CHALLENGES FACED BY COLLEGE STUDENTS IN SOLVING PROBABILITY OF EVENT PROBLEMS

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ABSTRACT

Many challenges are identified when students solve mathematic problems especially in solving probability word problems. This study was conducted to identify the major challenges faced by matriculation college students while solving the probability of an event word problem. Seven matriculation college students were the sample for this case study. Clinical interviews were employed as data collection. This data collection technique was selected based on the researcher's observation on the participants as they answered the task. The probability word problem task was given during the interview session. Semi-structured interviews are used to obtain in-depth information. The think-aloud analysis involves observations leading to individual behaviours in the oral or nonverbal form of participants and the researcher's field notes. Participants were found to have difficulty interpreting probabilities. Three categories of difficulties were identified, unfamiliar with the meaning of the word, not well versed in the nature of the probability, and unable to identify the goal of the probability word problem. The implication of this study can contribute ideas to instructors in developing the pedagogical techniques practiced in probability word problems.

Keywords: *Challenges, Probability of an Event, Think-Aloud Analysis*

INTRODUCTION

Problem-solving is a very important learning process in the mathematics curriculum. Based on the Malaysian Education Development Plan, problem-solving is an element contained in 21st-century skills, which is one of the focuses of mathematics learning (Malaysia Education Ministry, 2013). Therefore, problem-solving skills among students must be taught from young as problem-solving is closely related to word problems.

The development of students' mathematical learning depends on the type of word problem both verbally and in the written form (Daroczy, Wolska, Meurers, & Nuerk, 2015; Schley & Fujita, 2014). In general, word problems serve as contributors to students' achievement in mathematics generally. Many students experience difficulties in mastering problem-solving skills, specifically, problem-solving skills involving problem translation, problem integration, solution planning (Effandi Zakaria & Normah Yusoff, 2009; Gagatsis & Elia, 2004), as well as calculation, memory capacity, and problem solving (Guven &

Cabakcor, 2013; Phonapichat, Wongwanich, & Sujiva, 2014; Swensen, 2015; Ulus Kusdinar, Sukestiyarno, Isnarto, & Afit Istiandaru, 2017).

In mathematics, among the disciplines of knowledge that dominate the daily situation is probability. This is because, probability is a very important aspect to predict the outcome of future events. Examples of probability of an event that dominates daily life activities are controlling the flow of traffic through the highway system, predicting the number of people of all ages involved in an accident, and estimating the spread of rumours (Batanero, Chernoff, Engel, Lee, & Sánchez, 2016). Probability is not about predicting whether a particular event will occur but about determining how that probability is distributed over possible events (Baltaci & Evran, 2016; Galavotti, 2015).

Learning the concept of probability and solving the word problem of probability presents a challenge to the students. This is because students need to master the concept of probability, problem-solving process and understand the probability of problems simultaneously when solving probability word problems (Beitzel & Staley, 2015; Galavotti, 2015; Rusydah Usry, Roslinda Rosli, & Siti Mistima Maat, 2016). The review of previous studies has largely focused on problem-solving for a probability topic, such as conditional probability either manually or by using software (Beitzel & Staley, 2015; Gabriel, 2002; Gugga & Corter, 2014; Inzunza, 2006; Xing, 2016), joint events probability (Beitzel, Staley, & DuBois, 2011; Zahner & Corter, 2010) and the Bayes network (Ong & Lim, 2014).

The probability of an event seems simple when it involves the sample space, the probability of an event, and conditional probability. However, there are scarce empirical researches on probabilities and events (Corter & Zahner, 2007). Discussions about the difficulties or challenges faced by college students while solving probability problems are also limited as most of the studies focused on the skills and attitudes of students while solving problems (Zakaria, Yazid, & Ahmad, 2009). The performance of college students is still unsatisfactory although these students have learned the basics of probability at secondary level (Danisman & Tanisli, 2017).

Thus, this study was conducted to identify solution strategies and challenges encountered by students while solving sentence probability problems. This study focuses on the difficulties experienced by students to achieve the correct solution to the problem sentence of the probability of an event. The presentation of this study will only answer the question of what challenges are faced by matriculation college students while solving the probability word problem.

METHODOLOGY

This study employed a case study design using the clinical interview technique. This technique was developed by radical constructivism in the context of one-to-one interaction with observing the behaviour of participants as they solve mathematical problems (Lundgrén-Laine & Salanterä, 2010; von Glasersfeld, 2002). Therefore, the triangulation process used in the data collection comprised of interviews, observations, and document analysis. Triangulation process increases the credibility of this study.

Seven participants were selected from a matriculation college in Peninsular Malaysia. Sampling techniques aimed at maximum variation are used to meet the characteristics of participants who are required to obtain data from non-homogeneous study samples (Patton, 2015). Guba and Lincoln (1989) suggest that it is the responsibility of the researcher to ensure that sufficient contextual information about the fieldwork site is provided to enable the transferability of research information.

The instruments involved in the study include semi-structured interview protocols, observation protocols, and probability word problem tasks. The task given has three questions of probability word problems. The probability word problem focuses on the subtopics of Probability of Independent Events and Probability of an Event.

Clinical interview sessions were conducted after lectures and during participants' leisure time. Interview sessions were recorded so that each participant's behaviour could be observed and recorded for reference. All audio and visual data are transcribed in verbatim form. Data were encoded and distributed into appropriate categories after the refraction process. The analysis of observational data from field note entries was also coded. Comparative analysis techniques were used to record emerging themes to answer research questions.

The trustworthiness of this study was gained from three criteria namely credibility, transferability, and dependability which were discussed above. Merriam and Tisdell (2016) said that trustworthiness in the social sciences is a difficult aspect because human behaviour is not static and often changes. A replica of a qualitative study will not yield the same findings.

RESULTS AND DISCUSSION

This section presents the findings of the study obtained from the clinical interviews that were conducted. Based on the constant comparative analysis implemented, among the themes that emerged from this study is the difficulty of students to interpret probabilities. There are three categories of challenges that students faced when solving the probability word problem, namely unfamiliar with the meaning of the word, not well-versed with the nature of the probability, and unable to identify the goal.

The first challenge identified was that participants were unfamiliar with the meaning of the word. In this study, the participants did were unfamiliar with the meaning of some words, such as "subsequent", "given" and "perfect square number". Participants knew there was an underlying meaning of the words "subsequent" and "given", but they could not identify and understand in detail, although the participants tried to read the translations of the questions. Below are the responses by the participants.

Participant A:

> "…perfect square number is a number that can be a square root. The number should be an even number, right. No decimals…"

Participant B:

> "…given that …what does this mean…"
> "…I don't understand the meaning of subsequent. Next… test one…test two… both tests? …"

Participant C draws and labels the wrong tree diagram as the participant did not understand the meaning of "given" where the label of next branches depends on the first label of a tree branch (Refer Figure 1).

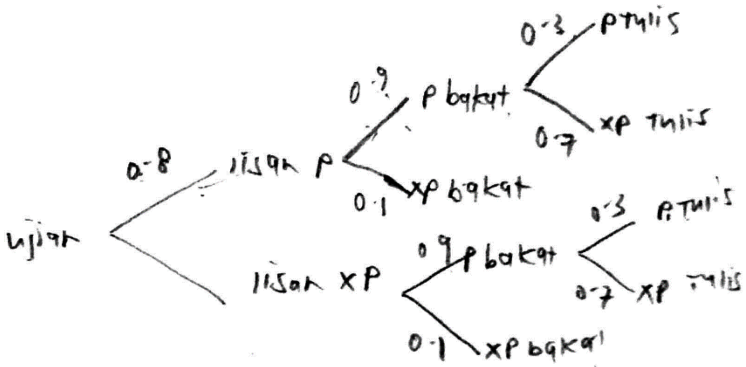

Figure 1. Diagram by Participant C

From the response, participant A assumed that the perfect square is a square root. The participant mistakenly identified the mathematical meaning. Meanwhile, participant B explained that he did not know the meaning of 'given' and 'subsequent'. Based on Figure 1, the diagram drawn by participant C indicates that the participant was unfamiliar with the meaning of words in the problem that must be exchanged to the mathematical terms before being substituted in the diagram to be used in the solution process.

When participants are unable to comprehend the implicit meaning of the word, the participants tend to ignore the information conveyed. When the terms "subsequent" and "given" in the specified word problem confuses the participants, the participants misunderstood the meaning by using future events, instead of employing the next event that occurs.

The findings of the interview explained that the participants understood the sentence "probability of event A occurs if event B also occurs " by relating the statement to the conditional probability formula. However, the findings of this study found that participants could not represent the statement "probability A, given B is 0.1" to the mathematical sentence " $P(A|B) = 0.1$ ". Below is the response by participant D.

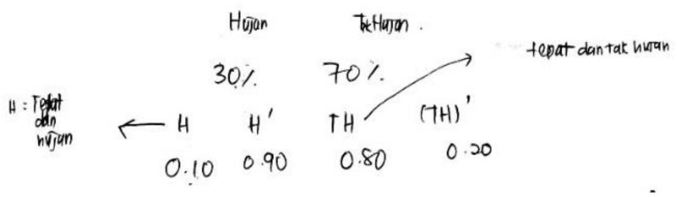

Figure 2. Diagram by Participant D

> "... I feel like I have to use the table. Rain, thirty percent and no rain, seventy percent. Then, on-time, maybe rain and no rain. Rain, on-time, zero point one zero. No rain, on-time, TH, zero point eight zero, so TH prime, not on-time, zero point two zero. So, the question is the probability of rain, given that the flight took off on time*..."*

Based on the responses by participant D, the diagram showed that the participant did not understand the probability terms. Therefore, participant D is unable to relate the statement in the problem to the mathematical terms as well as probability terms.

Conditional probability terms are very different than probability reasoning. Based on the findings of the study by Gugga and Corter (2014), participants will face difficulties and make mistakes when events are given in reverse, i.e. $P(B|A)$ versus $P(A|B)$ because of the sounds of both events are almost the same. When the word problem of conditional probability is given in the verbal form, participants will write conditional probabilities normally, i.e., events A to B as opposed to events B to A.

The second challenge is being unfamiliar with the nature of probability. The study also found that the participants did not understand the concept of probability involving the law of probability. The properties of probability involved sample space as well as set notation. Participants list the outcomes without set notation for questions involving the listing for a sample space. They are able to list all the desired numbers as a calculation path. They also know how to find the probability of the desired event but did not record or represent the probability term with the symbol "P". Participants are also careless when implementing the final solution in the solution process. They failed to re-explain the final value obtained by leaving the calculation result without any statement.

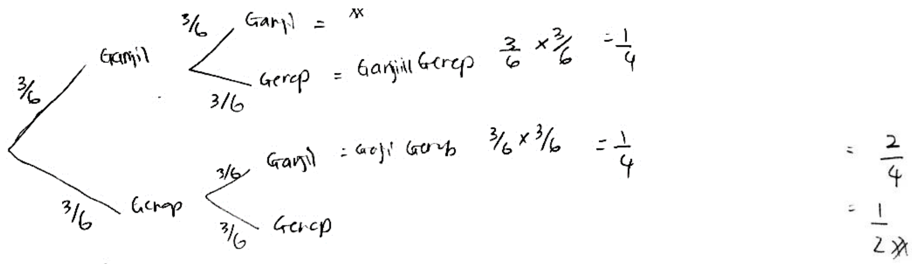

Figure 2. Diagram by Participant E

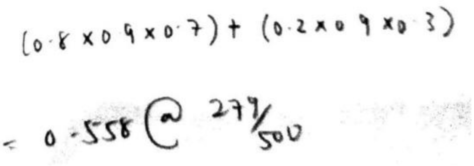

Figure 3. Diagram by Participant F

Figure 3 and Figure 4 showed solutions processed by the participants without any statement. They assumed that the examiner knew what they did to the problem given. From the interviews, the researchers understood that their explanations verbally contradict the presentations. However, the participants did not know that the presentations represent their thoughts.

Previous studies posit that the examiner was unable to identify the student's concept of probability if a solution is given without any statement (Bobek & Corter, 2010; Gugga & Corter, 2014; Ruzela Tapsir, Nik Azis Nik Pa, & Sharifah Norul Akmar bt Syed Zamri, 2018; Xing, 2016). Charles, Lester, and O'Daffer (2005) also suggested that systematic mathematical problem solving and having certain procedures give a good impression on a student, and teachers are able to evaluate the solution process smoothly. Thus, students need to be proficient with other numerical properties such as the use of probability symbols, set notations in the list of outcomes, and probability values between one and zero (Batanero et al., 2016).

The third challenge that participants faced is not being able to identify the goal in the problem. The participants were found to have difficulty interpreting probability events. Some participants are confused by the problem text. Even though students read the question repeatedly, they still failed to identify the goal of the problem. Participants who have difficulty interpreting probability events will read the question slowly and repeatedly even in front of their teacher. When students were unable to interpret probability events, they will assume the related questions are complicated to solve because they are unable to obtain information from the problem. This situation hinders the students to proceed to the next process of problem-solving.

Response by participant F is as follows:

> "...candidates pass at least two tests. Oh...so passing all three tests can't be taken. So we calculate this only (while pointing at the first and second tree branches). So, zero point eight, multiply zero point nine, multiply zero point seven, add zero point two, multiply zero point nine, multiply zero point three. The answer is zero point five five-eight. The probability is less than one so this answer is correct..."

The responses showed that participant F did not know the goal of the problem but confidently gave the solution after she read the problem. Participant F had difficulty in interpreting the probability events and solves the question using the law of probability as stated in Figure 4. Wrong interpretation caused the solution process to lead to the wrong answer.

Similarly, Arum, Kusmayadi and Pramudya (2018) discussed pertaining to the difficulty of understanding probability problems. The study found that students who could not identify the goal of the problem indicated that they did not understand the problem.

Based on the observations and interpretations by the researcher, participants assumed that probability terms are the same as the mathematical terms and vice versa. Inzunza (2006) stated that difficulty faced by students in interpreting and using correct probability terms will disrupt the problem-solving process. However, as for this study, participants are confused with the information provided to be represented to mathematical symbols only, and solutions are still implemented to obtain the value of the correct answer.

CONCLUSION

Many studies in the field of mathematical problem solving have focused on students' skills while solving problems. Thus, this study is expected to contribute to the lack of empirical studies in identifying weaknesses or difficulties faced by students while solving problems. This is because such studies provide information on the difficulties faced by students in learning and teaching probability, as well as contribute ideas to instructors in developing the pedagogical techniques practiced. Instructors can curate methods and approaches in addressing the issue of student difficulties in the mathematical problem-solving process before, during, or even after the learning and teaching sessions are implemented.

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