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Research Article

Investigating Preservice Teachers' Interpretations and Discussion of Real-Life Examples: Focusing on the Use of Percent

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ABSTRACT

The objective of this study is to examine the characteristics of PSTs' selection of real-life examples for discussion, the aspects they attended to during the discussion, and their reflection on facilitating the discussion. A total of 46 elementary PSTs in the US joined in the study. We gathered data from a three-phase task using an online course management platform and restricted the real-life contexts using percent. The collected data were analyzed using enumerative and ethnographic content analysis. The findings revealed that while we detected some shifting features in PSTs' selection of real-life contexts compared to prior studies, we found their static preference of contexts around consumer practices over others (e.g., contexts related to science and critical literacy). The findings also revealed that PSTs tended to pay attention to the contexts with critical views when the context was not straightforward and direct computations were not readily feasible to get the final answer. Moreover, the vague contexts helped encourage PSTs to view the situations critically, but they had difficulties having meaningful mathematical discussions in such contexts. We provided suggestions and implications for future research based on these findings.

Keywords: preservice elementary school teachers, real-life examples, teacher interpretation, group discussion, percent

INTRODUCTION

"Why are we teaching and learning mathematics in school?" Although it is hard to specify a short answer to this question, various scholars' studies for decades have converged several essential aspects of school mathematics: supplying a set of valuable skills and procedures and work-related knowledge (Ernest, 2016; Watson, 2004). Scholars offered the learning of mathematics necessary for every citizen in today's world, serving as a tool to ease successful societal participation (Ernest, 2016; Watson, 2004). Therefore, contextualizing mathematics is a reasonable means to engage students in meaningful mathematics learning as they start sensing the world (Blum, 2015; Bolstad, 2020; Freudenthal, 1991; OECD, 2019; Solomon et al., 2021) and offers a springboard for advancing mathematical understanding (Mason, 2016).

The National Council of Teachers of Mathematics (NCTM) underlined connection as a critical mathematical process standard (NCTM, 2000). It opined that teachers should ensure that their students connect school mathematics and real-life examples. However, studies reported that students were likely to exclude real-world knowledge when solving and interpreting real-life problems (e.g., Altay et al., 2017; Gellert & Jablonka, 2009). They delay real-world knowledge and solve word problems using mechanical processes and unrealistic knowledge (Inoue, 2005). Additionally, some students perceived that real-life problems in mathematics classrooms were artificial and unrelated to the authentic real world (Greer, 1997). For these issues, scholars have suggested that some teachers lack appropriate knowledge, skills, and beliefs for interpreting and using real-life examples to connect school mathematics and the natural world (Chapman, 2006; Garii, & Okumu, 2008; Rubel & McCloskey, 2021).

Teachers' interpretation of real-life examples is critical because "what teachers think and do essentially govern whether and how students will encounter real-world connections for the mathematics they learn in school" (Gainsburg, 2009, p. 265). That includes accomplishing the dual goals of connecting mathematics and the natural world—learning how mathematics is employed to solve real-life problems (teaching with mathematics) and practicing relevant mathematics through problem-solving (teaching mathematics)—teachers adopt an essential role by choosing meaningful contexts and orchestrating the mathematical learning process concerning the context (Depaepe et al., 2010; González, 2017; Popovic & Lederman, 2015). Therefore, teachers should bridge mathematics with real-life examples (Freudenthal, 1968) and contextualize them to help students connect mathematical ideas and real-life problems (NCTM, 2014).

If mathematics teachers should possess such competencies, teacher educators should comprehend preservice teachers' (PSTs) interpretation and discussion of real-life examples and prepare them as future mathematics teachers (Simic-Muller & Fernandes, 2020). Thus, it is critical to ask whether current PSTs have such competency. Many studies investigated PSTs' competency in interpreting real-life examples (e.g., Lee, 2012; Pirasa, 2016; Yilmaz, 2020). These studies collected and examined individual PSTs' written responses or interviews, including their formation or assessment of word problems. In these studies, while PSTs exhibited extremely positive beliefs about using real-life connections, the real-life contexts generated by PSTs were predominantly unproblematic that led to simply applying formulas (i.e., disguised routine computational problems). Noting this tendency, the present study asks PSTs to select existing real-life examples, as opposed to asking them to create real-life examples, and decipher the meaning behind the real-life contexts via a series of group discussions, as opposed to relying on one-shot responses in individuals' written texts or interviews. Moreover, this study examined PSTs' reflection on the group discussions to understand what affordances and limitations were presented when using real-life examples in mathematics classrooms (Ghousseini & Herbst, 2016). Therefore, the objective of our study is to examine the characteristics of PSTs' selection of real-life examples for discussion, the aspects they attended to during the discussion, and their reflection on facilitating the discussion. Focusing on PSTs' perspectives should enable the findings of this study to offer insights for mathematics teacher preparation. More specifically, the following research questions guide this study:

1. What contexts of real-life examples do PSTs choose for the group discussion?
2. What aspects of the selected real-life examples do PSTs attend to during the group discussion?
3. What affordances and limitations do PSTs notice when deciphering real-life examples via group discussion?

RELATED LITERATURE

Using Real-Life Examples for Mathematics Teaching and Learning

Many scholars stressed the relevance of using real-life examples for teaching and learning mathematics. For instance, Freudenthal (1968) proposed realistic mathematics education (RME) theory considering the connection between mathematics and reality (Beswick, 2011; Solomon et al., 2021). Freudenthal (1968) noted that people were likely to generalize their experiences and actions into abstracted laws and rules to learn them. Activities mathematizing the real world led to the development of mathematics. Thus, teachers should not impose invented mathematical knowledge on their students. Instead, they should provide students with opportunities to rediscover mathematical concepts and procedures by investigating real-life examples. Those learning experiences help students achieve accurate mathematical understanding and employ the acquired knowledge in real-life whenever they need it (Freudenthal, 1991).

Like Freudenthal (1968, 1991), other researchers stressed using real-life examples for learning mathematical modeling (e.g., Blum & Niss, 1991; de Lange Jnz, 1996). Blum and Niss (1991) suggested that a mathematical model comprised real-life situations, mathematical content, and a definite relationship. Hence, real-life examples could be simplified and formalized with mathematical modeling (Blum, 2015). Consequently, learning mathematics with real-life examples helps students develop competencies of describing, explaining, and predicting real-world (Blum, 2015). Similarly, Programme for International Student Assessment (PISA) defined mathematical literacy as an "individual's capacity to formulate, employ and interpret mathematics in various real-life contexts" (OECD, 2019, p. 14). Therefore, de Lange Jnz (1996) suggested that real-life examples were starting points for learning mathematics. Teachers should provide real-life examples and guide their students' mathematical investigations, changing into abstract mathematical reasoning.

Beswick (2011) suggested the five benefits of using real-life examples in mathematics education: (1) meeting the economic needs of society, (2) improving students' understanding of society, (3) boosting students' mathematical performance, (4) extending students' appreciation of the nature of mathematics, and (5) developing positive attitudes toward mathematics. The first two elements showed the utility value of mathematics, while the last three elements were about learning mathematics. Moreover, all these elements positively affected students' effort, engagement, participation, and achievement of mathematics (Beswick, 2011; Laurens et al., 2017; Solomon et al., 2021). In a word, scholars have stressed the relevance of using real-life examples for mathematics teaching and learning and teachers' active roles in using real-life examples for their students.

Preservice Teachers' Interpretation and Discussion of Real-Life Examples

PSTs could bridge real-life examples and school mathematics (Freudenthal, 1991). Nevertheless, previous studies have reported that PSTs report overly optimistic beliefs about real-life connections. Furthermore, vast discrepancies existed when PSTs interpreted word problems for real-life relationships (e.g., Lee, 2012; Verschaffel et al., 1997). For instance, Verschaffel et al. (1997) asked 332 elementary PSTs to solve word problems and assess students' written answers. The researchers found that PSTs resisted using real-world knowledge and did not consider realistic elements when interpreting word problems. Likewise, in a study researching elementary PSTs' competencies to connect geometric concepts and real-life examples, Pirasa (2016) found difficulties connecting them due to a lack of mathematical knowledge and experiences designing word problems considering the real world. Moreover, many PSTs gave typical real-life examples they had learned since elementary schools when asked to select real-life examples. Studies analyzing secondary PSTs have reported similar findings (e.g., Yilmaz, 2020). Lee (2012) requested PSTs to collect real-life story problems and statements explaining why they chose those problems. Lee (2012) reported that some PSTs neglected real-life connectedness and focused on mathematical elements when selecting real-life story problems. Specifically, 42% of statements focused on the mathematical dimension, including whether the problems offered clear directions, appropriated for state standards, and used multiple modes of representation.

Other researchers examined PSTs' interpretation and discussion of real-life examples centering on critical issues, such as race, ethnicity, culture, and socioeconomic (e.g., Ensign, 2005; Simic-Muller & Fernandes, 2020). For instance, Simic-Muller and Fernandes (2020) reported that 24 out of 33 PSTs were not interested in those topics or were reluctant to use them. The PSTs thought introducing critical issues in mathematics classrooms was inappropriate for students and might distract their interest. Similarly, Ensign (2005) reported that PSTs felt overwhelmed and nervous about critical issues.

Conceptual Framework

Researchers have proposed various frameworks to investigate mathematics teachers' interpretation and discussion of real-life examples. Based on Bruner's (1985) paradigmatic and narrative modes of a knowing framework, Chapman (2006) investigated the US mathematics teachers' notion of contexts for teaching. Teachers had paradigmatic ways centered on context-free and universal logical aspects in word problems. In contrast, teachers with a narrative mode were sensitive to context and stressed human intention and action. For instance, the former teachers thought contextual factors were irrelevant and redundant to solve problems. However, the other teachers believed that contexts were meaningful and helpful for students' mathematics learning. Although there were some variations, Chapman (2006) reported that many teachers exhibited the paradigmatic mode in interpreting word problems. Using the same framework, Depaepe et al.

(2010) researched Flemish mathematics teachers and found that they were more likely to focus on a paradigmatic mode than a narrative mode. Likewise, González (2017) suggested two dimensions to analyze teachers' interpretation and discussion of realistic contexts: the mathematical and contextual. The mathematical dimension examined whether the example was in line with mathematical curriculum and knowledge and the contextual dimension discussed how relevant the example was to their daily lives. González (2017) examined mathematics teachers with group discussions and discovered that mathematics teachers considered both dimensions when evaluating word problems to a different degree.

More recently, in the study of investigating secondary mathematics teachers' contextualization of mathematics, Rubel and McCloskey (2021) utilized commonly cited affordances to analyze participants' lessons: (a) formative–supporting the learning of mathematics, (b) affective–motivating students to learn mathematics, (c) functional literacy–supporting the teaching of how to solve essential everyday problems, and (d) critical literacy–teaching mathematics for social justice. Rubel and McCloskey (2021) reported that while the formative, affective, and functional literacy rationales were evident in generic human experiences and marketplace contexts, few cases were related to critical literacy.

METHOD

In the present study, we primarily use the inductive content analysis approach (Grbich, 2013) to comprehend PSTs' perspectives of real-life examples. However, we also particularly pay attention to the prior studies, in particular two frameworks used in González's (2017) and Rubel and McCloskey's (2021) studies to develop theoretical sensitivity (Strauss & Corbin, 1990).

Participants and Context

A total of 46 PSTs in two sections of an elementary mathematics methods course in the Midwest US joined in the study. 25 PSTs enrolled in one unit and 21 in the other. Due to the COVID-19 pandemic, we used the asynchronous online format for both units. The first author was the instructor of both sections. All PSTs had to take two or three mathematics courses and one method class. One of the required mathematics content courses covered whole numbers and operations, number theory, extensions to integers, fractions, decimals, percent, real numbers, and proportional reasoning. Therefore, PSTs refreshed their prior mathematics knowledge. During the data collection, the PSTs were completing a cursory review of Common Core State Standards (National governors association center for best practices, council of chief state school officers, 2010) across elementary grades and effective mathematics teaching practices suggested by professional organizations (NCTM, 2014; TeachingWorks, n. d.). Additionally, they shared their past mathematics learning experiences as students. Making mathematics teaching more meaningful was one of the recommended teaching practices in the course and applicable by solving real-world mathematical problems.

Task Design and Data Sources

Our study restricts the real-life contexts using percent for a couple of reasons. First, percent is a type of fractional number meaning high-leverage content (National Mathematics Advisory Panel, 2008; van de Walle et al., 2013). Second, the percent is a practical topic utilized in various contexts in our daily lives (Pöhler et al., 2017). We gathered data from a three-phase task using an online course management platform. We asked individual PSTs to choose one real-life example employing percent during the first phase of the task. While the sources of examples were open, we requested PSTs to select an example meeting the following conditions: (a) an example that we frequently encounter in our daily lives, (b) an example PSTs wonder about its mathematical meaning, (c) an example allows various points of discussion, and (d) an example usable in their future classrooms.

Regarding the second phase, we demanded that individual PSTs facilitate and participate in the forums (i.e., group discussion), interpreting and discussing the real-life example chosen in the first phase by taking on two roles. As a facilitator, each PST encouraged their online discussion forum with the example they chose using various questions and prompts and presented the conclusive interpretation of the initial problem upon completion. While serving as facilitators, we asked PSTs to avoid judgmental comments or evaluations and use prompts to help the discussion move along (e.g., could you elaborate on that?). As forum participants in their peers' forums, PSTs interpreted and elaborated on the real-life examples selected by their peers. The meetings lasted for four weeks. After completing the four-week online forums facilitation and participation, PSTs sent their short reflections via an anonymous survey to report their experiences as facilitators and stressed insights or challenges they faced while participating in the forums, which was the third phase of the task. It provided opportunities to appreciate benefits and challenges when using real-life examples as mathematics teachers (Ghousseini & Herbst, 2016). The survey had one open-ended expression: **"**Please share any meaningful incidents or challenges you encountered while facilitating the forum using a real-life example of your choice.**"**

Data Analysis

We analyzed the gathered data pursuing the enumerative and ethnographic content analysis approach (Grbich, 2013). The enumerative content analysis stresses frequencies of individual categories, helping researchers understand the relevance level of each class; the higher frequency shows the more dominant class. The ethnographic content analysis examines the meaning of data in contexts. Therefore, this analysis highlights the explanatory importance, patterns, and nature of data. This study was mainly grounded in the data within the study by developing data-driven codes (DeCuir-Gunby et al., 2011; Grbich, 2013), but the analytical frameworks used in prior studies (e.g., González, 2017; Rubel & McCloskey, 2021) helped us develop theoretical sensitivity (Strauss & Corbin, 1990).

To examine the first research question, we developed emergent codes for the data in the first phase of the task. While we examined the codes used in the prior studies, including design, shopping, pricing, banking, transportation, games, and physics (Garii & Okumu, 2008; Pirasa, 2016; Rubel & McCloskey, 2021), we noticed that we needed a different set of codes for the data in this study. For instance, some PSTs chose examples related to vaccine efficacy and DNA tests, which the current events might significantly influence. Also, some PSTs selected typical examples that were readily available in textbooks and similar to drill-based problems. Therefore, we regrouped the codes into the following:

1. Straightforward contexts: The context with all necessary pieces of information are presented straightforwardly and likely leads to computations directly rather than the analysis of the context.
2. Scientific data for non-commercial purposes: The context that primarily uses scientific facts without advancing others' commercial interests (e.g., weather forecast, vaccine efficacy, DNA test, and mapping).
3. Information for consumers: The context related to consumers' decision-making (e.g., product labeling, comparative statements describing nutrients/ingredients, naming products, effectiveness of products, and consumer satisfaction survey).

For the second research question, we analyzed PSTs' contributions as forum participants (i.e., their comments on real-life examples). The facilitators' talk moves were not used in the data analysis (e.g., "interesting idea. Could you elaborate on that?", "Does anyone agree or disagree?"). We first explored the number of contributions based on contexts. Then, we analyzed the nature of PSTs' contributions employing González's (2017) framework. The individual contributions are classified into mathematical or contextual dimensions. Afterward, we conducted open coding to sense the purposes of individual contributions.

The emergent codes of mathematical dimension included identifying the whole (referent unit) and demonstrating computational procedures, while the emergent codes of contextual dimension included expressing wondering and questioning implicit messages in the context. The mathematical dimension was in line with the formative and functional literacy rationales in Rubel and McCloskey's (2021) work. The "expressing wondering" in the contextual dimension was in line with the affective rationale in Rubel and McCloskey's (2021) work. The entries coded in the "questioning implicit messages in the context" category were related to critical literacy rationale in Rubel and McCloskey's (2021). **Table 1** depicts categories and descriptions of codes explaining PSTs' contributions during the forums.

For the third research question, we analyzed PSTs' reflective responses to the anonymous survey upon completion of discussions. We determined three categories: (1) forum implementation, (2) the mathematical dimension, and (3) the contextual dimension. The review on forum implementation had benefits and limitations of forums and online forums and difficulties as facilitators. The mathematical dimension included

Table 1. Codes for analyzing PSTs' contributions

Category	Sub-category	Descriptions & examples
Focusing on the mathematical dimension	Identifying the whole (referent unit)	Asking for or explaining the whole in the context: *This phone company says it covers 99% of Americans. What does this mean? 99% of what?*
	Demonstrating computational procedures	Explaining how to compute the answer or give similar examples to explain computational procedures: *The original price was $100 and 40% off brought it to $60. The additional 25% off would bring it to $45. If the original price was $100, you get it for $45.*
Focusing on the contextual dimension	Expressing wondering	Showing wondering and curiosity by repeating the given problem, but without adding any mathematical ideas: *This is an interesting question. I hear all about this on the news and in pamphlets, but I've never really looked at what it means.*
	Questioning implicit messages in the context	Asking questions in interpreting the given context: *Does this mean that it [less-sodium product] is definitely healthier? What if it is still unhealthy but contains less sodium than other products?*

Table 2. Number of real-life examples selected by PSTs (N=46)

Category N(%)	Sub-categories	Examples
Typical straightforward contexts 5(10.9%)	–	*There is an exam with 50 questions. Each question is worth 2 points. If the minimum score to pass is 70%, how many questions does the student have to get right to pass the exam?*
Scientific data for non-commercial purposes 8(17.4%)	Weather forecast (n=3)	*My weather app says there is a 20% chance of rain. What does this mean?*
	Vaccine efficacy (n=3)	*I recently got my COVID vaccine. It says that the vaccine is 95% effective. What does this mean?*
	DNA test (n=1)	*I recently received my results back from a DNA test. When looking at my results, it says I am 23% Irish. I wonder what this 23% means.*
	Mapping (n=1)	*It is a statistic that more than 80% of the ocean has never been mapped or explored by humans. How did scientists come up with this number?*
Information for consumers 33(71.7%)	Product labeling (n=9)	*According to the label on my hair mask, it is made with 98% "naturally derived" ingredients. I wonder what this percentage is telling me about the ingredients.*
	Comparative statements describing nutrients/ ingredients (n=8)	*Lay's Oven Baked Original potato chips claim to have 65% less fat. What does that mean?*
	Nutrition facts/amount of ingredients (n=6)	*I was looking at the Nutrition Facts on my cereal box this morning. What does 12% Total Carb mean?*
	Naming products (n=5)	*We all know there are different kinds of milk (e.g., whole, 2%, 1%, skim). When looking specifically at 1% low-fat milk, what does the 1% mean?*
	Effectiveness of products (n=4)	*Labels of cleaning products often say, "Kills 99.9% of germs!" What does this mean? What germs are left over?*
	Consumer satisfaction survey (n=1)	*Based on overall rankings, Delta ranks #1 with 86.9/100 (86.9%). Delta only scored a 12/20 on "Loyalty." So what makes them the best airline?*

opportunities for further investigations and the lack of mathematical contributions. Moreover, the contextual dimension discussed students' thoughts about real-life examples, including surprise, uncertainty, and effectiveness. After establishing the categories, we used the two coders to independently code a sample of about 20% of the selected real-life examples (phase 1 data), forum entries (phase 2 data), and reflection responses (phase 3 data). The accord between the two coders was 92%. The two coders jointly coded for the rest of the data and resolved any coding discrepancies if they arose.

FINDINGS

Contexts of PSTs' Selection of Real-Life Examples

To respond to research question 1, **Table 2** shows what types of real-life contexts PSTs chose for their discussion forum. The information for consumers had the largest share. A total of 33 PSTs (71.7%) took examples from the consumers' information when buying products, such as information used in advertisements and nutrition fact labels. The examples in this category contained percent values but did not reveal the whole or part.

Table 3. Numbers of contributions by category

Category (number of forums)	Number of contributions	Focusing on the mathematical dimension (54.5%)		Focusing on the contextual dimension (45.5%)	
		Identifying the whole	Demonstrating computational procedures	Expressing wondering	Questioning implicit messages in the context
Typical straightforward contexts (5)	50	1 (2%)	40 (80%)	1 (2%)	8 (16%)
Scientific data for non-commercial purposes (8)	110	48 (43.6%)	4 (3.6%)	50 (45.5%)	8 (7.3%)
Information for consumers (33)	511	260 (51.0%)	13 (2.5%)	83 (16.2%)	155 (30.3%)
Total	**671**	**309 (46%)**	**57 (8.5%)**	**134 (20%)**	**171 (25.5%)**

Eight PSTs (17.4%) chose examples related to scientific data for non-commercial purposes, such as the data used in weather forecasts and vaccine efficacy relevant during the pandemic. The standard of a DNA test was similar to a commercial DNA test. Nonetheless, it belonged to this category because the content of the example was primarily factual without advancing others' commercial interests. The examples in this category did not indicate the whole or the part. Five PSTs (10.9%) selected straightforward examples similar to problems for drills in typical textbooks, such as calculating exam scores or discounts and sale prices. Although they met the selection criteria (i.e., an example that we frequently encounter in daily life) and some could have been in the category of information for consumers, we coded them differently due to the straightforward structure with a clear indication of the *whole, part,* and *percent*.

PSTs' Contributions During the Forums

To answer the second research question, we analyzed the PSTs' contributions to the discussion forums. There existed 46 discussion forums and 671 contributions, save for the facilitator's prompts. The findings showed versatile aspects of PSTs' handling of the presented real-life examples in terms of the number and the nature of their contributions. **Table 3** shows the number of contributions according to the individual category. The forums related to information for consumers had the largest contributions (76.1%), followed by scientific data for non-commercial purposes (16.4%) and typical straightforward contexts (7.5%). Most contributions were presenting the solution process and the answer to the given problem, examining contextual information or presenting modified problems and solutions for the problems. Of the 671 contributions, 46% of contributions (n=309) focused on identifying the whole of the problem. Moreover, 25.5% of contributions (n=171) expressed wanting to understand contexts of the problem, and contributions about demonstrating computational procedures were 8.5% (n=57). Questioning implicit messages in the context had a 20% share (n=134). Overall, there were more mathematical contributions (n=366, 54.5%) than contextual contributions (n=305, 45.5%).

Typical straightforward contexts

The five forums belonged to the typical straightforward context category, where the whole, part, and percent were apparent, resulting in 50 contributions. The majority of contributions in this category (80%) were about explaining or presenting the mathematical computation process to solve the given or modified problems. One contribution (2%) was about confirming the whole, and one contribution (2%) was expressing wondering about the context of problems. Two forums that utilized discount prices led to eight contributions (16%) raising additional issues. One forum used the following example: "*I was at the mall and saw a sign outside a store saying '40% off! Additional 25% off' What does that mean*?" After a PST stated that this meant that the customer would pay 65% off the price, which was incorrect, some PSTs questioned the intention behind this advertisement. Some excerpts are, as follows:

> "I see this a lot! I feel stores present this because it looks like a bigger discount. If one was looking at this, they might see 65% off, which is false."

> "That is only 55% off compared to the 65% customers might have been expecting from the advertisement. It is good to be cautious of advertisements because they can be misleading."

These results indicated that discourse for typical straightforward contexts centered on mathematical dimension (82%) than contextual dimension (18%).

Scientific data for non-commercial purposes

The eight forums in this category yielded 110 contributions. Unlike the typical straightforward contexts, scientific data for non-commercial purposes had almost equal distribution for the mathematical (47.2%) and contextual dimensions (52.8%). The most frequent contribution was expressing wondering about the context of problems (45.5%). For instance, the following excerpt was from a forum that discussed the meaning of "a 20% of chance of rain" in the weather forecast:

> "This is a great question that is just as confusing to me as it seems to be for others too. I have always wondered where the percent in the weather forecast comes from and how it is determined."

One of the highest contributions (43.6%) identified the whole in this context as it was unclear when we used this phrase in our everyday lives. PSTs presented many competing ideas. In the "20% of chance of rain" forum, the following thoughts appeared:

> "It means 20 percent of your area will get rain."

> "It means that 20% of the time that the weather looked this way in the past, it rained."

About 7.3% of contributions were not interested in problem-solving. Instead, PSTs shared their personal information or concerns about whether this example (DNA test) would suit class discussion from a critical perspective. Some examples are in the following excerpts:

> "I recently received my results back from a DNA test, and it shows percentages of different ethnicities that are in my DNA. When looking at my results, it says I am 23% Irish. I wonder what this 23% means."

> "By categorizing people racially or nationally, this type of test may cause tensions between people."

> "I disagree. This is a good way to confirm that we are all different and diverse."

Information for consumers

The remaining 511 contributions from 33 forums used real-life examples we face as consumers. Like forums for scientific data for non-commercial purposes, forums for information for consumers depict equal distribution across the mathematical (53.5%) and contextual (48.5%) dimensions. Moreover, more than half of the contributions (50.1%) emphasized identifying the whole. For instance, a forum used the following advertisement for discussion: "L's *Oven Baked Original potato chips claim to have 65% less fat. What does that mean?*" Below are some excerpts probing what is considered the whole:

> "65% less fat compared to what? Compared to an apple? Compared to a birthday cake? Labels should be more transparent."

> "I believe this is a comparison to their other chip products."

The second portion of contributions focused on questioning implicit messages in the context (30.3%) discussed how consumers could interpret this ad, including PSTs themselves. Some sample excerpts appearing in the above "potato chips" context are, as follows:

> "It guarantees that a bag of baked chips is the lower fat option than whatever they are comparing to it. However, we don't know if it is the healthier option. What other ingredients are in the baked chips and how it is processed could be two questions to investigate to see which is healthier. Low-fat does not necessarily mean healthy."

Table 4. Numbers of mathematically invalid contributions by category

Category	Number of contributions	Mathematically invalid contributions*
Typical straightforward contexts	50	1 (2%)
Scientific data for non-commercial purposes	110	16 (14.5%)
Information for consumers	511	24 (4.7%)
Total	**671**	**41 (6.1%)**

Note. *The percentage of mathematically invalid contributions was calculated by dividing the number of contributions of each category by mathematically invalid contributions

> "Consuming the less-fat option may be a good thing. However, we cannot say that it is definitely healthy. It depends on how much fat it had to begin with. For example, if it has 65% less fat than a really bad quality high-fat product, it may still be unhealthy."

Mathematically invalid contributions

Mathematically invalid contributions were also present. **Table 4** illustrates the discussions of those invalid contributions by context category. As individual contexts had different numbers of mathematical contributions, a direct comparison between the rates of invalid contributions in different categories may be inappropriate. However, some noticeable differences like invalid contributions by context category and the real-life example used existed. The forum for information for consumers had the most considerable invalid contributions (n=24), followed by scientific data for non-commercial purposes (n=16) and typical straightforward contexts (n=1). However, when we divided mathematically invalid contributions from the number of contributions of each category, the forum for scientific data for non-commercial purposes had the largest percentage (14.5%).

One invalid contribution in forums for typical straightforward contexts had the following example: "*I was at the mall and saw a sign outside a store saying "40% off! Additional 25% off." What does that mean*?" As for this question, a PST claimed that the final discount would be 65% off by adding 40% and 25%.

Of the 16 invalid contributions in forums for scientific data for non-commercial purposes, 15 contributions were from three forums regarding the COVID-19 vaccine efficacy. Clinical trials determine vaccine efficacy by comparing disease rates between a group that received the vaccine and another group that received a placebo. Illustratively, if a vaccine has proven efficacy of 80% in clinical trials, this suggests that the vaccinated people have an 80% lower risk of developing the disease than the unvaccinated group. This does not imply that 20% of the vaccinated group will become ill. However, PSTs' invalid contributions claimed just that. Some excerpts from invalid contributions in the forum asking about the 95% COVID-19 vaccine efficacy rate follow:

> "I think this means that for every 100 encounters with the virus, only 5 of those encounters could get you sick."

> "The 5% could just mean that 5% of those people in the trials may have gotten Covid-19 either a mild or stronger case."

Regarding the forums for information for consumers, one forum giving the highest number of invalid contributions used a context of an energy drink's nutrition facts label with some items noted with more than 100% of values (e.g., Vitamin B6: 240%, Vitamin B12: 490%). At the beginning of the discussion, 12 contributions suggested that this was not plausible:

> "None of these percentages make sense to me and how something can be over 100%."

Various forums had other invalid contributions. For instance, some misinterpreted the whole size (e.g., "*You can get 9% of suggested daily fat from the whole package*"; however, the serving size is only two cookies, and the nutrition facts depend on one serving size). In another example of interpreting the lean-to-fat ratio ("85% lean, 15% fat" ground beef), some PSTs made mistakes because they did not use the same unit of measurement (e.g., using oz for the serving size and gram for the amount of fat). In the same example, a PST drew a frequently used bar model when introducing fractional numbers in school as shown in **Figure 1** and asked, "If I eat the shaded portion, am I eating 100% fat?"

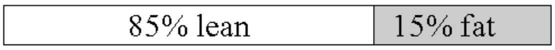

Figure 1. Bar model

Table 5. Numbers of reflections by category (N=60)

Category N (%)	Sub-categories	Examples
Forum implementation 27 (45%)	Benefits of forums (n=8)	*I found the whole forum to be meaningful. It was interesting to get the perspectives of my classmates, and their ideas contributed to my conclusion.*
	Limitations of forums (n=4)	*I encountered a situation where most people wrote very basic ideas or simply repeated my question.*
	Benefits of virtual forums (n=5)	*An asynchronous online forum was a plus because if this was done as a face-to-face conversation, I do not know if we would have had this type of information shared... We spent more time researching questions and thinking deeply about the questions.*
	Limitations of virtual forums (n=5)	*While I think virtual may be an easier environment to engage students... I still find it somewhat difficult to provide an opportunity for all to participate.*
	Role as a facilitator (n=5)	*Although I prepared a question, to be honest, I don't know exactly what math is involved in and the answer. I was unsure how to facilitate when I didn't know the answer and related knowledge.*
Mathematical dimension 20 (33.3%)	Opportunity for further investigations (n=11)	*When someone offered* [mathematical] *information that we didn't know or notice before, that was the perfect time to delve deeper into it or keep it in mind to research later.*
	Lack of mathematical discussions (n=9)	*I noticed that all the students were super focused on what counted as "naturally derived." This was important for their understanding, but they seemed to focus on this only instead of the actual percentage at hand. I wasn't sure how to move their thinking along.*
Contextual dimension 13 (21.7%)	Surprise & uncertainty (n=11)	*In my forum, there were mixed opinions on what nutrition facts labels meant. I was shocked that reading these labels is much more complicated than I had thought before.*
	Effectiveness (n=2)	*This was a good example to use because it got people to think about what those percentages mean on food labels.*

PSTs' Reflection on the Forums

To answer the third research question, a total of 46 PSTs shared the challenges they confronted while facilitating forums and other reflective comments. When some PSTs shared multiple ideas, their reflections were classified into smaller meaning units. Thus, their reflections yielded 60 meaningful units. **Table 5** shows the number of reflections according to the individual category. Forum implementation had the largest proportion (45%), followed by mathematical dimension (33.3%) and contextual dimension (21.7%).

DISCUSSION AND IMPLICATIONS

Researchers have emphasized the importance of PSTs' competency in interpreting real-life examples (Freudenthal, 1991; Simic-Muller & Fernandes, 2020). Therefore, we examined the characteristics of PSTs' selection of real-life examples for discussion, the aspects they attended to during the discussion, and their reflection on facilitating the discussion. Given the limitations of previous studies which examined PSTs' competencies by asking them to generate questions (e.g., Yilmaz, 2020) and evaluate students' written answers (Verschaffel et al., 1997), this study encouraged PSTs to select realistic examples from daily life that pique their curiosity. This process allowed PSTs to choose real-life examples that led to their active participation and discussion. This section discusses our findings, as well as the issues they illuminate for educating well-prepared novice teachers.

PSTs' Selection of Real-Life Context: Static and Shifting Features

To answer the first research question, we examined phase 1 of the task and noted that PST's tendency to choose contexts centered around consumer practices were still very prevalent; this is consistent with prior studies (Lee, 2012; Garii & Okumu, 2008; Pirasa, 2016; Rubel & McCloskey, 2021). However, we noted some differences in PSTs' choices. The consumer practices-related contexts in the current study were geared toward information needed for consumers' decision-making. In contrast, prior studies found that contexts based in consumer practices predominantly put more emphasis on computation rather than analyzing contexts. It is plausible that the shift in PSTs' attention compared to the prior studies might be influenced by the limitations of the scope of the mathematical topic (i.e., percent) and choices for selection (i.e., existing examples in our daily lives that pique a PST's mathematical curiosity).

Nonetheless, we noticed that some PSTs still preferred to utilize real-life contexts as the conceptual anchor to learning mathematics (c.f., formative rationale in Rubel & McCloskey, 2021). When imposing limitations to PSTs' selection of contexts (i.e., existing real-life examples using percent in which they wonder about the meaning), we had anticipated that this condition might encourage PSTs to look into various statistical data that possibly could lead to discussion related to critical mathematical literacy—teaching mathematics for social justice (Rubel & McCloskey, 2021). However, consistent with prior studies, our findings showed that the selection of contexts related to critical literacy was still rare (Rubel & McCloskey, 2021; Simic-Muller & Fernandes, 2020).

When compared to prior studies, we detected some shifting features in the ways PSTs selected real-life contexts; however, we also observed their static preference for consumer-based contexts over other contexts. This result may reflect PSTs' experiences and comfort level (Ensign, 2005). Also, it indicates that PSTs' capacity in this area is to be taught with intention rather than relying on natural development. Thus, the implications for mathematics teacher educators include the following: (a) the need for raising the consciousness of the various uses of real-life context in mathematics education, and (b) the need for offering purposeful opportunities for PSTs to engage in different approaches to real-life contexts.

Balancing Teaching Mathematics vs. Teaching with Mathematics

We identified several characteristics in analyzing PSTs' contributions to forum discussion to answer the second research question. The typical straightforward contexts yielded the least number of contributions and the least number of invalid contributions. This is likely because the cleanness of the contexts, whose main goal was teaching mathematics, left very little room for arguments (both mathematical and other debates). No more active participation occurred once PSTs reached a consensus on the "answers" to problems. Thus, the contributions in this category were mainly focused on the mathematical dimension (González, 2017).

The forums that utilized other contexts (scientific data and information for consumers) produced substantially more contributions and more mathematically invalid contributions. The vagueness of the contexts (for many cases of commercials and intentionally vague contexts) offered more spaces for questions, different interpretations, and debates. However, PSTs' engagement in meaningful mathematical discussion became scarce. It led them to ask more questions about the hidden/implicit messages the contexts had (e.g., deciphering the hidden messages in commercials), which related to critical literacy rationale of using real-life examples (Rubel & McCloskey, 2021). Our takeaways from these findings are two-fold. First, PSTs tended to pay attention to the contexts with critical views when the context was not straightforward, and direct computations were not readily feasible to get the final answer. Second, the vague contexts helped encourage PSTs to view the situations critically, but they had difficulties having meaningful mathematical discussions in such contexts. PSTs' reflective survey results we designed to answer the third research question also showed mixed perspectives. While PSTs responded positively to the surprises and uncertainties they experienced in deciphering hidden meanings in the contexts, they were also concerned about superficial debates on non-essential aspects and the lack of meaningful mathematical work in those forums.

These findings left some remaining questions for us. How can we further probe PSTs' understanding of what real-life contexts are appropriate for their mathematical spaces as learners and future teachers?

How can we help our future teachers balance their goal of teaching mathematics and teaching with mathematics in choosing real-life contexts? As we observed PSTs' lack of mathematical work in non-straightforward contexts, how can we help them build a more robust intellectual infrastructure? While our study planted seeds to fight the habitual use of real-life contexts for our future teachers by encouraging them to notice things other than computations, we suggest other teacher educators join us to probe our remaining questions.

We acknowledge a limitation of the study. This study analyzed PSTs' contributions during the asynchronous online group discussions to answer the second research question. While this approach helped us encourage all PSTs' participation and offerred ample time for them to think, the lack of in-the-moment interactions might have limited them from analyzing and building on the ideas of their peers more deeply. If PSTs had opportunities for offline, real-time interactions, they might have provided other meaningful contributions, and the findings of this study might have been different. Therefore, further studies can examine PSTs' thinking and interactions in the offline environment to see if this setting leads to any qualitatively different results. Despite this limitation, the series of online forums used in this study can provide teacher educators with ideas to implement online group discussions and examine PSTs' competency in interpreting real-life examples.

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