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Dialogical interactions mediated by technology in mathematics education

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Abstract
This paper discusses the results of a research[1] that integrates Digital Interactive Storytelling (DIST), competence-oriented mathematical activities, and argumentation called DIST-M. The general aim is to support a reflective knowledge of mathematical concepts by implementing a digital educational device based on collaborative and dialogical activities proposed by researchers. Within a dialogical dimension of interactions (Bakhtin, 1981), argumentative practice is considered a social activity, where the acquisition and elaboration of new knowledge take place within a social space with multiple interlocutors in a dynamic process. The participants are engaged in constructing and negotiating mathematical meanings within a specific context. This dialogical approach to argumentation tends to create an authentic argumentative culture that is a system of implicit and explicit rules where the exchanges and interactions among participants require a joint elaboration of new meanings, within a given mathematical context, through a dialogical exchange. Learning and development result from a dialogical negotiation process during which new knowledge is developed and those already possessed are re-organized and systematized (Bakhtin, 1981; Vygotskij, 1978). In the current pandemic circumstances, technologies are the main tools to uphold the educational processes. Despite the fact that the DIST-M was implemented and tested before the Covid-19 era, its epistemic bases of dialogism mediated by technology could significantly keep alive the dialogic interaction in educational settings that have been heavily affected by the social distancing and promote mathematical thinking. The articles focus on the United Nations Sustainable Development Goal n. 4," Ensure inclusive and equitable quality education and promote lifelong learning opportunities for all."

Keywords.
Argumentative culture, Dialogical Interaction, Digital Interactive Storytelling, Mathematics, Technologies

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Introduction

Previous psychological research (Arcidiacono, 2015; Rigotti & Greco, 2010; Schwarz & Baker, 2017) has explored the role of argumentation in knowledge construction processes in educational settings. This work has highlighted that, under certain conditions, argumentation contains a learning mechanism that allows people to change their position on a topic and expand and acquire new knowledge in interaction contexts (Doise, Mugny & Perret-Clermont, 1975; Doise & Mugny, 1981). Thus, argumentation enables people to change their point of view (Van Eemeren & Grootendorst, 1984; Leitao, 2000; Piro, 2015; Mollo, 2018).

In this paper, we analyse the link between argumentation and the development of new knowledge in teaching-learning contexts. We specifically focus on a dialogical dimension of interactions (Bakhtin, 1981). Our theoretical frame of reference is based on classical works of social genetic constructionism

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(Doise, Mugny & Perret-Clermont, 1975; Perret-Clermont, 1979; Iannaccone, 1984; Iannaccone & Ligorio, 2001) on the contributions of Vygotsky's (1978) cultural-historical school, and on previous research on argumentation (Van Eemeren & Grootendorst, 1984, 1992; Grossen & Perret-Clermont 1994; Perret-Clermont, Trognon & Marro, 2008; Piro, 2015), particularly in the area of mathematics education (Boero, 1999; Mariotti, 2006; Antonini & Mariotti, 2008; Boero, Douek, Morselli & Pedemonte, 2011; Mariotti, Durand-Guerrier & Stylianides, 2018).

The socio-constructivist perspective has developed a concept of learning that holds that the cognitive dimension (of learning) is connected to the social dimension. There is a strong link between the social condition of thought – understood as an activity located in sociocultural interaction – and cognitive activity (Iannaccone, 2010; Marsico, 2017, 2018, Marsico *et al*., 2015; Szulevicz *et al.*, 2016). The processes through which participants give meaning to what is happening around them (and their interpretation of the context in which these interactions occur) are indissolubly intertwined with how they perceive their involvement in a shared activity (Coppola *et al.,* 2015).

These studies reveal that cognitive activity – particularly the acquisition of new information – cannot be decontextualised or traced exclusively to activity in isolation. Instead, it depends on one's understanding of the world and the progressive (social) construction of the meaning of the events in which one is continuously involved (Iannaccone & Arcidiacono, 2017; Iannaccone, 2010; Perret-Clermont, 2004). Based on this landscape, argumentation takes on a central role; it is understood as a discursive and exchange process defined by the presence of others with different skills, specific cultural artifacts, and action practices. It requires quasi-formal reasoning and unfolds within certain interaction contexts (Baker, 2006; Iannaccone, 2010; Iannaccone & Arcidiacono, 2017; Manzi et al., 2020; Mollo, 2021; Plantin, 2015; Van Eemeren & Grootendorst, 1984). Emerging from these premises are dialogical and pragmatic dimensions of argumentation, where argumentation can be understood as action-oriented to an addressee and immersed in a specific context (Rigotti & Greco, 2010).

Furthermore, the dialogical and pragmatic dimensions of argumentation point to previous studies, including the work of Austin (1961, 1962), Searle (1973, 1975, 1978), and Grice (1957, 1975). All of them contributed to developing the modern argumentation theory as a particular type of dialogical interaction between speakers. They also represent the foundation of pragma-dialectics (van Eemeren & Grootendorst, 1984, 1992; van Eemeren, Grootendorst, Snoeck & Henkemans, 1996; Schwarz & Baker, 2017).

Pragma-dialectics adopts the critical perspective that finds its highest expression in the concept of critical discussion and implies the argumentative use of language to resolve a difference of opinion[3]. Within this model, a difference of opinion can only be resolved if the counterparts reach a common decision on the acceptability or unacceptability of an argument[4]. This will only be possible if the interlocutors are willing to critically examine all the elements of a particular thesis to determine whether there is an area of agreement – a sort of 'common ground' between the speakers – where they share values and knowledge. This condition is essential for the success of discussion[5]. In this sense, according to pragma-dialectics, argumentation activates a process of discursive negotiation using language (van Eemeren & Grootendorst, 1984, 1992).

[3] A difference of opinion on a thesis is the starting point for initiating discussion.

[4] Its primary objective is to bring out the weaknesses of a specific thesis or argument through a rational and intersubjective verification of its validity that highlights those elements that would lead to its refutation. In this perspective, attention to the discussion aims to highlight the defense strategies or persuasion stratagems used by the various interlocutors in order to bring out the dialectical procedure used to conduct the critical discussion: the linguistic act contained in the discourse.

[5] Argumentation is conceived as a tool of reasonable persuasion in which critical discussion is aimed at reaching a common and reasonable conclusion that arises from certain premises and the willingness of the interlocutors to continuously subject their thesis to critical testing (justification of opinions and consideration of oppositions).

In this paper, we focus on analysing the process of solving a mathematical problem that arises within a story proposed to student participants. Our qualitative analysis concerns the *critical discussion* activated in the forum by questions asked by the researcher, who is a character in the story. The research has two purposes: 1) to help the student build mathematical arguments and 2) to support, through the creation of a dialogic argumentative space mediated by technologies, reflective knowledge of mathematical concepts. The scripts and related mathematical tasks were designed to induce the production of argumentative chains. This work focuses on analyzing the solving process of a mathematical problem arose within a story proposed to the students. The students are engaged in the construction and negotiation of mathematical meanings. The qualitative analysis concerns the critical discussion activated in the forum by the questions asked by the researcher (who is a character of the story).

In education, considering argumentation only in terms of logical proof would be reductive to its potential for learning and development. In effect, logical proof[6] represents one aspect of argumentation, the product of a much more complex argumentative process. This process takes the form of a kind of critical engagement that involves reasoning, experience (or rather, our knowledge), context, and each other. From our perspective, critical engagement is defined as the ability to think critically – in other words, to reflect, process, and evaluate facts: to cognitively engage in meaning-making (Langer, 2005; Bonney & Stemberg, 2010; Gruenfeld, 2010). The participants must propound and justify their viewpoints for a dialogical exchange to turn into argumentation. The group member presents her viewpoint and defends it against counterarguments and the critical questioning raised by the researcher (who takes the role of an opponent). We have two dimensions: the dialogical and dialectical dimensions of argumentation. The dialogical dimension of argumentation points to the role of "the other" (the researcher). Here the argumentation and the dialectical dimension emphasize the role of systematic opposition and critical questioning in argumentation that comes from the researcher (Van Eemeren & Grootendorst, 1984; Leitao, 2000).

Argumentation as dialogue

It is clear from the above discussion that to consider argumentation in terms of language and, consequently, dialogue, it is fundamental to understand the potential of argumentation in educational contexts fully and within teaching-learning processes. According to Vygotsky (1978), language and dialogue represent fundamental tools in learning and development from early childhood.

Within collaborative social interactions, dialogue helps children organise and systematise knowledge. In fact, according to Vygotsky (1978), children already possess a number of spontaneous concepts (pseudo concepts) that are as yet disorganised and unsystematic. Vygotsky's pseudo concepts constitute the most widespread form of children's real thought, and in school-aged children, this form of thought prevails over all other forms. Through guided verbal exchanges, a child who already possesses such concepts can compare these concepts with an adult's logical and rational concepts. With this comparison, the child's knowledge will become, in turn, more logical and rational. Through dialogue with a child, an adult can determine the course (and effect) of the development of generalisations. Vygotsky emphasises that the adult does not impose '*on the child his way of thinking,*' but that this confrontation is instead part of a co-construction of knowledge that implies a negotiation of intersubjective meanings.

Learning and development result from a dialogic negotiation process through which new knowledge is developed, and knowledge that is already possessed is organised and systematised. From this perspective, argumentation cannot be considered exclusively in its role of logical proof or only as a form of communicative confrontation between parties. Instead, argumentation should be considered in its dialogical

[6] Argumentation as proof is understood as a discursive mode that has the purpose of demonstrating (evidence of fact) and justifying a certain type of reasoning and that is articulated in series of discursive moves such as: hypothesis, thesis or proof (Piro, 2015; Rigotti & Greco, 2010).

dimension as a form of organisation and development of knowledge evolving within a process of intersubjective discursive negotiation that uses language for intrapersonal construction of new levels of knowledge.

Concept and meaning development represent a complex act of thinking that require multiple functions (such as voluntary attention, logical memory, abstraction, comparison, etc.) and high levels of awareness. On the other hand, thinking can be conceptualised as communication with oneself. It 'arises as a modified private version of interpersonal communication' (Sfard, 2001). The argumentative practice, considered within a dialogical dimension of interactions (Bakthin, 1981), is configured as a social activity through which one may acquire and process new knowledge within a social space and with multiple interlocutors in a dynamic process that sees participants engaged in the construction and negotiation of meanings within a given context. This dialogical approach to argumentation tends to create an 'argumentative culture' – a system of implicit and explicit rules that contains built-in exchanges and interactions involving the active participation of the participants in jointly elaborating potential meanings within a given context through a dialogical exchange. Beyond that argumentative space, any meaning loses its meaning (Iannaccone, 2010; Perret-Clermont, 2014; Bakhtin, 1981).

An important element that comes into play in learning processes is context (Walton & Krabbe, 1995; Muller Mirza, 2012; Muller Mirza & Perret-Clermont, 2009). Studies of educational contexts (Iannaccone, 2010) have highlighted how institutional frames give shape to social interactions and intertwine with cognitive activity by influencing how individuals within those frames perform the different participatory activities (Iannaccone & Perret-Clermont, 1993; Perret-Clermont, 2001; Iannaccone, 2010). Modern theories of argumentation also include the interpersonal reasoning model[7] (Walton & Krabbe, 1995), in which reasoning plays a central role in argumentation. Reasoning is immersed in communication, and resides within a strong dialogical and pragmatic dimension. The argumentative moves implemented by the interlocutors are guided by reasoning that follows the discursive relationship. In fact, its development depends on the course of the conversation and considers the social dimension, the purpose of the dialogue, and contextual differences. Interpersonal reasoning evolves within interpersonal discursive space; every cognitive decision is tied to the relational and contextual situation.

Walton and Krabbe (1995) found out that dialogue rules, which are defined by different communicative contexts, emerge from argumentative practice. The rules within the dialogue define the argumentative moves or the actions that conform to the rules of the particular communicative context. The violation of a rule (shift) corresponds to a fallacy, or non-cooperative contribution (Grice, 1957, 1975), within the discursive space that relates to the purpose of the dialogue.

It is clear then that reasoning, which is inherent in argumentation is dominated not so much by rationality but by how reasonable the arguments are in a particular context. Within a very strong pragmatic dimension, every argumentative move makes sense and can be used within a given context, while outside of that context, every discursive move loses its function. It is essential to take into account different communicative contexts, despite the fact that they cannot be considered variables external to the cognitive activity – but still they play an important role. Such argumentative dynamics, in different interaction contexts, may activate a process of knowledge transformation through a game of social roles and negotiated positions. Through these sorts of interactions, an argumentative space that continuously evolves in a temporal dimension (in the dimension of the micro-history of the dialogue) is delineated; this argumentative space becomes, in turn, a space of thought (Bakhtin, 1981; Perret-Clermont, 2014). The creation of an argumentative space within which students may confront what is unknown (to them) or elements that may

[7]For an in-depth discussion of the model, see Walton D., Krabbe E., 1995, Commitment in Dialogue. Basic concepts of interpersonal reasoning, State University of New York Press, New York.

destabilise their opinions represents for the participants a source of reflection and awareness of themselves and their own knowledge (Savarese et al., 2019). This perspective demonstrates the importance of creating educational devices that involve cooperative activities that centre on argumentation as a tool for conceptual evolution. We assume that through learning activities that involve argumentation, we can not only facilitate access to new knowledge but also improve students' argumentative skills.

Research purpose

The overall goal of this research is to support the reflective knowledge of mathematical concepts. We aim to develop the participants' reasoning and argumentative skills in mathematics through a training and research device that integrates digital interactive storytelling (DIST), called DIST-M. Reasoning and argumentative skills are defined as those skills that involve '*logic-based mental processes that allow one to analyze and connect the elements of a problem in such a way that one can draw conclusions, verify a given justification, or provide justification for statements or solutions to the problem*' (Niss, 2015). We presume that argumentation is an important scaffolding tool – a dialogic space of collaborative interaction modulated on the students' abilities within which they co-construct knowledge. To this end, we present our analysis of an activity carried out with participants from the first class of the secondary school.

Participants[8]

Thirty students from the first class of a human sciences high school took part in the activity. The students were divided into seven groups of four or five members (in the case of five members, two students took the 'Pest' role). We left the composition of the groups to the discretion of the class teacher, asking that they ensure that the groups were of a homogeneous level among themselves.

DIST-M educational device

The methodological architecture for this study develops around a digital story in the form of comics (Marsico, Mollo, Albano & Pierri, 2019; Albano, Pierri & Polo, 2019), which evolves through multiple episodes and revolves around five animated characters (four teenagers and an adult, the uncle of one of the teenagers) who face mathematical problems that are born spontaneously from the story itself.

Each student participant is a character in the story and, within and through the narrative, is given a role and actions to perform – sometimes alone, sometimes in collaboration with others. Unlike previous research and practices, this study does not place the participants as either the one who listens and reproduces a story or as the one who invents a new one. Instead, we integrated the students into the scenario that was presented through the comics, assigning them the position of the protagonist by giving them the role of one of the characters. Through this approach, the students direct the development of the story through their interactions with each other. We used the same approach for the mediating role of the expert, who was also a character in the story and who acts on the basis of the stimuli/responses/silences of the other actors. The teacher and student actions evolve on the following two interrelated levels.

Mathematical storytelling

This refers to the context within which the mathematical problem arises, develops, and evolves through various phases of action that correspond to many episodes in the story. The storytelling is related to specific objectives that are fundamental for constructing argumentative and communicative competencies and for initiating mathematical proof. These objectives are as follows:

[8] Informed consent was obtained from all individual participants included in the study

1. Exploring (episode 1): observe and produce a summary description of what was found;
2. Conjecture (episode 2): refine the wording of the previous description;
3. Formalise (episode 3): manipulate the conjecture to pave the way for proof;
4. Prove (episode 4): identify and organise arguments in an appropriate deductive chain, justifying each step of the deduction.

Each episode is structured into two working phases:

a) peer work, where the students interact among themselves, trying to reach a shared answer to a sub-problem posed within the episode; here, the role of the expert is played by the vignettes;
b) asymmetric work, where the students interact with the character representing the expert, with the aim of discussing, refining, and institutionalizing what they found by themselves; the starting point of this phase is the answer shared by the students in phase a).

Roleplaying

Within each episode of the story, the characters' work is governed by the specific roles, which can be associated with the main cognitive functions that come into play in the mind of a mathematician in a problem solving situation (Albano, Coppola & Dello Iacono, 2021; Coppola et al., 2015; 2011; 2019).

Some of these roles have been played by the students: the Boss (the group's organiser and articulator of the work: social interaction and relationship in work); the Blogger (the verbaliser and speaker: memory and communication); the Pest (the devil's advocate: critical/constructive thinking); and the Promoter (the initiator and animator of the task performance: insight and knowledge).

Alongside these roles is a character who plays an asymmetrical role with respect to the peer group: the Expert (the adult in the story). The expert exercises his function as a mediator (Bartolini-Bussi & Mariotti, 2008) and intervenes as needed, both in autonomous moments of peer work and in moments dedicated explicitly to the confrontation between the peers and the expert. This role is important because the expert ensures the group's productivity and acts as a guide and facilitator.

The technological architecture of the story was based on an online digital platform (Moodle). This model involves the integration of various online technologies, some of which are general purpose (forum or chat) while others are specific to mathematics (spreadsheets, CAS, DGS). We also used technology developed specifically to support the development of some specific skills, such as language toolkits (Albano & Dello Iacono, 2019).

Methodology

We analysed, at different levels, an activity concerning the resolution of a mathematical problem presented in one of the stories proposed to the pupils (Fig. 1) (Doise, 1982b). The analysis examined 1) the communicative interactions within the forum, identifying the resolution strategies that the pupils co-constructed in their interactions (dialogic dimension); and 2) the participants' individual reasoned responses before and after the discussion (monological dimension).

We will present the analysis of the dialogic dimension in this paper.

Dialogical dimension analysis

According to pragma-dialectics, in order for a confrontation of opinions to turn into true argumentation and, therefore, a space for dialogic interaction, students must propose and justify their points of view (confrontation stage) and let their responses be examined by others (opposing points of view and possible critical questions posed by other students) (Van Eemeren & Grootendorst, 1992; Mollo, 2018;

2021). Our qualitative analysis[9] examined the students' argumentative skills in collaborative learning situations. In particular, we focused on the students' ability to produce arguments for and against their own and others' positions[10]. This analysis also focused on how the critical questions posed by the researcher in one of the discussions in the general forum activated further elaborations, reflections, and possible conceptual changes in the students. Our intent is to highlight the transformations and micro-transformations that took place among the participants.

Activity

The activity presented to the participants concerned a fantasy story titled 'Aliens are coming,' which follows a group of four friends engaged in the enterprise of communicating with aliens from whom they have received mysterious messages composed of numbers and operations. The four friends in the story correspond to the four expected participant roles: group, speaker, devil's advocate, and technology; these were appropriately contextualised within the story as character profiles (Fig. n. 3). The activity, which was the subject of our analysis, began with a screen of numbers and operations sent by the aliens (see Fig. 1, below) that contained the following hidden mathematical question (Mellone & Tortora, 2015): '*Pick four consecutive natural numbers, multiply the middle two between them, multiply the extreme two, subtract the results. What do you get?*'

Figure 1

Starting from what is shown in Figure 1, the students are guided to explore and conjecture that the result of the mathematical question posed is 2, then they are required to prove it, and to this aim, they need to mathematically formalize their conjecture. In each phase, the students are expected to support with suitable arguments their findings.

The activities were designed and supervised by teachers and researchers. The students were asked to discuss the problem and possible solutions in chats until they arrived at a single shared solution. One member of the group, the Blogger, was given the task of communicating (through the forum) the answer that was discussed and chosen by the whole group.

The forum used by the participants was managed by the researcher or the teacher who filled the expert role (the uncle in the story). The expert's task was to collect the solutions presented, in argumentative form, by the different groups and *stimulate* the students through questions and suggestions to *activate* a reflection on the answers given and the concepts learned. This was performed in accordance with the Vygotsky's theory of the zone of proximal development, which holds that reasoning abilities increase in

[9] Here, there are no conditions for analysing the entire critical discussion using the rules of pragma-dialectics; the analysis was inspired by some concepts of the pragma-dialectical approach.
[10] The data in their entirety are available to researchers who may be interested.

interaction among peers and with more experienced people (Vygotsky, 1978). Each group member within the game took on a role, and our analysis examined the communicative interactions within the forum.

Data analysis

In this section, we will present an analysis of the arguments produced within the study's general discussion forum. The analysis of critical discussion was guided by some of the principles from the theory of language acts (Searl, 1979) used in the pragma-dialectical model (Van Eemeren & Grootendorst, 1992); some of the linguistic moves used by the students and the researcher in the different stages of critical discussion could be characterised as language acts (Searl, 1979). We considered the following acts, in particular[11]: *assertive, commissive,* and *directive*[12]. The focus of the analysis was on the relationship between the critical questions posed by the researcher and the emergence of any conceptual changes, elaborations, or reflections by the students – i.e., conceptual transformations and micro-transformations.

Dialogic dimension analysis

The starting point of an argumentative discussion is the shared answer to the sub-problem at stake reached by the students in the peer work phase. It is the expression of a point of view and the resulting non-acceptance[13]. The first excerpt presented below represents the start of a peer discussion, started by the expert inside the story by means of vignettes, as shown in the following excerpt from the story: '*But where do you think these aliens want to go? Why are they sending us these quaternary*[14]?' (Fig. 2).

Figure 2

After the peer work and submission of the shared answers, the focus of the discussion is the argued response provided by groups 6 and 1. The language act in this first phase of the discussion is assertive; the students expressed their theses with respect to solutions for the task.

Answer no. 1: Group no. 6

1. Any quaternary consisting of 4 consecutive natural numbers can be chosen because the difference between the product between the 1st and 4th numbers and the product between the 2nd and 3rd numbers results in 2.
2. A = natural number
3. B= A+1C= B+1D= C+1
4. B*C-A*D. The result found is 2, because the messages are probably coming from a planet 2 light years away.

[11] For an in-depth study of the theory of language acts, see Searl, J. R. (1969). Speech acts. An Essay in the philosophy of language. Cambridge: Cambridge University Press. Tr.it (1992) Linguistic acts. Essay on the philosophy of language. Turin: Bollati Borghieri.
[12] Assertive when the subject commits to the acceptability of a thesis; directive when the subject tries to push the listener to perform a verbal act (request, forbid, reply, recommend, etc.); commissive when the speaking (or writing) subject commits to the interlocutor to do or not do something (promise, accept or not accept, decide to start the discussion, etc.).
[13] Non-acceptance can take many forms; it can be expressed indirectly or directly (i.e. strong or weak) (Baker, 2015).
[14] Group of four.

Answer no. 2: Group no. 1

5. This is the answer of group 1
6. Taking 4 successive numbers (n+1), for example, 9,10,11,12, if we subtract the product of the averages by the product of the extremes, we always get 2: 9,10,11,12
7. (10×11) - (9×12) = 110 - 108 = 2
8. (n+1 * n+2) - (n * n + 3). n =9

In accordance with pragma-dialectics, the expert's questioning of the groups' proposed responses opens the discussion phase (excerpt no. 1):

Excerpt no. 1: Researcher (Gianmaria, as a character in the story)

9. Hi guys, I have read all your answers. But I am still in doubt. I think I have understood the way you have selected the quaternary. However, you have not convinced me that by choosing consecutive numbers, the result of the operations the aliens do will always be two.
10. I have seen that many of you call the four numbers a, b, c, d and then say that a*d-b*c=2.
11. But I am not sure that whatever the numbers a, b, c, and d are, in math, the result is 2. I seemed to remember that each letter can take on any value, So how do I know that the result is really 2?

The expert expressed his non-acceptance of the answers given by the students in two ways. First, he expressed it directly, but in a weak way (directive act) ('I still have a doubt,' 'you haven't convinced me'). Then he expressed it in the form of a question ('So how do I know that the result is really 2?'). In turn 11, he expressed his alternative point of view ('But I am not sure that whatever the numbers a, b, c, d are, in math the result is 2. I seemed to remember that any letter can take on any value.')[15]. In round 2, the researcher engaged the students in a comparison process ('I have seen that many of you call the four numbers a, b, c, d and then say that a*d-b*c=2').

At this point, the argumentation phase begins; as excerpts 2 and 3 show, the students presented arguments in support of the answers presented in the forum. The communicative act here is commissive; the students accepted the challenge and began to defend their argument. It is interesting to note the diversity of the arguments used by the different groups to defend the same thesis (that their answer is correct – i.e., that the result is always 2).

Excerpt no. 2: Student no. 1, Group no. 3

12. Group three thought to consider A as any number, and A+1, A+2, and A+3 its consecutives. In all quaternary, the numbers are arranged so that we get the subtraction between the product of the second and third numbers and the product of the first and fourth numbers, then:
13. (A+1)(A+2) - A(A+3)
14. If we solve this expression, we get:
15. (A+1)(A+2) - A(A+3)=.
16. =A^2 + 3A + 2 - (A^2 + 3A)=
17. = A^2 + 3A + 2 - A^2 - 3A= 2 And it is for this reason that whatever value we give to A (even a large number) the result will always be 2.

Excerpt no. 3: Student no. 2, Group no. 5

18. because, in my opinion, by writing the quaternary in a literary form, then replacing the smallest number of the quaternary with b and getting the others consecutive to him by adding the appropriate number, we get a small literary expression that gives us a result the number 2.
19. b;(b+1);(b+2);(b+3) b belongs to the set of natural numbers
20. [(b+1)*(b+2]-[b*(b+3)] =

[15]The expression 'I seemed to remember that any letter can take on any value' represents, moreover, the opening phase, within which we focus on the shared premises (plane of knowledge) that constitute the 'zone of agreement' between the participants in the discussion. The sharing of a common ground represents the essential element for the start of any critical discussion.

21. =b^2+2b+1b+2-b^2+3b=
22. =2 We can consider this expression as the formula to calculate the various expressions derived from the quaternary given to us by the aliens. And to explain that it works with any natural number, both big and small.

In excerpt 2, the student presents the answer as a product of collaborative group work (turn no. 12, 'group three thought'), while in excerpt 3, the student expresses himself individually, initially (turn no. 18, 'in my opinion), but later in the course of the argumentation, he switched to the first person plural (turn no. 22, 'We can'). This change is indicative of how the critical discussion took place in cooperative terms (Grice, 1957), which was certainly thanks to the establishment, in the opening phase, of a 'zone of agreement' among the participants[16].

In addition, the responses provided (compared to those provided initially by groups 1 and 6) are well-argued and consistent, and the propositions provided by the students to reach the conclusion (i.e., the result is 2) adequately supported the conclusion.

Excerpt no. 4: Researcher

23. does the same apply if we substitute a letter for the largest number in the quaternary?
24. Why are you so convinced that the value you assign to A is not important?

In excerpts 4 and 9, in order to make students' arguments more effective, the expert resorted to the use of a series of non-inductive technical relaunches and reformulations of the answers with the aim of facilitating the process of reflection on the knowledge. The linguistic act here is directive; the effect is further clarification of the position taken by the students (excerpt no. 5).

Excerpt no. 5: Student no. 3, Group no. 2

25. First, I thought of taking a letter, for example, C, as any number C+1, C+2, C+3.
26. In quaternary s, to get a result 2, we have to do the subtraction between the product of the second and the third number and also the product between the first and the fourth number, so taking C in consideration, we can do
27. (C+1) (C+2) - C(C+3). And if we want to solve this little expression, we will see that it will come out as a result 2 because:
28. (C+1) (C+2) - C(C+3) = C^2+ 2C +1C + 2 - b^2+3C=2
29. This expression, as we have seen, could be the right formula to calculate the various quaternary s given by the aliens. Also, we have seen that whatever value we give or attribute to C, with a small number or with a large number, the result, as we have seen, will always be 2

In excerpts 6, 8, 10, 11, and 12, we see that the students defended their classmates' positions ('I agree with...'). It is interesting to note that the students did not limit themselves exclusively to agree, that they also 'enriched' the answers given by their colleagues with further arguments supporting the position. This phenomenon is inherent in collaborative learning situations in which knowledge is supposed to be co-constructed; during such a process, students do not remain 'stationary' in a position[17] (Nonnon, 1996) as the construction of knowledge evolves within the argumentative space.

Excerpt no. 6: Student no. 4, Group no. 1

30. I agree with my classmates because I also think that [(n+1)*(n+2)]-[n *(n+3]= 2 is the operation needed to answer this question.

Excerpt no. 8: Student no. 5, Unidentified group

31. 31. In my opinion, the expression:(A+1)(A+2)-A(A+3) is an expression that can be applied with any number, even with relatively large numbers because the result of the expression will always be 2 with any number.

[16] This phenomenon, the shift from 'I' to 'we' (to the group), is present in most of the discussion.
[17] Such as, for example, simply stating that you agree with your partner or faithfully reproducing the thesis.

Excerpt no. 9: Researcher

32. So you mean that the value of the letter A is not important? And what did you get that from?

The question posed by the expert Excerpt 9 (turn 32, ‘So you mean it's not important...’) was interpreted as a challenge by the other students. Being challenged during a discussion is a relevant developmental experience that offers an opportunity to revise one's positions in order to deepen, understand and reflect (Backer, 2015). This challenge triggered a series of chain responses (examples excerpted from no. 10 and no. 11).

Excerpt no. 10: Student no. 6

33. The value we assign to A is not important because in the expression
34. (A+1)(A+2) - A(A+3) =
35. all the values of A are cancelled
36. = A^2 + 3A +2 - A^2 - 3A= 2
37. Because +A^2 - A^2 and +3A - 3A are equal to 0, and the only result of the products between the four consecutive numbers that cannot be ‘undone’ is 2.

Excerpt no. 11: Student no. 7

38. I agree with Zuleika's exposition because it doesn't matter what value we give to A because at the end of the expression the terms in A are cancelled out and at the end (as the final number always comes out 2).
39. In fact: A^2+3A+2-A^2-3A=2,
40. Because 3A and -3A cancel out and A^2 -A^2 cancel out, and so the result that remains will be 2.

This process of interpersonal knowledge triggered by the discussion led student no. 9 to self-reformulate (excerpt no. 12) the reasoned response provided by the group (answer no.1, group no.6). This was followed by a chain of other reformulations elaborated by other students[18].

Excerpt no. 12: Student no. 9

41. 41. If we consider the letter B as any number B+1, B+2, B+3
42. 42. As we know, in the quaternas if we do the subtraction between the product of the second and the third number, minus the product of the first and fourth number, we will see that it will come out for sure as a result always 2.
43. 43. If we make and unfold a small expression such as:
44. 44. (B+1) (B+2) – B(+3)= B^2 +2B+1B +2 – B^2-3B =2
45. 45. So as we have seen, the result is always 2, even if we give, in this case to B, a value with a larger or smaller number.

Excerpt no. 13 represents the beginning of the conclusion stage of the critical discussion, with the difference of opinion being resolved in a reasonable manner and with the researcher's acceptance of the proposed theses (turn 46, ‘Okay, guys, you've convinced me’).

The pragma-dialectical model emphasises that in the conclusion phase of the critical discussion, the resolution of the divergence of opinions is realised, though it may also represent the beginning of a new discussion. At this point, the participants might begin a new discussion on a modified version of the old divergence that begins from different premises. Examples of this are shown in excerpts 14 and 15, in which the challenge is accepted by student 2.

Excerpt no. 13: Researcher

46. Okay, guys, you've almost convinced me.
47. I am left with only one doubt. But then can we use the calculator or not?
48. I seem to remember that some of you used it to try with very large quaternas, and it didn't work. What was the

[18] This paper does not report all of the discussion – only some excerpts.

problem? I can't really remember...didn't 2 come out?
49. And if the calculator can't do operations with numbers that are too high (by the way, how high?), how do I check that 1,256,257890*1256257891-125625789*1256257892=2?
50. Or maybe you guys are saying I don't need to do any calculations?
51. Please don't keep me on my toes too much :-)

Excerpt no. 14: Student no. 2

52. you only need to change the signs
53. or example, A is the largest number in the quatern,
54. A; A-1; A-2; A-3
55. and do the same calculation.

Excerpt no. 15: Student no. 2

56. The problem that was found during the calculations is that the last numbers of these numbers, relatively very large, were not identified because there was the exponential notation,
57. But applying the expression, we used, the problem is solved.
58. Or if you want to be 100% sure, you have to do the calculations by hand with the help of the calculator.

In conclusion, we observed that the argumentative moves enacted by the students were guided by a form of interpersonal reasoning (Walton & Krabbe, 1995) whose development depended on the course of the conversation. This course is steered by the different kinds of the teacher-researcher’s interventions in managing the mathematical discussion (Albano, Coppola & Fiorentino, 2021; Bartolini Bussi M. G., Boni M., & Ferri F., 1995). For each episode of the story, the mathematical discussion guided by the researcher-teacher aims to support the key elements of the argumentative and proving competencies, which are the phases of exploration, conjecture, formalization, and proof (for instance, see excerpts 1, 4, 9). In addition, during the discussion, the students demonstrated argumentative skills, and the answers they argued for were the result of reasoning that was based on the problem and stimulated by the discussion itself.

Discussion of results and conclusion

Our analysis of the critical discussion reveals that it activated an interpersonal elaboration of knowledge in the students involved, which is in accordance with Vygotsky’s (1978) and Sfard's (2001, 2008) theories that state that dialogue represents the basis of the construction of knowledge. We observed a critical engagement on the part of the students in defending their point of view in the face of the critical examination operated by the expert, who assumed the role of the opponent. The students acted as one group within a shared argumentative space of meanings (Perret-Clermont, 2004) that fostered a process of co-processing the solution (Bruner, 1996; Vygotsky, 1978). Moreover, the discussion stimulated the students to freely express their ideas, which activated, on the one hand, a reflection on their own reasoning, and on the other hand, a process of error control that was regulated by the group (Vygotsky, 1978; Bartolini-Bussi & Mariotti, 2008). We were also able to identify differences in the students' argumentative skills earlier in the process when a greater commitment emerged, from a cognitive point of view to providing argumentative responses. It is likely that during the discussion, the students perceived freedom of thought and a certain degree of awareness regarding the possibility of being able to act freely within a conversational space (Iannaccone, Convertini & Perret-Clermont, 2016; Iannaccone & Perret-Clermont, 2014; Iannaccone & Zittoun, 2014; Inhelder, Sinclair & Bovet, 1974; Boero, 1999; Sfard, 2008). In convincing the others of their position, they were required to present arguments (which the other will also present). This would not only lead one (or more than one) of the participants to adhere to a position (as a process of knowledge negotiation would be triggered), but it can also make known and understandable any knowledge that is still opaque (Grize, 1982, 1983) and of course, by the interlocutors (Vergnaud, 2015; Wegerif, 2013; Traverso, 1990).

In the reasoning enacted during the argumentative practice, each cognitive decision evolved within the interpersonal discursive space (Walton & Krabbe, 1995; Sfard, 2001; Mollo, 2018; 2021) in a social dimension that considered the aim of the dialogue as well as any contextual differences. Reasoning (inherent in argumentation) emerged that was fully immersed in a social dimension, in a dialogical space of confrontation between the participants and the other, in which every cognitive decision was influenced by the relational situation (Rigotti & Greco, 2010). Valsiner and Wertsch have demonstrated the cultural roots of cognitive processes and have shown that action does not necessarily consist of the simple execution of previous cognitive decisions but that it constitutes the way in which social actors explore the specificity of the situation (Wertsch, 1998). Rationality is not sufficient to account for all reasoning (Rigotti & Greco, 2010).

In this study, the role of the researcher was to mediate between students and knowledge, orchestrate discussions in the forum, and lead the students to reflect upon their learning process. The students acted within a co-constructed dialogic space that was mediated by the technologies used in the study, which facilitated a process of co-elaboration of the solution. In addition, the discussion encouraged students to freely express their ideas and promoted the students' reflection on their own reasoning and the process of error control regulated by the group. This has been realized thanks to the fundamental role of the researcher-teacher and of the learning environment equipped with the potential of technology. Indeed the researcher-teacher plays the role of designer of the activity as well as of the expert-teacher in the DIST-M. Based on the a priori expected answers of the students, she is able to real-time moderate within the storytelling the advancement of the construction of the competencies, not only regarding the solution of mathematical questions but above all of the development of competencies of argumentation, proving and communication (Albano, 2017; Polo 2017).

In the current pandemic circumstances, technologies have become essential tools for upholding educational processes. Despite the fact that the DIST-M was implemented and tested before the COVID-19 era, its epistemic basis of dialogism mediated by technology could have a great effect on preserving dialogic interaction in educational settings, which have been heavily affected by social distancing, and in promoting mathematical thinking in general. The articles focus on the United Nations Sustainable Development Goal n. 4, which aims to '*ensure inclusive and equitable quality education and promote lifelong learning opportunities for all*.'

References

Albano, G. (2017). E-mathematics engineering for effective learning. In G. Aldon, F. Hitt, L. Bazzini, & U. Gellert (Eds.), Mathematics and Technology (pp. 349–370). Advances in Mathematics Education. Springer.

Albano, G., Coppola, C., Dello Iacono, U. (2021). What Does 'Inside Out' Mean in Problem Solving? *For the Learning of Mathematics* 41(2), 32-36.

Albano, G., Coppola, C., Fiorentino, G. (2021). The Role of Teacher in DIST-M Activities. International Journal for Technology in Mathematics Education (to appear).

Albano, G., Coppola, C., Dello Iacono, U., Fiorentino, G., Pierri, A., & Polo, M. (2020). Technology to enable new paradigms of teaching/learning in mathematics: the digital interactive storytelling case. *Journal of e-Learning and Knowledge Society*, Special Issue on "Smart Learning in Smart Cities", Vol 16 No 1, pp. 65-71

Albano, G., Dello Iacono, U. (2019). A scaffolding toolkit to foster argumentation and proofs in mathematics: some case studies. *International Journal of Educational Technology in Higher Education*, 16:4 (pp. 1-12)

Albano, G., Pierri, A., Polo, M. (2019). Engagement in mathematics through digital interactive storytelling. In U.T. Jankvist, M. Van den Heuvel-Panhuizen, & M. Veldhuis (Eds.): *Proc. of CERME 11* Utrecht, the Netherlands: Freudenthal Group & Freudenthal Institute, Utrecht University and ERME, pp. 1501-1508.

Arcidiacono, F. (2015). Argumentation and reflexivity. In G. Marsico, R. Andrisano-Ruggieri & S. Salvatore (Eds.), *Reflexivity and Psychology* (pp. 169-193). Information Age Publishing.

Antonini, S. & Mariotti, M.A. (2008). Indirect proof: what is specic to this way of proving? *ZDM Mathematics Education*, 40(3), 401-412.

Asterhan, C. S. C., & Schwarz, B. B. (2009). Argumentation and Explanation in Conceptual Change: Indications from Protocol Analyses of Peer-to-Peer Dialog. *Cognitive Science, 33*, 374–400.

Austin, John L. (1961). *Philosophical Papers*. Oxford University Press

Austin, John L. (1962). *How to do things with words*. Oxford University Press.

Arcidiacono, F. (2015). Argumentation and reflexivity. In G. Marsico, R. Andrisano-Ruggieri & S. Salvatore (Eds.), *Reflexivity and Psychology* (pp. 169-193). Information Age Publishing.

Asterhan, C. S. C., & Schwarz, B. B. (2009). Argumentation and Explanation in Conceptual Change: Indications from Protocol Analyses of Peer-to-Peer Dialog. *Cognitive Science, 33*, 374–400.

Bartolini Bussi M. G., Boni M., & Ferri F. (1995). Interazione sociale e conoscenza a scuola: la Discussione Matematica. Centro documentazione educativa.

Bartolini-Bussi, M. G., & Mariotti, M. A. (2008). Semiotic mediation in the mathematics classroom: Artifacts and signs after a Vygotskian perspective. In L. English, M. G. Bartolini-Bussi, G. Jones, R. Lesh, & D. Tirosh (Eds.), Handbook of International Research in Mathematics Education (2nd ed.) (pp. 746–783). Lawrence Erlbaum.

Bakhtin, M. M. (1981). *The dialogical imagination: four essays by M. M. Bakhtin*. University of Texas Press.

Boero P. (1999). Argumentation and mathematical proof: a complex, productive, unavoidable relationship in mathematic and mathematics education. *International Newsletter on the Teaching and Learning of Mathematical Proof, 7/8*.

Boero, P., Douek, N., Morselli, F., Pedemonte, B. (2011). Argumentation and proof: A contribution to theoretical perspectives and their classroom implementation. In: M. F. F. Pinto, T.F. Kawasaki (Eds.), Proceedings of the 34th Conference of the International Group for the Psychology of Mathematics Education (pp. 179-205). Belo Horizonte: Exact Sciences Institute

Coppola, C., Mollo, M., Pacelli, T. (2015). The development of logical tools through socially constructed and culturally situated activities. In M.V. Dazzani, M. Ristum, G. Marsico, & A.C. Bastos (Eds), Educational Contexts and Borders through a Cultural Lens. Looking Inside, Viewing Outside. Springer. ISSN 2364-6780 – DOI 10.1007/978-3-319-18765-5

Coppola C., Mollo M., Pacelli T. (2011). An experience of social rising of logical tools: the role of language. *International Journal for Mathematics Teaching and Learning*, ISSN 1473 – 0111.

Coppola, C., Mollo, M., Pacelli, T. (2015). The Development of Logical Tools Through Socially Constructed and Culturally Based Activities. In: Marsico, G., Dazzani, V., Ristum, M., de Souza Bastos, A. (eds). *Educational Contexts and Borders through a Cultural Lens. Cultural Psychology of Education, vol 1.* Springer.

Coppola, C., Mollo, M., Pacelli, T. (2019). The worlds' game: collective language manipulation as a space to develop logical abilities in a primary school classroom. *European Journal of Psychology of Education*. Vol. 34, Issue 4, October 2019. DOI: 10.1007/s10212-018-0401-1.

Ferrari, P.L. (2002). *Developing language through communication and conversion of semiotic systems*. In A.D. Cockburn & E. Nardi (Eds.), Proc. of the 26th Conf. of the Intern. Group for the Psychology of Math. Ed. Norwich (UK), 2, 353-360.

Friedrich, J. (2012). L'idée d'instrument psychologique chez Vygotski. *Rivista italiana di filosofia del linguaggio*, *6*(2), 189-201.

Grice, P. (1957), "Meaning", The Philosophical Review 66, pp. 377-388, ora in P. Grice (1989), pp. 213-223.

Grice, P. (1975). Logic and conversation. in P. Cole e J.L. Morgan (a cura di) Syntax and Semantics - Speech Acts, Academic Press, tr. it. in SBISA', M. (a cura di) 1978, ora in P. Grice (1989), pp. 22-40.

Grize, J.-B. (1982). *De la logique à l'argumentation*. Dolz.

Grize, J.-B., & Piéraut-Le Bonniec, G. (1983). *La contradiction: essai sur le opérations de la pensée*. Presse Universitaires de France.

Iannaccone, A., Convertini, J., & Perret-Clermont, A.-N. (2016). "Je te raconte une histoire pour t'aider à comprendre". Pluralité de perspectives conversationnelles et actualisation des pratiques argumentatives chez l'enfant. *Congress SSRE 2016 "Où s'arrête l'école? Transformations et déplacements des frontières éducatives"*. Lausanne, July 2016.

Iannaccone, A., & Perret-Clermont, A.-N. (2014). Regole ed eccezioni: a propositio del ruolo delle interazioni sociali nei processi di apprendimento e nello sviluppo cognitivo. In A. Smorti (Ed.), *L'eccezione e la regola: opposizioni, convergenze*, *paradossi* (pp. 97-122). SEID.

Iannaccone, A., & Zittoun, T. (2014). Overview: The activity of thinking on social spaces. In T. Zittoun & A. Iannaccone (Eds.), *Activities of Thinking in Social Spaces* (pp. 1-12). Nova Science Publishers.

Inhelder, B., Sinclair, H., & Bovet, M. (1974). *Apprentissage et structures de la connaissance*. Presses Universitaires de France.

Leitao, S. (2000). The potential of argument in knowledge building. *Human Development, 43*, 332-360.

Linell, P. (2009). *Rethinking Language, Mind, and World Dialogicality*. Information Age Publishing.

Manzi, F., Savarese, G., Mollo, M., Iannaccone, A. (2020) Editorial: Sociomateriality in Children with Typical and/or Atypical Development. FRONTIERS IN PSYCHOLOGY. Vol. 11.

Mariotti, M. A. (2006). Proof and proving in mathematics education. In A. Gutiérrez, & P. Boero (Eds.), Handbook of research on the psychology of mathematics education, (pp. 173–204). Sense Publishers

Mariotti, M. A., Durand-Guerrier, V., & Stylianides, G. J. (2018). Argumentation and proof. In T. Dreyfus, M. Artigue, D. Potari, S. Prediger, & K. Ruthven (Eds.), Developing Research in Mathematics Education: Twenty Years of Communication, Cooperation and Collaboration in Europe (1st ed., pp. 75–89). Routledge.

Marsico, G., (2017). Jerome S. Bruner: Manifesto for the Future of Education, *Infancia y Aprendizaje, Journal for the Study of Education and Development, 40*(4), 754-781, DOI: 10.1080/02103702.2017.1367597;

Marsico, G., (2018). The challenges of the Schooling from Cultural Psychology of Education. *Integrative Psychological and Behavioural Sciences. 52*(3),474-489 DOI: 10.1007/s12124-018-9454-6;

Marsico, G., Mollo, M., Albano, G., Pierri, A. (2019). Digital Storytelling and Mathematical Thinking: An Educational Psychology Embrace. *International Journal of Innovation in Science and Mathematics Education*, 27(6), 36-44.

Marsico, G., Dazzani, V., Ristum, M. & Bastos A.C. (Eds). (2015). *Educational contexts and borders through a cultural lens – Looking inside. Viewing outside*. Cultural Psychology of Education, *1*, Springer;

Mellone, M., Tortora, R. (2015). Ambiguity as a cognitive and didactic resource. In Krainer K., Vondrová N. (Eds.), Proc. of CERME 9, Praga, pp. 1434-1439.

Mollo, M. (2018). Il potere intrinseco dell'argomentazione nei processi di apprendimento. In P.Maturi, F. Piro, L.M. Sicca, M. Squillante, M. Striano (Eds.), *Sfide didattiche. Il pensiero critico nella scuola e nell'università.* Editoriale Scientifica.

Mollo, M. (2021). *Academic Cultures: Psychology of Education Perspective*. Hu Arenas (2021). https://doi.org/10.1007/s42087-021-00238

Muller Mirza, N. (2012). Interactions sociales et dispositifs de formation: Une perspective psychosociale. In V. Rivière (Ed.), *Spécificités et diversités des interactions didactiques* (pp. 67-90). Riveneuve.

Muller Mirza N., & Buty, C. (2015). L'argumentation dans les contexte de l'éducation: enjeux et questions vives. In N. Muller-Mirza & C. Buty (Eds.), *L'argumentation dans les contextes de l'éducation*. Lang.

Muller Mirza, N., & Perret-Clermont, A.-N. (Eds.) (2009) *Argumentation and education: Theoretical foundations and practices*. Springer.

Niss, M. (2015). *Mathematical competencies and PISA*. In K. Stacey & R. Turner (Eds.), *Assessing mathematical literacy*, pp. 35–56. Springer.

Nonnon, E. (2015). Préface. In N. Muller Mirza & C. Buty (Eds.), *L'argumentation dans les contextes de l'éducation*. Lang.

Perret-Clermont, A.-N. (2004). The thinking spaces of the young. In A.-N. Perret-Clermont, C. Pontecorvo, L. Resnick, T. Zittoun & B. Burge (Eds.), *Joining Society: Social Interactions and Learning in Adolescence and Youth* (pp. 310). Cambridge University Press.

Perret-Clermont, A.-N. (2015). The architecture of social relationships and thinking spaces for growth. In C. Psaltis, A. Gillespie & A.-N. Perret-Clermont (Eds.), *Social Relations in Human and Societal Development* (pp. 51-70). Palgrave Macmillan.

Perret-Clermont, A.-N., Perret, J.-F., & Bell, N. (1991). The social construction of meaning. In L. B. Resnick, J.-M. Levine & S. D. Teasley (Eds.), *Perspectives on socially shared cognition*. American Psychological Association.

Polo M, (2017). The Professional Development of Mathematics Teachers: Generality and Specificity. In G. Aldon, F. Hitt, L. Bazzini, U. Gellert (eds.), Mathematics and Technology, Advances in Mathematics Education, DOI, © Springer International Publishing, pp. 495-521.

Rigotti E., Greco S. (2010) Comparing the Argumentum Model of Topics to Other Contemporary Approaches to Argument Schemes: The Procedural and Material Components, Argumentation 24(4): 489-512

Robin, B. (2008). The effective uses of digital storytelling as a teaching and learning tool. *Handbook of Research on Teaching Literacy through the Communicative and Visual Arts, Vol.* 2, (pp. 429-440). Lawrence Erlbaum Associates.

Schubauer-Leoni, M. L. (1986). Maître-élève-savoir: analyse psychosociale du jeu et des enjeux de la relation didactique. Ph.D. Thesis, Faculty of Psychology and Educational Sciences, University of Geneva.

Searle, John R. (1973), "Austin on locutionary and illocutionary acts", in I. Berlin et al. (1973), pp. 141-59.

Savarese, G., Iannaccone, A., Mollo, M., Pecoraro, N., Fasano, O., Carpinelli L. (2019). Academic Performance-related Stress Levels and Reflective Awareness: The Role of the Elicitation Approaching an Italian University's Psychological *Counselling. British Journal of Guidance and Counselling*. Vol. 47. Pag.1-10 ISSN:0306-9885.

Searle, John R. (1975a), "A taxonomy of illocutionary acts", in K. Gunderson (a cura di), Language, Mind and Knowledge, Minneapolis, University of Minnesota Press, ora in J.R.

Searle (1979), tr. it. in M. Sbisà (a cura di) (1978), pp. 168-198.

Searle, John R. (1975b), "Indirect speech acts", in P. Cole e J. L. Morgan (a cura di), Syntax and Semantics III: Speech Acts, New York, Academic Press, ora in J.R. Searle (1979), tr. it. in M. Sbisà (a cura di) (1978), pp. 252-280.

Sfard, A. (2001). Learning mathematics as developing a discourse. In *Proceedings of 21st Conference of PME-NA* (pp. 23-44). Columbus, OH: Clearing House for Science, mathematics, and Environmental Education.

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses, and mathematizing*. New York: Cambridge University Press

Sfard, A. (2011). There is more to discourse than meets the ears: looking at thinking as communicating to learn more about mathematical learning. *Educational Studies in Mathematics*, 46, 13-57.

Szulevicz T., May Eckerdal R., Marsico G., Valsiner, J. (2016). When disruptive behaviour meets outcome-based education*. Psihologij*a. Vol. 49(4), 447‑468, DOI: 10.2298/PSI1604447S;

Traverso, V. (1990). *L'analyse des conversations*. Nathan.

van Eemeren, F.H., & Grootendorst, R. (1984). *Speech Acts in argumentative discussions.* Foris Publications.

van Eemeren, F.H., & Grootendorst, R. (1992). *Argumentation, communication and fallacies. A pragma-dialectical perspective*. Lawrence Erlbaum Associates.

van Eemeren, F.H., Grootendorts, R., & Snoeck Henkemans, F. (1996). *Fundamentals of argu- mentation theory: A handbook of historical backgrounds and contemporary developments*.mLawrence Erlbaum Associates.

Vergnaud, G. (2015). Argumentation et conceptualisation: commentaires. In N. Muller Mirza & C. Buty (Eds.), L'argumentation dans les contextes de l'éducation (pp. XX-XX). Lang.

Vygotsky, L.S. (1930/1978). *Mind in society: e development of higher psychological processes*. Ed. by M. Cole, V. John-Steiner, S. Scribner, & E. Souberman. Harvard University Press.

Vygotskij, L. (1978). *Mind and society: The development of higher psychological processes*. Harvard University Press.

Walton D., Krabbe E. (1995). *Commitment in Dialogue. Basic concepts of interpersonal reasoning*. State University of New York Press.

Wegerif, R. (2013). *Dialogic: Education for the Internet Age*. Routledge.

Wertsch, J. V. (1998). *Mind as action*. Oxford University Press

Zan, R. (2012). La dimensione narrativa di un problema: il modello C&D per l'analisi e la (ri)formulazione del testo. Parte I. *L'insegnamento della matematica e delle scienze integrate*. Vol.35 A N.2 marzo 2012.

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