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International Journal of Curriculum and Instruction 14(3) (2022) 1800–1812

IJCI
International Journal of Curriculum and Instruction

Cognitive development of children's understanding of counting numbers using a rope

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Abstract

This qualitative case study aims to address the ways that pre-service mathematics teachers (PSTs) used a rope in a daylong research meeting for cognitive development of children's understanding of counting numbers in Tanzanian elementary schools. Three university mathematics pre-service teachers volunteered participating in this study. Collective notes were taken as data to present the ways that PSTs used a rope in the research meeting for children understanding of counting numbers. Data was analyzed using Bruner's (1966) work on cognitive development. The results show that using a rope, it was found that counting numbers (0, 1, 2, ….,9) were formed, represented visually and symbolically, and read. The finding has implications in teaching and learning counting numbers, including teachers can use a rope to teach the concept of counting numbers among children in elementary schools for a relational understanding of the concept.

Keywords: Counting numbers; elementary schools; real object; cognitive development.

1. Introduction

This paper starts by introducing the challenges of teaching and learning counting numbers among children in Tanzanian elementary schools. Then, the paper presents the theoretical framework used in this study. After that, the paper addresses teaching and learning strategies that have been used on counting numbers in elementary schools. Then, the paper presents research design, participants and the methods used for data collection. After that, the paper presents results of using a rope for teaching and learning counting numbers in the research meeting. The paper ends by providing concluding thoughts on using real objects for teaching and learning counting numbers in elementary schools.

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1.1. *The challenges of teaching and learning counting numbers in Tanzanian elementary schools*

In Tanzanian elementary schools, teachers usually use number cards to teach children in lower grades (grades 1, 2 and 3) the concept of counting numbers (0, 1, 2, 3,4, 5, 6, 7, 8, 9). However, teachers need to have a lot of cards because each card need to have only a single number let say 1 written on the card. Also, this numbering starts from symbolic representations, including reading counting numbers on cards followed by enactive representations such as children writing numbers on either individual small boards or exercise books. In doing so, the systematic organization of the concept of counting numbers is not cognitively developed among children's in schools (Ben-Hur, 2006; Bruner, 1966; Deogratias, 2020). Children need to learn the concept of counting numbers from enactive, iconic to symbolic representations (Dubinsky, 1991; Tall & Vinner, 1981; Sierpinska, 1994). Hence, using a rope as an example of real objects used in this study, the concept of counting numbers from enactive, iconic and finally to symbolic representations is developed and understood among children in elementary schools.

1.2. This study is framed under the lens of Bruner's theory of cognitive development

This study has used Jerome Bruner's cognitive development while focusing on the concept of counting numbers in elementary schools. Bruner provides teaching strategy by addressing three stages of cognitive representations: enactive which focuses on knowledge representations to the children through actions, iconic which focuses on knowledge representations through visual/picture, and symbolic which focuses on knowledge representations through symbols and words (Bruner, 1966). A rope was used to develop the concept of counting numbers in the research meeting where by it was transformed through knowledge representations in three stages by manipulating the rope through actions to form counting numbers, representing the formed counting numbers using the same rope into figures/pictures, and finally representing the counting numbers into symbols (0, 1, 2, ….., 9) as well as into words (zero, one, two, ….., nine).

1.3. Teaching and learning strategies that have been used on counting numbers in elementary schools

The concept of counting numbers has been taught to children in grades 1 to 3 using different teaching strategies. The strategies that have been used for teaching counting numbers include using: music and lyrics (for instance, Fenwick, 2009); bead string (for instance, Young-Loveridge, 1999); worksheets for pictures (Young-Loveridge, 1999); playing cards (for instance, Fuson, 1988); object counting (for instance, Baroody, 2006; Clements & Sarama, 2014; Department of Basic Education, 2003); Verbal counting (for instance, Baroody, 2006); picture (for instance, Aker, 1990; Dahl, 2005); counting sets (for instance, Department for Children, Schools and Families, 2009); number grid (for instance, Department for Children, Schools and Families, 2009); models and images (for

instance, Department for Children, Schools and Families, 2009); songs (for instance, Brombacher & Associates, 2013); number cards (for instance, Brombacher & Associates, 2013); counting body parts such as eyes, ears, hands, fingers etc (for instance, Brombacher & Associates, 2013); counting frames (for instance, Numeracy Handbook, 2003); counting in a pile (for instance, Brombacher & Associates, 2013); flard cards and cards with pictures on them (for instance, Numeracy Handbook, 2003); playing matching and sorting games using index cards withi 1-3 stickers on them (Baroody & Benson, 2001); math toolkits (for instance, Frykholm, 2008; Ministry of Education Republic of Singapore, 2013).

From the reviewed literature, it is found that many strategies have been used for teaching and learning counting numbers in elementary schools. Moreover, many websites have highlighted various strategies for teaching and learning counting numbers. It is found that counting objects (artificial and local objects), pictures, music, songs, video, and games have been used to engage students in learning counting numbers. However, no studies that have been conducted to Tanzanian university mathematics pre-service teachers on how they can use a rope to develop children's understanding of counting numbers. In particular, this study focused on exploring the research question: How do university mathematics pre-service teachers (PSTs) develop the concept of counting numbers using a rope available in children's living environment for their cognitive development of the concept?

2. Method

2.1. Research design

This qualitative case study, carried out in one of the Tanzanian public universities, explored the ways that PSTs used a rope to develop the concept of counting numbers in a daylong research meeting. The PSTs participated on the activity in the research meeting using Bruner's three stages for children's cognitive development. The data was collected focused on PSTs' participation in the meeting.

This research meeting focused on one "case" (Yin, 2014, p. 4). This design helped the researcher to get in-depth understanding how PSTs in a single group used a rope (as one of the real objects available in children's living environment) for children's cognitive development of counting numbers. In this study, PSTs in a single group is a case about using a rope in a Tanzanian context as a real object available in children's living environment for teaching and learning counting numbers. A rope and counting numbers are bounded systems to ensure that the "study remained reasonable in scope" (Baxter & Jack, 2008, p. 547).

This study was convenience (Merriam, 1998) because research site was chosen based on the availability of resources including participants as well minimizing cost during working with pre-service teachers (PSTs) in a daylong weekend.

The meeting was conducted on Saturday to avoid collision with other schedules such as university teaching timetable, tests and examinations. The meeting started at 9:00 am

and was scheduled into two sessions. The first session was from 9:00 am to 1:00pm, and the second session from 2:00pm-5:00pm.

2.2. Participants

Three university mathematics pre-service teachers in Tanzania volunteered participating in this study. The PSTs worked on the concept of counting numbers while using a rope available in children's living environment. They worked on the activity in a single small group which consisted of three members in the research meeting.

2.3. Data collection tools

Three tools were used to collect data; activity, observation and PSTs' collective notes. The following activity was used to collect data and to initiate PSTs' work on the activity: "imagine that you are a teacher of a second-grade class. At a parent-teacher conference, a parent comes to you and strongly suggest that you should be giving the children lots of objects to understand the concept of counting numbers. Describe how you could oppose this suggestion by developing the concept of counting numbers using a single object (i.e., rope) available in children's living environment?" Observation was used during a single group discussion among PSTs in the research meeting on the activity about counting numbers. Collective notes were the work that PSTs discussed in the group and written on the manila sheet, and later on was collected as data.

2.4. Analysis

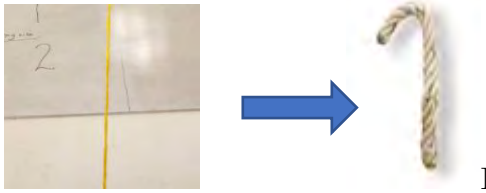
Bruner's (1966) was used as analytical framework in this study. Collective notes were used in the analysis to answer the research question: How do university mathematics pre-service teachers (PSTs) develop the concept of counting numbers using a rope available in children's living environment for their cognitive development of the concept?

3. Results

The PSTs in a small group developed the concept of counting numbers (0, 1, 2, …., 9) using a rope by starting with number 1 followed 2, then 3, 4, …9, and finally 0. Then, they demonstrated how numbers were formed using a rope through using enactive, iconic and finally to symbolic representations. Finally, PSTs demonstrated in small group the process of getting each counting number in the meeting. While demonstrating how to teach the counting numbers using a rope, the rope was transformed into numbers from 1 to 9 and then 0 without cutting or gluing it. The following steps were followed while forming the numbers using a rope.

Step 1: From the collective notes, the rope was transformed into a number 1 without any form of deformation.

FIGURE 1: *Forming a number 1 using a rope and writing a number 1*

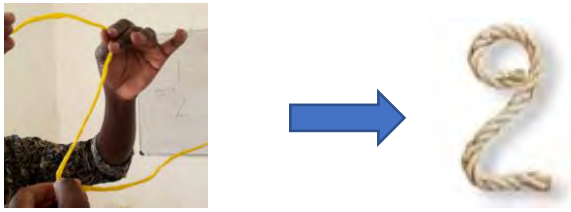


Read as "one"

From Figure 1, PSTs used a rope to form a number 1 through enactive. They started by demonstrating how a rope formed number 1 visually. Then, PSTs represented the number pictorially followed by symbolic representation (1). Finally, they could read as "one". In doing so, PSTs in a single small group were able to develop number 1 using a rope available in children's living environment from enactive, iconic to symbolic representations.

Step 2: From collective notes, the rope was further transformed into a number 2 as shown.

FIGURE 2: *Forming a number 2 using a rope and writing a number 2*



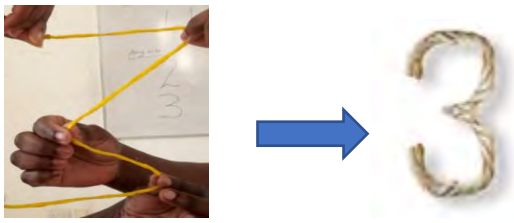
Read as "two"

"Demonstrating how 2 was formed using a rope"

From Figure 2, PSTs used a rope to form a number 2 through enactive. They started by demonstrating how a rope formed number 2 visually. Then, PSTs represented the number pictorially followed by symbolic representation (2). Finally, they could read as "two". In doing so, PSTs in a single small group were able to develop number 2 using a rope available in children's living environment from enactive, iconic to symbolic representations.

Step 3: The rope was transformed into a number 3 as shown.

FIGURE 3: *Forming a number 3 using a rope and writing a number 3*



Read as "three"

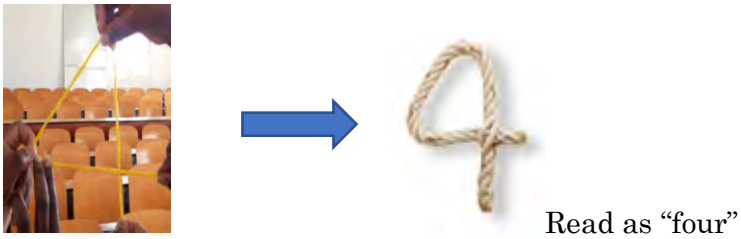
"Demonstrating how 3 was formed using a rope"

From Figure 3, PSTs used a rope to form a number 3 through enactive. They started by demonstrating how a rope formed number 3 visually. Then, PSTs represented the

number pictorially followed by symbolic representation (3). Finally, they could read as "three". In doing so, PSTs in a single small group were able to develop number 3 using a rope available in children's living environment from enactive, iconic to symbolic representations.

Step 4: The rope was transformed into a number 4 as shown.

FIGURE 4: *Forming a number 4 using a rope and writing a number 4*



Read as "four"

"Demonstrating how 4 was formed using a rope"

From Figure 4, PSTs used a rope to form a number 4 through enactive. They started by demonstrating how a rope formed number 4 visually. Then, PSTs represented the number pictorially followed by symbolic representation (4). Finally, they could read as "four". In doing so, PSTs in a single small group were able to develop number 4 using a rope available in children's living environment from enactive, iconic to symbolic representations.

Step 5: The rope was transformed into a number 5 as shown.

FIGURE 5: *Forming a number 5 using a rope and writing a number 5*

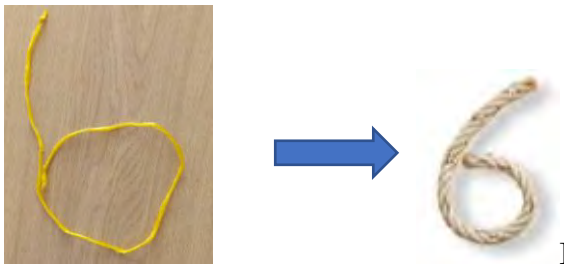
Read as "five"

"Demonstrating how 5 was formed using a rope"

From Figure 5, PSTs used a rope to form a number 5 through enactive. They started by demonstrating how a rope formed number 5 visually. Then, PSTs represented the number pictorially followed by symbolic representation (5). Finally, they could read as "five". In doing so, PSTs in a single small group were able to develop number 5 using a rope available in children's living environment from enactive, iconic to symbolic representations.

Step 6: The rope was transformed into a number 6 as shown.

FIGURE 6: *Forming a number 6 using a rope and writing a number 6*

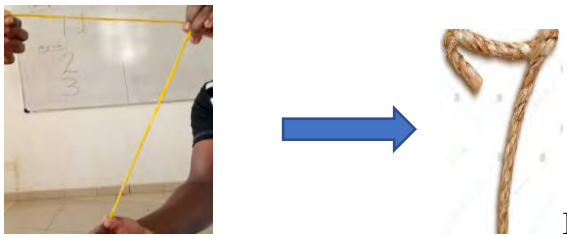


“Demonstrating how 6 was formed using a rope”

From Figure 6, PSTs used a rope to form a number 6 through enactive. They started by demonstrating how a rope formed number 6 visually. Then, PSTs represented the number pictorially followed by symbolic representation (6). Finally, they could read as “six”. In doing so, PSTs in a single small group were able to develop number 6 using a rope available in children’s living environment from enactive, iconic to symbolic representations.

Step 7: The rope was transformed into a number 7 as shown.

FIGURE 7: *Forming a number 1 using a rope and writing a number 1*

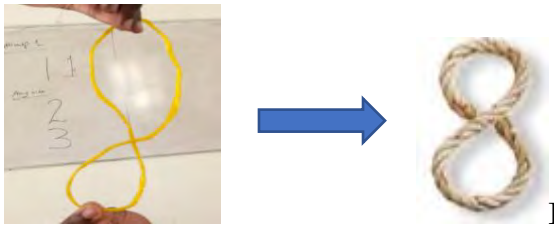


“Demonstrating how 7 was formed using a rope”

From Figure 7, PSTs used a rope to form a number 7 through enactive. They started by demonstrating how a rope formed number 7 visually. Then, PSTs represented the number pictorially followed by symbolic representation (7). Finally, they could read as “seven”. In doing so, PSTs in a single small group were able to develop number 7 using a rope available in children’s living environment from enactive, iconic to symbolic representations.

Step 8: The rope was transformed into a number 8 as shown.

FIGURE 8: *Forming a number 8 using a rope and writing a number 8*



“Demonstrating how 8 was formed using a rope”

From Figure 8, PSTs used a rope to form a number 8 through enactive. They started by demonstrating how a rope formed number 8 visually. Then, PSTs represented the number pictorially followed by symbolic representation (8). Finally, they could read as "eight". In doing so, PSTs in a single small group were able to develop number 8 using a rope available in children's living environment from enactive, iconic to symbolic representations.

Step 9: The rope was transformed into a number 9 as shown.

FIGURE 9: *Forming a number 9 using a rope and writing a number 9*

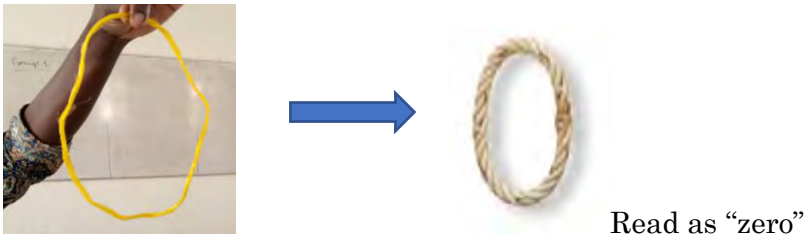
Read as "nine"

"Demonstrating how 9 was formed using a rope"

From Figure 9, PSTs used a rope to form a number 9 through enactive. They started by demonstrating how a rope formed number 9 visually. Then, PSTs represented the number pictorially followed by symbolic representation (9). Finally, they could read as "nine". In doing so, PSTs in a single small group were able to develop number 9 using a rope available in children's living environment from enactive, iconic to symbolic representations.

Step 10: The rope was transformed into a number 0 as shown.

FIGURE 10: *Forming a number 0 using a rope and writing a number 0*



Read as "zero"

"Demonstrating how 0 was formed using a rope"

From Figure 10, PSTs used a rope to form a number 0 through enactive. They started by demonstrating how a rope formed number 0 visually. Then, PSTs represented the number pictorially followed by symbolic representation (0). Finally, they could read as "zero". In doing so, PSTs in a single small group were able to develop number 0 using a rope available in children's living environment from enactive, iconic to symbolic representations.

4. Discussion

Through working in a single small group how to use a rope to develop children's cognitive development counting numbers in the research meeting, it was observed that the rope could easily be transformed into numbers 0 to 9 without cutting or gluing it. This observation shows that real objects are critical for conceptual development and understanding of a mathematical concept among learners (Baroody, 2006; Clement & Salama, 2014; Deogratias, 2020, 2022).

We can also observe that there were symbolic representations of counting numbers. Bruner (1966) emphasizes that symbolic representations, as one of his three stages of children's cognitive development, are critical in teaching and learning mathematics. Children need to understand counting numbers by giving symbolic and verbal counting of the numbers (Baroody, 2006). In doing so, the following symbolic representations of counting numbers (symbols and verbal counting) existed in the research meeting with PSTs while using a rope. The counting numbers are arranged in ascending order.

Symbol for number 0 which is read as "zero"

Symbol for number 1 which is read as "one"

Symbol for number 2 which is read as "two"

Symbol for number 3 which is read as "three"

Symbol for number 4 which is read as “four”

Symbol for number 5 which is read as “five”

Symbol for number 6 which is read as “six”

Symbol for number 7 which is read as “seven”

Symbol for number 8 which is read as “eight”

Symbol for number 9 which is read as “nine”

Hence counting numbers (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) were formed by PSTs in the research meeting while using a rope. The rope was improvised in the meeting which was used by PSTs to develop the concept of counting numbers by focusing on Bruner’s three stages of cognitive development; enactive, iconic and symbolic representations.

5. Conclusions

In this study, I have tried to reveal how a rope can be used to develop the concept of counting numbers among children in lower grades. In the process of exploring the ways that PSTs used a rope to develop the concept of counting numbers in the research meeting, I paid special attention to the learning activity. The activity involved the representations of a rope (physical object) through enactive to iconic, and finally to symbolic representations.

Using a rope in children's day to day living environment to form counting numbers is critical for children-teachers' engagement with the concept especially in classrooms, including children are actively engaged in learning because they learn counting numbers by doing, visualizing, and representing numbers symbolically. Because of that teachers can also improvise a rope in teaching and learning environment to help children write, read and understand counting numbers and numbers above 9 and give symbolic representations through enactive followed by iconic representations. Adopting Bruner's (1966) stages of cognitive development: enactive, iconic and symbolic enabled a targeted focus on a rope in the research meeting.

Understanding what we communicate and what we want to communicate mathematically while using a rope and other real objects is critical, given what a powerful real objects are to present for conceptual development and understanding of a mathematical concept (Deogratias, 2020, 2022; Dubinsky, 1991; Tall & Vinner, 1981; Sierpinska, 1994). Because of that mathematics teachers and educators need to think and look for real objects and know how they can use the objects for effective teaching and learning a mathematical concept in the classrooms.

Acknowledgements

The author acknowledges university pre-service mathematics teachers who volunteered participating in this study.

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