



Achieving Equity and Excellence in Mathematics Teaching

International and national assessments suggest that U.S. students' mathematics performance declines drastically as they progress to higher grades.¹ Improvement in K-12 performance remains elusive

despite efforts to set more rigorous academic standards, align curriculum materials more closely with those standards, and test more frequently. Such reforms have not changed how math is

States should revamp how teachers are equipped to deliver effective instruction.

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taught. Math instruction in the United States still focuses more on rules than on making sense of concepts.² Until that changes, student performance is unlikely to change.

Boosting Content Expertise

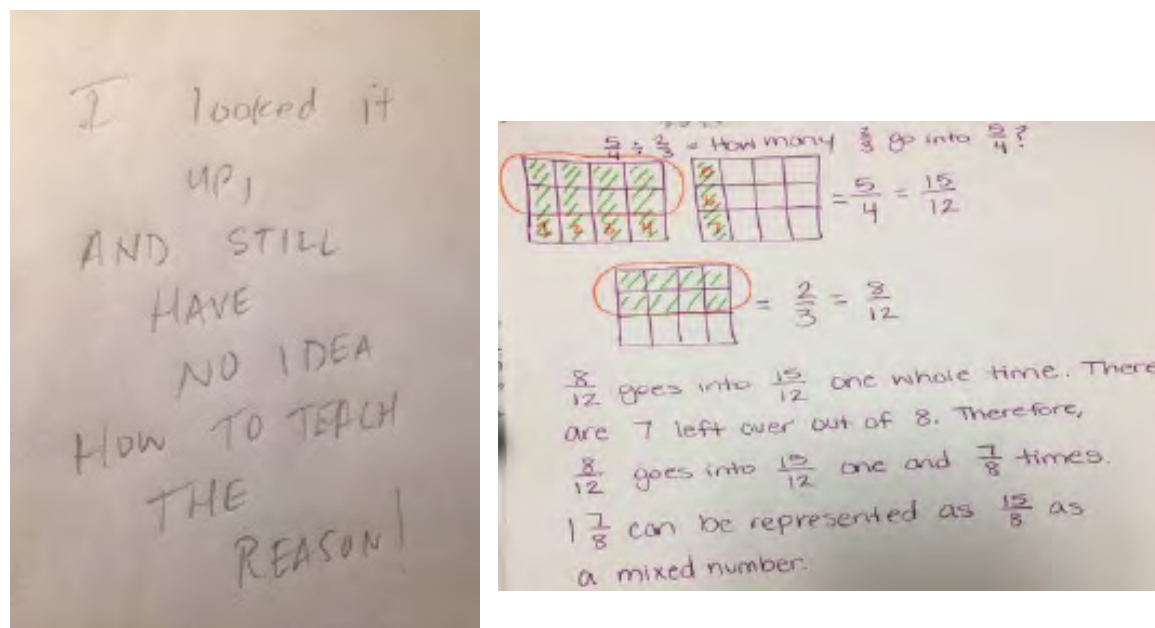
One way to improve instruction is through a more systematic approach to teacher training. Researchers have learned much over the last two decades about the nature of math teaching expertise and how to develop it.³ A robust understanding requires teachers to have mastered the conceptual underpinnings of the math rules they are teaching (e.g., why do you need to create a common denominator when adding two fractions with unlike denominators?), and they should know how different concepts taught within and across grade levels are connected (e.g., fractions can be connected to division).

When teachers' math understanding is fragmented and disconnected, the learning environment they create fails to be as meaningful as it could be. Studies dating to the early 1990s have shown that teachers' robust understanding is closely related to the way they help their students

make sense of math.⁴ Yet teachers in general, and elementary school teachers in particular, still do not routinely master conceptual underpinnings of the topics they teach. In a recent study with more than 300 grade 4 and 5 teachers, I found that 58 percent reported either not knowing how to explain fraction division conceptually or they explained it incorrectly.⁵ Only 26 percent provided a correct conceptual explanation (figure 1).

It is important to examine what these results mean for students, particularly students of color and students from low-income families, who typically have less access to teachers with strong mathematical knowledge.⁶ These students will learn rote procedures and probably struggle to remember them because they have no understanding of why a procedure works. Alternatively, students whose teachers help them make sense of the algorithm by making connections to key ideas will have a better learning experience (e.g., the number of groups of $\frac{2}{3}$ that can be made from $\frac{5}{4}$). These students can divide fractions even if they do not remember the invert-and-multiply algorithm and are more likely to see a connection between the division of fractions and making groups, a concept learned in earlier grades.

Figure 1. Sample Responses Teachers Provided as Conceptual Explanations for the Division of $\frac{5}{4}$ by $\frac{2}{3}$



Another important element of teachers' expertise is knowing how students learn concepts and building a repertoire of effective tools and strategies to aid conceptual understanding. Despite research that says students' math struggles are rooted in a lack of such understanding,⁷ teachers tend to assume these struggles with particular concepts are mainly related to an inability to remember rules and formulas.⁸

In turn, instructional responses often focus on helping students master procedural skills that teachers assume they lack. For instance, when students err in comparing fractions (say $\frac{3}{8}$ and $\frac{3}{4}$), teachers tend to assume students have forgotten how to compare fractions by using a benchmark fraction ($\frac{3}{8}$ is less than $\frac{1}{2}$, but $\frac{3}{4}$ is more than $\frac{1}{2}$) or by creating a common denominator. However, the root cause of the error may be a student's conceptual misunderstanding: viewing numerators and denominators as separate whole numbers rather than as a single number with a value. Appropriate, effective instructional responses will thus depend on the teacher's interpretation of why a student struggles.

When professional development for math teachers focuses on how concepts are developed and connected across the learning standards and across grades, teachers are more likely to develop the robust understanding they need to improve math instruction.⁹ Such programs should also include opportunities for teachers to learn more about students' mathematical thinking and their common struggles, as well as which tools and representations best support their learning. Evidence suggests that one way that teachers can build mathematical knowledge for teaching is by analyzing students' responses.¹⁰

One-shot workshops or fragmented learning opportunities will not significantly change teachers' practices. They need a series of opportunities organized around key mathematics domains, and they need to learn evidence-based practices to help students with common struggles.¹¹

State education leadership will be instrumental for ensuring that teachers receive this support. I also encourage state boards of education to partner with universities to offer funded programs and to fund cohorts of in-service educators to receive the support they need to develop content-specific expertise.

Bias as a Barrier to Equity

Improving teachers' expertise in teaching math

can improve all students' math performance while also addressing the historic inequity in students' access to expert teachers. However, latent cultural stereotypes also undermine the math performance of students from particular groups and thus must also be confronted. Such stereotypes link performance to natural rather than acquired ability and assign this ability based on gender or race. Students are particularly vulnerable to such cultural messages, which can harm their academic self-concept and performance.

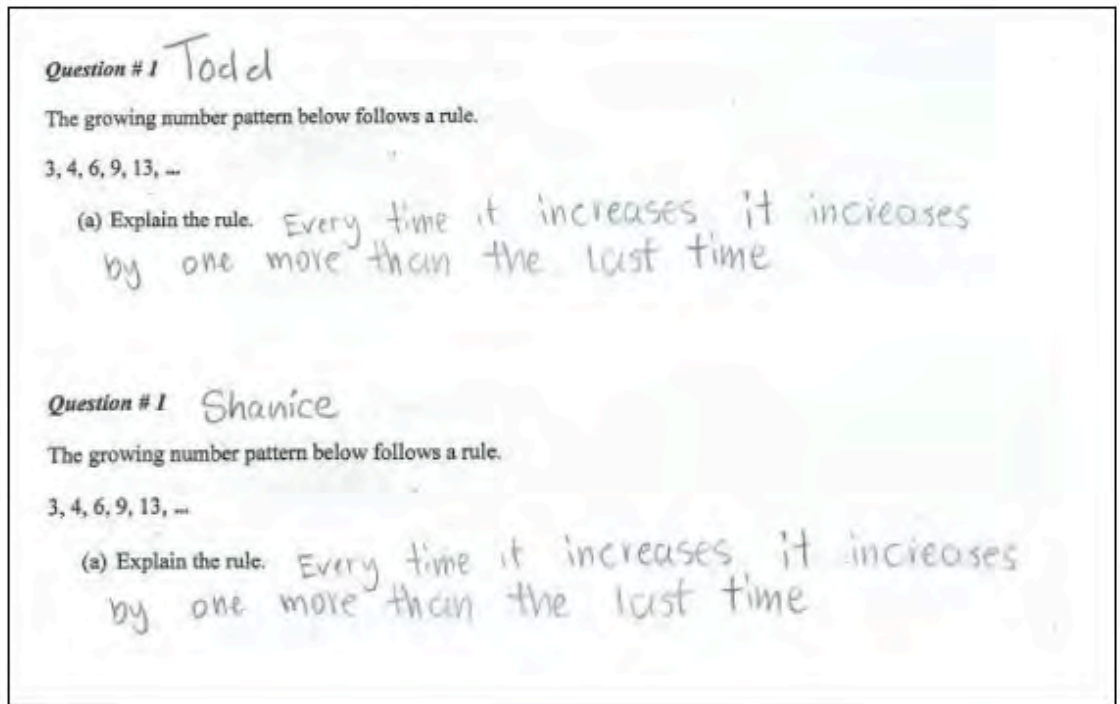
Like everybody else, teachers are shaped by these stereotypical beliefs, which might inadvertently affect their expectations of students and their interactions with them. Teachers' unconscious biases about their students' cognitive abilities stymie students' academic growth, as such biases shape instructional decisions and recommendations of students for gifted education programs.¹²

Distinguishing between teachers' unconscious biases and their accurate assessments of student ability is not easy, given that there are differences in subgroup performance. My colleagues and I conducted experiments in which teachers show implicit biases regarding the mathematical ability of students based on race and gender.¹³ In one study, teachers were given the same set of student solutions and asked to (1) grade the student's work based on its correctness and (2) estimate the student's math ability based on the student's response. The only difference was that teachers saw gender- and race-specific names linked to each solution, such as "Todd" (a White male-sounding name) or "Shanice" (a Black female-sounding name), as shown in figure 2. Our rationale for this study design was that if teachers lacked biases, they would not rate the same solution differently based on the different names.

Yet in data collected from 390 teachers, we found that teachers assumed students had higher math ability when they saw White-sounding names than when they saw Black-sounding names. Implicit bias was also observed for girls, who were perceived as having lower abilities than boys. Our findings showed that although teachers' evaluations of students' work did not change based on the students' race or gender, race and gender affected their perceptions of students' capacity to learn math. Both White and non-White teachers showed some type of implicit bias, which underscored the fact that no one should be assumed to be free of it.

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Figure 2. Sample Student Solutions



Implicit bias affects students' success, the careers they pursue, and academic self-concept.

National Association of State Boards of Education • May 2022

There are several implications from these findings. When female students and students of color struggle with math, it might signal to their teachers that they inherently lack the ability to do math. A recent study showed that male students who were not academically successful still majored in mathematics-heavy STEM fields such as engineering and computer science at a dramatically higher rate than female students with similar achievement levels.¹⁴ It could be that when girls are not doing well in math, they are receiving implicit messages to suggest it is because of low mathematical ability. Inherent biases may also explain why students of color are likewise underrepresented in math-heavy STEM fields.

Evidence-Based Implicit Bias Training

Implicit bias affects students' success, the careers they pursue, and academic self-concept. Thus efforts to boost teachers' content-specific expertise alone will be insufficient to increase math performance.¹⁵ They should also be offered specific training to enable them to identify the

conditions under which they might be relying on their implicit biases and how to overcome them.

State education officials can ensure that the training teachers receive is evidence based and specific to their work as teachers. More generic training may fail to address the impact of implicit biases on teaching. For example, knowing that teachers tend to draw on their biases in more ambiguous situations,¹⁶ teacher implicit bias training should include strategies the teachers can use to gather more information from their students before they make instructional decisions or recommendations. Training programs should also help teachers learn more about pedagogical practices that facilitate learning about their students, such as asking students to explain their thinking. Such practices might help teachers attend to students as individual learners rather than as members of an underrepresented group.

Teacher Pipeline and Licensure

Implicit bias training is an important lever by which state policymakers can improve students' math performance. But state boards can also

take strategic actions to improve the teacher pipeline. I propose modifying the teaching licensure requirements in the following ways: content licensure tests that capture teachers' robust math understanding, performance assessments for teacher candidates, and math content licensure and performance assessments for elementary school teachers. Given the efforts states are making to address teacher shortages, state board members might assume their hands are tied on licensure requirements. However, these shortages are closely related to the low salaries the teaching profession offers as well as working conditions.¹⁷ Thus I argue that changes in licensure will have little impact on teacher shortages. Rather, such changes will lead teacher education programs to refine their curricula to equip preservice teachers with the skills they need, skills that are also measured on the licensure tests. More important, these changes will improve the quality of student learning.

Content Licensure. Licensure exams leverage states' efforts to improve the quality of their teacher preparation programs. Yet evidence suggests that what is measured on existing content tests provides little information regarding future teacher effectiveness.¹⁸ State boards should investigate the extent to which the content licensure test they use measures the mathematical knowledge that matters for teaching. Indeed, Massachusetts' investigation of its licensure tests through an independent study is an exemplary approach to investigating whether what is measured on these tests matters for student outcomes. Similarly, other states can partner with experts to evaluate their current licensure tests or revise them accordingly.

Performance Assessments. Given that students of color and students from low-income families are more likely to be taught by novice teachers, it is an essential component of equity to ensure that all teacher candidates have grasped the rudimentary aspects of math teaching before they enter the classroom. Such pedagogical knowledge, along with a robust understanding of the content math teachers are teaching, leads to an increase in students' math achievement.¹⁹ State policies should not overlook the critical need for teachers to know how their students learn particular concepts and to acquire a repertoire of tools and practices that promote student learning. Currently, 16 states require a performance assessment that measures teachers'

knowledge of how to teach, which is a necessary step forward.²⁰

Elementary Teachers. Another way to improve students' math performance is to create strong mathematical foundations for all students in the early grades. Currently, elementary teachers have limited academic preparation in math and a less robust understanding of math concepts. Yet only a little more than half of U.S. states require elementary teachers to pass a content licensure test in mathematics.²¹

Even in states that require performance assessments, teacher candidates who hold multiple subject credentials are not required to have a separate passing score on how to teach mathematics. Nevertheless, elementary education plays a tremendous role in shaping students' math learning experience as well as their attitudes toward the subject. Thus requiring elementary teacher candidates to demonstrate mastery of mathematics teaching can help break the cycle of students struggling in mathematics.

Conclusion

Reforms to improve student math achievement and math curriculum have not yielded the hoped-for outcomes because improving math performance demands more systematic changes in how math teachers are equipped at the preservice and in-service levels. Research advances on the expertise needed in mathematics teaching have identified distinct knowledge and skills math teachers need. Policies to ensure that teachers and teacher candidates are equipped with this knowledge and these skills is the most viable solution to the nation's ongoing problem with math learning. Furthermore, it is essential to understand that no one, not even a teacher, is immune to societal stereotyping. Providing a system of supports to overcome teachers' reliance on implicit biases could help erase inequity in math instruction and close the performance gap of students from different groups. ■

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¹⁰Ibid.

¹¹Rossella Santagata and Wendy Bray, “Professional Development Processes That Promote Teacher Change: The Case of a Video-Based Program Focused on Leveraging Students’ Mathematical Errors,” *Professional Development in Education* 42, no. 4 (2016): 547–68, doi: 10.1080/19415257.2015.1082076.

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¹³Yasemin Copur-Gencturk et al., “Teachers’ Bias against the Mathematical Ability of Female, Black, and Hispanic Students,” *Educational Researcher* 49, no. 1 (2019): 30–43.

¹⁴Joseph R. Cimpian, Take H. Kim, and Zachary T. McDermott, “Understanding Persistent Gender Gaps in STEM,” *Science* 368, no. 6497 (2020): 1317–19.

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