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Understanding "proportion" and mathematical identity: A study of Japanese elementary school teachers

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Studies have found that problems exist with respect to elementary school teachers' understanding of proportions and their knowledge of the appropriate methods for teaching the concept. This study aims to help aspiring elementary school teachers form a healthy mathematical identity and deepen their understanding of mathematics. This quantitative study employed the descriptive-research survey method, surveying 86 students in 2019 and 110 students in 2021. Data were gathered using a survey questionnaire designed by Kumakura et al. (2019), with minor modifications made by the author. A major finding was that many students want to become elementary school teachers but are uncomfortable with the concept of proportions. Another important finding is that it is a challenge for students who wish to become elementary school teachers at a traditional school (University A) to hone their ability to use mathematical expressions and develop their sense of quantity. The findings suggest that it is important to help such students understand the content and refine their expressions.

Keywords: elementary teacher training, mathematical identity, proportion

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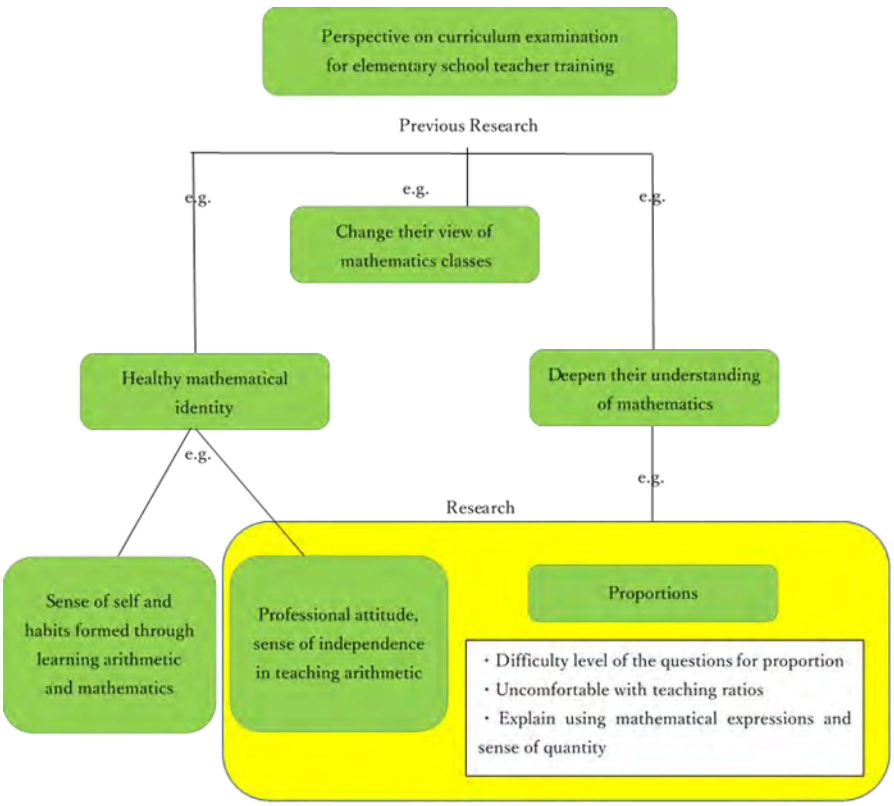

1 Introduction

On December 26, 2019, an editorial in the *Mainichi Shimbun*, a daily newspaper in Japan, asserted that the sinking teacher applicant ratio is putting the future of Japanese children at risk: "*There were on average 2.8 applicants competing for each available position at Japan's public elementary schools in this academic year's employment exam cycle, tying a record low ratio set in 1991, according to the Education Ministry*."

This demonstrates the high expectations regarding quality assurance in elementary school teacher training in Japan. In particular, the role of private universities is significant because, as of 2017, the number of universities with accredited programs for a first-class license to teach in elementary school stood at 183 private universities, 52 national universities, and four public universities. However, the teacher training programs at private universities in Japan face several challenges; in particular, few students choose mathematics as part of their university entrance exams. There is also a shortage of subjects related to mathematics and arithmetic at these universities, and the ratio between employment examinations and other examinations available to prospective teachers is decreasing. For these reasons, a method for examining the mathematics education curricula which is being used to train elementary school teachers at private universities in Japan is urgently needed.

In Japanese elementary school curricula, simple unit fractions, such as 1/2 and 1/3, are taught in the second grade. The meaning of fractions and decimals—other than unit fractions—is taught in the third grade. In the fourth grade, addition and subtraction of equal denominators, doubling with decimals, addition and subtraction of decimals, and proportions for simple cases are taught. Multiplication and division of decimals, addition and subtraction of different denominators, proportions of two different quantities, and percentages are taught in the fifth grade. Finally, multiplication and division of fractions, proportions, and ratios are taught in the sixth grade. More specifically, in the fifth grade, students systematically learn multiplication and division of decimals, addition and subtraction of different denominators, the ratio of two different quantities, and percentages, whereas, in the sixth grade, students learn multiplication and division of fractions, proportions, and ratios. In the fifth grade, the concept of multiplication can be used in a broader range of situations and meanings by considering its relationship with division and when the multiplier is a decimal. In other words, students learn that $A = B \times p$ (second usage). However, middle school students and beyond do not have any units specifically on proportions. Instead, they work only on problems involving fractions, decimals, ratios, and proportions in units that focus on equations, shapes, and the use of data. Because these older students no longer study proportions, the level of understanding proportions among prospective teachers in Japan may be inadequate, which can affect their mathematical identity. This study seeks to explore and clarify the level of conceptual understanding of “proportions” among students who hope to become elementary school teachers in the future and who are attending a private university to undergo training and obtain suggestions for forming students’ mathematical identities.

2 Theoretical framework

2.1 Mathematical identity

Studies on identity in mathematics education include those that view identity from the perspective of participation and positionality—constructed through participation and involvement in social groups (Rabe & Wenger, 1993). Others regard identity as a narrative (Sfard & Prusak, 2005), while a final group includes affective constructs such as emotions, attitudes, and beliefs (Bishop, 2012). Aguirre et al. (2013) define mathematical identity as “*dispositions and beliefs about the development of the ability to use mathematics in mathematical and life contexts*” (p. 14).

In Japan, Takahashi (2020) defines this identity as the self-awareness of arithmetic and mathematics held by elementary school students, while Nishi (2017) hypothesized that a positive identity would be an outcome of mathematics education, in a study conducted among first-year university students in the faculty of education at Hiroshima University. This study examines mathematical identity among current and prospective elementary mathematics teachers in Japan in two contexts: the context of understanding arithmetic and mathematics for students who aspire to become elementary school teachers; and the context of their transition from being a student to a teacher. Mathematical identity is a concept that includes self-awareness of arithmetic and mathematics, the subjective awareness and sense of what the job of a teacher of arithmetic and mathematics entails, how one is performing or wants to perform, confirmation of identity through one’s occupation as a teacher, and professional attitude and ability to utilize and nurture one’s identity. Kambara (2021) defines the mathematical identity of students who want to become elementary school teachers as “a sense of self and habits formed through learning arithmetic and mathematics, as well as a professional attitude that makes the most of one’s own personality and sense of independence in teaching arithmetic as an elementary school teacher" (p. 334). It is also necessary to understand mathematical identity as a concept that includes confirmation of identity through one's profession as a teacher and professional goal, to maximally benefit from and nurture the identity. This study identifies the following needs: (1) to help students develop a healthy mathematical identity that allows them to positively experience mathematics and find new ways to learn the subject; (2) to transform their attitude about mathematics from a teacher-driven, standardized view of the classroom; and (3) to deepen their understanding of mathematics.

In this study, I use the case of "proportions" to explore some of the issues pertaining to points (1) and (3) in more depth. First, the teaching of proportions has been a longstanding issue in arithmetic education in Japan, and despite the availability of many studies and practices, a few problems with respect to the understanding of proportions have been noted (Kumakura et al., 2019; Yoshizawa, 2019). Second, for those who aim to become elementary school teachers, understanding the meaning of proportions and its appropriate teaching methods is mandatory. The purpose of this study is to clarify and investigate the level of understanding regarding the concept of "proportions" among students who want to become elementary school teachers and are undergoing training at a private university and obtain suggestions for the formation of students' mathematical identities.

2.2 The concept of proportions

In mathematics, the concept of proportions is fundamental to many topics. A proportion, p, expresses the number of times when quantity A is compared with quantity B, where A and B are two similar types of quantities. B is called the base quantity, and A is the quantity to be compared. Proportions include the following relations:

- $p = A/B$ (first usage),
- $A = B \times p$ (second usage),
- $B = A/p$ (third usage).

The topic of proportions includes sub-topics, like percentages and "buai," and number representations, such as decimals and fractions. To express A as a percentage of B, base quantity B is considered in terms of 100 units. A percentage is a ratio that compares a number to 100, and its symbol is %. Under the sub-topic "buai," base quantity B is considered in terms of 10 units; and special terms like "wari,", "bu,", and "rin,", are used. In Japan, students learn that 1% is 0.01 and do not relate it to fractions. This study will mainly focus on proportions expressed as percentages (%).

Understanding the knowledge level of mathematics teachers is an international endeavor. Even in the last decade, there has been much research on teachers' knowledge of proportions and ratios (e.g., Howe, 2013; Monteiro, 2003; Olanoff et al., 2014). In Japan, most studies on the semantic understanding of proportions have been conducted in elementary schools. A keyword search on CiNii for "understanding of proportions, junior high school students, high school students, and university students" demonstrated only two papers by Higuchi (2005) and Kumakura et al. (2019). The former was a study of college students, but it compared the results of one written test to find the rate of increase using the first usage of the ratio with the results of additive calculus of different denominators and fractions, and the basic knowledge of information literacy. It did not investigate the semantic understanding of percentages among college students. The latter study was conducted on junior high school and high school students, and the items were developed based on previous studies to investigate students' "deep understanding of proportion," and detailed discussions were carried out. Following this study, I decided to investigate the situation regarding the understanding of the concept of "proportion" among students who want to become elementary school teachers through survey questions taken from the survey conducted by Kumakura et al. (2019; Tables 1 and 2).

3 Research method

3.1 Measures

I investigated the situation regarding understanding percentages among students who want to become elementary school teachers. In doing so, I referred to the questionnaire and survey questions by Kumakura et al. (2019; Table 1). This quantitative study employs a descriptive-research survey method. The survey questionnaire by Kumakura et al. (2019) was used to collect data pertaining to students' understanding of proportions.

The questionnaire by Kumakura et al. (2019; Table 1) consisted of eight items (I) that measured understanding of "proportions" on the following dimensions: the need to understand proportions or utility of proportions (I1 and I2), the meaning of proportions (I3–I5), and attitude toward proportion problems (I6–I8). The investigator added a question on "confidence in teaching proportions," so the final questionnaire had a total of nine items, which were used to investigate the students' mathematical identities with a special focus on proportions. The author then compared the data from the college students with the data from high school students in Kumakura et al. (2019) to clarify university students' understanding.

Table 1. Mathematical identities with a special focus on proportions

For the following nine items related to proportion, please select the option that best expresses your response to the questions: (a) Strongly agree (b) Agree (c) Don't think so (d) Don't think so at all
(1) The concept of percentages (%) is applicable to subjects other than mathematics. (2) Knowledge of percentages (%) is necessary for daily life. (3) If you express something as a percentage (%), you can see how much of the whole it is. (4) Expressing results in terms of percentage (%) shows changes such as increases and decreases. (5) We can compare two quantities by expressing them as percentages. (6) Solving percentage (%) problems is fun. (7) I am good at solving percentage (%) problems. (8) I may try to solve problems in daily life by using the concept of percentages I have learned (%). (9) I am confident in teaching percentages (%).

Source: Author's addition to Kumakura et al. (2019)

The survey also included a questionnaire consisting of six major questions related to different types of proportions: Questions 1 and 2, respectively, correspond to the second and third usages described in section 2.2. Question 3 asks the respondent to contrast quantities. Questions 4 and 5 are pp-type questions (these are questions in which the reference quantity [A] is multiplied by the percentage [p] to obtain the comparison quantity [B]; then, the reference quantity [B] is multiplied by the percentage [p'] to obtain a new comparison quantity [C]). Finally, question 6 is a p/p-type question (in this type, the comparison quantity [B] is divided by the percentage [p] to obtain the reference quantity [A]; then, the comparison quantity [A] is divided by the percentage [p'] to obtain the reference quantity [C]). The questions are illustrated in Table 2.

Two survey rounds were conducted. In the first survey (2019), I analyzed the percentage of correct answers to explore the understanding of percentages among students who want to become elementary school teachers. The second survey (2021) used the same questionnaire as the first, and I qualitatively analyzed the writing of survey question 6. Through these surveys, I tried to get a deeper understanding of the students' percentage understanding and gain a perspective to guide them. Furthermore, I tried to obtain suggestions for fostering their academic identity.

Table 2. Survey questions

Q1. If a cake with a regular price of 2000 yen is sold at a 30% discount, what is the price after the discount? Solve the question and mention all the steps.

Q2. Answer the following question:

A company is selling 180 g of canned salmon, the weight of the salmon is 20% more than that sold in the previous year. What was the weight of the canned salmon sold in the previous year?

(i) Solve the question mentioning all the steps, with special reference to how you calculated the content of the can sold in the previous year.

(ii) If you were to give an easy-to-understand explanation to a friend who did not understand how to solve the question, how would you explain it using a diagram, table, or figure? Draw/write this below. However, it is not necessary to use all the figures, tables, and pictures.

Q3. The following table shows the approximate land areas of Finland and Japan:

Country name	Finland	Japan
Land area	34 (million km^2)	38 (million km^2)

(i) Write an equation to find the approximate percentage of the land area of Finland with respect to the land area of Japan. However, you do not need to find the answer.

(ii) Write an equation to find the approximate percentage of the land area of Japan with respect to the land area of Finland. However, it is not necessary to find the answer.

Q4. At Junior High School A, 30% of the students commute to school by bicycle, and 60% of them are boys. What percentage of the school students are boys who ride bicycles to school? Solve the question with complete steps.

Q5. We looked at the annual number of visitors to the zoo from 2015 to 2017; the number of visitors in 2016 increased by 10% compared to the number of visitors in 2015. How did the number of visitors in 2017 compare to the number of visitors in 2015? Choose one correct answer from the options given below, circle it, and write the reason for your choice.

a. Increased b. No change c. Decreased

Q6. What is the ratio of forest area to total area in town A? Write the answer and how to find it in a way that elementary school students can understand. The forest area in the present year is the same as it was 10 years ago.

Source: (Kumakura et al., 2019)

3.2 Procedure

The first survey was a collective survey conducted in May 2019. It was administered in person at the university where it was also answered and collected. The data collected from university students were then compared to high school students who want to become teachers. In the second round, a survey was conducted in July 2021, with a slight modification to question 6 from “Write the method and answer” to “Write the method and answer in a way that elementary school students can understand.” This was changed to a more pedagogical and practical expression to measure the mathematical identity of prospective elementary school teachers. However, the answers to the question remained the same. Handouts containing practice exercises were distributed to the students present on campus. The survey questionnaires were distributed among the students. The students then prepared their answers at home and submitted them a week later. Both surveys were administered to students who were attending the same university.

3.3 Participants

The first survey was conducted with 86 third-year students at private universities studying to be elementary school teachers who agreed to participate in the research. The only arithmetic course taken by these students was arithmetic content theory in their first year. Some of the results of this first survey were reported by Kambara (2019). The second survey was conducted with 110 third-year students working toward a Bachelor of Education at the same private university who agreed to participate in the study. These were from a total of 135 students aspiring to become elementary school teachers at that private university. Private university A is a traditional school that has produced many teachers, and the number of graduates from here who find jobs as teachers is one of the highest among universities in the Kansai region of Japan.

In accordance with the code of ethics, the participants were asked to submit a consent form. All participants were provided details regarding the purpose of the research and confidentiality.

3.4 Statistical analysis

First, I calculated the percentage of positive responses for each question item (1–8) in the questionnaire. Each item was answered using a four-point Likert scale. Because the data were obtained on an ordinal scale and not on an interval scale, the scores were assigned as follows: (a) Strongly agree = 6; (b) Agree = 5; (c) Don't think so = 2; and (d) Don't think so at all = 1. A multiple regression analysis using the stepwise method was then conducted. Referring to Table 1, item 9, "I have confidence in teaching percentages (%)," was used as the objective variable, and items 1–8 were the explanatory variables. The implications for identity formation were then discussed.

For the survey questions on percentage comprehension, we calculated the percentage of correct answers and the average number of correct answers for the college student participants. We then compared these to the percentage of correct answers among high school students in Kumakura et al. (2019). Because we did not have access to the primary data from Kumakura et al. (2019), we did not test for differences in the means.

4 Results and discussion

4.1 Mathematical identity survey (2019)

4.1.1 Results

The percentage of affirmative responses (option a or b) in the questionnaire survey is illustrated in Table 3.

Table 3. Positive responses to the questionnaire survey (2019) (n = 86)

Item No.	Items	Percentage
1	Concept of percentages (%) finds application in subjects other than mathematics.	94.2
2	Knowledge of percentages is necessary for daily life.	97.7
3	Expressing it as a percentage (%) to see how much of the total it is.	96.5
4	Expressing the results in terms of percentage (%) shows the changes such as increase or decrease.	91.8
5	We can compare two quantities by expressing them as percentages.	82.5
6	Solving percentage problems is fun!	48.3
7	I am good at solving percentage problems.	26.7
8	I may try to solve problems in daily life by using the concept they have learned.	60.4
9	I am confident in teaching percentage (%).	15.2

The above table illustrates the percentage of affirmative responses to items 1 and 2, which pertain to the "necessity of proportions," and items 3 to 5, which pertain to the "meaning of proportions." Each of these items, with the exception of item 5, was found to be above 90%. The reason that the positive responses to item 5 (82.5%), which pertains to "comparing two quantities," tended to be lower than those for items 3 (96.5%) and 4 (91.8%) could be because proportions are not used as often when comparing two quantities on a daily basis. The affirmative responses to items 6 to 8, which pertained to "attitude toward solving proportions," were low, ranging from 20 to 60%. This indicates that the target students, those aspiring to become elementary school teachers, dislike proportions.

The percentage of affirmative responses to item 9, "I am confident in teaching percentages (%)," was also low at 15.2%. On this item, 29.1% of the respondents answered, "I don't think so at all," revealing that they have a strong sense of discomfort with respect to teaching percentages. A stepwise multiple regression analysis was carried out with item 9 as the objective variable and items 1–8 as explanatory variables (Table 4). The results revealed that item 7, "I am good at solving percentage problems," had a significant positive effect on item 9, while the others had no effect.

Table 4. Determinants of confidence in teaching percentages

Variable	Item 9	95% Lower limit	95% Upper limit	VIF
Item 7	$.404^{**}$	.213	.594	1.045
R^2	$.253^{**}$			

Note: **p < .01, VIF: Variance Inflation Factor

4.1.2 Discussion

Students who aspire to become teachers mainly encounter mathematics and arithmetic during primary school education, and they form their current "mathematical identity" through various experiences, such as meeting instructors and other students who are also pursuing the study of mathematics and arithmetic along with them. This mathematical identity is not immutable. Rather, it develops because of the motivation to become a teacher, the relearning of arithmetic and mathematics at university (by engaging in learning through activities requiring mathematical inquiry), and through relationships with others. Therefore, focusing on enabling students to feel confident about being good at solving proportion problems will lead to the formation of their identity as instructors. Nevertheless, the knowledge gained through rote memorization—training students to repeatedly derive the correct answers to problems they learned in elementary school—will eventually be forgotten. Thus, it is necessary to improve students' skills in understanding the essential meaning of proportions. To achieve this, it is important for instructors to devise an appropriate method to teach proportions. It is necessary to have a university education plan that integrates a unit on the concept of proportions that includes developing each hour's instructional plan, mock lessons, and handouts for practice exercises.

4.2 Survey on the understanding of the "percentages" (2019)

4.2.1 Results

In this portion of the survey, we investigated the students' understanding of the concept of percentages (%). The percentage of correct answers and the average number of correct answers are illustrated in Table 5.

Table 5. Percentage (%) of correct answers

Problem	University students (n = 86)	Second-year high-school students (n = 536)
Q1. Second usage	100.0	95.4
Q2. Third usage	67.4	74.0
Q3. Contrastive type	80.2	69.9
Q4. pp type	80.2	73.0
Q5. pp type	66.3	61.2
Q6. p/p type	32.6	41.6
Average number of correct answers	4.3	4.2

Note: Data presented in Table 5 on second-year high-school students (high school sophomores [n = 536 students]) were derived from the results of the survey conducted by Kumakura et al. in 2019.

The percentage of correct answers to questions 1, 3, and 4 was more than 80%, while the percentage of correct answers to questions 2, 5, and 6 was lower than 70%. Questions 4 and 5 both refer to the same pp-type, but as Kumakura et al. (2019) stated, the percentage of correct answers to question 5 was lower than that of question 4. Pp-type problems are written problems that can be solved by multiplying a percentage by a proportion, while p/p-type problems are written problems that can be solved by dividing a percentage by a proportion.

About 25% of the students interpreted the percentage increase/decrease "10% increase/10% decrease" in the same way as increase/decrease in quantity/number "10 person increase/10 person decrease." In other words, a quarter of the students seemed to be unable to distinguish between percentages and physical units, such as liters (L), grams (g), or pieces (pcs). Question 6 had the lowest percentage of correct answers (32.6%), and there were many wrong answers (e.g., the percentage of deforestation is 60% or 20%). Students who answered "60%" did not correctly understand the meaning of the percentage, while those who answered "20%" did not sufficiently understand the meaning of the percentage because they arrived at the answer by simply subtracting the percentages (50% – 30% = 20%).

4.2.2 Discussion

In Japan, ratios are taught in elementary school but not at the secondary level. The trend in the percentage of correct answers of university students and high school students, who differed in terms of age, region, and academic distribution, was highly similar. It seems unlikely that the understanding of proportions will improve naturally when these elementary school students become adults with more life experience. Additionally, the difficulty level of the problems increased with respect to the question order: second usage [(Q1)] → contrastive type [(Q3)], pp-type (no change in standard quantity) [(Q4)] → third usage [Q2], pp-type (with change in standard quantity) [Q5] → pp-type [(Q6)]. This indicated that students might not naturally deepen their understanding as they progressed through the grades. Therefore, it is necessary to ensure that students understand that proportion problems are of varying difficulty levels and can be used for teaching "proportions."

4.3 The understanding of "question 6" (2021)

4.3.1 Results

Among the 110 students, 93 (84.5%) answered question 6 correctly (i.e., 40% of the deforested area). Table 6 depicts the qualitative classification of the students' answers, focusing on their expressions.

Table 6. Evaluation criteria for categorizing students (n = 110)

Evaluation criteria	**Number of respondents**
Type A: The meaning of proportions is expressed correctly and described without logical leaps.	15 (13.6%)
Type B: The meaning of proportion is expressed correctly and described almost logically. Some leaps are made, with some examples not being close to reality.	12 (10.9%)
Type C: There is some problem understanding the meaning of proportions, and the explanation is insufficient.	26 (23.6%)
Type D: There is a problem understanding the meaning of proportions, and there are many inadequate explanations.	40 (36.4%)
Type E: There is a problem understanding the meaning of proportions, and the answer is wrong.	17 (6.4%)

A total of 27 students (24.5%) were included in Types A and B, while 66 students were in Types C and D, which is not a small number. This indicates that many students could answer the question correctly but could not provide an appropriate explanation.

A typical example of a Type A response (13.6%) is as follows:

> Let the total area of town A be 1. Ten years ago, the forest area was 50% of the total area, and thus it was 0.5. This year's forest area is 0.3 because it is 30% of the total area of Town A. Based on the area of the forest 10 years ago, we know that this year's forest area has decreased by 0.5-0.3 = 0.2. Based on the area of the forest 10 years ago, we know that the forest area has decreased by 0.2/0.5 = 0.4, or 40%. The forest area decreased by 40%.

Students whose responses were categorized as Type A were able to state reference quantities in their explanations, for example, "Let the total area of town A be 1."

A typical example of a Type B response (10.9%) is as follows:

> Let the total area of town A be 100 m^2. Ten years ago, 50% of the total area was 50 m^2. This year, it is 30% which is equal to 30 m^2. Based on the area of the forest 10 years ago, the area of the forest this year is 50-30 = 20, indicating a decrease of 20 m^2. If we express 20 m^2 as a percentage, we obtain 20/50 × 100 = 40, that is, a 40% decrease.

Thus, the 11 members of Type B proceeded to discuss the total area of Town A as 100 m^2. The remaining members assumed that the total area of Town A was 200 m^2. This indicates that these students described the area of a town as being about the same size as a classroom. Additionally, the description ignored the fact that the discussion of the assumed area was not generally applicable.

A typical example of a Type C response (23.6%) is as follows: “Based on the forest area 10 years ago, this year’s forest area is 30% of that, 0.3/0.5 = 0.6, which is 60%, and 100 - 60 = 40, which is 40%.” The example demonstrates that the standard quantity is misrepresented; the standard quantity value is unclear, and the calculation procedure is not explained. Most of the responses in Category C ask for the solution as “percentage - percentage = amount of decrease.” However, it is not clear whether the fact that the reference quantity A has not changed is implicit or due to a lack of understanding; in any case, it is not expressed correctly.

A typical example of a Type D response (36.4%) is as follows: “Let 50% be 1. If we replace 50% with 100%, then 30% is 60% of 50%; 100-60=40, thus a 40% decrease.” Thus, the explanations in Type D do not adequately state the relationship between proportions, reference quantities, or comparison quantities. Additionally, students do not express themselves with the awareness that they are explaining to elementary school students. A lack of explanatory language is another characteristic of Type D. Type E (15.5%) refers to those who answered 60%, 20%, or 50% as their answers. There were nine, seven, and one student in this category, respectively.

4.3.2 Discussion

As described above, p/p-type problems are not easy, even for students who wish to become elementary school teachers. Even those students who answered correctly had difficulty providing appropriate explanations and presenting the right amount of information for the situation. In arithmetic and mathematics learning, the ability to use proper mathematical language and have an appropriate sense of quantity are important qualities and competencies that we want children to acquire. Deepening the understanding of ratios among students aiming to become instructors and fostering the ability to explain the use of mathematical expressions and sense of quantity are issues that need to be addressed. Since the ability to provide correct explanations is inextricably linked to understanding mathematics, we must provide guidance for students to help them understand the content and refine their mathematical expressions. We believe that such guidance will help students develop a sound mathematical identity. In particular, proportion is an important topic because it is a significant concept when considering quantities per unit and in the functional domain. Additionally, “proportions” involve a mathematical concept that most students find difficult; thus, using this topic as a reference for refining students' understanding and methods of expression might be an effective way of helping them form a sound mathematical identity.

5 Conclusion

The purpose of this study was to clarify and investigate the level of understanding of the concept of "proportions" among students who want to become elementary school teachers and are undergoing training at a private university and obtain recommendations for the development of students' mathematical identities.

Of the survey participants, 15.2% responded positively to the question about teaching proportions. When it comes to teaching proportions, it was found that many students have a negative mathematical identity as instructors. Therefore, we investigated the determinants of "confidence in teaching proportions." A stepwise multiple regression analysis was carried out with item 9, "I am confident in teaching percentages (%)," as the objective variable and items 1–8 as explanatory variables (Table 4). The results revealed that item 7, "I am good at solving percentage problems," had a significant positive effect on item 9. In other words, it was found that being good at solving proportions could lead to a positive mathematical identity as a teacher.

Next, because it is necessary to understand the status of students' understanding of proportions, I examined the status of proportion-related problem solving and found the following: Examining mathematical identity and understanding proportions among college students who wanted to become elementary school teachers at University A revealed that the difficulty level of the numerical problems increases in the order of the second usage → contrast type, pp-type (no change in standard quantity), third usage, pp-type (with change in standard quantity), and p/p-type. Particularly, students had problems understanding the third usage and cases where the standard quantity was unknown (p/p-type). As such, students must acquire knowledge about these problems. This is similar to the results obtained by Kumakura et al. (2019) for high school students, suggesting the need for universities to provide students with opportunities to relearn proportions. In particular, for p/p-type problems, many students could not explain the correct reasoning, even when they could derive the correct answer. This suggests that there is a need for students to learn how to explain the problem-solving process. Because the ability to provide correct explanations is inextricably linked to understanding mathematics, it is essential that we provide guidance to students on the content and how to refine their expression. We believe that such guidance will help students develop a positive mathematical identity. Deepening the understanding of ratios among students aiming to become instructors and fostering their sense of quantity and ability to explain the use of mathematical expressions are also issues that need to be addressed.

This survey was limited to students at a private university in Japan who wanted to become elementary school teachers. In the future, the survey should be conducted at other universities worldwide to assess if the results differ depending on the differences in curricula in each country. Additionally, considering that the descriptive survey for question 6 was a collection and distribution survey, it is expected that the results would have been lower than the present results if a group survey had been conducted.

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References

Aguirre, M. F., Mayfield-Ingram, K., & Martin, B. D. (2013). *The impact of identity in K-8 mathematics learning and teaching: Rethinking equity-based practices*. National Council of Teachers of Mathematics.

Bishop, J. P. (2012). "She's always been the smart one. I've always been the dumb one": Identities in the mathematics classroom. *Journal for Research in Mathematics Education, 43*(1), 34–74. https://doi.org/10.5951/jresematheduc.43.1.0034

Higuchi, K. (2005). A study of college students' understanding of 'ratio.' In *Proceedings of the Conference on Mathematics Education, Vol. 38* (pp. 795–796). Japan Society of Mathematical Education.

Howe, C. (2013). Ratio and proportion: Research and teaching in mathematics teachers' education (pre- and in-service mathematics teachers of elementary and middle school classes). *An International Journal of Teachers' Professional Development*, *17*(4), 577–579. https://doi.org/10.1080/13664530.2013.793059

Kambara, K. (2019). A study on the understanding of proportions by students who want to be elementary school teachers: Implications for teaching 'proportions.' *Proceedings of the Annual Meeting of the Japanese Society for Science Education 43*, 181–184.

Kambara, K. (2021, September 20–22). *A survey study on the mathematical identity of prospective teacher students—Targeting second-year college students before learning pedagogy* [Paper presentation]. 53rd Research Conference of Japan Academic Society of Mathematics Education

Kumakura, K., Kunimune, S., & Matsumoto, S. (2019). A survey study on the understanding of proportion among junior and senior high school students. *Bulletin of the Center for Educational Practice*, (29), 80–89. Shizuoka University.

Monteiro, C. (2003, July 13–18). *Prospective elementary teachers' misunderstandings in solving ratio and proportion problems* [Paper presentation]. 27th International Group for the Psychology of Mathematics Education Conference Held Jointly with the 25th PME-NA Conference. International Group for the Psychology of Mathematics Education, Honolulu, HI, United States.

Nishi, S. (2017). A study of identity formed through mathematics education: A presentation of the hypothesis of identity identified through habit. *Journal of the National Association for Mathematics Education*, *23*(2), 117–128.

Olanoff, D. E., Lo, J.-J., & Tobias, J. M. (2014). Mathematical content knowledge for teaching elementary mathematics: A focus on fractions. *The Mathematics Enthusiast*, *11*(2), 267–310. https://doi.org/10.54870/1551-3440.1304

Rabe, J., & Wenger, E. (1993). *Learning embedded in situations*. (F. Saeki, Trans.). Industrial Books.

Sfard, A., & Prusak, A. (2005). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. *Educational Researcher*, *3*(4), 14–22. https://doi.org/10.3102/0013189X034004014

Takahashi, T. (2020). What we want from future arithmetic and mathematics teachers: What we have seen through identity research. *Joetsu Mathematics Education Research* (35), 1–28. Mathematics Department, Joetsu University of Education.

Yoshizawa, M. (2019). *University students who don't understand '%.'* Kobunsha.