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Enactment of mathematical agency in 'the mangle'

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Abstract
In this paper, we argue that during collaborative classroom interactions, '*the mangle*' is entangled in different forms of agency that (dis)enable mathematical agency. We first unpack student's agency in the context of collaborative classroom interactions, and then use excerpts of interactions to illustrate how '*the mangle*' is entangled in resistant agency and relational agency. The excerpts were sourced from a Grade 11 mathematics classroom, facilitated by the first author, as students worked through an activity on factorization of quadratic expressions. We organize our analysis of the critical incidents of only student-student interactions using Mueller et al.'s (2012) framework to re-story how '*the mangle*' is entangled.

Keywords: student's agency, mathematical agency, '*the mangle*', collaboration

INTRODUCTION

The notion of an agent can be generally viewed as an individual that influences or changes the evolution of their circumstance by taking action. In the context of classrooms, the action taken pertains to "learning in learning activities, in order to change the trajectory of theirs and their peers' learning" (Clarke et al., 2016, p. 32). In mathematics education research, a person's agency is understood as initiating ideas, agreeing with others, to elaborate and critique, to question or disagree with others (Gresalfi et al., 2009). This as noted means that the students are not only influenced but can adopt, adapt and ignore contributions by others (Pyhältö & Keskinen, 2012). Thus, agency should not just be viewed as collaborative action on an object (Edwards & D'arcy, 2004) but "as a capacity to recognize and use the support of others in order to transform the object" (p. 149).

In a classroom context, agency is not how the students act in particular situations, instead it is how they have control over the ways in which they act in the classroom. In this way, agency in the classroom should not be perceived as an individual's attribute. When agency is considered in relation to mathematics, it is referred to as mathematical agency. Although Meyer (2012) posits that agency is a slippery word to define, he contends that mathematical agency is defined by a positive self-concept towards mathematics. Mathematical agency is (dis)enabled by other forms of agency such as relational agency and resistant agency while engaging in mathematics. The purpose of this paper is to illustrate enactment of mathematical agency in '*the mangle*' during collaborative classroom interactions.

STUDENT'S AGENCY IN THE CONTEXT OF COLLABORATIVE CLASSROOM INTERACTIONS

Research on a student's agency in mathematics education draws on Pickering's (1995) metaphor of the '*dance of agency*' (Boaler & Greeno, 2000; Cobb et al., 2009; Wagner, 2007). In the '*dance of agency*', Pickering (1995) claims not to use science to explain knowledge but to explain how things are done. Attributes of the dance are observable when the learners cooperate, coordinate, interact, improvise, and respond to different styles and rules of doing things. In science, the mathematics included, the dance of agency can draw from how scientists draw on their agency by initiating and extending their scientific ideas. Pickering (1995) talks about this as a conceptual agency. But, there are times when scientists yield their agency of the discipline, disciplinary agency, to account for how science is studied and the changes that result.

Pickering (1995) talks of the performative or agency idiom, which is achieved through a process that he conceived of as '*the mangle*'. '*The mangle*' is interpreted as

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Contribution to the literature

- Student's agency in the context of collaborative classroom interactions.
- An illustration of how '*the mangle*' is entangled in other forms of agency that disable or enable mathematical agency.
- An illustration of how mathematical agency is enacted through resistant agency and relational agency during classroom interactions.

a dialectic of resistance and accommodation, where resistance is experienced when something either does not work or goes wrong. Furthermore, resistance is clarified as a block between current position and intentionality, which is the destination. On the other hand, accommodation happens when adjusting performance because of resistance. Thus, in '*the mangle*', there is exploration of what individuals might become without knowing where they are going; a new sense of self comes into being in '*the mangle*'. It is this explanation of '*the mangle*' that justifies Pickering's (1995) view of the extension of culture as open-ended in the sense of not being prescriptive.

In presenting the case of the '*dance of agency*', Boaler (2003), argued for the "need to study classrooms practices in order to understand relationships between teaching and learning" (p. 3). She contends that the practices of classrooms need to be captured in order to cross a divide between research and practice. "The capturing of some of the practices of teaching and converting them into a set of carefully documented records of practice assist researchers in producing artefacts that encourage a special kind of analysis grounded in practice" (p. 15). Correspondingly, the latest literature review on the concept of agency, focused on resistant agency in the context of a classroom (Winkler & Rybnikova, 2019). As contemplated earlier, defining agency is slippery, but in terms of resistant agency, this paper adopts Goodboy's (2011) notion, which describes resistant agency as the agency that is characterized by a rebellious character.

The review reports on research that focused on resistant agency characterize resistance as alienation from learning, rejection of the contents and skills taught, and criticism of the knowledge and values transmitted by schools (Mameli et al., 2020; Winkler & Rybnikova, 2019). Criticism of knowledge may include resistance that comes with teaching mathematics that enables students to learn new ideas by connecting those ideas to what they already know. Commenting on learning new ideas, Bråten (2016) posits the view that students may not adjust appropriately to specific classroom tasks based on their existing epistemic resources. There are studies that show that such inappropriate adjustments may be illustrated as resistance, especially when students choose to be indifferent, silent, or openly reject learning material or teaching approaches that are used (Alpert, 1991; Vetter et al., 2012).

In their literature review, Winkler and Rybnikova (2019) concluded that there are three approaches to resistance, namely, functional-instrumentalist understanding of student resistance–(where resistance is seen as a rebellious phenomenon), critical-emancipatory understanding of student resistance–(where resistance capital is valued for possible social change), and critical-functionalist understanding of student resistance that comes with challenging students' identities. They suggest that these types of resistance can be used to realize underlying assumptions regarding students' resistance and further suggested for "intensified future research of students' resistant behavior as related to the enhancement of on-task behavior" (p. 531).

In the context of Pickering's (1995) mangle of practice, such resistance can be in the form of refusing to interact with people or things. This resistance may be observable when students refuse to try new things in order to see what will happen.

Hence, we argue that during collaborative classroom interactions, '*the mangle*' is entangled in different forms of agency that (dis)enable mathematical agency. These different forms of agency includes resistant agency as discussed earlier as well as relational agency. Edwards (2005) defines relational agency as "a capacity to offer support and to ask for support from others" (p. 168).

In summary, the different forms of agency referred to in this paper are listed and briefly described in **Table 1**.

STUDENT-STUDENT COLLABORATION MODES

The modes of student collaboration that we focused on are those found in Mueller et al.'s (2012) framework,

Table 1. Forms of agency and their brief explanation

Forms of agency	Description
Mathematical agency	Positive self-concept towards mathematics
Conceptual agency	The way in which [mathematicians] draw on their agency by initiating and extending their ideas
Disciplinary agency	Agency of the discipline, that accounts for how the discipline is studied and changes that result
Resistant agency	Agency that is characterized by a rebellious character
Relational agency	A capacity to offer support and to ask for support from others

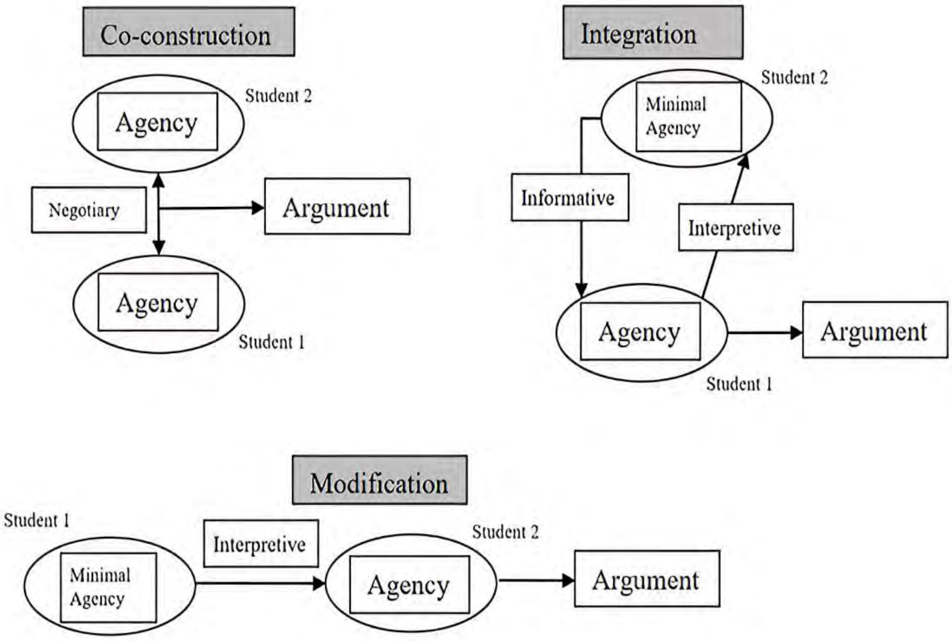

Figure 1. Three modes of collaboration (Mueller et al., 2012, p. 379)

namely *co-construction*, *integration* and *modification* of ideas. Mueller et al. (2012) built on Alrø and Skovsmose's (2006) theory on dialogue, which includes reformulating, challenging, and evaluating, to construct three modes of student collaborations. They looked at the discursive nature of each mode, its significance, and the interplay between agency and neighbor interactions that take place during instances of collaboration. All the latter traits of each mode of collaboration that were considered by Mueller et al. (2012) are illustrated by **Figure 1**.

Co-construction of ideas is a form of collaboration in which the dialogue occurs in a back and forth nature until the argument is built. Furthermore, co-construction is typified by negotiated discourse, and all participants equally share agency in the discussion.

Integration of ideas, which is similar to Alrø and Skovsmose's (2006) reformulating; is a form of collaboration that is identified when a student's argument is strengthened using ideas obtained from their peers. In other words, the ideas, explanations, or representations of others are assimilated into their original argument. Integration is typified by an informative and interpretive discourse. As a result, the original argument is interpreted by the second participant, who then enhances the argument in a way that informs the originator and affords the second participant an opportunity to assimilate the information in a meaningful way. Therefore, the originator of the argument is the principal agent, whereas the second participant influences the mathematical outcome and, thus, has minimal agency in the discussion.

Modification of ideas, which is similar to Alrø and Skovsmose's (2006) challenging and evaluating, occurs as students attempt to correct a peer or assist them to make sense of a model or argument that was originally expressed in an unclear or incorrect way. Modification is typified by interpretive discourse, as one student attempts to makes sense of another's flawed argument. Moreover, this sense-making student also has the primary agency in the discourse, since they have the ultimate control in the mathematical outcome of the discussion (Mueller et al., 2012). These modes of collaboration do not occur in a linear way, as discussed here, but instead occur in an integrated way and, therefore, their occurrence should be viewed as such.

BACKGROUND OF THE ILLUSTRATIVE CASE

The case that we use to illustrate how '*the mangle*' is entangled in resistant agency and relational agency is the excerpts of classroom interactions from a Grade 11 mathematics classroom. The classroom consisted of 49 students who worked in groups, which were neither predetermined nor fixed, but were dependent on students' preferences and the task at hand. None of the groups had membership of less than four or more than six. All groups were of mixed gender and mixed ability.

The classroom was facilitated by the first author, as students worked through an activity on factorization of quadratic expressions. The facilitation was part of construction of data for his master's dissertation on enactment of mathematical agency (Mokwana, 2017). The facilitator's teaching philosophy was centered around a teaching strategy that involves four stages which are not linear in operation and are not necessarily allocated equal times, but are seen as equally important and normative (Masha, 2004). They involve student–material interaction, student-student interaction, student-facilitator interaction and whole-class interaction.

In this paper, we draw data from the students' interaction with materials and student-student interactions. This is because during student–material interaction stage, students are given an opportunity to work individually. They are given tasks to work on and respond in writing. This stage provides the facilitator with knowledge that students have when they start working on some tasks. On the other hand, students' writings are used to start discussions either with peers or with him. Student-student interaction is also referred to as "on the desk conferencing". It usually takes place in groups with a maximum of six members per group. This is the initial stage of sharing ideas and negotiation of meaning.

For better illustration of our argument, in the excerpts we recorded voices of students who talked while other group members were silent. The students' interactions reported on, are critical incidences that illustrate interactions where *'the mangle'* was at play while the facilitator monitored whether the students were on task. This was done by noticing how the students carried out the given tasks, how they explained what they were doing and how they responded to questions. In particular, we illustrate our argument through two related excerpts of student-student interactions typified by the three modes of collaboration (Mueller et al., 2012) discussed earlier. The first excerpt typifies modification and co-construction modes of collaboration, where the students first challenged each other's understanding of the material given. After that, they co-constructed a shared meaning of the material through a dialectic negotiation. The second excerpt typifies integration and modification modes of collaboration. In this case, the students interpreted each other's utterances and developed a new argument, which led to them modifying their understanding of how to carry out the activity in the presented material.

In analyzing the excerpts, we focused on the critical incidents of students' interactions using Mueller et al.'s (2012) framework to re-story how '*the mangle*' is entangled. The framework allowed us to consider different learners' collaborative instances during the lesson in order to examine how mathematical agency was enacted. We found this framework relevant for two reasons, first, it focused on specific student-student engagements and secondly the perception on agency that was adopted in developing the framework draws from Pickering's (1995) conceptualization of agency. Therefore, we used the framework to illustrate forms of agency that are at play with mathematical agency. Verbatim quotes of critical incidents were used as a truthful account of what emerged during classroom interactions (Tripp, 2012). We then rewrote this account into a story through interpretations of interactions in order to bring meaning that is not explicit in the interaction. This process was done through arbitrary intra-subjectivity where we compared our storied version of the interactions.

ENTANGLEMENT OF '*THE MANGLE*' IN RESISTANT AGENCY AND RELATIONAL AGENCY

The illustration of the entanglement is based on a lesson which focused on the revision of factorization of quadratic expressions. The revision was done by first getting the students to engage in student-material interaction by reading the notes (**Figure 2**). The notes were based on factorization of quadratics whose coefficient of the squared variable is one.

Excerpt 1: Modification and Co-Construction

The critical incident captured in excerpt 1 started with a pin drop silence as the students read the given material. Frida broke the silence by a loud utterance (1.1): "Why do they have to complicate this? Isn't it that it can just be done by inspection?" Using Pickering's (1995) mangle, Frida's utterance may be an enactment of resistance caused by a block between what she knew and the intended new knowledge in the material.

On the other hand, Frida's utterance might have been an enactment of resistant agency in two ways. Firstly, the utterance could be an open resistance to new knowledge as it differed from how quadratic expressions were factorized previously. Frida's utterance might have been a suggestion to the teacher to withdraw the reading as it did not suit students' needs, particularly those who already know how to factorize quadratic expressions. Had she succeeded in such intentions, then her resistance would not have benefitted her learning.

Secondly, Frida's resistant agency could have been a productive plea for help from the group members to collaborate in modifying what, at the time, seemed complicated. She may have, at the time, realized that she did not benefit from interacting with the material given and wanted to trade that off for either student-student interaction or whole class interaction. Such a view of resistant agency is constructive and potentially benefits students (Mameli, et. al, 2020).

The conversation that followed after a pause shows how Frida's utterance can be viewed as an invitation to clarify a procedure followed in order to factorize a quadratic, which was different from the procedure she knew.

Excerpt 1

1.1-Frida: Why do they have to complicate this? Akere (isn't) it can just be done by inspection.

Pause.

Factorizing quadratics

To learn how to factorize, let us study again the removal of brackets from $(x+3)(x+2)$.

$$(x+3)(x+2) = x^2 + 2x + 3x + 6 = x^2 + 5x + 6$$

Clearly the number 6 in the final answer comes from *multiplying* the numbers 3 and 2 in the brackets. This is an important observation. The term $5x$ comes from *adding* the terms $2x$ and $3x$.

So, if we were to begin with $x^2 + 5x + 6$ and we were going to reverse the process we need to look for two numbers which multiply to give 6 and add to give 5.

$$? \times ? = 6$$
$$? + ? = 5$$

What are these numbers? Well, we know that they are 3 and 2, and you will learn with practice to find this by inspection.

Using these two numbers which add to give 5, we split the $5x$ into $3x$ and $2x$. We can set the calculation out as follows.

$x^2 + 5x + 6 = x^2 + 3x + 2x + 6$

$= x(x+3) + 2x + 6$ by factorizing the first two terms

$= x(x+3) + 2(x+3)$ by factorizing the last two terms

$= (x+3)(x+2)$ by noting the common factor of $x + 3$

The quadratic has been factorized. Note that you should never get this wrong, because the answer can always be checked by multiplying-out the brackets again!

Example

Suppose you want to factorize the quadratic expression $x^2 - 7x + 12$.

Starting as before we look for two numbers which multiply to give 12 and add together to give – 7. Think about this for a minute and you will realize that the two numbers we seek are – 3 and – 4 because

$-4 \times -3 = 12$, and $-4 + -3 = -7$

So, using the two numbers which add to give – 7 we split the $-7x$ term into $-4x$ and $-3x$. We set the calculation out like this:

$x^2 - 7x + 12 = x^2 - 4x - 3x - 12$

$= x(x-4) + 3x + 12$ by factorizing the first two terms

$= x(x-4) - 3(x-4)$ by factorizing the last two terms extracting a factor of – 3 in order to leave $x - 4$

$= (x-4)(x-3)$ by noting the common factor of $x - 4$

Once again, note that the answer can be checked by multiplying out the brackets again.

Example

Suppose we wish to factorize the quadratic expression $x^2 - 5x - 14$.

Starting as before we look for two numbers which multiply together to give – 14 and add together to give – 5. Think about this for a minute and you will realize that the two numbers we seek are – 7 and 2 because

$-7 \times 2 = -14$, and $-7 + 2 = -5$

So, using the two numbers which add to give – 5 we split the $-5x$ term into $-7x$ and $+2x$. We set the calculation out like this:

$x^2 - 5x - 14 = x^2 - 7x + 2x - 14$

$= x(x-7) + 2x - 14$ by factorizing the first two terms

$= x(x-7) + 2(x-7)$ by factorizing the last two terms

$= (x-7)(x+2)$ by noting common factor of $x - 7$

So the factorization of $x^2 - 5x - 14$ is $(x-7)(x+2)$.

Figure 2. Factorizing quadratics (Mathcentre, 2003)

1.2-Patrick: I think it's because if you factorize like this over and over again eventually you will be able to do it by inspection.

Pause (as students read through the material again).

1.3-Moganedi: Guys look at this example (referring to the factorization of x^2-7x+12) they say we are looking for two numbers which we multiply to get 12 and then add up to -7. They decided the numbers are -3 and -4 I don't understand.

1.4-Frida: Which one? Wait let me see, what is confusing you actually?

1.5-Moganedi: I don't understand why they say they are adding since a positive times a negative is a negative (referring to -3+-4). I think they must say they subtract.

1.6-Frida: Moganedi, actually it does make sense, the idea here is to come up with sort of a general way in which factorization can be done. You see in the first example they spoke of addition, even now they still stick to addition.

1.7-Patrick: Frida is right we are always going to add but you see our numbers are integers. Actually they were supposed to write (-3)+(-4) to show the addition of two negative integers.

1.8-Moganedi: Ok.

1.9-Frida: Is it ok now?

1.10-Moganedi: Mmm.

Pause.

1.11-Frida: So what are you doing now?

1.12-Moganedi: I am factorizing another one to see if I understand the explanation.

1.13-Frida: Ok.

1.14-Patrick: Explain it to us as you go so we can also see that you really understand.

Patrick seemed to have sensed Frida's utterance as resisting what Pickering (1995) calls disciplinary agency. He discouraged that (1.2) by encouraging Frida to continually work on the steps that were given to transform the given expression into its factors until she could perform the transformation without external stimuli. Patrick seemed to be aware of the need to yield to the mathematical agency, hence his advice to Frida. Patrick intentionally influenced Frida's thinking by inviting her to factorize according to the given information "over and over again" hence enacting relational agency.

Patrick's utterance was followed by a pause, after which Moganedi invited the group to collaborate in order to bring clarity to the ideas presented. This was an act of intentionality (Pickering, 1995) because he was confused by how the material presented addition of signed numbers (-3+-4). Patrick interpreted '+ -' to mean 'positive multiplied by negative gives negative'. This is not surprising because it is a typical classroom 'rhyme' that is memorized when working with signed numbers. His continued utterances suggest that he believed that -3+-4 should have been written as -3-4. This is, again, an indication of the tension between resistance and accommodation in '*the mangle*'. Further cross talks among the students showed that they were able to recognize the error in Moganedi's interpretation and modified his interpretation to mean (-3)+(-4), which resulted in the formation of new mental structures for Moganedi. This kind of talk is coined 'interpretive discourse' in which students attempt to make sense of others' faulty arguments (Mueller et al., 2012). In this excerpt, Patrick exercised primary agency as a sense-making student who controlled the mathematical outcome of the discussion (Mueller et al., 2012).

Using Edwards' (2005) relational agency as referent, i.e. the capacity to ask for support from others and offer support, Moganedi and Patrick enacted relational agency, respectively. Ultimately, the dialectic of resistance and accommodation stabilized when Moganedi 'factorized another quadratic' to check whether he had indeed accommodated the mathematical agency (1.12). Ultimately, with the enactment of agency, there was a joint agency when agentic students cooperated to achieve a shared goal.

At the end of the excerpt, Moganedi indicated his need to experience what he learned on a different quadratic expression. Learners need to experience an idea more than ones in order to "crisscross the intellectual landscape from different angles" (Nuthall, 2007, p. 161). By extending the new idea of factorizing expressions, we claim that Moganedi conceded his agency to another procedure that is accepted within the mathematics community–disciplinary agency. Frida and Patrick were interested in what Moganedi was doing. Patrick regarded himself as a knowledgeable other and invited Moganedi to use him as a resource to confirm that Moganedi understood; acting out relational agency.

Excerpt 2: Integration and Modification

As the lesson progressed, both Patrick and Frida left their group and each joined new groups. At that time, Moganedi remained with the group to explain the work to the other three members of the group. It was not clear, at the time, why Patrick and Frida left the group while they were resourceful agents. It later became apparent that they were called out to help the other groups with factorization of expressions.

Excerpt 2

2.1-Patrick: You cannot just say you don't know how to factorize and you don't even understand the examples given here. We did factorization last year; can you explain how you understood it then?

2.2-Potego: I told you I didn't understand it then and even now I don't understand.

2.3-Thomas: You (referring to Patrick) explain to us how factorization can be done easily.

2.4-Patrick: I am going to use the approach in the handout, even though it is long but it is simple to understand. Ok let us look at how x^2-7x+12 was factorized…

In his utterance in (2.1) one gets the impression that Patrick viewed Potego as agentic, with the capacity to act in ways that resulted in learning. This view of affordance is what Dokic (2010) viewed as allocentric perception of affordances, a situation where an agent perceives that another agent can do something. On the other hand, Potego's conduct of not understanding quadratic expressions may be viewed as counterproductive. Such conduct is exemplary of resistant agency (Goodboy, 2011) as it characterized a rebellious character. In this particular case, Potego's resistance seems to originate from her previous experience with factorizing quadratics – an indication that she did not make sense of the concept the first time she worked on it. Unfortunately, that seemed to affect the expected outcome that she comprehends the material at hand. In her utterance "I didn't understand …. I don't understand", Potego dismissed her authority in the collaboration. The interlocution, as such can be viewed as a hindrance to enacting agency.

But, using Gresalfi et al. (2009) reference of agency to include an individual's refrain from acting, and how this refrain contributes to joint action of a group, Potego's resistance to factorize may be her way to indicate that she was still willing to learn. This could clarify Thomas' utterance that Patrick should explain how factorization is done. Using Edward's (2005) definition of relational agency, "a capacity to offer support and to ask for support from others" (p. 168), Potego and Thomas' call for assistance was illustrated. Patrick reciprocated this form of agency by offering to explain factorization as presented in the given material as he comparatively found it easier to conceptualize. Potentially, Potego and Thomas may individually gain from collaborating with Patrick as he explained what did not seem salient to them, and hence enrich the entanglements in '*the mangle*'.

DISCUSSION

The purpose of this paper is to illustrate enactment of mathematical agency in '*the mangle*' during collaborative classroom interactions. We attempted to unpack student's agency in the context of collaborative classroom interactions and to use excerpts of interactions to illustrate how '*the mangle*' entangles with other forms of agency.

We argue that during collaborative classroom interactions, '*the mangle*' is not only experienced through resistance and accommodation; it also entangles with other forms of agency that (dis)enable mathematical agency. In our case, we illustrated its interplay with resistant agency and relational agency. The enactments of mathematical agency occurred when we perceived agency as an engagement, distributed authority and identity in mediation, discourse and collaboration. Hence, we conclude that in a classroom where students enact mathematical agency, there is no unique form of agency at play; instead, agency emerges as multifaceted.

In excerpt 1, Frida's resistant agency broke the silence and provided opportunity for collaboration between the students. Frida's plea for modification occurred as a result of the tension she experienced between her existing knowledge on factorization of quadratics and mathematical agency, as presented in the learning material. Although the teacher could have intervened, in this instance there was no intrusion. The teacher's voice was represented by a different approach to learning, as captured in the learning material provided. The absence of the teacher's voice in a way provided learners with "wait time" (Ingram & Elliott, 2016) and allowed them to exercise agency in deciding on how to engage in the activity at hand.

Patrick encouraged sense making in mathematical agency and thus, together with Frida, he acted on the learning material and co-constructed ideas (Mueller et al., 2012). Moganedi was observed inviting the group to collaborate in order to modify -3+-4 as in the reader to -3-4. It was through interpretive discourse (Mueller et al., 2012) that Moganedi's dilemma resulted in the formation of new mental structures. Patrick, emerged as a resourceful agent and, along with Moganedi, enacted relational agency (Edwards, 2005).

In excerpt 2, a call for integration was made as a result of Potego's declaration that she did not know how to factorize. This call also emanated from Thomas' petition to Patrick to explain how factorization could be done. Although, in this excerpt, we missed the group's earlier interactions and, therefore, could not identify earlier forms of agency, it could be claimed that Thomas' decision portrayed elements of relational agency. Thomas was also mathematically agentic because of his eagerness to make sense of the given mathematical task through assistance. This positive self-concept is what Meyer (2012) refers to as mathematical agency. We observed Patrick assuming the role of a teacher, which is confirmation of an achievement of mathematical agency.

What emerged from the two excerpts was the fact that mathematical agency is not enacted in isolation from other forms of agency. The forms of agency that were also at play during students' classroom interactions were mainly resistant agency and relational agency. These excerpts revealed that students engage in '*the mangle*' when they decide either to act on a mathematical task, as outlined in a mathematical text, or not. Not acting on a mathematical task does not mean not doing the task but, rather, doing the task differently.

Hence, in Frida's case, the issue was not about factorizing quadratics per se, but factorizing quadratics in a "particular" way. Although there was no instruction which specified that students should factorize the given mathematical expressions using the approach in the text, Frida's interpretation appeared to have indicated exactly that.

Agency is enacted through seeking assistance, as demonstrated by Thomas' positive self-concept. He saw himself having the potential to make sense of the mathematical task with the assistance from Patrick, and not from the teacher. This is also one of the striking features of both excerpts where on the one hand, the teacher's contribution to the interactions could be perceived as absent; a situation which resulted from the nature of the classroom from which the interactions were sourced. The teacher observed but held back from active participation.

Thus, it could be argued that, in order for students to enact mathematical agency in the classroom, the teacher should trade off their authority (Erfjord et al., 2015), and serve as a guide at the side of the learners, if and when needed. This way of letting go of the teacher's role by encouraging students to participate in, and give directions to, their discussion should be seriously considered, and encouraged, in classrooms where *'the mangle'* is at play. As a result, this allows for the achievement of agency to become a tool for use in mathematics learning through the promotion of student-student interactions. It could be argued that it is not a waste of time during collaborative classroom interaction to provide room for an interplay of different forms of agency because they have capital that contributes qualitatively to *'the mangle'*.

CONCLUSION

Reflecting on the illustrative case used in this paper, we conclude that enacted mathematical agency is not isolated from other forms of agency. The enactments of mathematical agency occurred when we perceived agency as an engagement, distributed authority and identity in mediation, discourse and collaboration. Hence, we conclude that in a classroom where students enact mathematical agency, there is no unique form of agency at play; instead, agency emerges as multifaceted. For students to be mathematically agentic, there are three imperative aspects that a teacher should take care of. The first imperative is to allow the students to interact without the teacher's intrusion. However, similar to Fuentes (2018), we are cognizant of the silences that come with student-student interactions. Therefore, our suggestion is that the role of a teacher is not total disengagement but is that of a '*Guide-on-the-side*' (Sfard & Kieran, 2001, p. 202). The second imperative is that learning should be facilitated in a manner that will afford the students an opportunity to make their own decisions about engaging in the tasks at hand. The third imperative is that teaching and learning support material needs to be made available in order to initiate interactions. These imperatives suggest for research that focuses on how teachers engage in students' interactions to support the development of student agency without disenabling it.

The argument of this paper is independent of the structures of groups that worked together on the task given. However, the argument depends on the teacher's relegation of their authority as knowledge developers to the authority of learners as they work in collaborative groups. This relegation of control is a strength that practicing teachers can draw from when working with small collaborative groups. We note that material agency in the dance of agency is not included in the argument of this paper. In the field of material agency people "capture, seduce, download, recruit, enroll, or materialize that agency, taming and domesticating it, putting it at service, often in the accomplishment of tasks" (Pickering, 1995, p. 6). The absence of material agency in this paper can be fertile ground for studies that illustrate how it affords the enactment of mathematical agency in collaborative classroom interactions across different sittings.

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