducted with secondary school students studying in a public school in a province of the Eastern Anatolia Region of Turkey in the academic year 2020-2021. All of the 6^{th}, 7^{th} and 8^{th} grade students in secondary school participated in the study voluntarily. Necessary permissions were obtained from Bayburt University and the relevant institution directorates in order to carry out this research.

Random sampling method was used to determine the sample of this study. The research was conducted with 82 of the 6^{th} (number of students:33), 7^{th} (number of students:21) and 8^{th} (number of students: 28) grade students at the school where the sample was selected. 5^{th} grade students were not included in this study because there is no outcome related to algebra learning field in the 5^{th} grade curriculum. 38 of the students in the study were male and 44 of them were female students. The economic status of the families of the students is moderate. Algebraic Thinking Level Determination Form was examined and the algebraic thinking levels of the students were revealed. According to the answers given by the students, interviews were conducted with some of them.

In the study, the identities of the students and participants were not disclosed, and codes were used for the students while presenting the findings. In order to facilitate the research, abbreviations related to the study group were used. For example, E601 coded student represents the first student studying in 6th grade.

2.3 Data Collection

During the data collection process, two data collection tools were used. The data collection tools used are the Algebraic Thinking Level Determination Form and the form consisting of the questions asked in the interview. The tools used in the data collection process are explained in the following headings.

2.4 Algebraic Thinking Level Determination Form

In this study, it was aimed to determine the learning status of the concept of algebra in secondary school and the levels of algebraic thinking by considering the mathematics curriculum. For this, the grade levels with algebra concepts at secondary school levels were determined and common questions were included for each of these levels, addressing the 6^{th}, 7^{th} and 8^{th} grade levels. Since there is no acquisition related to the concept of algebra in the 5th grade, this grade level is excluded. Care was taken to ensure that the questions to be used in the study were not multiple-choice questions. When multiple choice questions are used alone, they may be incomplete in revealing the real levels and knowledge of the students. For this reason, care was taken to choose questions that require written answers.

In accordance with the desired criteria, it was decided to use the Algebraic Thinking Level Determination Form developed by Usta and Gökkurt Özdemir (2018). This test, abbreviated as ATLDF, is included in this study because it covers all 6^{th}, 7^{th} and 8^{th} grade levels. In ATLDF, which has 8 questions in total, it was first presented to the opinions of 2 teachers working in secondary schools and experts. The feedback was received that the algebraic thinking levels Determination Form (ATLDF) submitted for evaluation would serve the purpose. It was decided that it would be appropriate to apply algebraic thinking in the form of 30+30 minutes. For the ATLDF, it was determined that questions 1 and 2 will meet Level 1, questions 3 and 4 will meet Level 2, questions 5 and 6 will meet Level 3, and finally, questions 7 and 8 will correspond to the last level, Level 4. These test questions were applied to the students by their own mathematics teachers, and it was aimed that the students would not be hesitant, not remain without making a comment, and not be excited. ATLDF was applied to students in 60 minutes, which is 2 lesson hours, and the test was used to determine students' algebraic thinking levels, misconceptions and learning levels. Each lesson hours were applied separately.

2.5 Interviews

Interviews can be classified as structured, unstructured and semi-structured according to the status of the study (Seggie & Bayyurt, 2015). In this study, semi-structured interview was applied because the form applied to determine algebraic thinking levels consisted of open-ended questions and these interviews were conducted individually. The interviews were carried out in accordance with the situations determined by the researcher. In semi-structured interviews, there are questions that the researcher has planned in advance, and these questions can affect the course of the interview with different questions according to the flow of the interview (Ekiz, 2003).

The mistakes made by the students were marked on the Algebraic Thinking Level Determination Form. Algebraic Thinking Level Determination Form was kept with us during the interview so that the students could see their answers and talk about the answers given. Some questions were asked so that the students could reveal their thoughts. In order for students to express their thoughts, questions were asked to them such as “Why did you think that?”, “How did you do this question?”, “How did you decide to do it like that?”, “Do you think your answer is correct?”. %It was tried, through the interview, to reveal how healthy the answers given by the students were, how much of the answers were given consciously and how many of them were randomly answered. The interviews in the research were conducted individually.

2.6 Data Collection Process

During the data collection process, first legal permissions were obtained. The right time was determined by contacting the administration and the teacher at the school where the research would be conducted. Information was given about the Algebraic Thinking Level Determination Form, which will be made after the subjects and concepts related to the concept of algebra are processed. Information about the application and duration of this test was given.

It was confirmed by the practice teacher that the 30+30 minutes given was appropriate. The students were given 2 lesson hours (30+30=60 minutes) and the test was applied. The students were given general information about this process, and they were informed by their teachers that this application is not an exam, that there will be no scoring, that personal information will not be shared with anyone, and that the application should be taken seriously.

In the process after the application, the data were examined and evaluated. Algebraic thinking levels, misconceptions, and algebraic understanding levels of the students were determined by making evaluations, and interviews were conducted with the students determined during the determination of these levels. Before the interviews were held, contact was made with the school administration and the practice teacher, and the Algebraic Thinking Level Determination Form was kept ready with us during this process. Interviews were conducted 1 week after the test date. In the interviews, it was paid attention that the students were not affected by each other.

2.7 Data Analysis

In this research, similar concepts were brought together and arranged in a way that readers could understand. There are 4 levels of algebraic thinking in the research. The sub-steps of these levels are grouped under the same title. Opinions of experts in the field were taken. In addition, algebraic thinking levels were examined in detail according to gender, frequency, grade levels and percentages. In the table below, the categories and contents used in the analysis of the questions in the algebraic thinking levels determination form are given.

Table 1. Categories and Contents Used When Analyzing the Questions in ATLDF

Understanding Levels	Scoring Criteria
Not understanding	- Not answering, and answers with expressions such as "I don't know", "I don't understand",
	- Repeating the question in a same way
	- Irrelevant and unclear answers
Misconception	- Scientifically incorrect answers
Partial Understanding by Special Misconception	- Answers with partial understanding of concepts but also with misconceptions
Partial Understanding	- Answers that contain some, but not all, of a valid answer
Full Understanding	- Answers that include all aspects of a valid answer

2.8 The Role of the Researcher

In descriptive research, the role of the researcher to be involved in the event, that is, the researcher's spending time in the application area, spending one-on-one time with the participants, collecting the experiences and observations in the environment by meeting with the participants directly and using them in the analysis of the data further increases the role of the researcher. The researcher kept the tests, which he had done before, with him in the interviews and had the chance to examine and observe the subject in detail by asking the students one-to-one questions.

3. Findings

At this stage of the research, the findings related to determining the level of understanding of concepts related to algebraic expressions and algebraic thinking levels, identifying misconceptions and revealing algebraic thinking levels were discussed in ATLDF, which was applied to a total of 82 students from the 6th, 7th and 8th grade levels in the secondary school in the study group.

According to the grade levels, the students' level of understanding the questions will be shown in the tables below. In Table 2 below, the understanding level of all of algebraic thinking levels determination form questions by the 6th grade students are given in a single table.

Table 2. Level of Understanding of ATLDF Questions by 6th Grade Students

Question No.	Not understanding	Misconception	Partial Understanding by Special Misconception	Partial Understanding	Full Understanding
1.	21%	27%	6%	12%	34%
2.	18%	21%	12%	18%	31%
3.	27%	15%	15%	9%	34%
4.	31%	18%	21%	18%	12%
5.	64%	33%	0%	0%	3%
6.	61%	12%	27%	0%	0%
7.	43%	24%	15%	3%	15%
8.	64%	36%	0%	0%	0%
Mean	41%	23%	12%	8%	16%

The understanding levels of the questions in the ATLDF applied to 33 students by the 6th grade were tried to be given in a single table. As it can be understood from here, it is seen that the percentages of partial understanding and full understanding, in which 6th grade students have high levels of misunderstanding and misconceptions, are 8% and 16%. While the percentages of the first three questions were high, this rate decreased in the following questions. The rate of 15% in the 7th question draws attention. When the means are examined, we see that the highest mean is at the level of not understanding with 41%, followed by Misconceptions with 23%, and the lowest percentage is partial understanding with 8%. Question 8 consists of only not understanding and misconception for students. One of the reasons for this is that students have difficulty in thinking about sets other than the natural number set.

In Table 3 below, the understanding level of all of algebraic thinking levels determination form questions by the 7th grade students are given in a single table.

Table 3. Level of Understanding of ATLDF Questions by 7th Grade Students

Question No.	Not understanding	Misconception	Partial Understanding by Special Misconception	Partial Understanding	Full Understanding
1.	10%	33%	0%	0%	57%
2.	5%	9%	5%	19%	62%
3.	5%	9%	5%	38%	43%
4.	29%	19%	19%	9%	24%
5.	33%	33%	5%	5%	24%
6.	29%	33%	38%	0%	0%
7.	38%	29%	29%	0%	4%
8.	71%	24%	0%	5%	0%
Mean	28%	24%	13%	9%	26%

With the ATLDF applied to 21 students in the 7th grade, algebraic relations, levels of understanding algebraic thinking and algebraic thinking levels were revealed. As can be seen from Table 3, 7th grade students' understanding levels were high in the first 3 questions, and then the levels declined to 0% in the 8th question. The rate of students' general misconceptions is 24% and their level of not understanding is 28%. It is noteworthy that the full understanding and partial understanding in the 6th question are 0%. When the means are examined, we see that the highest mean is at the level of not understanding with 28%, and the lowest average is partial understanding with 9%. It is quite remarkable that none of the students were able to achieve either full or partial understanding in the 6th question. The decrease in the level of full understanding as the questions progressed shows that the algebraic thinking of the students also decreased.

In Table 4 below, the understanding level of all of algebraic thinking levels determination form questions by the 8th grade students are given in a single table.

Table 4. Level of Understanding of ATLDF Questions by 8th Grade Students

Question No.	Not understanding	Misconception	Partial Understanding by Special Misconception	Partial Understanding	Full Understanding
1.	28%	25%	11%	0%	36%
2.	21%	18%	11%	14%	36%
3.	28%	18%	4%	7%	43%
4.	36%	14%	11%	0%	39%
5.	29%	32%	0%	7%	32%
6.	39%	11%	39%	4%	7%
7.	39%	22%	14%	0%	25%
8.	50%	32%	10%	4%	4%
Mean	34%	21%	12%	5%	28%

We can see that algebraic thinking levels and understanding levels are partially higher in 8th grade, the last year of secondary school. ATLDF applied to 28 students was analyzed in the table. As can be seen, while the percentages of full understanding in the 6th and 8th questions are the lowest, partial understanding is 0% in the 7th question. When we look at the means, the highest mean of 8th graders is not understanding with 34%. Then, it is seen that while the highest percentage is in full understanding, misconceptions are 21%. It is expected that the learning levels of 8th grade students are higher than the others. It can be said that the reason why this class level is partially higher than other classes is that the subject of algebraic expressions is afforded more time than in the other classes. When we look at the mean of 8th grades, it is a thought-provoking result that the sum of not understanding and misconceptions is 55%.

In addition, in Table 5 below, the levels of 82 students, to whom algebraic thinking levels Determination Form was applied, are given as percentages.

Table 5. Distribution of Students' Algebraic Thinking Levels by Grade Levels

ATL	Grade	Level 0		Level 1		Level 2		Level 3		Level 4		Total
		N	%	N	%	N	%	N	%	N	%	N
	6	18	55	8	24	6	18	1	3	0	0	31
Grade	7	7	33	7	33	5	24	2	10	0	0	21
	8	14	50	2	7	4	14	5	18	3	11	28
Total		39	46	17	21	15	19	8	10	3	4	82

When Table 5 is examined, It is seen that almost half of the students on these levels are at Level 0. It is understood that the total number of students who can reach the 1st Level is 17 and the 1st Level students are mostly in the 7th grades as a percentage. It is seen that the number of students who can reach the 2nd level is 15 and the percentage of those who can reach this level is mostly in the 7th grades. The number of students who can reach Level 3 is quite low and the total number is 8. When analyzed as a percentage, those who reach the 3rd level the most out of the three classes are the 8th grades with 18%. When we look at the fourth and last level, we see that only 8th graders can reach this level. The rate of 8th grade students who reached the 4th level as a proportion of all the students who took all the tests is 4%. The number of 8th grade students who can reach Level 4 is 3. When we look at the table in general, it is seen that most of the students are at Level 0 and the rate is 46%, and that Level 1 students are 21%, Level 2 students are 19%, Level 3 students are 10%, and the last level, Level 4, students are 4%. When we examine the algebraic expressions according to the grade levels, we see that the 8th grade students who have studied this subject for the longest time have the highest levels compared to the other grades, followed by the 7th grade students, and the 6th grade students who have the least time-based algebraic expressions, respectively.

In Table 6 below, algebraic thinking levels of 82 students, to whom algebraic thinking levels determination form was applied, are given as a percentage according to gender.

Table 6. Distribution of Students' Algebraic Thinking Levels by Gender

ATL		Level 0		Level 1		Level 2		Level 3		Level 4		Total
		N	%	N	%	N	%	N	%	N	%	N
Gender	F	18	41	11	25	7	16	5	11	3	7	44
	M	21	55	6	16	8	21	3	8	0	0	38
Total		39	46	17	21	15	19	8	10	3	4	82

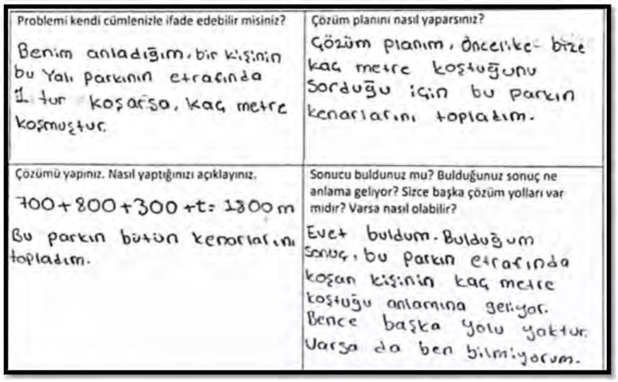
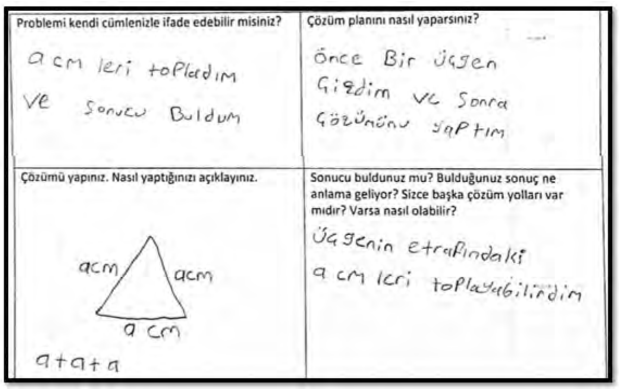
When Table 6 is examined, in the distribution of algebraic thinking levels by gender, it is seen that the rate of male students at Level 0 is 55% and the rate of female students is 41%. While the rate of male students at Level 1 is 16%, the rate of female students is 25%. In Level 2, where there are 8 male students, the percentage of males is higher than the percentage of female students. At the third level, it is seen that female students outnumber male students, and while the percentage of females is 11%, it is observed that male students remain at 8%. When we reached the final level, it was observed that no male student could reach the 4^{th} Algebraic Thinking Level and that all students who reached this level were female students.

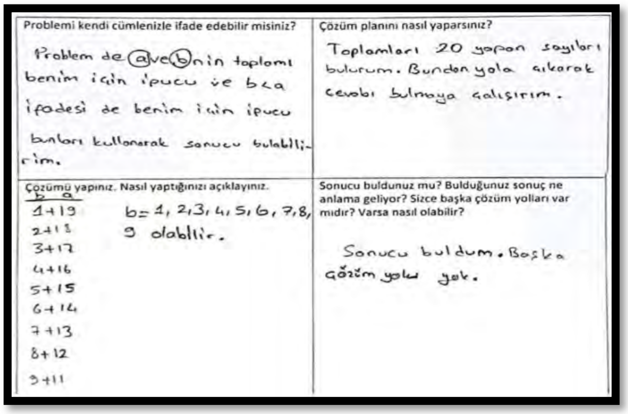
Finally, in Table 7, all the misconceptions of the students in this study are given in a single table.

Table 7. Misconceptions of Students Determined by ATLDF

	Misconceptions
•	Thinks that the variable is the sum of all the numbers given in the question,
•	Variables don't matter in addition,
•	The variable is not taken into account when performing operations in algebraic expressions,
•	Students believe that letters have no place in mathematics,
•	Every operation given in an algebraic expression must produce a result,
•	While writing an algebraic expression, the student tries to get the result by multiplying the known value with the variable,
•	Thinks that the result of the algebraic expression is the sum of the numbers given in the question,
•	The inability to collect like terms when converting a verbal expression to an algebraic expression,
•	Letters have no meaning in math,
•	Addition always produces results,
•	When expressing an algebraic expression in its simplest form, like terms are multiplied,
•	The result is the same in addition if like terms are exactly the same,
•	Students do not pay attention to order of operations,
•	Thinks that the value of the algebraic expression is the sum of all the coefficients and values given in the question,
•	Every operation given in an algebraic expression must produce a result,
•	Inability to add like terms while expressing an algebraic expression in its simplest form,
•	It is believed that letters have no place in mathematics,
•	Inability to combine like terms,
•	Variables do not matter in addition and subtraction,
•	Inability to combine two different variables,
•	Not accepting 1 and -1 as coefficients,
•	Students think that each letter has only one value,
•	Students believe that letters can only represent natural numbers,
•	Thinks that the algebraic expression is the sum of all the numbers given in the question,
•	In algebraic expressions, variables do not matter in addition,
•	When expressing an algebraic expression in its simplest form, the terms are multiplied and the coefficients are added,
•	Inability to compare the result according to different values of the unknown,
•	Believes that in algebraic expressions, multiplication will always magnify the result,
•	Thinks that the values taken by the variables are limited,
•	Generalizing a value given to a variable to the result of an algebraic expression.

Table 8. Some Examples of Students' Answers to Questions

 Problemi kendi cümlenizle ifade edebilir misiniz? Benim anladığım, bir kişinin bu Yalı parkının etrafında 1 tur koşarsa, kaç metre koşmuştur. Çözüm planını nasıl yaparsınız? Çözüm planım, öncelikle bize kaç metre koştuğunu sorduğu için bu parkın kenarlarını topladım. Çözümü yapınız. Nasıl yaptığınızı açıklayınız. 700+800+300+t: 1800 m Bu parkın bütün kenarlarını topladım. Sonucu buldunuz mu? Bulduğunuz sonuç ne anlama geliyor? Sizce başka çözüm yolları var mıdır? Varsa nasıl olabilir? Evet buldum. Bulduğum sonuç, bu parkın etrafında koşan kişinin kaç metre koştuğu anlamına geliyor. Bence başka yolu yoktur. Varsa da ben bilmiyorum.	The perimeter of the rectangle given in the question and the side whose length is given as a variable did not give any numerical value, but the variable was not taken into account when processing with algebraic expressions.
 Problemi kendi cümlenizle ifade edebilir misiniz? a cm leri topladım ve sonucu buldum Çözüm planını nasıl yaparsınız? Önce bir üçgen çizdim ve sonra çözümünü yaptım Çözümü yapınız. Nasıl yaptığınızı açıklayınız. acm acm a cm a+a+a Sonucu buldunuz mu? Bulduğunuz sonuç ne anlama geliyor? Sizce başka çözüm yolları var mıdır? Varsa nasıl olabilir? Üçgenin etrafındaki a cm leri toplayabilirdim	It has been observed that they have problems with writing algebraic expressions. It is seen that while writing an algebraic expression suitable for a verbally given situation, they cannot collect its variables properly.
 Problemi kendi cümlenizle ifade edebilir misiniz? sekiz iks eksi altı y eksi iks eşittir soru işareti. Çözüm planını nasıl yaparsınız? Önce soruyu iyice anlarım sonra soruya odaklanırım. Yani bilinenleri bir araya bilinmeyenleri bir araya yazarım ve işlem yapıp cevabı bulurum. Çözümü yapınız. Nasıl yaptığınızı açıklayınız. Yani 8x-6 -x diyor ya işte sekiz iksile altının arasında eksi vardır ve sekizden altıyı çıkartıp bilinmeyenleri yazarım. 8x-6y-x=2xy-x 2x²y Sonucu buldunuz mu? Bulduğunuz sonuç ne anlama geliyor? Sizce başka çözüm yolları var mıdır? Varsa nasıl olabilir? iki iks y eksi iks eşittir iki ikskare eksi y anlamına gelir. evet vardır. (2xy)·(2x-y)=4x²y	It is seen that he could not understand the algebraic expression given in the question and could not develop the right strategy for the solution. It is seen that he thinks that different features related to algebraic expressions will be used in the question since it is 8th grade and similar terms are mixed with dissimilar terms.#

 Problemi kendi cümlenizle ifade edebilir misiniz? Problem de a ve b nin toplamı benim için ipucu ve b<a ifadesi de benim için ipucu bunları kullanarak sonucu bulabili- rim. Çözüm planını nasıl yaparsınız? Toplamları 20 yapan sayıları bulurum. Bundan yola çıkarak cevabı bulmaya çalışırım. Çözümü yapınız. Nasıl yaptığınızı açıklayınız. b a 1+19 2+18 3+17 4+16 5+15 6+14 7+13 8+12 9+11 b= 1, 2, 3, 4, 5, 6, 7, 8, 9 olabilir. Sonucu buldunuz mu? Bulduğunuz sonuç ne anlama geliyor? Sizce başka çözüm yolları var mıdır? Varsa nasıl olabilir? Sonucu buldum. Başka çözüm yolu yok.	The student explained the expression given in the question, which includes the unknown, in a logical way and tried to find the other unknowns. However, although the algebraic expression given in the question is not from any set, it is seen that the student only selects numbers from the set of natural numbers and makes the result.

4. Discussion

In the first two questions belonging to Level-1 of the algebraic thinking levels Determination Form, students are expected to write an algebraic expression suitable for a verbally given situation. In the first question, it was observed that the most misconceptions were seen in the 7th grade, while in the second question, it was observed that the most misconceptions were seen in the 6th graders. It is seen that the level of full understanding at the first level is higher for the 7th graders compared to the other grades. The fact that both misconceptions and full understanding are seen mostly in the 7th grades at the first level is due to the fact that the students were not selected according to a certain order and the test was administered to all students. It is understood that half of the 8th graders and 55% of the 6th graders could not pass to Level 1 and remained at Level 0. Kaya (2017) examined the success levels of 143 6th grade students in the field of learning algebra and emphasized that the mean score of the students was even behind the medium level. Therefore, the result is similar to the result of this study.

In the findings obtained for Level-2, it is observed that the students decrease except for the 8th grades and their misconception and not understanding situations increase. In the first of the questions at this level, the students were asked to write the numerical value given in the question instead of a letter and to find the value of the other unknown, and in the second, to find the perimeter of the shape whose side lengths are unknown but equal. It was observed that approximately half of the students at each grade level could not understand these questions or had misconceptions. At this level, it is desired to use equality in algebraic expressions, but half of the students could not do this correctly. Yaman, Toluk, and Olkun (2003) state that students use the concept of equality as a result of an operation rather than a relation. In this study, it was seen that the students could not make sense of equality properly.

In Level-3, which includes the 5th and 6th questions, the students were asked to find the other unknown according to the value given in cases where there are unknowns, and to combine like and unlike terms correctly in algebraic expressions. When the questions at this level were examined, it was determined that only 3% of the 6th-grade students fully understood the 5th question, while 24% of the 7th grade students understood it. On the other hand, 32% of the 8th grade students were able to fully understand this question and give the desired answer. Considering the outcomes in these questions, it can be considered that one of the reasons why 6th graders have difficulty in dealing with like and unlike terms is that students see algebraic expressions for the first time at this grade level and have difficulty remembering them. When we came to the 6th question, these rates decreased considerably and it was revealed that the only class that could show full understanding in the 6th question was 8th. Very few of the students knew that natural algebraic expressions do not only consist of natural numbers.

Level 4 includes Questions 7 and 8. Questions at this level include bringing together like terms in cases involving unknowns, multiplying numbers with unknowns, and finding other unknowns according to the given values. It was determined that the students are quite unsuccessful in the final level questions and the number of students who can pass to the final level is quite low. When the understanding levels of the 7th question are examined, it is seen that the 6th graders surpass the 7th grades at the level of full understanding. The class with the highest success in the 7th question is the 8th grade. It was also stated in the study of Çelik and Güneş (2013) that the level of algebraic thinking of the students increased as the grade level increased as they progressed towards the final level, and it shows parallelism with this study. In this study, it is observed that the upper classes are more successful as the levels increase.

One of the other findings in this study is that students cannot understand the different uses and meanings of letters and symbols correctly as they pass from arithmetic to algebra. Dede (2004) stated that students had difficulties in understanding the concept of variable and the transition process to algebra. In this context, it is similar to this study.

It was revealed that 8^{th} grade students were more successful in algebraic thinking and full understanding compared to other grades. Gülpek (2006) emphasized that as the years passed and the grade levels increased, the algebraic thinking of the students increased and the 8^{th} graders could give more accurate answers to the fourth level questions. This shows parallelism with this study. In another result, it was emphasized that most of the 7^{th} grade students were at the zero and first level, and the 8^{th} grade students were equally distributed across the levels. In this study, it was determined that more than half of the 7^{th} grade students were at level zero and at level one, while the 8^{th} grade students were mostly at level zero and were unevenly distributed across the levels. When the results are evaluated, it can be said that they differ with this study. In addition, the fact that the algebraic thinking levels of the eighth students in the study of Oral et al (2013) concentrated at level 0 is similar to this study.

In the algebraic expression study of Öner Sünkür et al. (2012), it was determined that the majority of 7^{th} grade students could find the value of a letter as a result of arithmetic, but they had difficulty in performing operations on unknowns. In this study, it was revealed that students had misconceptions while finding the values of algebraic expressions and had difficulties in operations with unknowns.

In Birgin and Demirören's (2020) study, an algebra test was applied to 120 8^{th} grade students and the students' mistakes in simple visual and algebraic expressions were mentioned, and it was seen that it was caused by incorrect algebraic operation selection, misinterpretation by ignoring shape patterns, ignoring parenthesis in algebraic expressions, and arithmetic operation error. Similarly, in this study, it was determined that the students chose algebraic expressions incorrectly, ignored the order of the operation, and had problems in the transition from arithmetic to algebra.

When the answers given by the students to the algebraic thinking levels Determination Form were examined, it was revealed that almost half of the students could not pass from arithmetic to algebra and could not understand the questions. It is understood that the reason for this is arithmetic problems from the past and that the transition to algebra cannot be built on solid foundations with these problems. Akkan, Baki and Çakıroğlu (2011) emphasized the learning of algebra concepts in the pre-algebra period and stated that arithmetic knowledge directly affects algebra learning.

Yenilmez and Avcu (2009), in their study on 6^{th} grade students' successful levels in algebra learning areas, revealed that the students did not have any problems with the representation and conservation of equality, but they had problems in establishing and solving the equation. The fact that that study was conducted with 6 students pointed out that the results may vary, but in this study, students thought that algebraic expressions about equality had to produce results. In addition, although there is no achievement of establishing equations in the 6^{th} grade, it is the class in which the students introduce algebra.

Usta and Gökkurt Özdemir (2018) conducted a study with 12 6^{th}, 7^{th} and 8^{th} grade students, and it was observed that all of them answered the first level questions correctly, that in the second level questions, they gave numbers to the unknown sides and could not solve the question correctly while finding the perimeter of the figure whose side lengths are not given, that the fifth and 6^{th} graders had difficulties in the questions about the third level, and that 1 7^{th} grade student and 4 8^{th} grade students were able to pass the fourth level. The algebraic thinking levels Determination form applied to 12 students is the same as the test applied to 82 students in this study. The reason for the difference in results is thought to be due to the high number of students. In this study, it is seen that almost half of the students are at level-0 and 21% of them are at level-1.

In this study, when algebraic thinking levels and understanding levels were compared according to gender, it was determined that female students were more successful at all levels except level-2. It is seen that all three students who can reach the fourth level are female students. Similarly, Kaya (2017) applied the "Algebraic Thinking level Determination Tool" test consisting of 10 questions to 7^{th} grade students and observed that female students were more successful than male students in each question. Likewise, in the study of Acar (2019), there is a significant relationship between the sense of number and algebraic thinking levels, depending on gender, in favor of girls, which supports this study.

5. Conclusion

This study reveals the algebraic thinking levels, learning situations and misconceptions of secondary school 6th, 7th and 8th grade students. In this study, algebraic thinking levels Determination Form was applied to 82 students and then interviews were made.

In general, it has been observed that misconceptions are seen at almost all grade levels. Some of the reasons for the high number of misconceptions are that students try to answer the questions even though they do not understand them, add numbers to situations where numbers are not available, algebraic expressions are not understood enough, the transition from arithmetic to algebra is not healthy, the number of materials and activities used during teaching is not sufficient.

When the learning situations are examined, it is seen that more than half of the students cannot understand the questions or they have misconceptions and they are deficient in algebraic expressions as a result of the findings. It can be said that 6th grades' lagging behind in algebraic expressions compared to other grades is related to their less receiving education of the subject compared to other grades. The fact that the 8th grades' level of understanding is higher than the other classes can be related to the fact that they have more algebraic expressions, subjects and outcomes than other classes. It is because grades 6 have algebraic expressions for one year, grade 7 for two years, and grade 8 for three years.

When the algebraic thinking levels of the students were examined, it was concluded that almost half of them remained at level-0 for all grade levels, that is, they did not have a level. As the levels increase, the number of students decreases. Level-4 has only 3 8th grade students. No student from the other grade level could reach the last level. In addition, it is understood that only female students can reach the 4th level and this is similar as a result of other studies. In general, female students' understanding levels are higher than males.

6. Recommendations

Based on the results of this study, in which 6th, 7th and 8th grade students' algebraic thinking levels were determined, their misconceptions were investigated, and the status of Learning Algebraic Expressions were examined, some recommendations were made below.

1) In this study, which is observed to have a lot of misconceptions at all grade levels, predetermining the transition processes of students from arithmetic to algebra and accordingly explaining the subjects and outcomes of algebraic expressions in 6th grade with more care and visualization can reduce misconceptions.

2) Subjects and outcomes related to algebraic expressions can be taught to students by using active learning methods (Töman and Yarimkaya, 2018; Yarimkaya, Ünsal and Töman, 2018) that contribute to students' learning in a more meaningful way by embodying the subjects and outcomes.

3) It is seen that learning algebra both provides convenience in solving problems in daily life and makes great contributions to the development of mathematics and other disciplines for further education and training activities. Therefore, identifying misconceptions and planning solutions to eliminate them both in the pre-algebraic period and during the transition to algebra will contribute to the development of algebraic thinking.

4) Similarly, by determining the arithmetic understanding levels of primary school students, development studies can be done for them; and also, algebraic thinking levels can be determined for high school students and compared with the results of this study.

5) Misconceptions and learning difficulties experienced during the transition from arithmetic to algebra can be identified and activities and studies can be carried out for them.

6) Starting from the 5th grade in secondary school, it can be ensured that the students have a sufficient level in arithmetic and are ready for algebra when they graduate from the 5th grade, by taking the priority concepts into consideration and ensuring continuity during the transition from arithmetic to algebra, establishing a relationship with planned and daily life information.

Note: This study was obtained from the master's thesis of Ömer Gökburun (Gökburun, 2021)

References

Acar, S. (2019). *Sayı hissi ile cebirsel düşünme becerisi arasındaki ilişkinin farklı değişkenler açısından incelenmesi* (Yüksek lisans tezi). Yükseköğretim Kurulu Ulusal Tez Merkezi'nden edinilmiştir. (Tez No. 562723).

Akkan, Y., Baki, A., & Çakıroğlu, Ü. (2011). Aritmetik ile cebir arasındaki farklılıklar: cebir öncesinin önemi. *Elementary Education Online*, *10*(3), 812-823.

Birgin, O., & Demirören, K. (2020). Sekizinci sınıf öğrencilerinin basit görsel ve cebirsel ifadeler konusundaki hata ve kavram yanılgılarının incelenmesi. *Uluslararası Sosyal ve Eğitim Bilimleri Dergisi*, *7*(14), 233-247. https://doi.org/10.20860/ijoses.797472

Çelik, D., & Güneş, G. (2013). Farklı sınıf düzeyindeki öğrencilerin harfli sembolleri kullanma ve yorumlama seviyeleri. *Kuram ve Uygulamada Eğitim Bilimleri*, *13*(2), 1157-1175.

Çepni, S. (2007). *Araştırma ve proje çalışmalarına giriş*. Trabzon: Celepler Matbaa.

Dede, Y. (2004). Değişken kavramı ve öğrenimindeki zorlukların belirlenmesi. *Kuram ve Uygulamada Eğitim Bilimleri Dergisi*, *4*(1), 24-56.

Ekiz, D. (2003). *Eğitimde Araştırma Yöntem ve Metotlarına Giriş*. Ankara: Anı Yayıncılık.

Gökburun, Ö. (2021). *Cebirsel düşünme kavramının ortaokul seviyesinde öğrenilme durumunun araştırılması (Yüksek lisans tezi).* Yükseköğretim Kurulu Ulusal Tez Merkezi'nden edinilmiştir. (Tez No. 690821).

Gülpek, P. (2006). *İlköğretim 7. ve 8. sınıf öğrencilerinin cebirsel düşünme düzeylerinin gelişimi* (Yüksek lisans tezi). Yükseköğretim Kurulu Ulusal Tez Merkezi'nden edinilmiştir. (Tez No. 206195).

Kaya, D., & Keşan, C. (2017). İlköğretim seviyesindeki öğrenciler için cebirsel düşünme ve cebirsel muhakeme becerisinin önemi. *International Journal of New Trends in Arts, Sports & Science Education*, *6*(1), 29-38.

Kieran, C. (1992a). The Learning and Teaching of School Algebra. In: Grouws DA (Ed.), *Handbook of Research on Mathematics Teaching and Learning*. New York: Macmillan Publishing Company.

Kieran, C. (2004b). Algebraic thinking in the early grades: what is it. *The Mathematics Educator*, *8*(1), 139-151.

Munn, P., Johnstone, M., & Holligan, C. (1990). Pupils' perceptions of "effective disciplinarians". *British Educational Research Journal*, *16*(2), 191-198. https://doi.org/10.1080/0141192900160207

Oral, B., İlhan, M., & Kınay, İ. (2013). İlköğretim 8. sınıf öğrencilerinin geometrik ve cebirsel düşünme düzeyleri arasındaki ilişkinin incelenmesi. *Pamukkale Üniversitesi Eğitim Fakültesi Dergisi*, *34*(34), 33-46. https://doi.org/10.9779/PUJE469

Öner Sünkür, M., İlhan, M., & Kılıç, M. A. (2012). Yedinci sınıf öğrencilerinin cebirsel düşünme düzeyleri ile zekâ alanları arasındaki ilişkinin incelenmesi. *Erzincan Üniversitesi Eğitim Fakültesi Dergisi*, *14*(2), 183-200.

Seggie, F. N., & Bayyurt, N. (2015). *Nitel Araştırma Yöntem Teknik, Analiz ve Yaklaşımları*. Ankara: Anı Yayıncılık.

Töman, U., & Çimer, O. S. (2014). Investıgatıon by level of pre-servıce scıence teachers' reflectıve thınkıng. *Route Educational and Social Science Journal, 1*(2), 116-125. https://doi.org/10.17121/ressjournal.55

Töman, U., & Yarımkaya, D. (2018). 7. sınıf ışık konusunun öğretiminde akran öğretimi tekniği kullanımının öğrencilerin başarı düzeyleri üzerindeki etkisi. *Hitit Üniversitesi Sosyal Bilimler Enstitüsü Dergisi*, *11*(1), 499-514. https://doi.org/10.17218/hititsosbil.418586

Türkoğlu, D. (2017). *Cebirsel düşünme becerisi üzerine bir meta - sentez çalışması* (Yüksek lisans tezi). Yükseköğretim Kurulu Ulusal Tez Merkezi'nden edinilmiştir. (Tez No. 481289).

Usiskin, Z. (1999). *Conception of School Algebra and Uses of Variables,* 7-13, Ed: B. Moses, Algebraic Thinking, Grades 9-12: Readings from NCTM's School Based Journals and Other Publications, Reston, Va: NCTM.

Usta, N., & Gökkurt Özdemir, B. (2018). Ortaokul öğrencilerinin cebirsel düşünme düzeylerinin incelenmesi. *Eğitimde Nitel Araştırmalar Dergisi - Journal of Qualitative Research in Education*, *6*(3), 427-453. https://doi.org/10.14689/issn.2148-2624.1.6c3s20m

Ünsal, Y., Kızılcık, H. Ş., & Yarımkaya, D. (2018). Fizik eğitimi kongrelerinde sunulan bildirilerin analizi: Türkiye örneği. *Fen Bilimleri Öğretimi Dergisi, 6*(2), 173-196.

Yaman, H., Toluk, Z., & Olkun, S. (2003). İlköğretim öğrencileri eşit işaretini nasıl algılamaktadırlar? *Hacettepe*

Üniversitesi Egitim Fakültesi Dergisi, *24*(24), 142-151.

Yarımkaya, D., & Ünsal, Y. (2019*). Fizik eğitiminde akran öğretimi*. Ankara: İksad Yayınevi.

Yarımkaya, D., Ünsal, Y., & Töman, U. (2018). Kuvvet ve hareket konusunun öğretiminde akran öğretimi tekniği kullanımının 9. sınıf öğrencilerinin fizik dersine yönelik tutumları üzerindeki etkisi. *Eğitim ve Toplum Araştırmaları Dergisi*, *5*(1), 16-32.

Yenilmez, K., & Avcu, T. (2009). Altıncı sınıf öğrencilerinin cebir öğrenme alanındaki başarı düzeyleri. *Ahi Evran Üniversitesi Eğitim Fakültesi Dergisi*, *10*(2), 37-45.