MATHEMATICS TEACHING RESEARCH JOURNAL
SPRING 2022
Vol 14 no 1

Developing Conceptual Understanding of Irrational Numbers Based on Technology through Activity System

Abolfazl Rafiepour[1], Kazem Abdolahpour[1], Danyal Farsani[2]

[1]Mahani Mathematical Research Center, and Department of Mathematics Education, Faculty of Mathematics and Computer, Shahid Bahonar University of Kerman, Iran, [2]Facultad de Educación, Psicología y Familia, Universidad Finis Terrae, Santiago, Chile

Rafiepour@uk.ac.ir & Drafipour@gmail.com, k_a_abdollahi@yahoo.com, dfarsani@uft.cl

Abstract: The main purpose of this study is to develop a conceptual understanding of the irrational number of the square root of 2 ($\sqrt{2}$ *). Participants in the study were 20 ninth-grade male students. Activity Theory was used as a framework to show the development of the conceptual understanding. Since this study was conducted during the COVID-19 pandemic; online teaching method was adopted. In this teaching method, WhatsApp messaging and calculator were used as our basic technology. Virtual education lasted 2 sessions (120 minutes) for the development conceptual understanding of the irrational number of square root of 2. To produce data, WhatsApp Export Capability was used. For data analysis, the online teaching activity system was used. By analyzing this activity system, three tensions were understood. Modifying these tensions, has led to their students making the concept of the irrational number of the square root of 2 (* $\sqrt{2}$ *) and reach a single definition.*

INTRODUCTION

In mathematics education, concepts have historical roots. The studies show that teachers and students have problems in understanding irrational numbers such as *square root* (Sirotic & Zazkis (2007b). As many students are not able to provide a uniform and accurate definition of square root (Sirotic & Zazkis, 2007a), because school mathematics has focused on problem-solving techniques. This has caused irrational numbers, such as square root, to have been neglected in school mathematics (Fischbein, 1995).

A review of studies on teaching irrational numbers shows that teachers use a variety of methods to teach it. One of the methods that teachers use to introduce an irrational number is to use the example $\sqrt{2}$ and π (Zazkis, 2005). The second method that studies show is to use the definition of the perfect root, in other words, the irrational number is a number that does not have a perfect root (Shinno, 2018; Patel & Varma, 2018). But the formal definition of the irrational number that most people use is as follows:

A real number that is not a rational number is said to be irrational (Rosenthal, Rosenthal, & Rosenthal (2014), p.66)

On the other hand, the irrational number 2 can be considered an irrational number by the triangle chord, where the point of intersection of a circular arc with radius 1 with the axis of real numbers indicates its geometric location that is shown in Fig. 1.

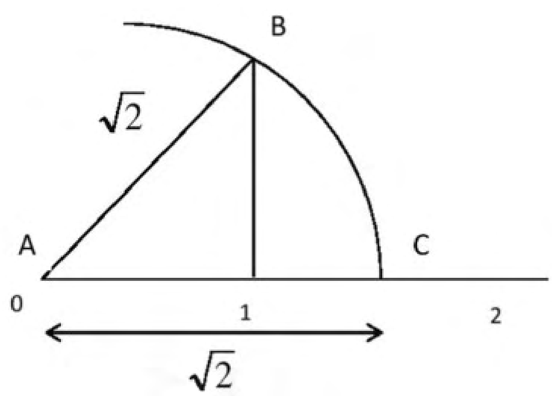

Figure 1. Geometric drawing of the irrational number 2 (Kidron, 2018, p.66)

In other words, school mathematics places more emphasis on instrumental understanding and less emphasis on conceptual understanding of mathematical concepts. This focus has led research to show that students have difficulty understanding this set of irrational numbers. In addition to students, teachers have difficulties understanding the concept of irrational numbers. In this regard, Sirotic & Zazkis (2007b) in their study showed that even mathematics teachers have a misunderstanding of the concept of irrational numbers. Hence, the concept of irrational numbers is perceived to be a challenging concept for teachers and students. On the other hand, the importance of irrational numbers can be described in such a way that without understanding it, one cannot understand the meaning of a set of real numbers (Sirotic & Zazkis, 2007a). Therefore, understanding the concept of irrational numbers is essential for students, especially in junior and high school (Voskoglou & Kosyvas, 2012). Given the importance of this issue, the present study focuses on understanding one of the irrational numbers, the *square root* of the number 2 ($\sqrt{2}$).

From what has been said in the previous section, it can be concluded that teaching the concept of irrational numbers is a fundamental issue for mathematical education. But the most important issue that is gripping most societies today is the prevalence of the Covid-19 pandemic. The Covid-19 has affected the political, technical, and pedagogical aspects of education worldwide (Breda et al., 2020; Cahapay, 2020). The challenge posed by the Covid-19 pandemic has affected face-to-face teaching, causing teachers and educators turning to online teaching. This issue has opened a new field in education (Borba, 2021). Education in this area has become very difficult due to its complexities and has become an important issue for teachers and students (Cahapay, 2020). Hence, teaching concepts such as irrational numbers, in which teachers and students themselves have difficulty, has doubled the difficulty of teaching during COVID-19 pandemic era. On the other hand, the threat posed by education due to the spread of the Coronavirus has become an opportunity to use technology that can be used to teach mathematical concepts (Attard & Holmes, 2020). Therefore, the issue of technology is very important learning tool in this era. In this regard, the importance of technology has led NCTM (2000) to consider it as one of the important principles and standards of school mathematics and to express it as follows:

> ***Technology:*** *Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning (p.11).*

What highlights the role of technology in this era is its interactive nature between teachers and students (Kaput & Thompson, 1994) that can be used to engage students and improve education. Students are perceived to have a positive attitude towards the use of technology (Eyyam & Yaratan, 2014). This positive attitude can be led to more interaction. In addition to a positive attitude, technology can improve students' understanding of mathematical concepts. In this regard, Dogruer and Akyuz (2020)'s study showed that using technology could improve students' understanding of difficult mathematical concepts and improve their learning comprehension. For example, one of the simplest technologies that students use is the calculator, according to Satianov (2015), review of studies about the calculator, shows that few studies have used calculator to teach mathematical concepts. But with the outbreak of the Coronavirus, real-world classroom space has given way to virtual spaces (Mulenga & Marbn, 2020). Teachers and students in virtual space use chat-enabled software to interact with each other. The latest published statistics related to the most popular global mobile messaging apps 2021 shows WhatsApp Messenger with 2 billion users, has the most users among messengers in the figure below:

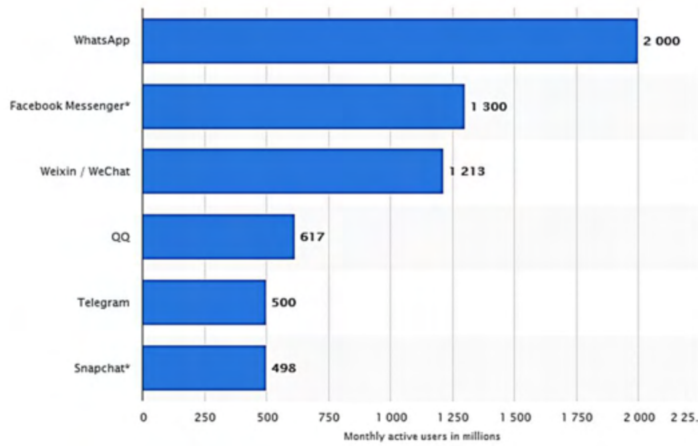

Figure 2. Percentage of users of the most famous messengers in the world *(*cited *in web site: (*https://www.statista.com/statistics/258749/most-popular-global-mobile-messenger-apps*)*

In this regard, Barhoumi (2015) showed that WhatsApp Messenger has good features for teaching such as interaction, creating a class group, sharing. It can be concluded that there is a good interactive space in this messenger that can be used for educational purposes. Therefore, the main purpose of this study is to answer the following question:

- How can technology be used in a mathematics classroom to develop a better understanding of the concept of *square root* of 2 ($\sqrt{2}$)during the Covid-19 era?

Theoretical Framework

As mentioned in the previous section, the purpose of this study is to develop a conceptual understanding of the irrational number. For this purpose, it is necessary to examine the structure of students' collective activity. In this regard, Engstrom 1987 defines the structure of collective activity in the form of activity system of Activity Theory. The activity system consists of 6 components: Tools, Subject, Rules, Community, and Division of labor and Object. The result of the interaction of these 6 components with each other is the output of the activity system. These 6 components, along with the system output, can be seen in the following Figure:

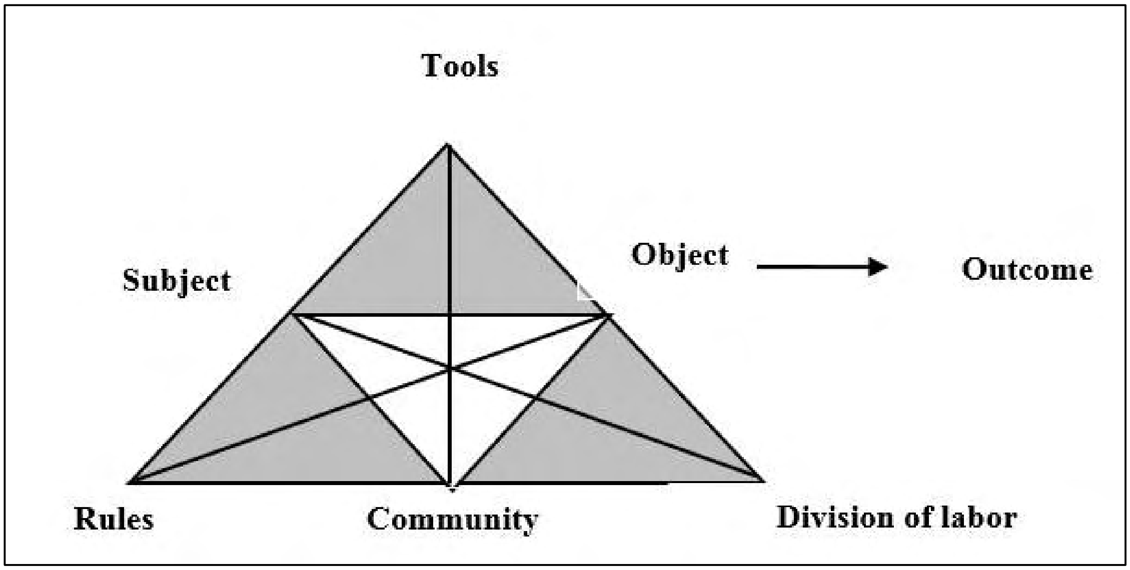

Figure 3. The structure of human activity (Engestrom, 1987 cited in Jurdak, 2006, p. 288).

As mentioned in the introduction, this study was conducted during the Coronavirus period, in which teaching was temporarily conducted online. On the other hand, studies show that the activity system can be used to analyze classroom interactions (Salloum and BouJaoude, 2020, Huang & Lin, 2012)

Therefore, it is necessary to define an activity system for online teaching that includes classroom interactions. In this regard, Barhoumi (2015) considered three levels of activity system for online teaching: The technological level, the individual level, and the community level which showed in below Figure:

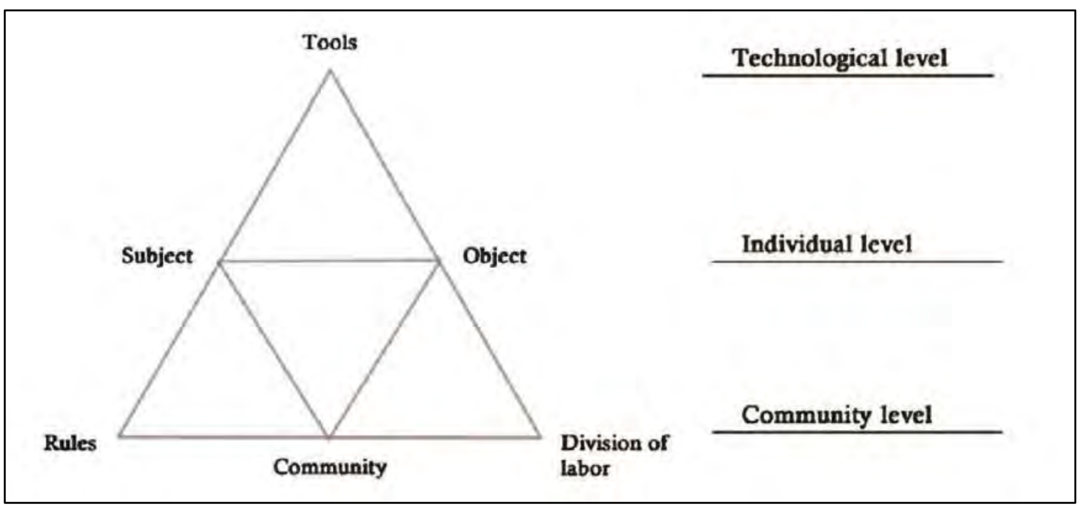

Figure 4. three levels of Activity Theory for Online teaching (Barhoumi, 2015, p.221)

On the other hand, before defining these three levels, it is necessary to define the 6 components of the activity system of Figure 4 The definition of these 6 components is defined as the following 6 principles:

- *The first principal is the orientedness of the object. The objective of the activity system has social and cultural properties in the system, such as collaborative or cooperative learning in an online course.*
- *Subjects are actors engaged in activities. This is considered the individual level of Activity Theory; students are contextual subjects engaged in collaborative learning.*
- *Community or externalization is considered a social context of the system and a community level of Activity Theory; all actors are involved in the activity system (e.g., a group of students engaged in learning based on social interaction for constructing and sharing of knowledge is an example of a learning community).*
- *Tools are considered a technological level of Activity Theory. In the system, communication between communities is mediated by tools that transmit social knowledge. It includes the artifacts used by actors in the system. Tools influence actor-structure interactions and are influenced by culture.*
- *The division of labor is a considered a hierarchical structure of activity or the division of activities among actors in the system.*
- *Rules are the conventions and guidelines regulating activities in the system, such as rules of discussion between students in collaborative learning (Barhoumi, 2015, p.221)*

Having defined the components of the activity system, it is necessary to define the three levels mentioned in Figure 4 in the field of online teaching that is the subject of the study. In Figure 4, the first level is the technology level. In the present study, this level was defined as the use of WhatsApp Messenger to conduct virtual classes and calculator to perform mathematical calculations. The second level of Figure 4 is the individual level. In fact, the individual level is the same activity that students do in the WhatsApp virtual classroom. The third level is the collective level. Students can benefit from each other's comments, questions, and whole classroom discussions to better develop an understanding of the concept of irrational numbers and make it a comprehensible concept. Given these levels that were defined for Online teaching activity system, then, this activity system needs a driving force to use it to develop this concept of irrational number. In studies of Activity Theory, researchers cite tension as the driving force behind the system and see it as a source of development.

Given that the topic of this study is the irrational number and this particular topic is perceived to be challenging, and thus it is necessary to analyze the tensions that occur in the operating system. In an activity system, researchers state that tensions occur either within the components of the system or between its components (Engestrom, 2001). The tensions that occur within the components of the activity system are called level one tension and, the tensions that occur between the components of the activity system are called level two tensions (Engstrom, 2015). Therefore, in this study, students are involved with the concept of irrational number by creating level 1 and 2 tensions of the activity system (*square roo*t).

Methodology

The theoretical framework of the study is based on Activity Theory. On the other hand, Activity Theory is the nature of qualitative work and is used to analyse in qualitative methodological issues (Yamagata-Lynch, 2010). Hence, Activity Theory is a framework for qualitative data analysis (Hashem & Jones, 2007). Researchers use the method of collecting interview data, observing ethnographic methods, and case studies for the activity systems of Activity Theory (Russell, 2009). They provide evidence for perceived tensions in operating systems by referring to interview quotes and descriptive narratives (Hashem & Jones, 2007). On the other hand, data collection method is related to the time when face-to-face training was available to the research participants. But nowadays, with the spread of the coronavirus, conventional data collection methods cannot be used. Instead, the capacity of cyberspace can be used to collect data. In this regard, WhatsApp Messenger has a great ability to generate text data with images. This is how the Export chat WhatsApp is more, the production process by which data will be described. In this regard, first, the virtual classroom environment is described as follows:

> In this virtual class, the first author of the research was a teacher who has 9 years of experience teaching mathematics in junior and high school. The teaching method adopted in this virtual class was based on the constructivist learning method. In other words, in this virtual classroom, the classroom environment was such that students built their knowledge by talking, discussing, and typing their views on mathematics. All students' activities in the group were recorded by WhatsApp Messenger. Students responded to the teacher's question in a written format (using symbols, letters, and pictures). All of the students' responses to the teacher's questions were documented as a source of data collection. The data was generated as a Word File under the following process by WhatsApp export chat messenger. The stages of production of study data were identified step by step with the following numbers:

Step 1

In this step you click on the three dot symbol in the right corner, this image appears with 6 items that you click on more

Step 2

In this step, click on option 4, ie Export chat

Step 3

In this step, you can choose one of two options, which is usually the first option

Step 4

At this point, it will ask you how to save data, you can choose the same WhatsApp.

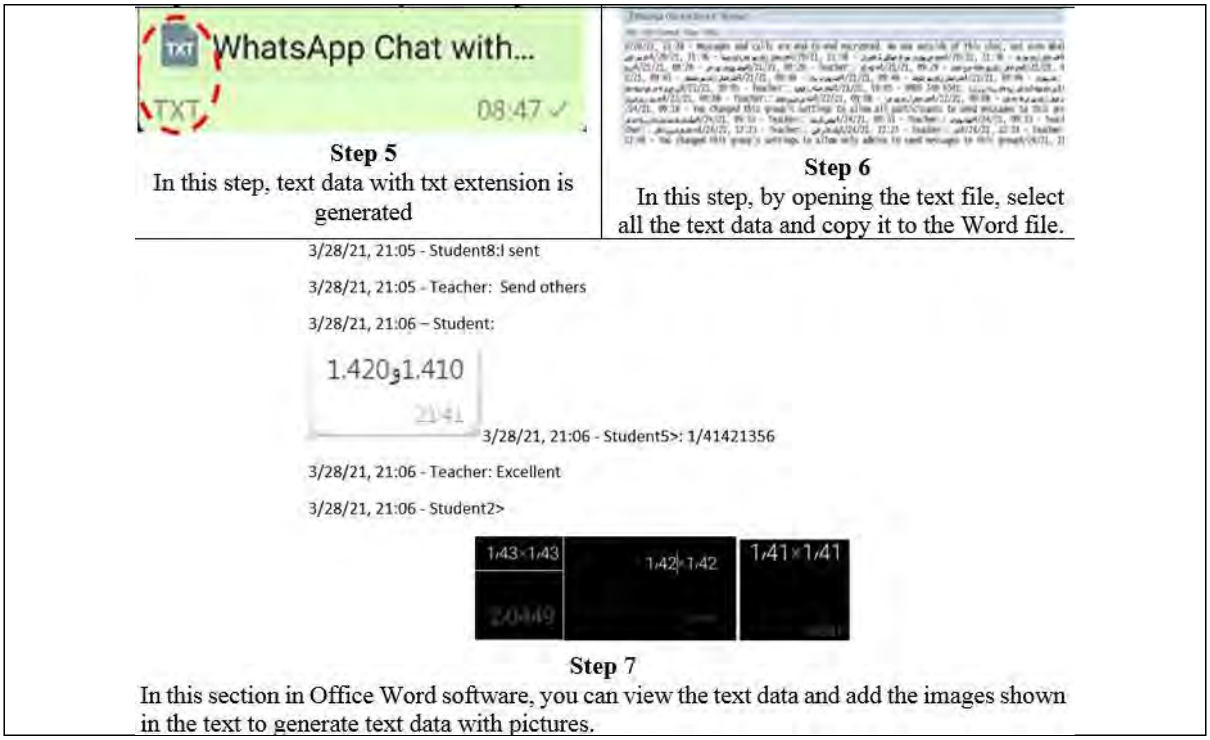

Figure 5. The process of generating text data with images by using WhatsApp Export

Since the study uses the activity system of Activity Theory as a tool of analysis, in this regard, each activity system is defined as a unit of analysis (Engstrom, 2001). The process of data analysis by activity systems can be described as follows:

- **Step 1:** First, the elements of the activity system are defined according to the topic of the study (definition of the components of the activity system).
- **Step 2**: After defining the components of the activity system, the researchers refer to the study data to understand the tensions that occur in the activity system. Finally, tensions are defined at this stage (definition of tensions).
- **Step 3**: In this step, the sources of tensions are identified (identify sources of tensions)
- **Step 4:** After identifying the sources of tensions, the researchers refer to the study data again to find clues from the tensions modification so that they can refer to it in the study findings (finding clues from the tensions modification).

Therefore, in this study, these steps were performed in the order shown above:

- **Step 1**: First, according to the study topic, the study activity system in the field of online teaching was defined.
- **Step 2:** At this stage, the entire study data were read several times to understand the tensions that had occurred in the classroom for the students.
- **Step 3:** In the next step, the sources of tensions were identified.

- **Step 4:** Tensions, evidence related to their correction was collected. Given that the subject of the study is the development of the conceptual understanding of the irrational number. In this regard, evidence of modification of tensions was collected, which in the findings of the study shows how students using They were able to modify these tensions and develop the concept of the irrational number.

After stating how to analyze the study data, it is necessary to explain the details of the research participants, the virtual class (time, number of sessions, subject of teaching, etc.) through which the study data was generated. Participants in this study were 20 ninth-grade male students from three schools who volunteered to participate and their work be used for the study. Students attended this virtual class for one hour from 9 pm to 10 pm for 14 nights from April 3 to 17, 2021. A total of 840 minutes of virtual class was held for the total irrational numbers. In these 14 sessions, students were taught the concept of irrational numbers: Square and Cube roots, addition and subtraction of them. Since the volume of study data was very high, this study has focused only on a part of this data related to the development of the concept of *square root* of 2. The development of this concept took two sessions. These two sessions are related to April 3 and 4, 2021. The times of these two sessions are illustrated minute by minute in the study findings section. In this regard, the output of the conversations that the teacher and the students had at that time, turned into a 120-page Word Document, which due to the high volume of this data, only a selective part is selected in the study.

Results

In this section, the findings related to the analysis of perceived tensions and their modifications related to the online teaching activity system of the concept of *square root* of 2 are presented. In this regard, the elements of this system of activity are defined as follows:

- **Tools:** The technological tool in this activity consisted of both WhatsApp Messenger and students' calculator.
- **Community:** the community of this system is all classroom that all the people in the class together form.
- **Subject:** In this system, the subject is the students of the class who, by dividing the work, advance the classroom in a participatory manner.
- **Object:** The object is the understanding of the concept of the irrational number by all students of the system.
- **Rules:** The teacher used WhatsApp to control locking and opening the group at pre-determined intervals).
- **Division of labor:** WhatsApp Messenger as a technology tool enabled the classroom teacher to be able to manage the virtual classroom teaching, meaning, who when students are able to contribute writing in the group chat and when the chat was momentarily disabled.

In addition to the components of the activity system that were defined, this system has two elements of tension and its modification, which are shown in Figures 6-1 and 6-2.

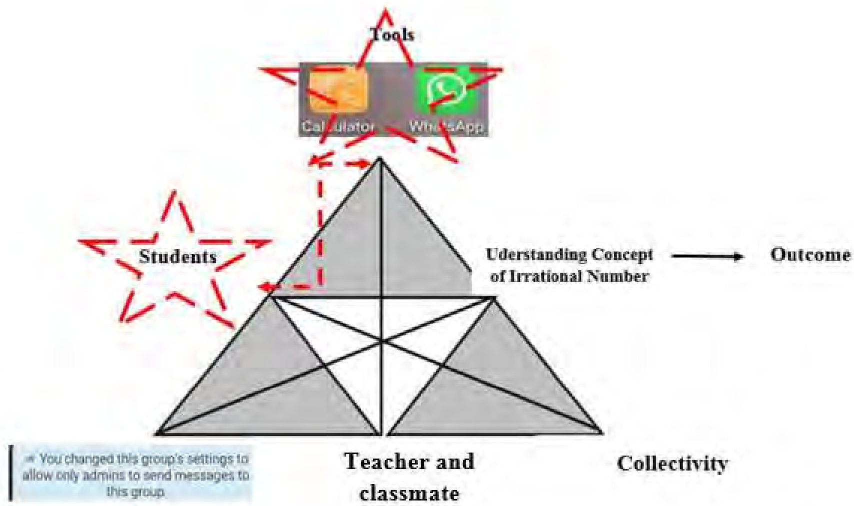

Figure 6-1. Creating tensions of levels one and two in the system activity of Online teaching the concept of the irrational number of $\sqrt{2}$

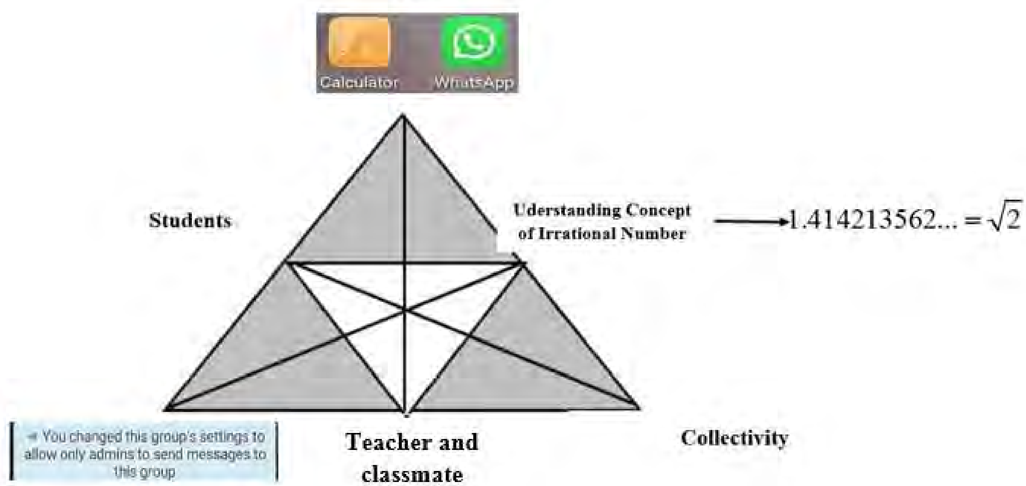

Figure 6-2. Modifying tensions of levels one and two in the system activity of Online teaching the concept of the irrational number of $\sqrt{2}$

1- 21:06 - Teacher: Find a number that when multiplied by itself result in 1?

2-$(21:06, 21:08)$ All students: number one and the negative one

3-21:08 -Teacher: Excellent. Next question: What is ‘the number’ that when multiplied by itself would give us 4?

4-21:08-Students 3, 1, 15,10, 20 and 8: 2, -2 and 2^2
5-21:09 - Teacher: Excellent. So far, you have noticed that the answer is both positive and negative. Our focus is on positive numbers:

$$(2)\times(2)=4 \quad ,(-2)\times(-2)=4$$

6-21:12 - Teacher, think more about the next question and do not answer too soon. Find a number that when it is multiplied by itself twice, will result in 2?
7-21:16 - Student3: 2×1
8-21:16 -Teacher: Very well done. This number is good, but since the two numbers are not the same, the answer is incorrect. The question is whether the two numbers are the same. Please others say their view of point about this question, even if it is not true.
9-21:16- Student8: $(-2)\times(-2)$
10-21:17 - Studentt20: 1.2×1.2
11-21:17 - Students 1 and 8: 4, 2^1
12-21:21 - Teacher: These 4 and 2^1 are good. But the problem is that multiplying by itself twice is 4, if the question is asked; the product of 2 numbers is 2.
13-21:21 - Student 1: 1.3×1.3
14-21:22 - Student20: 2^0
15-21:22 - Teacher: It is good hearing all your answers. I just observed that among the answers that are given, someone said we shall multiply 1.2 by 1.2. What is the product of this?
16-21:23 - Student20: 1.04
17-21:24 - Teacher: You can use your calculator
18-21:24 – Students 8,20,3 and 1: 1.44
19-21:25 - Teacher: This is a good number, but find a number whose product is closer to 2
20-21:26 – Students 8, 20, 15 and 3: $1.4\times1.4=1.96$
21-21:27 - Student1: $1.2\times1.2=1.44$
22-21:28 - Studen8: $1.5\times1.5=2.25$
23-21:28 - Teacher: Congratulations to all. So far, most of you have said 1.4 and one person has said 1.5. ($1.4\times1.4=1.96 \qquad 1.5\times1.5=2.25$)
24-21:29 - Teacher: Now tell me what is the next number in between. I will lock the group and open it for another two minutes. You can send your answer once the group is opened
25-21:29 - Close to this group
26-21:31 - Open to this group
27-21:31 - Studen8: This number is 1.42 .
28-21:31 - Student 1: 1.41
29-21:31 - Teacher: Let everyone to express their opinion.
30-21:31 – Students 3 and 20: it is between 1.40 and 1.50.
31-21:31 -Student15: What do you mean? I do not understand anything.

32-21:32 - Student20: I mean the next number that we multiply twice by itself is the result of two is between two numbers $1.40, 1.50$.
33-21:32 - Student3: Multiply by itself, see it approaching two?
34-21:32 -Student15: Ah!
35-21:32 - Teacher: So the next number is between $1.40, 1.50$. Now choose a number from one point four to one point five. Multiply it twice by itself to get two.
36-21:32 - Studen10:, this number $1.41 \times 1.41 = 1.9881$ is closer to two
37-21:32 - Studen8: I suggest these two numbers $1.412, 1.416$:

$$1.416 \times 1.416 = 2.005056 \quad 1.412 \times 1.412 = 1.993744$$

38-21:32 - Teacher: For now, say up to two decimal places. You do not need three decimal places now.
39-21:33 – Students 20, 3 and 10: 1.41
40-21:33 - Teacher: Why did you all choose one number? I choose 1.43
41-21:34 -Student15: This message was deleted
42-21:34 - Studen10: Because the other numbers that are multiplied by themselves are greater than2
43-21:34 - Student20: Because the product of other numbers is more than two.
44-21:34 - Teacher: So now, calculate the product of the numbers and see which of these numbers are closer to two? $1.41 \times 1.41 = ? \quad 1.42 \times 1.42 = ? \quad 1.43 \times 1.43 = ?$
45-21:34 - Close to this group
46-21:35 - Open to this group
47-21:36 – Students 3, 20 and 1: $1.41 \times 1.41 = 1.9881$
48-21:36 - Teacher: Send the answer to all these three multiplications above
49-21:37 - Student20: $1.42 \times 1.42 = 2.0164$
50-21:37 – Students8 and 5: $1.43 \times 1.43 = 2.0449$
51-21:37 - Teacher: Now that you multiplied these three numbers, the product of which number is closer to two?
52-$(21:37, 21:38)$ - All students: The product of the first number is closer to the number 2

$$1.41 \times 1.41 = 1.9891 \quad 1.42 \times 1.42 = 2.0164 \quad 1.43 \times 1.43 = 2.0449$$

53-21:38 - Close to this group
54-21:39 Teacher: Now tell me, the product of which two numbers gives us 2?? I will open the group in two minutes

$$1.41 \times 1.41 = 1.9891 \quad 1.42 \times 1.42 = 2.0164 \quad 1.43 \times 1.43 = 2.0449$$

55-21:41 - Open to this group
56-21:41 - Teacher: Please let everyone know, what are the two numbers whose product is the closest to 2?
57-$(21:41, 21:32)$ All students: Between these two numbers $1.41, 1.42$

58-21:44 - Teacher: So choose a number between these two numbers. Multiply twice by it so that its product is even closer to two. Use your mobile calculator: $1.410, 1.420$

59-21:47 – Students 8, 20 and 3: $1.414 \times 1.414 = 1.999396$

60-21:48 – Students 1 and 15: $1.415 \times 1.415 = 2.002225$

61-21:48 - Student 4: $1.4142 \times 1.4142 = 1.99996164$

62-21:49 - Teacher: So now multiply 1.415 by itself, the result is as follows?

$$1.415 \times 1.415 = 2.002225$$

63-21:50 - Student 1: gets bigger than 2

64-21:50 -Teacher: Well, $1.415 \times 1.415 = 2.002225$ and $1.414 \times 1.414 = 1.999396$. Let us choose the number 1.414. We are examining a number that is slightly bigger that 1.414 that when we multiply it by itself would result us in obtaining a number even closer to 2. Now let us discover the fourth digit of this number? What could it possibly be?

65-21:52 – Student 3: 1.4142

66-21:54 - Teacher: Others please feel free to express opinion. We are not looking for the right answer. We want all your opinions to reach a conclusion. Remember your opinion in important for all of us

67-21:55 - Students 1, 15, 20 and 6: $1.4142 \times 1.4142 = 1.99996164$ $1.4142 \times 1.4142 = 2.00034449$

68-21:56 - Teacher: Well, then you all got the fourth digit. Excellent, amazing. Can we still get closer to 2 by finding the fifth digit?

69-21:56 – Students 10, 20, 1, 15, 3: one or two

70-21:57 - Teacher: Now let everyone try test their guess

71-21:57 -Student3: If we choose the fifth digit of be two, then the product $1.41422 \times 1.41422 = 2.0000182084$ will be more than two

72-21:57 Student 6: I got the same answer

73-21:57 Students 3,1 and 20:The fifth decimal place $1.41421 \times 1.41421 = 1.99998992$ becomes1

74-21:59-Student15:If the fifth decimal place is doubled, this $1.41422 \times 1.41422 = 2.0000182084$ product is greater than two, so the fifth decimal place becomes one, that is 1.41421

75-21:59 - Teacher: Excellent. So the fifth decimal place (1.41421) is the number one

76-22:00 - Teacher: Now can you guess the sixth digit? I will open the group chat in two minutes

77-22:02 -Student15: $1.414211 \times 1.414211 = 1.99999275$. The sixth digit becomes one

78-22:02 Student 6: the number becomes 2 ($1.414212 \times 1.414212 = 1.99999558$)

79-22:02 - Student20: $1.414213 \times 1.414213 = 1.99999841$

80-22:02 - Student 1: So the sixth digit of the decimal point gets even closer to two

81-22:02 – Students 8 and 3: The sixth digit becomes three

82-22:03 - Teacher: So the sixth digit is 3: 1.414213

83-22:03 - Teacher: Find the seventh decimal place?

84-$(22:00, 22:08)$. All students: The seventh digit of the decimal is 5:

$$1.4142135 \times 1.4142135 = 1.99999982$$

85-22:08 - Teacher: 1.4142135. Calculate the eighth digit in the next session

$$1.4142135 \times 1.4142135 = 1.99999982$$

Session 2: at 4 April 2021

86-$(21:00, 21:06)$: All students in the eighth digit become 6:

$$1.41421356 \times 1.41421356 = 1.99999999$$

87-21:06 - Teacher: Excellent. Now find the ninth digit of this number

88-$(21:07, 21:11)$.All students: The ninth decimal digit is two:

$$1.414213562 \times 1.414213562 = 1.9999999989$$

89-21:11 - Teacher: Excellent. You are all coming up with an answer, fine

90-21:12 - Teacher: The ninth digit was obtained. Now, what would be the tenth digit?

91.$(21:11, 21:13)$ All students: The tenth digit of the decimal number becomes 3: 1.4142135623 $(1.4142135623 \times 1.4142135623 = 1.99999999979325598129)$

92-21:15 - Teacher: Excellent, very well! Now, find the eleventh digit of 1.4142135623

93-21:15 - Student4: No

94-21:15 - Student10: The calculator gives an error

95-21:15 - Student4: I cannot count with the calculator, the calculator gives an error

96-21:15 -Student15: No more than me. He says enough.

97-21:15 - Student3: I cannot calculate more than ten decimal places with this calculator

98-21:15 - Teacher: How was the calculator?

99-21:16 -Student15: the calculator tired!!!!

100-21:16 - Student3: No more than 10 digits

101-21:16 - Student4: No, not the eleventh digit. Unless we have a different program to utilize.

102-21:16 - Teacher: Did your calculators give an error?

103-21:16 - Student 1: Yes, the calculator stopped me

104-21:16 -Student15: made worse than me

105-21:17 - Teacher: Well, you finally challenged the calculator

106-21:17 - Student 1: The calculator gives an error

107-21:17 - Teacher: Now how do we get the eleventh digit?

108-21:18 - Student3: I do not know. This is an imaginary number

109-21:18 - Student10: We've got to use our minds

110-21:18 -Student15: How many decimal places do you want to get?

111-21:18 - Student4: Let's go, we do not have the patience to calculate this number

112-21:19 - Teacher: Try calculating it without a calculator

113-21:19 - Student 1: No matter how much we multiply, we will not reach two

114-21:19 -Student15: By the time we finish multiplying will be morning.

115-21:19 - Student4:?
116-21:19 - Student3: Very hard. How do we find the eleventh decimal place?
117-:20 -Teacher: Do you think the eleventh digit of the number 1.4142135623 can be guessed or not? What is this number called when no matter how much depth we go into, we cannot multiply it by itself to get 2? Think for two minutes
118-21:22 - Student3: No, it can't
119-21:23 -Student15: irrational number
120-21:23 - Studen4: I think this number is an irrational number
121-21:23 - Teacher: What does an irrational number mean?
122-21:23 - Student 1: We cannot talk about it with certainty
123-21:24 - Student20: It means a number whose decimal point becomes infinite
124-21:24 - Student4: irrational number means unknown or does not tell you the exact meaning.
124-21:24 -Teacher: Do you think this number is the last decimal place of that number 1.4142135623?"
125-21:25 – Students 4, 20, 15, 3 and 1: No
126-21:25 - Student4: Because it is an irrational number
127-21:26 - Student4: to think for the rest of our live
128-21:26 - Student17: This number is vague
129-21:26 -Student15: Not known. This number is vague for us
130-21:27 - Teacher: The result. Multiply a number by itself. The result is two. An unknown number. We only know its decimal places. We have ten decimal places for this account number. The calculator can no longer count this number, as some calculators cannot count more than ten decimal places. So this number is unknown or it is the same as the irrational number. Write the symbol of this irrational number.
131-21:31 – Students 4, 1,6, 15,17 and 20: *Square root* of 2
132-All students say *Square root* of 2: $\sqrt{2}$

Figure 6. Creating tension and modifying it in the online teaching activity system the concept of the irrational number of $\sqrt{2}$

In this section, tensions and their modification are discussed in detail by mentioning the evidence. Students gradually understood and constructed the concept of irrational number, which is described in the process illustrated in Figure 6. At the beginning of this process in line 1, the teacher asks the students the first question,*"* **Find a number that when multiplied by itself result in 1?***"*. Asking this question engaged students to collaborate in the virtual classroom, and to came up with the answer of the number one and the negative one. Next, the class teacher raises the level of the previous question a little, and this time asks the question, "**What is 'the number' that when multiplied by itself would give us 4*?*" (line3). Students offered their answers in line 4.

The first tension is created by the teacher's question "Find a number that when it is multiplied by itself twice, will result in 2**?**" (line6). This tension is between students, which is level 1 from tensions of the activity system. This question engages students in the classroom and causes students to come up with different answers that are not close to the answer to the question. Students express various answers such as 2^0, 2^1, 1.3 and 1.2. Among the answers given by the students, the answer of 1.2 (line10) provides a clue to make the square root of 2, which the teacher uses it to ask students to multiply 1.2 by 1.2 “It is good hearing all your answers. I just observed that among the answers that are given, someone said we shall multiply 1.2 by 1.2 "What is the product of this?" (line15). Students calculate the product of these two numbers as 1.44. In line 19, the teacher asks the students to “**find a number whose product is closer to 2”.** Most students have chosen 1.4 as their answer with the exception of only one student obtaining 1.5 (line22). The same different answers of the students8 helped to complete the class discussion and redirected the whole class’s attention to get closer to the answer of Q_1 question which subsequently reduced the intensity of the tension that this question had created among the students.

In line 29, the teacher asks the student to get the second digit of 1.4. Students give answers of 1.41, 1.42. "I do not understand anything," says one of the students (student 15, line31). This opposite answer of the student helps to clarify the answer of the question and causes one of the students to guide the above student, which means the next number, the number between 1.40 and 1.50 (line32). The same guidance causes the student to notice and say the word "Ah" (line34). Letting students talk can address any mis-conceptions or clarify their understandings (Farsani, 2015).

This causes other students to express the three numbers 1.41, 1.42 and 1.43. Next, the teacher asks them to calculate the product of these three numbers 1.41, 1.42 and 1.43 when multiplied by themselves and to explore which product is closer to 2 (line44). All students calculated (using calculator) and suggested that only the product of 1.41 is the closest to 2, and the products of both 1.42 and 1.43 are greater than 2. Therefore, all students on line 52 choose 1.41. So far, students have discovered two decimal places in the answer to the question. Next, for the third decimal place, students present two numbers 1.415 and 1.414, which by calculating the multiplication of these two numbers by themselves and comparing their answers; students choose the next number to be 1.414. Students then obtained the fourth decimal place of this number in 5 minutes (lines65 to 68), which became 1.4142. As can be seen, the students were able to calculate the fourth decimal place in less time by collaborating.

The teacher asked students to test their guess to get the fifth decimal place of this number. Most students suggest the numbers 1 and 2 for the fifth decimal place, which was eventually, reported the number 1 as the fifth decimal place. Calculations were a key to the students in obtaining the fifth place value (lines71 to 75). By getting the number 1.41421, the teacher asked the students to find the sixth decimal place. The students, with the spirit of their collaborative participation achieved the seventh, eighth, ninth and tenth digits in a much shorter time. It is important to note that students obtained the first 4 digits (first, second, third, and fourth) in 39 minutes, while they obtained the next four digits (fifth, sixth, seventh, and eighth) in 20 minutes. Student’s flow of progression consists of their use of collaboration, the use of WhatsApp messenger as a platform where students could both express and view other peers' responses. As a consequence, they could evaluate their responses in relation to their peers' responses:

- Students gain the number 5(1.4142135) as the seventh digit of the number 1.414213, in 8 minutes $(22:00, 22:08)$.
- Students gain the number 6(1.41421356) as the eighth digit of the number 1.4142135, in 6 minutes $(21:00, 21:06)$.
- Students gain the number 2(1.414213562) as the ninth digit of the number 1.41421356, in 4 minutes $(21:07, 21:11)$.
- Students gain the number 3(1.4142135623) as the tenth digit of the number 1.414213562, in 2 minutes $(21:11, 21:13)$.

Students were able to overcome the first tension (T_1) created by question **Q_1** and obtain the approximate answer to question **Q_1** at 1.4142135623 with the help of a calculator. This answer shows that students have been able to gain a greater understanding of the concept of square root of 2 by modifying this tension to a large extent. Next, the teacher created the second tension (T_2) in the system by asking the second question (Q2), "*Calculate the eleventh digit of 1.4142135623*" (line137). This tension (T_2) is of the first level tension, e.g. it is of the intra-component tension of the activity system. This tension has challenged technology. When the teacher asks the students to count the eleventh digit of the number as 1.4142135623, the answer given by the students indicates the tension in the technology itself.

For example, phrases such as "the calculator give an error, the calculator tired! (line99). The calculator gives an error (line106), the calculator stopped me, etc. (line103)" illustrates that the calculator as an instrument of technology itself is challenged and which subsequently created a technological tension. On the other hand, the design of the second question (**Q_2**), in addition to creating a second tension in technology, has caused in Figure 6-1 the third tension (T_3) between students and technology (calculator) which is a type of second level tension of the activity system. This tension can be expressed in sentences such as "I can't calculate more than ten decimal places with this calculator, I can't count with the calculator," (lines95) is apparent throughout the lesson. The teacher uses the same sentences of the students to correct the tensions and asks them to answer the question, "Do you think the eleventh digit of the number 1.4142135623 can be guessed or not? (line117)". By asking this question enabled showing students' understanding by expressing comments like:

- *21:22 - Student3: No, it can't (line 118)*
- *21:23 -student15: irrational number (line 119)*
- *21:23 - Studen4: I think this number is irrational number (line 120)*
- *21:23 - Teacher: What does an irrational number mean? (line121)*
- *21:23 - Student 1: We cannot talk about it with certainty (line122)*

- *21:24 - student20: It means a number whose decimal point becomes infinite (line1623)*
- *21:24 - Studen4: irrational number means unknown or does not tell you the exact meaning*. *(line124)*

Students refer to an important point in these sentences: the word "irrational number" which they explicitly refer to and define. Students define this number as follows*:" An irrational number is an unknown number that cannot be talked about with certainty; its decimal number becomes infinite" (line123)*. Next, the teacher asks the students the question, "Do you think this number is the last decimal place of that number 1.4142135623?" (line124), causes students to complete the concept of an irrational number that is unknown (lines125). In general, in this process, the student concluded that the answer to question Q_1 is an irrational number. In the last part of this process, the teacher asks the students to specify the mathematical symbol of this irrational number (line130). The symbol that the students introduce is the symbol of the square root of two ($\sqrt{2}$). Students were empowered to construct the conception of the irrational number of $\sqrt{2}$.

Discussion and Conclusion

This study explored the concept of the *square root* of two during the Coronavirus pandemics that he algebraic proof is as follows:

Agarwal, & Agarwal (2021) use inverse reasoning to prove it. Suppose radical two is an expressive number. Therefore, there are two integer numbers, p / q, where q is zero and p, q are co-primes that have the following relationships.

$$\sqrt{2} = \frac{p}{q} \Rightarrow p = \sqrt{2}q \Rightarrow p^2 = 2q^2 \Rightarrow \begin{cases} p = 2k \\ q = 2k \end{cases} \quad (1)$$

Given that Equation (1), p, q are multiples of two numbers, this is in contradiction with their being prime. Therefore, radical two is an irrational number.

For this purpose, technology was used to explore this concept. In this regard, students using the concept of irrational numbers (lines1, 3, and 44) could determine the approximate amount of the square number of two. This is consistent with the findings of Agwu (2007). He used the upper and lower bounds to teach the irrational number in his study. However, the difference between the two studies can be expressed as follows: In the present study, the upper and lower bounds were used to develop a conceptual understanding of the irrational number, but in Agwu (2007), the upper and lower bounds were used to develop the procedural understanding of the irrational number.

As mentioned in the introduction, researchers use different methods to teach irrational numbers (Sirotic & Zazkis, 2007a, Shinno, 2018; Patel & Varma, 2018; Protasov et al., 2009; Zazkis, 2005). Despite such showed teaching methods, students have difficulty with irrational numbers (Sirotic & Zazkis, 2007a). But the findings of this study showed that with the help of each other and with the teacher's guidance, students can reach a comprehensive and uniform understanding of the definition of the irrational number.

The findings showed that by combining the two technologies of calculator and WhatsApp messenger, the first in the field of mathematical calculations and the second in the field of virtual classroom and class discussion, can teach hard and complex mathematical concepts. This finding is consistent with Dogruer and Akyuz (2020)'s the results of the study. Their study showed that the use of technology can improve students' understanding of difficult mathematical concepts and improve their learning comprehension. This finding is in line with Attard & Holmes, (2020)'s the results of the study. In their study, they found that using technology provides an opportunity for students to interact with each other in a variety of ways.

In this study, the Activity Theory activity system was used to analyze classroom interactions in order to better classroom interactions that is formed in the development of the concept of an irrational number. A review of other studies shows that researchers used the activity system to analyze classroom interactions (Salloum & BouJaoude 2020; Huang & Lin, 2012). In their study, they used Activity Theory to examine the understanding of classroom interactions and showed that through these activity systems of this theory, classroom interactions can be analyzed and understood, and developed in the field of education can be created. Huang & Lin (2012) used Activity Theory to analyze classroom activities.

Mulenga & Marbn (2020) showed that Covid-19 is the gateway to digital learning in mathematics education. But the problem with teaching during Covid-19 pandemic is that teachers have limited skills in using technology such as mobile phones and other Medias such as WhatsApp for the teaching and learning of mathematics. Therefore, addressing the issue of technology in teacher education is a necessity that should be addressed to improve the skills of teachers. Another point that was shown in the methodology of the study was the use of WhatsApp messaging software to generate study data. With the outbreak of the Coronavirus, it is practically impossible to use the traditional form of research collection that was in the past today, and it is not possible to use this method to collect research data. For example, researchers used to meet face-to-face with research participants to conduct data and conduct audio and video interviews in the past. In addition, in this method, the researcher had to spend a long time producing study data to be able to produce textual study data from audio and video interviews. But this study introduced a new way of generating study data in which all classroom conversations between teacher and student, along with time and image, were presented in a coherent text. Therefore, this study showed that technology can be used to teach mathematical concepts, but it can also be used to study data collection and speed up the research process.

Another important point that this study showed was the challenge of technology. The findings of this study showed that when students could not obtain the eleventh decimal digit of the radical number two ($\sqrt{2} = 1.4142135623....$) using the multiplier of the calculator. In this regard, this study used the same issue to create tension among students and tension in technology (calculator). Subsequently, with the guidance of this teacher, modification of tensions was directed towards the development of an understanding of the concept of square root. Eventually, students were able to construct the concept of the irrational number. In constructing the concept of the irrational number, students constantly tested their conjectures about radical two decimal places using a calculator. This process continued up to ten decimal places of the irrational number ($\sqrt{2}=1.4142135623....$), but from the eleventh decimal place of this number, that is $\sqrt{2}=1.4142135623?$, the students realized that due to the inability of the calculator to multiply, this number and the next digits of this number cannot be guessed and tested. Hence, the students realized that the answer to the math problem, "Find a number that is multiplied twice by itself, is the result of two" is an irrational number whose whole numbers cannot be guessed and shown. In the following, the students' definition of the irrational number is similar to what is found in the research literature as the definition of the irrational number. According to Patel and Varma (2018), the irrational number is a number that, unlike the rational numbers, cannot be measured; its decimal digits are non-repetitive and infinite. Similar to this definition, students in the study concluded that the decimal number of this number is continuous or in other words infinite. On the other hand, when students realized that this number could not be guessed, they realized that this number could not be measured. Therefore, it can be said that the interactive and constructive atmosphere in the virtual classroom was provided to students by technology, which played a significant role in the development of this mathematical concept.

In the end, the challenges we faced in using the technology were as follows: the limitation of the calculator as a technology tool in multiplying numbers above 11 decimal places, the limitation of WhatsApp messenger in showing images of study participants, the limitation students have access to a computer and use a cell phone instead, not seeing how students feel when they cannot do the calculation.

Acknowledgments:

This work has been supported by Mahani Mathematical Research Center, Shahid Bahonar University of Kerman, Kerman, Iran and the Chilean National Agency for Research and Development ANID/PAI 77200008.

References

[1] Attard, C., Holmes, K. (2020). An exploration of teacher and student perceptions of blended learning in four secondary mathematics classrooms. *Educational Studies in Mathematics.* https://doi.org/10.1007/s13394-020-00359-2

[2] Agwu, N. (2007). Using the Concept of Upper and Lower Bounds to Find Square Roots. *Mathematics Teaching-Research Journal*. Vol 2, pp. 1-10

[3] Agarwal, R. P., & Agarwal, H. (2021). Origin of Irrational Numbers and Their Approximations. *Computation*, *9*(3), 29

[4] Barhoumi, C. (2015). The Effectiveness of WhatsApp Mobile Learning Activities Guided by Activity Theory on Students' Knowledge Management. *Contemporary Educational Technology*, 6(3), 221 – 238.

[5] Borba, M.C. (2021). The future of mathematics education since COVID-19: humans-with-media or humans-with-non-living-things. *Educational Studies in Mathematics* (2021). https://doi.org/10.1007/s10649-021-10043-2

[6] Breda, A., Farsani, D., Miarka, R. (2020). Political, technical and pedagogical effects of COVID-19 in mathematics education: An overview of Brazil, Chile, and Spain. *INTERMATHS: Revista de Matemática Aplicada e Interdisciplinar*, 1 (1), 3-19

[7] Breda, A., Seckel, M. J., Farsani, D., Silva, J. F., and Calle, E. (2021). Teaching and learning of mathematics and criteria for its improvement from the perspective of future teachers: a view from the onto semiotic Approach. *Mathematics Teaching Research Journal*

[8] Cahapay, M.B. (2020). Rethinking education in the new normal post-COVID-19 era: A curriculum studies perspective. *Academia*, 4(2). https://doi.org/10.29333/aquademia/8315

[9] Chesnais, A. (2021). Enhancing classroom discourse about measure to foster a conceptual understanding of geometrical practices. *ZDM Mathematics Education*. https://doi.org/10.1007/s11858-021-01255-0

[10] Dogruer, S.S., Akyuz, D. (2020). Correction to: Mathematical Practices of Eighth Graders about 3D Shapes in an Argumentation, Technology, and Design-Based Classroom Environment. *International Journal of Science and Mathematics Education*, 18, 1507 .https://doi.org/10.1007/s10763-020-10055-z

[11] Eyyam, R., & Yaratan, H. S. (2014). Impact of use of technology in mathematics lessons on student achievement and attitudes. *Social Behavior and Personality: An International Journal*, 42(1), 31S–42S

[12] Engestrom, Y. (2001). Expansive learning at work: Toward an activity theoretical reconceptualization. *Journal of Education and Work*, 14(1), 133–156.

[13] Engestrom, Y. (2015). *Learning by expanding: An activity-theoretical approach to developmental research (2nd ed.).* Cambridge, UK: Cambridge University Press.

[14] Fischbein, E., Jehiam, R. and Cohen, D. (1995). The concept of irrational numbers in high-school students and prospective teachers. *Educational Studies in Mathematics* 29, 29–44.

[15] Hashim, N., and Jones, M. (2007). *Activity Theory: A framework for qualitative analysis*. Paper presented at the 4th International Qualitative Research Convention (QRC).

https://www.statista.com/statistics/258749/most-popular-global-mobile-messenger-apps/. Accessed January 20, 2021

[16] Huang C-H, Lin F-L (2013) Using activity theory to model the Taiwan Atayal students' classroom mathematical activity. *International Journal of Science and Mathematics Education* 11(1):213–236. doi:10.1007/s10763-012-9381-9

[17] Jurdak, M. E. (2006). Contrasting perspectives and performance of high school students on problem solving in real world, situated, and school contexts. *Educational Studies in Mathematics,* 63(3), 283–301. doi:10.1007/s10649-005-9008-y

[18] Kidron, I. (2018). Students' conceptions of irrational numbers. *International Journal of Research in Undergraduate Mathematics Education*, *4*(1), 94-118

[19] Patel, P., and Varma, S. (2018). How the abstract becomes concrete: Irrational numbers are understood relative to natural numbers and perfect Squares. *Cognitive Science*. Published online. https://doi.org/10.1111/cogs.12619

[20] Satianov, P. (2015). Using Calculators in Teaching Calculus. *Mathematics Teaching Research Journal Online*, *7*(4). Retrieved from https://commons.hostos.cuny.edu/mtrj/wp-content/uploads/sites/30/2018/12/v7n4-Using-Calculators-in-Teaching-Calculus.pdf

[21] Sirotic, N., Zazkis.R. (2007a). An Irrational Numbers: The Gap between Formal and Intuitive Knowledge. *Educational Studies in Mathematics* .65, 49–76.

[22] Sirotic . N. and Zazkis. R. (2007b) Irrational numbers on the number line – where are they? *International Journal of Mathematical Education in Science and Technology*, 38:4, 477-488, DOI: 10.1080/00207390601151828

[23] Salloum, S., and BouJaoude, S. (2020). Understanding Interactions in Multilingual ScienceClassrooms through Cultural-Historical Activity Theory(CHAT): What Do Contradictions Tell Us? *International Journal of Science and Mathematics Education,* https://doi.org/10.1007/s10763-020-10114-5

[24] Shinno, Y. (2016). Reification in the Learning of Square Roots in a Ninth Grade Classroom: Combining Semiotic and Discursive Approaches. *International Journal of Science and Mathematics Education,* 16, 295–314 (2018). https://doi.org/10.1007/s10763-016-9765-3

[25] Kaput, J. J. and Thompson, P. W. (1994). Technology in mathematics education research: The first 25 years in the JRME. *Journal for Research in Mathematics Education, 25*(6), 676-684

[26] Mulenga, E. M., and Marbán, J. M. (2020). Is COVID-19 the gateway for digital learning in mathematics education? *Contemporary Educational Technology*, 12(2), ep269. https://doi.org/10.30935/cedtech/7949

[27] NCTM (2000). Principles and standards for school mathematics. Reston, Va.: NCTM

[28] Protasov, V., Applebaum, M., Karp, A., Kasuba, R., Sossinsky, A., Barbeau, E., et al. (2009). Challenging problems: Mathematical contents and sources. In E. J. Barbeau and P. Taylor (Eds.), *Challenging mathematics in and beyond the classroom: The 16th ICMI study (pp. 11–51)*. New York: Springer

[29] Rosenthal, D., Rosenthal, D., & Rosenthal, P. (2014). *A readable introduction to real mathematics*. Springer

[30] Russell, D. R. (2009). Uses of Activity Theory in written communication research. In A. Sannino, H. Daniels, and K. D. Gutiérrez (Eds.), *Learning and expanding with Activity Theory (pp. 40–52)*. New York, NY: Cambridge University Press

[31] Surowiecki, J. (2004) *The Wisdom of Crowds*. New York: Doubleday. Anchor Books

[32] Yamagata-Lynch, L. C. (2010). *Activity systems analysis methods.* Boston: Springer

[33] Zazkis, R. (2005). 'Representing numbers: Prime and irrational', *International Journal of Mathematical Education in Science and Technology* 36(2–3), 207–218