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Mathematics Mobility in the Middle Grades: Tracking the Odds of Completing Calculus

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Abstract

High school calculus has become indispensable for students seeking a college degree in a STEM field. However, in the present study, we argue that the mathematics opportunities that students seize (when afforded) in middle grades are the key to earning calculus credit in high school. To take calculus in high school, students usually need to take advanced mathematics in middle school to take the prerequisite courses. We analyzed the probability of earning credit in calculus based on a sample of (n = 17,765) students and their eighth-grade mathematics courses. Using descriptive statistics and odds ratio effect sizes we found that taking advanced (i.e., algebra in eighth grade) mathematics courses greatly increased a student's chances of earning calculus credit in high school. However, the results indicated that taking double advanced (i.e., geometry in eighth grade) only statistically significantly improved the odds of Asian students earning calculus credit in high school. While participating in double advanced mathematics statistically significantly lowered the odds of earning calculus credit compared to the advanced track for Black and Latinx students. These findings have major implications for providing equitable math instruction for all students, especially in light of the many math students who are tracked into courses with limited ways to earn the calculus credit necessary for STEM-related college and career success.

Introduction

Early access to advanced mathematics learning opportunities remains an important consideration for persistence and success in mathematics. Specifically, access to advanced mathematics in the middle grades increases a student's *mathematics mobility*, defined as the ability to move between different levels of mathematics. When a student takes advanced mathematics in the middle grades, they are free to move through the entire mathematics course sequence available in high school; however, if they do not take advanced mathematics in middle school, there are often logistical constraints that will prevent them from completing the entire mathematics course sequence. Thus, their mathematics mobility is limited. For example, prior research has shown that students who took algebra in eighth grade were between three and 11 times more likely to be in the high school honors or college mathematics track. However, the authors note that African American and Latinx students were underrepresented, and Asian American students were overrepresented compared to White students even after controlling for prior achievement (Ballon, 2008). In addition, the effects of taking advanced mathematics in the middle grades are well documented, yet previous studies have focused on differentiating effect sizes between groups rather than within groups. Thus, the present study focuses on middle school mathematics and student ability to earn high school calculus credit in light of mathematics ability grouping/tracking and its effects on student opportunities. Specifically, we hope to illuminate the long-term effects of the opportunity gap and provide recommendations to support equitable access to high school calculus for students of color. To this end, the purpose of this study was to characterize the unique effects of advanced and double advanced eighth-grade mathematics participation on calculus success across racial and ethnic groups.

Regrettably, many students lack early opportunities to receive rigorous mathematics instruction from well-prepared mathematics instructors. Yet, in *This We Believe: Keys to Educating Young Adolescents*, the National Middle School Association (2010) emphasized the importance of offering curriculum that is challenging, exploratory, integrated, and relevant along with the need for educators to employ multiple learning and teaching strategies. It is important to note that the National Middle School Association pushed not only for challenging curriculum, but *This We Believe* specifically involved the offering of advanced math courses earlier. This shift in focus, coupled with the “Algebra for All” movement, corresponds to the increased number of middle schools offering algebra and geometry. In addition, these movements increased the number of students enrolling in advanced mathematics courses. Today, eighth-grade algebra is considered advanced mathematics, and many schools have begun to offer geometry at the eighth-grade level as a double advanced option in response to the "Algebra for All" movement. However, while the number of high school students who earned calculus credit doubled between 1982 to 2004, the race-based gaps in opportunities for calculus completion did not change (Domina & Saldana, 2012). Given the importance of calculus as a prerequisite for engineering and science success, closing these gaps in opportunities is an important consideration for science, technology, engineering, and mathematics (STEM) workforce diversification.

Missed opportunities to take algebra in the middle grades often exclude Black and Latinx students from earning calculus credit in high school well before ninth grade. Moreover, there are noticeable differences in participation when disaggregating these data by race. According to data from the National Center for Education Statistics (2016), Asian students participate in high school calculus at a rate of 45%, followed by White student participation at 18%, Latinx student participation at 10%, and Black student participation at 6%. Disproportionate access to high school calculus creates academic and career challenges for students lacking these opportunities to learn. To address these issues, we will investigate the participation trends for different racial/ethnic groups of learners across the most common middle grades mathematics tracks.

Literature Review

Tracking

Tracking, or the placement of students into differentiated courses due to their perceived ability, is an enduring practice in the United States despite copious research on its adverse effects on students experiencing poverty and minoritized students (Oakes, 2005). For example, K. Tyson (2011) argued that tracking often separates students along racial lines in ways that create within-school segregation that negates the gains made by African American students after desegregation, and Clotfelter (2004) also found tracking associated with less interracial contact in schools.

Although research on academic tracking led to a decrease in tracking practices in the 70s and 80s, the practice has surged more recently, especially in mathematics, where about three-fourths of eighth-grade students during the 21st century have attended tracked mathematics courses (Loveless, 2013). Unfortunately, these tracking policies have led to a lack of access to eighth grade algebra for traditionally marginalized and minoritized students of color in middle schools. Ngo and Velasquez (2020) found that students experience limited mathematics mobility once they enter a tracking system, with the repetition of mathematics courses significantly linked to race/ethnicity. Unfortunately, most administrators and policymakers do not focus on structural issues like tracking that prevent all students from equitable access to algebra (Herbel-Eisenmann et al., 2018). Hence, the resurgence of a need to examine how tracking can hinder a student's mathematics mobility.

Although the proportion of eighth graders in advanced or double advanced mathematics courses doubled between 1990 and 2011, sizeable between-school variation remains for the chances of a student taking those advanced courses, which mathematics ability grouping in elementary school may explain (Domina, 2014). Black students have reduced odds of taking middle school algebra even when they have the same level of achievement as White students, often because teacher recommendations have a greater negative effect on Black students' placement in eighth-grade algebra (Faulkner et al., 2014). This lack of high-track middle school mathematics coursework limits students' ability to take high-track mathematics courses in high school. Due to lower levels of achievement, prior coursework, and lower socioeconomic status, Black students are more likely to take low-track mathematics courses in the tenth grade than White students. However, individual-level differences between students, like coursework and family backgrounds, do not completely explain the Black-White gap in mathematics course taking (Kelly, 2009).

Importance of Calculus

Calculus is an important form of mathematics for all students, yet access and availability remain scarce. Only 20% of high school students take calculus, which has become a prerequisite for admission into elite universities. Disproportionate access to high school calculus creates academic and career challenges for students lacking these opportunities to learn. For instance, many American Association of University schools require non-STEM degree-seeking students to complete Calculus 1 before graduation (Coleman et al., 2019). Completing calculus in high school would support the career success of many non-STEM majors by allowing these students to achieve their calculus requirements before college.

For students seeking a STEM degree, completing high school calculus increases the probability of obtaining a STEM degree and succeeding in a STEM career by increasing their college readiness and STEM interest. For example, students who took Advanced Placement calculus courses in high school were more likely to select STEM careers (Robinson, 2003). High school calculus was also the strongest predictor of grades in STEM gatekeeper courses such as college physics and calculus (Cullinane, 2011; Redmond-Sanogo et al., 2016; W. Tyson, 2011). Eagen et al. (2014) discovered in their sample that STEM majors had an 11-percentage point higher AP calculus course taking rate than other majors, with over half of engineering students having taken calculus in high school. Maltese and Tai (2011) also found that taking high school calculus was a significant predictor of pursuing a STEM major and earning a STEM degree. However, STEM interest was an even stronger factor. This relates to Sadler et al.'s (2014) finding that high school calculus predicted increased STEM interest.

These benefits of high school calculus are especially relevant for Black and Latinx students. Avery and Goodman (2022) argued that the racial gaps in advanced mathematics block access to STEM careers. Similarly, W. Tyson et al. (2007) found that racial STEM disparities occur because there are fewer options for Black and Latinx students to engage in STEM preparation in high school through courses like calculus. Fry et al. (2021) found that the STEM workforce has an underrepresentation of Black and Latinx workers that they argue will not close because of the gap in STEM degrees for Black and Latinx college students. However, Smith et al. (2018) determined that AP STEM course taking led to higher STEM grades and STEM major completion for underrepresented minority students.

In addition to lowering access to STEM careers, economically, these missed or absent opportunities create long-term wage disparities. For instance, Battey (2013) found that differences in mathematics coursework for White students led to a wage gap that totaled hundreds of billions of dollars.

Access to Calculus

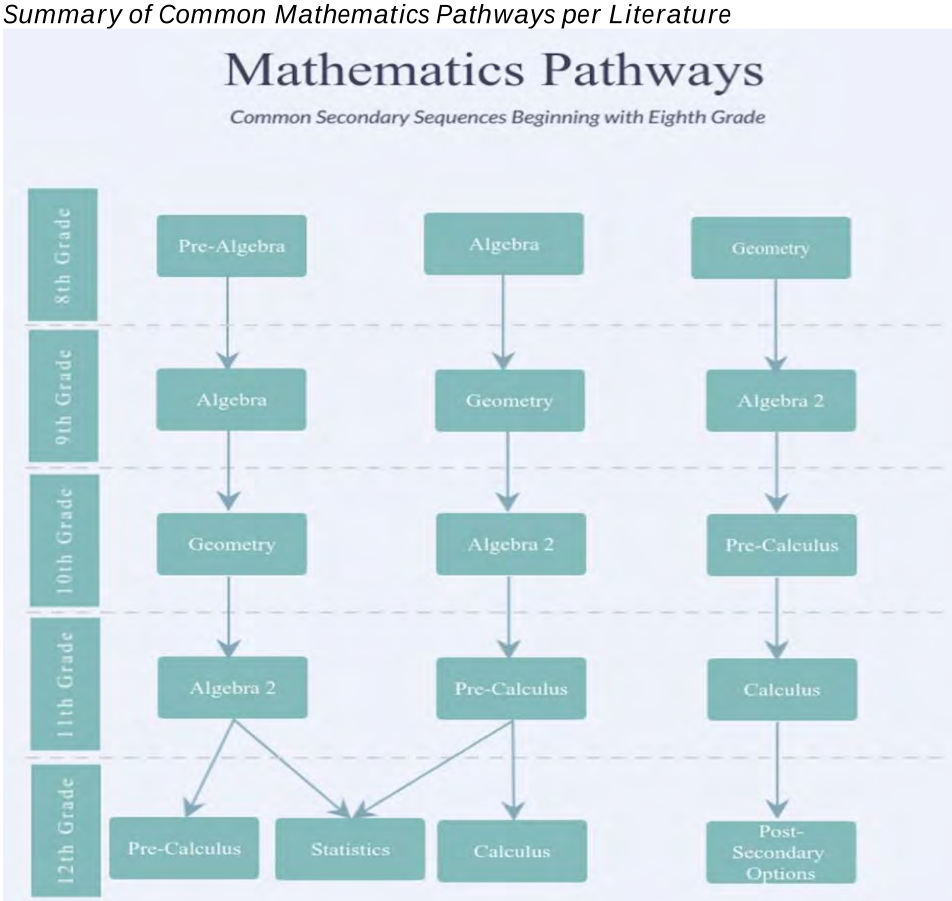
Addressing the access and opportunity challenges is complicated by the systematic barriers created by the traditional mathematics curriculum sequence structure used in most schools across the country. The disparity in funding for high schools limits the opportunities for less privileged students to take calculus coursework in high school (Bressoud, 2021). The preparation for calculus begins before high school, as students must complete a standard sequence of courses before attempting calculus. The United States' most common mathematics courses sequence is pre-algebra, Algebra I, geometry, Algebra II, trigonometry/pre-calculus, and calculus (Irizarry, 2021). As shown in Figure 1, there are three common paths for mathematics coursework in the traditional mathematics sequence. All three mathematics pathways depend on the student's eighth-grade course placement. The first pathway is the traditional or regular middle grades sequence that includes pre-algebra in eighth grade and then Algebra 1 in ninth grade. Note that this pathway is the first of the three and concludes with the student taking pre-calculus or statistics in 12th grade.

Depending on prior academic attainment in middle school and teacher recommendations, many U.S. students begin their high school mathematics sequence with Geometry and then progress to Algebra 2, pre-calculus, and calculus. This mathematics sequence is what we deemed the *advanced mathematics track*. The center of Figure 1 illustrates this common pathway. However, as noted in Figure 1, the student would not have access to calculus without the mathematics education intensification in eighth grade from prealgebra to Algebra 1. Lastly, an emergent trend for some school districts is to allow students to take Geometry in eighth grade.

In this double advanced case, these students are likely to progress on the path depicted on the far right-hand side of Figure 1. Although Figure 1 shows several pathway options for students, we must consider that mathematics courses are also horizontally differentiated into leveled courses, such as standard or accelerated (W. Tyson & Roksa, 2016). This additional consideration means that the sequential nature of the mathematics curriculum is both vertical and horizontal.

Due to the sequential nature of mathematics, students can only advance to the subsequent mathematics course after completing the previous course in the sequence. In some instances, students can "double-up" on classes, such as taking geometry and Algebra 2 simultaneously; however, researchers have found few favorable results to support simultaneous enrollment (Schiller & Hunt, 2011). Moreover, doubling up creates tight course trajectories and ultimately limits access to high school calculus.

Figure 1
Summary of Common Mathematics Pathways per Literature



Given the number of courses present in the standard course sequence for calculus readiness, there remains a dominant concern to increase student access to rigorous mathematics courses early. Across the US there are continuous debates and suggested strategies to support access to and success in eighth-grade algebra for all students. Based on the standard sequence presented above, if all schools required students to take algebra in the eighth grade, they would have sufficient time to take the remaining requisite classes to attempt calculus as high school seniors. However, efforts to provide eighth-grade algebra for all students have had mixed results. Although many students find success in calculus after completing advanced mathematics courses in middle school and high school, other students struggle in calculus despite earlier mathematics success and, consequently, develop negative mathematics attitudes (Domina & Saldana, 2012; Lee & Mao, 2020; Morton & Riegle-Crumb, 2019). One plausible explanation for the observed differences in the effectiveness of early access to algebra on calculus success is the quality of those early mathematics experiences. Previous research discovered that early accelerated mathematics helped encourage students from disadvantaged backgrounds to take more advanced high school mathematics courses and improved college readiness (Dougherty et al., 2017; Ma, 2010). Regrettably, many students lack early opportunities to receive rigorous mathematics instruction from well-prepared mathematics instructors.

Theoretical Framework

To conceptualize, theorize, and interpret the results of our study, we will use the inequality regimes concept that Hanselman et al. (2021) adapted from workplace inequality literature to analyze variations of inequality of opportunity across schools. They argued that two primary organizational practices create educational inequality regimes: (1) "practices that determine how many desirable educational opportunities schools provide" (Hanselman et al., p. 2), and (2) gatekeeping practices. To explore their theory, they examined the different ways schools responded to California's "Algebra for All" effort to synthesize micro-level tracking studies and macro-level theories of maintained inequality to "advance a meso-level theory of educational inequality" (Hanselman et al., p. 2). They found differences in opportunities were consistent and related to the organizational characteristics of the schools, with some consistently reproducing inequalities of opportunities while others worked against those inequalities.

Tracking was the primary mechanism for realizing inequality regimes (Hanselman et al., 2021). Schools made decisions that played two interrelated roles that created unequal opportunities through tracking: (1) provision decisions about what environment to create and how many high-status spots to create, and (2) allocation decisions about how to distribute those opportunities. Even with external pressure to offer algebra to more students, local regimes continued reproducing inequality of opportunity for students to take middle school algebra. Inequality regimes explain how tracking reproduces inequality, especially in mathematics, where different tracks serve as gatekeepers to student opportunities. Our study extends the work of Hanselman et al. on inequality regimes by examining the effects caused by those observed inequalities of opportunities to access algebra for middle school students on calculus credit obtained. The following research questions guided this endeavor:

1. What proportion of different racial/ethnic groups earns calculus credit in high school across middle grades mathematics tracks (i.e., regular, advanced, or double advanced mathematics)?

2. How does academic tracking in middle grades mathematics (i.e., advanced/double advanced mathematics participation versus regular mathematics participation) affect the likelihood of earning calculus credit in high school for different racial/ethnic groups?

Methodology

The present exploratory quantitative study used descriptive statistics and odds ratio (OR) effect sizes. The present study used a sample of students (N = 23,503) who participated in the High School Longitudinal Study (HSLS) of 2009/2012. According to the HSLS data collection protocol, students were randomly selected from over 21,000 students from 944 public, charter, and private schools in the US. This population consisted of 47% male and 53% female students. In 2009, data were collected from participants in the base year. The base-year data collection included online surveys administered to students, parents, mathematics teachers, and administrators. Similar online surveys were administered to parents and students in subsequent follow-up sessions. These variables represent the independent and dependent variables examined in this study. R Version 4.1.0 was used to analyze the data. The relevant variables were S1M8 (8th-grade mathematics course), X1RACE (student race/ethnicity), and X3T1CREDCALC (at least one credit earned in Calculus).

Our independent variables were three common middle grades mathematics course offerings: regular (i.e., mathematics 8/pre-algebra); advanced (i.e., algebra I); and double advanced (i.e., geometry). These were based on data collected from student transcripts. We then coded calculus credit as a binary dependent variable. It is important to note that we only focused on courses and racial groups with over 500 students in this study to minimize biases related to representation differences based on sample size variation, leaving us with a sample size of 17,765. First, we calculated descriptive statistics, such as means and percentages. Using the sample sizes from that analysis, we then calculated the ORs with the course of interest (i.e., mathematics 8/pre-algebra, algebra I, or geometry) as the treatment and obtaining credit in calculus as the response (i.e., dependent variable).

Analysis

To analyze the differences in the effects of academic tracking in middle grades mathematics on long-term mathematics learning outcomes (i.e., earning calculus credit), we calculated the 95% confidence intervals of percentages of students who earned calculus credit by race and middle grades mathematics track. In the current study, confidence intervals were selected because they provide point estimates for population parameters and a measure of the precision of these point estimates (Cumming & Finch, 2001; Young & Young, 2016). Two of the most commonly used families of point estimates are summary statistics (e.g., means, percentages) and effect sizes (i.e., *d, r,* OR) (Young et al., 2019; Zientek et al., 2010). In this study, we utilized one statistic from each family of point estimates: (1) percentages (i.e., summary statistics); and (2) ORs (i.e., effect sizes).

We used percentages to assess the representation of calculus credit earning trends across middle grades mathematics tracks and racial/ethnic subpopulations. ORs were used to assess the likelihood of earning calculus credit across middle grades mathematics tracks and racial/ethnic groups. Essentially, these two-point estimates provide related yet uniquely different pieces of information. The percentages describe how prevalent calculus credit earning is amongst students on middle-grade tracks across different ethnic groups. A sample question related to these point estimates is, "What percentage of Asian students who completed an advanced middle grades mathematics course earned calculus credit in high school?" Contrarily, the ORs provide data related to the predictability or likelihood of earning calculus credit in high school associated with participation in different middle grades mathematics tracks and racial/ethnic groups. Here the question shifts to, "What is the likelihood that an Asian student who took advanced mathematics in the middle grades will earn calculus credit in high school?"

When using confidence intervals, we refer to sample percentages or effect sizes as point estimates because they approximate population parameters. These estimates are unique for each academic track and racial/ethnic sample and are compared to establish trends across outcomes. This is important because each outcome (i.e., percentage of students earning calculus credit versus the odds of students earning calculus credit) is unique, and with this uniqueness comes various amounts of error. The confidence intervals allow the amount of error to be quantified across groups for comparison (Young & Young, 2021). The level of precision in each point estimate controls the error associated with each point estimate, represented by the width or the distance the confidence interval extends in both directions from the point estimate. The width of the confidence interval represents the precision associated with each point estimate. Specifically, the smaller the width of the confidence interval, the more precise the measurement, and the wider the confidence interval, the less precise the measurement (Finch & Cumming, 2009). The width of the confidence interval directly relates to the standard deviation of the point estimate and inversely relates to sample size. Appropriately, if the variability (standard deviation) is small, then the point estimate is more precise, and likewise, larger sample sizes are more representative, which also increases the precision of the point estimate.

In the present study, we utilized percentages as point estimates. To form a sample proportion, we took *X* (binominal random variable; e.g., earn calculus credit yes or no) and divided it by *n* (student sample size). The random variable *P′* is the sample proportion $P' = \frac{x}{n}$. Here *p′* is the estimated proportion or sample proportion of students that earned calculus credit, while *x* is the exact number of students earning calculus credit and *n* is the sample size. Finally, the confidence interval is then calculated using the formula below:

$$p = p' \pm \left[Z\left(\frac{a}{2}\right) \sqrt{\frac{p'(1-p')}{n}} \right]$$

Where $Z\left(\frac{a}{2}\right)$ is set according to the desired 95% confidence level and $\sqrt{\frac{p'(1-p')}{n}}$ represents the standard deviation. We calculated the confidence intervals for the percentage of students who earned calculus credit across the three different types of tracks (i.e., regular, advanced, and double advanced). The confidence interval tables include the sample and subsample sizes for each group of interests.

We calculated ORs for all racial groups in the analysis by middle grades mathematics course completed, using the students' tracked course (i.e., regular, advanced, or double advanced course) as the treatment. The OR is one of several statistics used to assess the risk of a particular outcome if a certain factor is present (Schmidt & Kohlmann, 2008). ORs indicate the likelihood of occurrence of a certain event versus its absence in relation to a particular treatment or variable (Allen, 2017). An OR provides information about effect sizes for two groups, and it is commonly used in medical research and cross-sectional studies. Values for the OR range from 0 to infinity; however, a value of 1 represents no relationship. An OR greater than 1 indicates a better chance of a favorable result when exposed to a treatment. An OR less than 1 translates into a lower probability of a positive outcome when exposed to the treatment. The effect size increases or decreases in magnitude, as a result moves further from 1, with no result less than zero (Allen). In our study, the OR facilitated estimates of how much more likely it is that students placed in an advanced middle grade mathematics course will earn calculus credit in high school.

We used the above relationship between advanced course tracking and calculus credit earned in the following manner in this study:

a. OR = 1, no relationship between middle grades mathematics track and calculus credits earned
b. OR < 1, students in this middle grades mathematics track are less likely to earn calculus credit in high school
c. OR > 1, students in this middle grades mathematics track are more likely to earn calculus credit in high school

We calculated 95% confidence intervals to determine whether or not the OR values were statistically significantly different from 1. If the OR statistically significantly diverges from 1, then the 95% confidence interval will not include 1. The debate continues surrounding the interpretation of effect size magnitude. Moreover, given the limited use of ORs in educational research, we provide a comparison strategy that utilizes benchmarks similar to those proposed for interpreting Cohen's *d* to minimize confusion and to reduce ambiguity. According to Chen et al. (2010), OR values of 1.68, 3.47, and 6.71 are equivalent to Cohen's *d* = 0.2 (small), 0.5 (medium), and 0.8 (large), respectively.

Results

The present study indicates that students who take regular mathematics in eighth grade have an extremely low chance of earning calculus credit in high school. For eighth graders who took regular mathematics, only 3.89% received credit in calculus. The OR for receiving calculus credit was 0.8, and an OR of less than 1 indicates reduced chances of obtaining credit in calculus. Given the traditional course sequence presented earlier, this is somewhat understandable given that most students only take one mathematics course per year. Thus, if they begin with Algebra I in ninth grade, they will need to take two mathematics courses in one year to catch up with their peers who took algebra in eighth grade. Unfortunately, data indicate that Black, Latinx, and students experiencing poverty represent the majority of students who do not participate in advanced mathematics courses in eighth grade.

Contrarily, students who take advanced or double advanced mathematics courses in eighth grade have a much higher chance of earning calculus credit in high school. For students who took advanced mathematics in eighth grade, 32.20% received credit in calculus, while of those who took a double advanced mathematic course, 57.80% received credit in calculus. Table 1 presents the percentages of each student group that earned calculus credit organized by eighth grade mathematics course completed. The OR for advanced students was 6.17, and for double advanced students, it was 8.23. Thus, by taking algebra in eighth grade, a student is approximately six times more likely to earn calculus credit in high school than a student who took pre-algebra. Likewise, by taking geometry in eighth grade, a student is approximately eight times more likely to earn calculus credit in high school than a student who took pre-algebra. It is important to note that there were nuanced differences in race disaggregated data (see Figure 2).

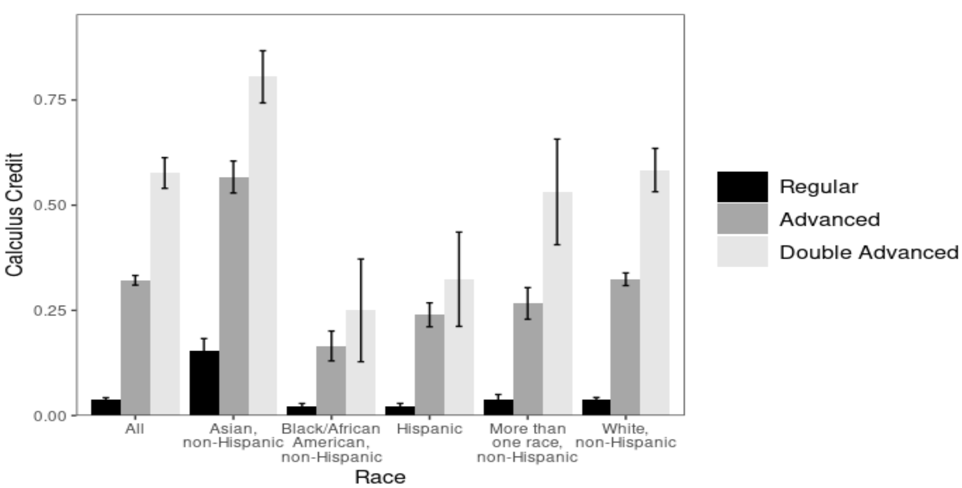
Table 1

Percentage of Students Who Gain Credit in Calculus by Eighth-Grade Mathematics Course

Race/Ethnicity	Math Level	*n*	Percentages
Asian, non-Hispanic	Regular	543	15.30%
Asian, non-Hispanic	Advanced	665	56.70%
Asian, non-Hispanic	Double Advanced	159	80.50%
Black/African-American, non-Hispanic	Regular	1330	2.11%
Black/African-American, non-Hispanic	Advanced	423	16.50%
Black/African-American, non-Hispanic	Double Advanced	52	25.00%
Hispanic	Regular	1960	2.24%
Hispanic	Advanced	872	24.00%
Hispanic	Double Advanced	71	32.40%
More than one race, non-Hispanic	Regular	994	3.82%
More than one race, non-Hispanic	Advanced	540	26.70%
More than one race, non-Hispanic	Double Advanced	64	53.10%
White, non-Hispanic	Regular	6044	3.81%
White, non-Hispanic	Advanced	3693	32.40%
White, non-Hispanic	Double Advanced	355	58.30%
All	Regular	10871	3.89%
All	Advanced	6193	32.20%
All	Double Advanced	701	57.80%

Figure 2

Percentage of Students Who Received Calculus Credit by Eighth-Grade Mathematics Course



*Error bars represent 95% confidence intervals

After completing the regular eighth grade mathematics track, the odds of earning calculus credit were less than 1 for all racial groups, indicating reduced odds of earning calculus credit. This result corresponds with the less than 4% of all students who earned calculus credit after completing the regular mathematics track. Aggregate results indicate that students who take advanced and double advanced mathematics in eighth grade have a statistically significantly higher likelihood of earning calculus credit. However, for all independent racial groups except Asian, there was no significant difference between the odds of earning calculus credit for students who took double advanced compared to advanced mathematics in eighth grade, based on overlapping 95% confidence intervals for most groups (excluding the more than one race category).

The odds of Asian students earning calculus credit after taking advanced mathematics in eighth grade were 3.05 compared to 6.71 when Asian students took double advanced mathematics in eighth grade. This is more than double an increase in the odds of earning calculus credit. On the other hand, African American students who took advanced mathematics were about six times more likely to earn calculus credit than African American students who completed the regular mathematics course. However, the odds were reduced for African American students when they completed double advanced mathematics, decreasing from 6.49 to 5.63. This reduction in odds suggests that when an African American student takes geometry in eighth grade and then enters high school, they do not earn calculus credit at a higher rate than the regular track. Despite having to take at least three more years of mathematics (Algebra 2, pre-calculus/trigonometry, or calculus) and limited options for the third year of mathematics besides calculus or statistics.

Our analysis revealed a similar trend for Latinx students, albeit the magnitude was far more extreme (see Figure 3). For example, Latinx students' odds of earning calculus credit after taking advanced mathematics were 9.24 but reduced to 4.88 when taking double advanced mathematics. In addition, students who identified with more than one race that took advanced mathematics had an OR of earning calculus credit of 4.98 compared to 8.42 when they took double advanced mathematics. Finally, White students who took advanced mathematics in eighth grade had an OR of 6.54 compared to 8.14 when they took double advanced mathematics. Table 2 provides the complete list of ORs by race and eighth grade mathematics course completed.

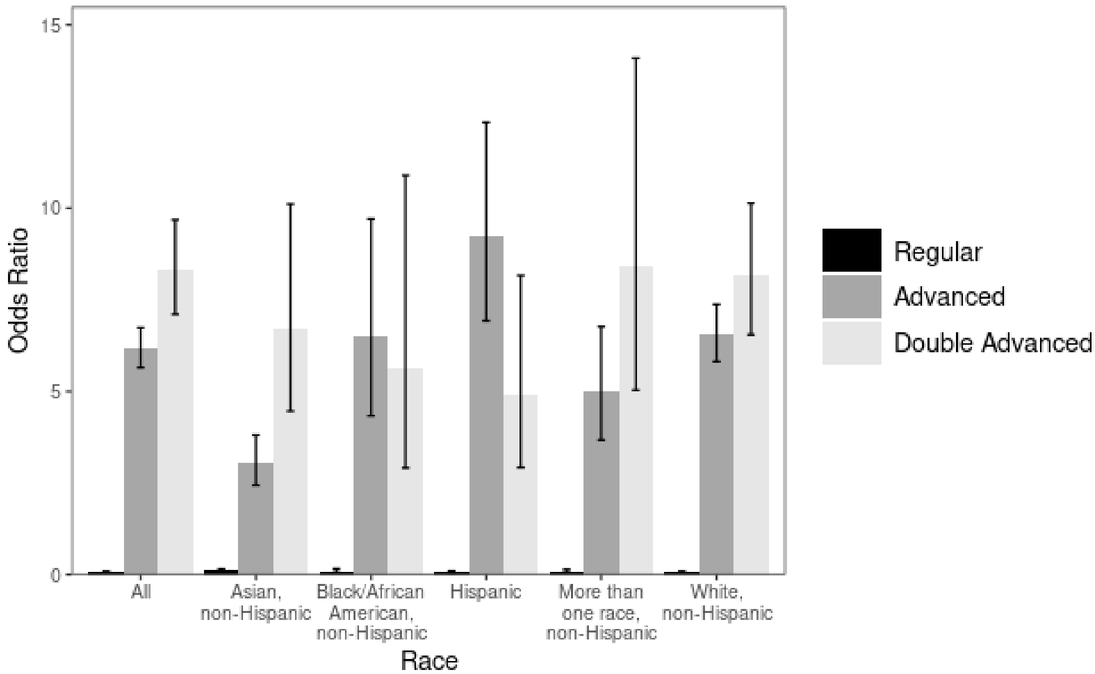
Table 2

Odds Ratios for Chances to Gain Credit in Calculus

Race/Ethnicity	Math Level	Odds Ratio	95% CI LL	95% CI UL
Asian, non-Hispanic	Regular	0.11***	0.09	0.15
Asian, non-Hispanic	Advanced	3.05***	2.44	3.81
Asian, non-Hispanic	Double Advanced	6.71***	4.46	10.11
Black/African-American, non-Hispanic	Regular	0.10***	0.07	0.16
Black/African-American, non-Hispanic	Advanced	6.49***	4.34	9.70
Black/African-American, non-Hispanic	Double Advanced	5.63***	2.91	10.89
Hispanic	Regular	0.07***	0.05	0.10
Hispanic	Advanced	9.24***	6.92	12.34
Hispanic	Double Advanced	4.88***	2.92	8.16
More than one race, non-Hispanic	Regular	0.10***	0.07	0.14
More than one race, non-Hispanic	Advanced	4.98***	3.67	6.76
More than one race, non-Hispanic	Double Advanced	8.42***	5.03	14.09
White, non-Hispanic	Regular	0.07***	0.06	0.09
White, non-Hispanic	Advanced	6.54***	5.81	7.37
White, non-Hispanic	Double Advanced	8.14***	6.55	10.13
All	Regular	0.08***	0.07	0.08
All	Advanced	6.17***	5.65	6.73
All	Double Advanced	8.29***	7.10	9.68

***p<0.001

Figure 3
Odds Ratios for Calculus Credit Earned Based on Eighth-Grade Mathematics Course



*Error bars represent 95% confidence intervals

Limitations and Future Considerations

The present study cannot explain why students in different mathematics tracks have different calculus credit earning rates. However, the present study focused on quantifying one long-term learning outcome of advanced mathematics participation in the middle grades. Future studies should address both the many opportunity- and propensity-related factors that can contribute to these differences. For instance, many schools require that students receive recommendations to take advanced middle grade mathematics courses, while other school districts have algebra for all policies. This differential level of access warrants further examination. Moreover, researchers could address how the differential learning experiences in advanced middle grades mathematics and student prior knowledge based on grade point average or standardized testing could affect long-term learning outcomes despite access to advanced mathematics in the middle grades.

The present analysis uses student race and middle grades mathematics track as the main units of analysis. According to emergent intersectional research, it is important to consider the many different aspects of student identity (e.g., race, gender, income). Thus, future studies should consider the effects of student gender, socioeconomic status, and other previous research associated with middle grades mathematics placement and calculus completion. To our knowledge, the present study is the first to quantify the relationship between middle grades mathematics placement and high school calculus learning outcomes for different racial groups. Although many have alluded to this relationship (e.g., Barger & McCoy, 2010; Champion & Mesa, 2018; Useem, 1992), we contend that providing summary statistics and related effect sizes for these trends represents an essential first step toward deeper investigations.

Discussion

The present study results have several important implications for middle grades mathematics teaching, learning, and policy. First, students taking regular mathematics courses in eighth grade have an extremely low chance of earning calculus credit in high school. Second, the reduced proportions and odds of earning calculus credit in high school by taking advanced mathematics in the middle grades across all racial groups have noticeable implications for the calculus pipeline. Third, there is a noticeable mathematics course sequence present in nearly all U.S. high schools (Riegle-Crumb, 2006; Schneider et al., 1997). The most common sequence is a progression from general mathematics to algebra I, geometry, algebra II, trigonometry or pre-calculus, and calculus. This traditional mathematics course sequence may create systematic and logistical challenges for students who did not take an advanced mathematics course in middle school.

Although there has been much debate surrounding the sequential nature of this progression, one challenge that receives limited attention is the ability to complete this sequence without acceleration. For instance, 94% of U.S. states require at least three years of high school mathematics for graduation. However, more specifically, three (6%) states only require two years, 32 (64%) states require three years, and 15 (30%) states require four years of mathematics (Gao, 2017). Given that most students will only complete three years of mathematics in high school, taking an accelerated path is the only way many students can complete the sequence. Because algebra I is considered ninth grade mathematics in most high schools, and most high schools only require three years of mathematics, most students complete their mathematics requirements after taking algebra II in their junior year. This trend directly aligns with the findings from the present study, as students who do not take advanced or double advanced mathematics in the middle grades are statistically significantly less likely to earn calculus credit in high school. One important consideration related to this trend is that calculus is not required to graduate in most high schools; moreover, high school calculus is unattainable without acceleration and the completion of an additional mathematics course requirement for most students. However, the double accelerated path, which requires that algebra I be completed in the seventh grade and subsequently geometry in eighth grade, provides a path to high school calculus that does not require taking an additional year of mathematics that is not required to graduate.

Unfortunately, the second major takeaway from this study is that the double advanced middle grades mathematics option might be less efficacious for all racial groups except Asian students. Based on the data presented in the present study, taking geometry in eighth grade only statistically significantly increased the odds of earning calculus credit for Asian students. For African American and Latinx students, the odds of earning calculus credit were lower when these groups of students took double advanced middle grades mathematics, or geometry, in eighth grade. For White students, the odds increased, but for all groups, except Asian students, the differences were not statistically significantly different. Because the double advanced option requires a student to complete algebra I in seventh grade, the traditional middle grades mathematics course content must be covered in its entirety in sixth grade, along with the prerequisite algebra knowledge, which could pose a substantial challenge for most students, as indicated by the lack of statistically significant improvement across most racial groups.

As eighth grade geometry becomes more common amongst middle schoolers, it is vital to clarify why this mathematics pathway is less effective for African American and Latinx learners. One explanation could be that because algebra and geometry are considered high school mathematics courses, some states may allow credits earned in the middle grades to count towards the required number of high school courses needed for graduation. Subsequently, some students may then choose to forgo taking any additional mathematics courses after meeting the minimum requirement, which is reached after the ninth grade for students on the double advanced track in this scenario. However, further research is needed to address why double advanced mathematics participation seems to not benefit African American and Latinx students in the same manner as it does Asian and White students.

The rationale for the difference in efficacy of the double advanced middle grades track warrants further consideration. Thus, we provide the following potential future lines of inquiry. Future research should examine possible opportunity gaps that may have a more devastating effect on the highest performing African American and Latinx learners. Unfortunately, only 50% of U.S. high schools offer calculus (U.S. Department of Education Office of Civil Rights, 2014). Moreover, national reports indicate that most schools lacking calculus and other advanced STEM courses typically serve large populations of African American and Latinx learners (May & Chubin, 2003; W. Tyson et al., 2007). Thus, we argue that examining the opportunity structures that afford and constrain access to calculus for double advanced learners warrants further investigation. Based on the observed trends in the present study, we identified specific implications for policy and instructional practice:

1) In order to increase the odds of Black and Latinx students earning calculus credit in high school, we must provide more than simply access to eighth grade algebra; rather, there is a need for more support and enrichment in grades sixth and seventh as well as subsequent support in high school to build on mathematics knowledge and skills. Moreover, we recommend that policy and programming include more interventions and support for the development of mathematics identity, interest, and self-efficacy, as these psychological mathematics dispositions tend to be highly correlated to STEM persistence (Young et al., 2021; Young et al., 2019).

2) We must consider the need for bridge programs to make up for the lack of algebra knowledge and skills for those who take regular mathematics in the sixth, seventh, and eighth grades to provide such students the opportunity for calculus in high school. In addition, out-of-school time enrichment activities are a good initial consideration to support early STEM interest in the middle grades (Young & Young, 2018; Young et al., 2017). However, accountability policies that promote four-year mathematics requirements for graduation as well as a variety of mathematics acceleration opportunities in the middle grades are also warranted (Young et al., 2019). These policies would help create a more inclusive and stable trajectory to calculus that facilitates mathematics mobility.

3) We must increase our knowledge related to the effects of double advanced middle grades mathematics tracks on subsequent mathematics success. The results of the present study indicate that taking algebra in seventh grade and geometry in eighth grade only statistically significantly affects the odds of completing calculus in high school for Asian students. According to a famous quote by Bobby Unser, "Success is where preparation and opportunity meet." The data here may suggest that many Latinx and Black students were not fully prepared for the opportunity to take double advanced middle grades mathematics courses. However, the lack of statistically significant differences between regular and double advanced mathematics tracks could also indicate that Black and Latinx students lacked a critical mass of ethnically matched peers on the double advanced path and thus chose to forgo pursuing calculus due to feelings of isolation, as documented in the literature (Jeffries & Silvernail, 2017; Young et al., 2020). Subsequently, because earning calculus credit is only one learning outcome, scholars must examine other outcomes to better understand this phenomenon.

Conclusion

In conclusion, our results suggest that many African American and Latinx students may be victims of *elusive opportunities*. We define elusive opportunities as a lack of access to the full scope of the opportunities that are warranted for high-achieving underrepresented students due to a lack of resources at the schools they attend. For example, prior research indicates that some African American and Latinx students choose not to participate in calculus in high school, even when afforded access to advanced STEM courses at their local high schools (Riegle-Crumb, 2006). The literature commonly cites two explanations.

First, some argue that African American and Latinx students choose not to enroll in advanced STEM courses in high school due to a lack of a critical mass of African American and Latinx peers in advanced STEM courses (Solorzano & Ornelas, 2004). For example, according to Taliaferro and DeCuir-Gunby (2008), African American students tend to have issues related to belonging and operational citizenship within AP courses in the 10 high schools participating in their study, a common result of the absence of a critical mass of ethnically-matched peers. Similarly, inductive and constructive analysis of interview data reveals that peer relations and parental support were the primary determining factors of Latinx student underrepresentation in

AP coursework (Walker & Pearsall, 2012). In contrast, others cite a lack of explicit counseling on which high school courses students should take to attend competitive colleges and universities (Davis et al., 2013). Students experiencing poverty and students of color lack access to information about the courses needed for acceptance at different types of colleges (Schneider et al., 2012). Nonetheless, our results indicate that participating in an advanced mathematics track (i.e., algebra in eighth grade) in the middle grades is essential to earning calculus credit in high school from a logistical as well as a pedagogical perspective. Future research should look into school-, district- and state-level factors that could contribute to these trends. We recommend first examining the trends related to the number of state-required mathematics courses for graduation and how these requirements are impacted by the courses students have access to in the middle grades. Then we suggest that researchers examine the relationship between calculus availability in high schools across the nation that serve large populations of Black and Latinx learners who completed advanced mathematics in the middle grades and the subsequent calculus credits earned by this group of students. Finally, we hope that educational stakeholders will consider the above implications as they make future decisions related to middle grades mathematics tracking practices and policies that can influence a student's mathematics mobility.

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