ACTA DIDACTICA NAPOCENSIA
Volume 15, Number 1, 2022 - DOI: 10.24193/adn.15.1.10

THREE-DIMENSIONAL GEOMETRIC PERCEPTIONS IN ENEM: A CONTRIBUTION FROM GEOGEBRA FOR MATHEMATICS TEACHERS IN BRAZIL

**Francisco Régis Vieira ALVES, Renata Teófilo DE SOUSA,
Francisca Cláudia Fernandes FONTENELE**

Abstract: This article presents a didactic situation focused on geometric reasoning using 3D GeoGebra, using the concepts Theory of Didactical Situations (TDS). The objective of this work is to bring a didactic proposal for the teaching of Spatial Geometry, working with the students' geometric perception, oriented and elaborated based on the dialectic of TDS - action, formulation, validation, and institutionalization. For its construction, a question from the National High School Examination (ENEM) was used about polyhedra, worked in an intuitive way. Finally, it is pointed out that TDS associated with technology has great potential to leverage the student's geometric reasoning, satisfying what is sought to develop in school curricula.

Key words: GeoGebra 3D. Geometric perception. Theory of Didactical Situations.

1. Introduction

The Teaching of Geometry is a recurring theme in several studies in the scope of the Teaching of Mathematics, thanks to its importance to humanity since ancient times. According to Lorenzato (1995, p. 5) "without knowing Geometry, the interpretive reading of the world becomes incomplete, the communication of ideas is reduced, and the view of mathematics becomes distorted".

Based on the relevance of this subject to the school curriculum of basic education, the Common National Curricular Base (BNCC, acronym in Portuguese) points out the primordiality of an integrated view of Geometry with real applications, considering the students' experiences and encouraging them to recognize and grasp concepts and create problem solving strategies, through not only what is offered in textbooks, but above all using digital media and technological resources.

The BNCC mentions that "the use of technologies allows students to deepen their active participation in this problem-solving process" (Brazil, 2018, p. 528). This is because the technology incorporated into the educational context can help the student, optimizing processes and offering tools for him to explore different learning methods, in addition to the visual stimuli that can be discovered with resources such as educational software, mobile applications, websites, social networks, among others.

GeoGebra software has the potential to develop students' visual perception, with respect to Spatial Geometry. Because it is dynamic geometry software with a 3D window available on its interface, it enables the viewing figures on a xyz plane. As a way to develop students' geometric skills, we bring in this work a methodological proposal that involves the use of a didactic situation for the teaching of Spatial Geometry aided by this software.

The choice of 3D GeoGebra is verified by the fact that this software enables the creation of dynamic objects, serving as a pedagogical tool for building and exploring mathematical concepts. In addition to its easy handling, it serves as a differential in addition to the two-dimensional drawings present in textbooks. According to Breda, Trocado e Santos (2013) the alternative of a three-dimensional visualization is presented as a way to facilitate the student's understanding of concepts, which favors his learning.

Received November 2021.

Cite as: Alves, F. R. V. de Sousa, R. T., Fontenele, F. C. F. (2022). Three-dimensional geometric perceptions in ENEM: a contribution from Geogebra for mathematics teachers in Brazil. *Acta Didactica Napocensia,* 15(1), 114-123, https://doi.org/10.24193/adn.15.1.10

Given this, this work aims to build a didactic situation using the GeoGebra software for the teaching of Spatial Geometry, guided by the need to visualize geometric elements such as vertices, faces and edges and the relationship between them. Thus, the process of assimilation of this theme by the student becomes viable, as well as the development of his geometric thinking. To resolve this didactic situation, the assumptions of the Theory of Didactic Situations are used as support for this work, about the conception of a planned application.

In the following sections we will discuss the Theory of Didactic Situations as a teaching theory, as well as its contribution to the structuring of this work. We will also show the GeoGebra software in its 3D version as a technological resource and didactic situation resolved with its support and we close with the final considerations about this proposal.

2. Geometric perceptions in ENEM questions

Euclidean Geometry is part of the school curricula of Brazilian basic education and its postulates serve as the basis for Spatial Geometry, core of this work. Thus, with respect to geometric perception, to establish connections between vertices, faces and edges and the relationships between these entities autonomously, the student has the possibility to formally understand what a polyhedron is and even other spatial figures, as well as its applicability in different situations.

The inevitability of perceiving oneself within space points to the importance of geometric knowledge for man. However, even with all this proven relevance, Geometry as a curricular component of the mathematics discipline is still an obstacle for students. In particular Spatial Geometry, because there is a greater difficulty in perception and association of fundamental geometric entities and their respective association with the composition of spatial figures. For Lorenzato (1995, p. 3) “many teachers do not have the geometric knowledge necessary to carry out their pedagogical practices”. The author also states that this gap is due to teacher training, consequence of the Modern Mathematics Movement, which occurred in the 60s and 70s, which minimized the importance of teaching geometry in schools.

According to Camilo, Alves e Fontenele (2020) there is a certain rejection on the part of the students to this curricular component, possibly due to the difficulty that mathematics teachers face in comprehensively presenting visual exposure, because in many cases, they only have pedagogical resources limited to traditional means. However, this curricular component is of great relevance for assessments and entrance exams, such as ENEM, for example.

Regarding ENEM, it is a large-scale evaluation, which verifies the level of proficiency of students in the final stage of Basic Education. This exam allows students access to higher education courses, both in Brazil and in universities abroad, such as Portugal, France, England, and Canada. The ENEM reference matrix brings a set of competencies and skills expected from students at the end of this teaching stage, being measured via a standardized test.

The National High School Exam (ENEM) commonly uses questions of Spatial Geometry in its tests to explore potentialities about students' geometric reasoning. Due to its practical applicability, it is a very recurrent subject in its tests, being necessary its development so that there is a good performance in this area of knowledge. Its Reference Matrix, with regard to the subject in question, presents as competences and skills:

> Area Competence 2 - Use geometric knowledge to read and represent reality and act on it.
> H6 - Interpret the location and movement of people / objects in three-dimensional space and their representation in two-dimensional space.
> H7 - Identify characteristics of flat or spatial figures.
> H8 - Solve the problem situation that involves geometric knowledge of space and shape.
> H9 - Use geometric knowledge of space and form in the selection of arguments proposed as a solution to everyday problems. (BRAZIL, 2009, p. 5, authors' translation).

Analyzing the ENEM Reference Matrix, this brings the importance of the association of geometric thought to reality and reinforces the issue of building models for solving problems, which often does not need formulas or ready mathematical expressions. It is worth mentioning that due to the subjective and interdisciplinary character of ENEM, its statements usually relate the contents and disciplines within their skills and abilities.

In this way, we try to use a didactic situation based on this matrix and organized based on the Theory of Didactical Situations, with the contribution of GeoGebra as a resource for didactic transposition, aiming to collaborate with the mathematics teacher, enabling visual perception and understanding of spatial figures by high school students.

3. Theory of Didactical Situations

The Theory of Didactical Situations (TDS) offers a way to understand the relationship between the triad student, teacher, and learning, as well as the *milieu* in which the didactic situation occurs, this being called by Brousseau the Didactic Triangle, which has its vertices as a representation of the relationships between these three pillars.

For student learning to occur, Brousseau (2008) brings that there must be a link between knowledge and the interaction between two or more people, where such interactions may occur for what he calls the didactic situation, that causes the apprehension of knowledge. It can be a game, a challenge or any type of device designed for learning to take place effectively. Thus, the didactic situation is characterized by “a model of interaction of a subject with a determined environment” (Brousseau, 2008, p. 20).

The didactic situation, to function effectively, it must be preserved in what Brousseau calls as a didactic contract. Brousseau (2008) defines a didactic contract as a verbal contract that determines the role of the subjects – teacher and student –, places and functions of everyone involved in the didactic situation, in a system of obligations whose reciprocity is necessary, and this relationship being mediated by knowledge.

Starting from that point, TDS' scope is to bring the student's work closer to that of a researcher, through the formulation of hypotheses, construction of concepts, validation of strategies and theories, having the teacher as mediator of the process, by promoting didactic situations or sequences previously prepared for the student to act on knowledge and turn it into knowledge for yourself. For Brousseau (2008), teaching situations must be developed by the teacher so that the student builds and appropriates knowledge.

TDS is divided into phases or dialectics that can be modeled according to the situations of action, formulation, validation, and institutionalization, enabling student learning. In this way, we can summarize such situations as:

Action situation: initially, it is taking a position on the proposed problem.

Formulation situation: in this phase, there is an exchange of information between the student and the environment, along with the verbalization of ideas and conjectures.

Validation situation: the student presents his solution strategy to the class and tries to argue based on his reasoning, checking if what he conjectured is, in fact, valid.

Institutionalization situation: phase in which the figure of the teacher enters the scene to synthesize everything that was exposed in the previous stages, in a formal manner and with appropriate mathematical language.

Alves (2019) shows that the dialectics mentioned above demand a detailed and careful file of attention and analysis, as we are interested in the perspective and understanding of the teacher's action, mediated, and supported by fundamentals that support and indicate the adoption of a teaching methodology for Mathematics.

The moment when the student builds knowledge occurs in the adidactic situation, being built for the student to interact with an environment without the teacher’s intervention in that interaction, and this includes the first three stages of the TDS. Institutionalization, on the other hand, is an integral part of the transformation of knowledge – simple familiarity, but not intimacy with the object of study – in knowing – intellectual, which admits concepts and judgments –, through the return process, which occurs throughout the entire didactic situation. Brousseau (2008, p. 89) provides that "devolution is the act by which the teacher makes the student accept responsibility for a learning situation (adidactic) or a problem and assumes the consequences of that transfer himself". According to Margolinas (2015), wisdom, when it does not correspond to knowledge, when not institutionalized, is fragile.

The intention to teach represents the process of seeking to transmit knowledge, that is, generating knowledge of a situation (Margolinas, 2015). Thus, the return process in the case of adidactic situations is considered a more general process, not linked in a particular way to the teaching organization.

Due to these assumptions, TDS becomes a teaching theory that meets the needs of this work.

4. GeoGebra as a tool for Didactic Transposition

Didactic transposition allows the transformation of scientific knowledge into school knowledge (Polidoro & Stigar, 2010), enabling mediation between this knowledge to facilitate the student's understanding. The different ways in which such transformations can occur allow a range of methodological possibilities for the teacher. Chevallard (1991), presents the following definition for didactic transposition:

> A content of knowledge that has been defined as knowing how to teach, undergoes, from then on, a set of adaptive transformations that will make you able to occupy a place among teaching objects. The 'work' that makes an object of knowledge to be taught, an object of teaching, is called didactic transposition. (Chevallard, 1991, p. 39, authors' translation).

The search for methods that facilitate the student's understanding and learning has been the focus of many academic works. Didactic transposition through technologies has been studied by several authors, some of them: Lemos and Carvalho (2010), Díaz-Urdanetta, Kalinke and Motta (2019), Abar (2020a; 2020b).

Lemos and Carvalho (2010) emphasize the importance of using software that provides interaction with the real world, stimulating the student's perception and making the teaching and learning process more reflective, arousing interest.

Díaz-Urdanetta, Kalinke and Motta (2019) reinforce that GeoGebra is a dynamic mathematics tool and due to its accessibility and low complexity in the use of its tools, it becomes a resource that allows a differentiated approach, enabling the presentation of several topics in Mathematics in a single interface. This allows experimentation and visualization of Mathematics with great potential to develop the student's knowledge.

There is a big challenge for the school when it comes to achieving the goals of learning in Mathematics. Despite the evolution over the years, there is still a long way to go, mainly about the use of technology in the classroom. GeoGebra is a resource that comes to add to the teacher and facilitate his practice, mainly in the presentation of content of complex assimilation, however, many teachers still have difficulties in handling this tool. A justification for this, according to Abar (2020a), is that to develop innovative strategies on the part of the teacher requires greater dedication to his professional development, demanding more time so that he can absorb all the information, study, analyze and transpose all these ideas into his practice.

It is important to note that computational resources should be explored in the school environment more actively and more frequently to follow the technological evolution of society in general. From this perspective, Abar (2020a, p 33) emphasizes that "the development of information and communication technologies, as well as their introduction in schools and training environments, is accompanied by phenomena of the same order as those of didactic transposition". That is, the student's understanding is facilitated since the technological support has great dynamism, providing subsidies for the didactic transposition to occur in a meaningful way.

For the GeoGebra resource to provide effective learning, it is suggested that the teacher is aware of the best way to carry out the didactic transposition of the content associated with this resource. Thus, "it is important to understand the complex transformation process that mathematics goes through until it becomes an element to be taught" (Abar, 2020b, p. 2).

To try to explore these theories (TDS and Didactic Transposition) within the didactic proposal and elaboration of the didactic situation, the following section presents a proposal for the application of this theory.

5. An application proposal

For the organization of the teaching situation through the proposed question, we will use the GeoGebra software to assist the visualization and structuring of geometric perceptions. According to Alves and Borges Neto (2012), the exploration of GeoGebra as a technological tool allows the visualization of unimaginable situations, when restricted to pencil and paper.

For this analysis we present a question extracted from ENEM (Figure 1) following the learning process from the TDS, being modeled by action, formulation, validation, and institutionalization situations. From the didactic contract signed between the teacher and the class, it will be agreed that students will be able to use the geometric construction, produced in GeoGebra, as a way of supporting the understanding of the didactic situation presented. It is necessary that these students have gone through an initial immersion and have knowledge about the tools of GeoGebra. This construction is available in: http://www.geogebra.org/m/usy6w4wh.

For a trophy model, a polyhedron P was chosen, obtained from cuts at the vertices of a cube. With a flat cut at each corner of the cube, the corner is removed, which is a tetrahedron with edges smaller than half the cube's edge. Each face of the polyhedron P is painted using a different color from the other faces.

Based on the information, what is the amount of colors that will be used in painting the faces of the trophy?

A) 6

B) 8

C) 14

D) 24

E) 30

Figure 1. *ENEM question about polyhedra.*

Objective of the problem: Identify that each vertex of the cube turns into a triangular face, changing the total number of faces in the figure and, therefore, the student must indicate the new total quantity.

Below, we present a mathematical and cognitive analysis of this issue from the dialectic of TDS.

Action situation: It is expected that the student do a thorough reading of the problem, seeking in their previous knowledge some geometric elements necessary for the solution of the issue (vertices, faces, edges, tetrahedron, etc.), establishing a relationship between the data presented in the question and seeking to mentally visualize what must be done to solve it.

Formulation situation: At this stage, the student is expected to identify the existence of a convex polyhedron in the geometric approach proposed in the question. Such identifications can be conjectured from the construction manipulation in GeoGebra, made available by the teacher. Figure 2 shows the initial screen viewed by the student.

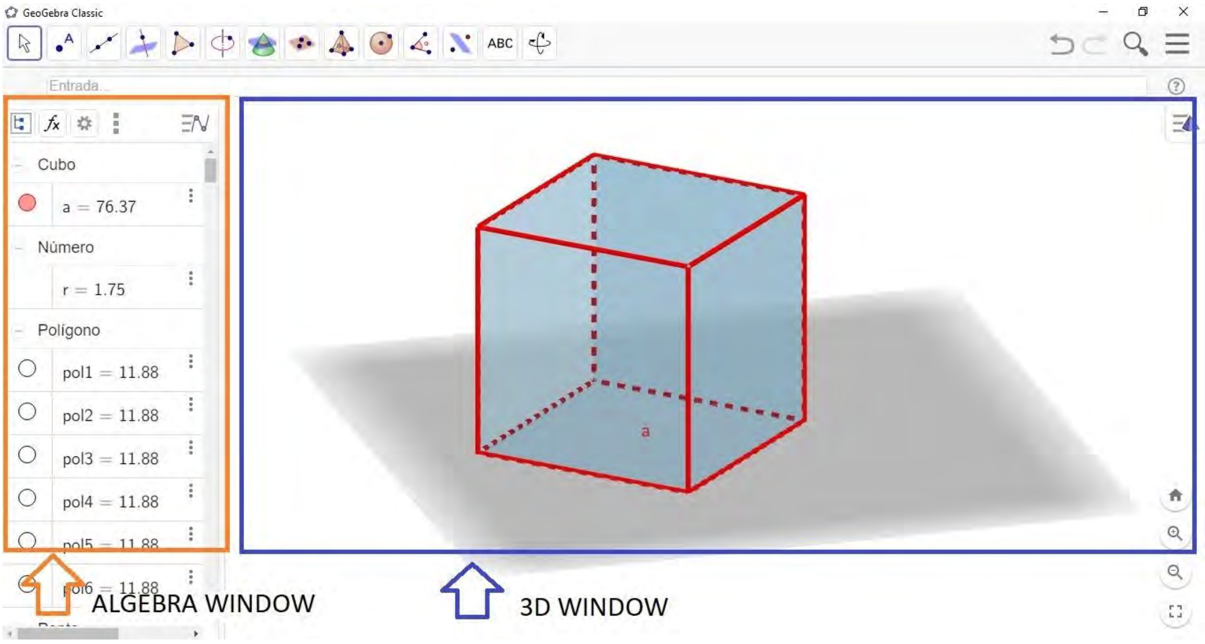

Figure 2. *Cube construction view, in algebra window and 3D window.*

From the home screen, the student will have access to the tools in the algebra window and the geometric construction of the cube in the 3D window, as shown in Figure 2. The algebra window brings the necessary tools for handling the construction, where the student will be able to manipulate tools, which in turn are hidden, in search of the necessary elements to visualize the construction that solves the presented question.

It is hoped that the student will understand mathematically that a cube has 8 vertices, 6 faces and 12 edges and, from tetrahedral markings at its vertices, visualize that the new figure will have a greater number of faces. After this, the student must manipulate the algebra window and activate the "display label" coming to Figure 3. In addition, there is the possibility of manipulating the colors to see the new faces built, facilitating the visualization of the truncated highlighted faces, as shown in Figure 4.

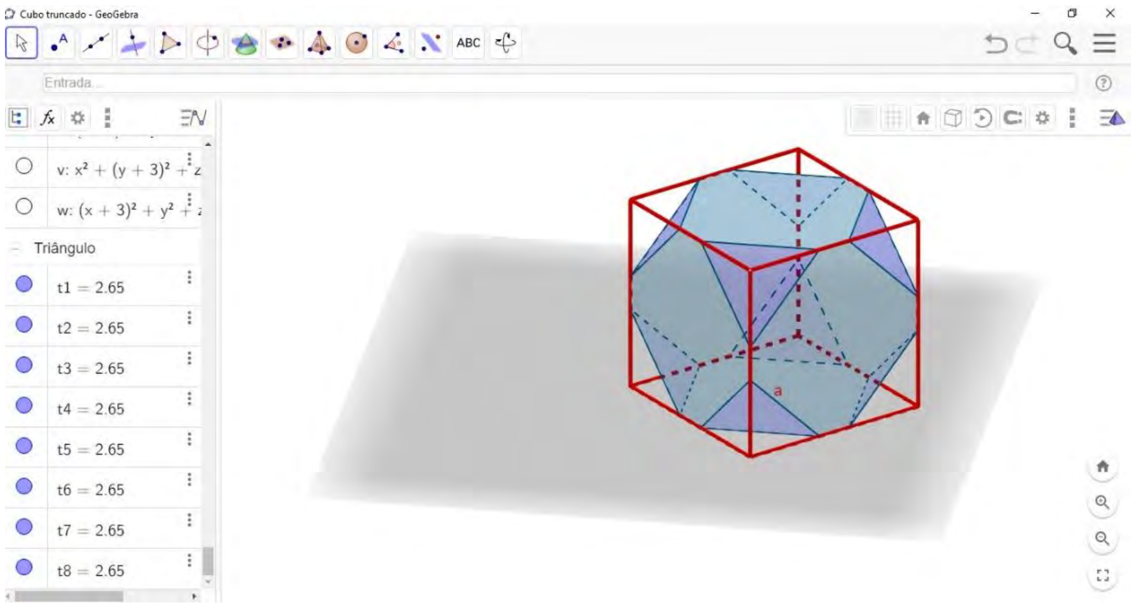

Figure 3. *Tetrahedron markings on the "corners" of the cube.*

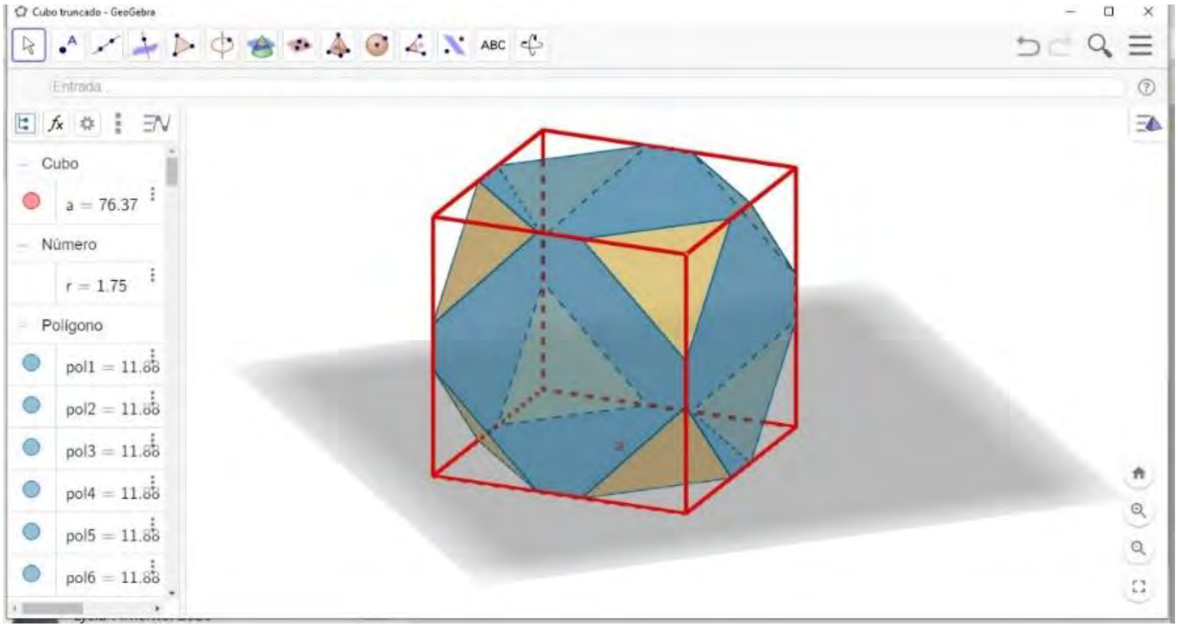

Figure 4. *Tetrahedron markings on the "corners" of the cube with different colors.*

Upon reaching this stage of construction, the student will notice that by sectioning the tetrahedrons from the vertices of the cube, a new convex polyhedron can be obtained, with a different number of faces, solving the question. According to Alves (2019, p. 109) "With GeoGebra software, students can develop a global and local analysis ability of numerical properties extracted from the computational and geometric environment".

Validation situation: From the geometric perceptions conjectured in the previous steps, when hiding the cube initially presented using the "hide object" function, the student must find the construction shown in Figure 5. In Figure 6, the truncated tetrahedron can be seen in a frontal view.

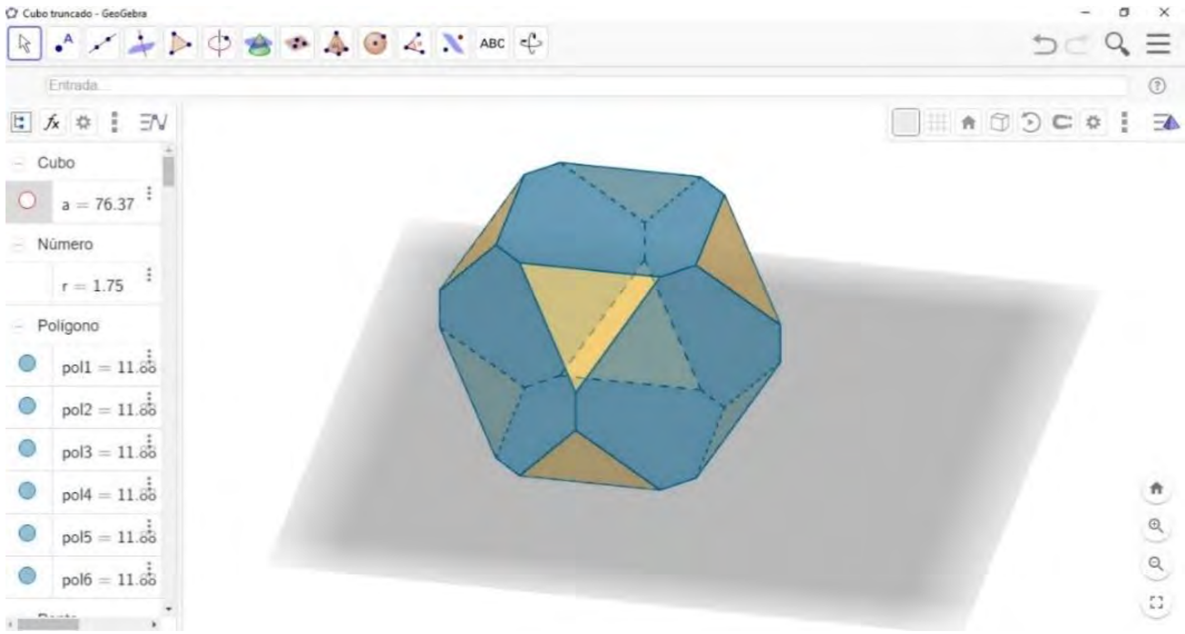

Figure 5. *Side view of the truncated cube.*

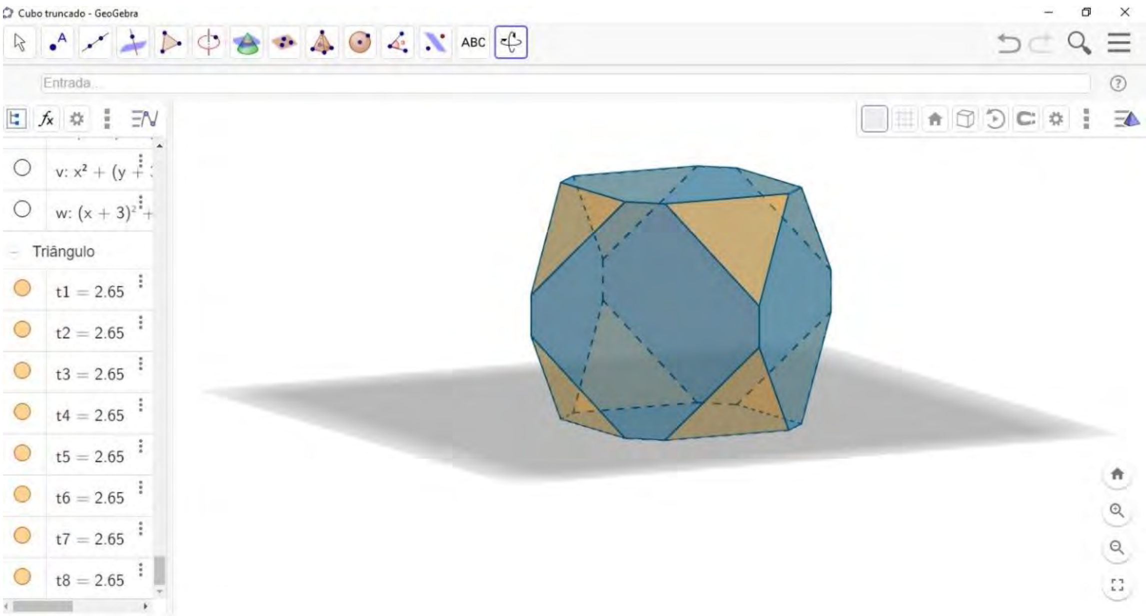

Figure 6. *Front view of the truncated cube.*

A possible way of validating for the student will occur when he perceives the 8 additional triangular faces from the truncations made at the vertices of the cube, which are added to the 6 existing faces, verifying that the new polyhedron that forms the trophy in question has a total of 14 faces.

Institutionalization situation: In the final stage of the didactic situation, the teacher must take into account all the observations generated by the students, establishing a comparison between the model built in GeoGebra and the proposal of the question presented. The purpose of this question is to make the student understand the vertices, faces and edges in an intuitive way, from that point on, explore the relationship between such entities in the presented polyhedron, generalizing to the other convex polyhedra.

According to Margolinas and Drijvers (2015) the initial usability of knowledge, when it has meaning for specific situations, it usually becomes less explicit or even hidden, and mathematical knowledge can become a type of formal knowledge. This institutionalization process should not be avoided, because it serves to strengthen and summarize the initial knowledge, which is an aspect of the didactic transposition pointed out by Chevallard (1991).

The question presented explores the subject of polyhedra, inducing the student to visualize the elements of vertices, faces and edges, from an exploration of geometric vision. According to Lorenzato (1995) it is common for students to have no reaction when faced with this type of question, because no numbers or measurements were given, which denotes the strong arithmetic tendency of reasoning, through ready formulas, making mechanical learning. In some situations, only a geometric perception, geometric reasoning and geometric language are needed to solve them.

Alves (2019) points out that using the potential of GeoGebra software, the Brazilian teacher has the possibility to instigate student engagement with regard to a dynamic exploration of geometric properties, thus working with student visualization and perception, with software having an important role in the evolution of their learning. Thus, the indicated question explores manipulation with GeoGebra, in order to develop geometric perceptions about the relationship between the elements of a convex polyhedron to then introduce the subject, before presenting the construction of a ready formula, stimulating the student's geometric thinking.

6. Final considerations

With this work we conclude that TDS contributes significantly to the elaboration of a research coherent with our proposal, considering the relevance of using technologies for the teaching of Spatial

Geometry, enabling the construction of a didactic situation with the potential to develop students' geometric perception.

TDS develops both the autonomy of the student, who happens to be a researcher and construct their own knowledge, as the teacher's praxis, that through the mediation of didactic situations is able to evolve in its pedagogical and methodological resourcefulness. It is worth emphasizing its importance, because this theory allowed us to construct a didactic situation with property, making attitudinal predictions of the student, supporting the teacher's work in identifying difficulties and possible cognitive, epistemological and didactic blocks in the teaching of this subject.

GeoGebra 3D has a difference with respect to Classic GeoGebra, by allowing visualizations and three-dimensional constructions, allowing the construction of objects for experimenting and exploring concepts within Spatial Geometry, as a dynamic and interactive resource. Alves (2016) shows that technology can affect the mediation process in the teaching of certain topics, however, its use in a complementary character emphasizes a dimensional change, in order to identify elements of a qualitative nature, which was in fact the objective of the question presented.

Therefore, technology in the teaching of Spatial Geometry is a relevant proposal for improving the assimilation of this subject, through the development of the student's geometric perception, without the use of ready formulas.

References

Abar, C A. A. P. A. (2020a). Transposição Didática na criação de estratégias para a utilização do GeoGebra [Didactic Transposition in the creation of strategies for the use of GeoGebra]. *Revista do Instituto GeoGebra de São Paulo,* 9(1), 59-75.

Abar, C. A. A. P. (2020b). Teorias da Transposição Didática e Informática na criação de estratégias para a prática do professor com a utilização de tecnologias digitais. [Theories of Didactic and Informatics Transposition in the creation of strategies for the teacher's practice with the use of digital Technologies]. *Revista Sergipana de Matemática e Educação Matemática,* 5(1), 29-45.

Alves, F. R. V. (2019). Visualizing the Olympic Didactic Situation (ODS): teaching mathematics with support of the GeoGebra software. *Acta Didactica Napocensia*, 12(2), 97-116.

Alves, F. R. V. (2016). Teoria das Situações Didáticas (TSD): sobre o ensino de ponto extremantes de funções com arrimo da tecnologia. [Theory of Didactic Situations (TSD): on the teaching of extreme point of functions supported by technology]. *Revista Eletrônica Sala de Aula em Foco,* 5(2), 59-68.

Alves, F. R. V. & Borges Neto, H. (2012). Engenharia Didática para a exploração didática da tecnologia no ensino no caso da regra de L'Hôpital. [Didactic Engineering for the didactic exploration of technology in teaching in the case of L'Hôpital's rule]. *Educação Matemática Pesquisa,* 14(2), 337 - 367, 2012.

Brazil. (2009). Instituto Nacional de Estudos e Pesquisas Educacionais Anísio Teixeira (INEP). [National Institute of Educational Studies and Research Anísio Teixeira (INEP)]. *Matriz de referência para o ENEM.*

Brazil. (2018). *Base Nacional Comum Curricular.* [Common National Curriculum Base]. Retrieved from http://basenacionalcomum.mec.gov.br/.

Breda, A., Trocado, A., & Santos J. (2013). O GeoGebra para além da segunda dimensão. [GeoGebra beyond the second dimension]. *Indagatio Didactica*, 5(1), 61-84.

Brousseau, G. (2008). *Introdução ao estudo das situações didáticas: conteúdos e métodos de ensino*. [Introduction to the study of didactic situations: contents and teaching methods]. São Paulo: Ática.

Camilo, A. M. S., Alves, F. R. V., & Fontenele, F. C. F. (2020). A Engenharia Didática articulada à Teoria das Situações Didáticas para o ensino da Geometria Espacial. [The Didactic Engineering articulated to the Theory of Didactic Situations for the teaching of Spatial Geometry]. *Revista Iberoamericana de Educación Matemática*, 16(59), 64-82.

Chevallard, Y. (1991). *La transposition didactique:* du savoir savant au savoir enseigné. Paris: Ed. La pensée Sauvage.

Díaz-Urdanetta, S., Kalinke, M. A., & Motta, M. (2019). A Transposição Didática na elaboração de um objeto de aprendizagem no GeoGebra. [The Didactic Transposition in the elaboration of a learning object in GeoGebra]. *#Tear - Revista de Educação, Ciência e Tecnologia*, 8(2), 1-12.

Lemos, B. M., & Carvalho, C. V. A. (2010). Uso da Realidade aumentada para apoio ao entendimento da Relação de Euler. [Use of Augmented Reality to support the understanding of Euler's Relation]. *Revista Renote - Novas Tecnologias na Educação,* 8(2).

Lorenzato, S. A. (1995). Por que não ensinar Geometria? [Why do not teach Geometry?] In: A Educação Matemática em Revista. Blumenau: *Sociedade Brasileira de Educação Matemática,* 4(3), 3-13.

Margolinas, C. (2015). Situations, savoirs et connaissances… comme lieux de rencontre? *Formation et pratiques d'enseignement en questions*, 2(19), 31-39.

Margolinas, C., & Drijvers, P. (2015). Didactical Engineering in France; an insider's and an outsider's view on its foundations, its practice, and its impact. *ZDM Mathematics Education,* 47, 893-903.

Polidoro, L. F., & Stigar, R. (2010). A Transposição Didática: a passagem do saber científico para o saber escolar. [The Didactic Transposition: the passage from scientific knowledge to school knowledge]. *Ciberteologia - Revista de Teologia & Cultura,* São Paulo, 27, 153-159.

Authors

Francisco Régis Vieira Alves, Federal Institute of Education, Science and Technology of Ceará, Fortateza (Brazil). E-mail: fregis@ifce.edu.br

Renata Teófilo de Sousa, Federal Institute of Education, Science and Technology of Ceará, Sobral (Brazil). E-mail: rtsnaty@gmail.com

Francisca Cláudia Fernandes Fontenele, Vale do Acaraú State University, Sobral (Brazil). E-mail: claudiafontenele05@gmail.com

Acknowledgement

We would like to thank the National Council for Scientific and Technological Development - CNPq for the financial support for the development of this research in Brazil.