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A Systematic Review on Geometric Thinking: A Review Research Between 2017-2021

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Abstract: Geometric thinking affects success in learning geometry. Geometry is studied from elementary school to university level. Therefore, in higher education and basic education, it is necessary to carry out a systematic review in order to obtain tips for improving geometric thinking skills. A systematic review of geometric thinking was done in this study. In this study from 2017 to 2021, geometric thinking was investigated in the form of a synthesis review of the effect size of the given treatment. This is a comprehensive discussion of theories, models, and frameworks on the topic of geometric thinking from 36 articles. The research findings revealed that the interventions used were predominantly effective, with effect sizes ranging from "small" to "very large," with the "very large" effect obtained in the intervention of van Hiele's learning phase and various technology-based-media and concrete manipulative media. The research trend was reflected through twelve clusters of interrelated keywords. The results of this literature review suggested that it is necessary to carry out a specific study on how to achieve the highest level of geometric thinking, a more detailed form of scaffolding, and concrete manipulative media and technology that can be explored for a certain level of the participants' geometric thinking.

Keywords: *Geometric thinking, pre-service teachers, technology based-media.*

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Introduction

Proportion of geometry material is always present at every level of basic education and is almost the same at every level (National Council of Teachers of Mathematics [NCTM], 2000). When compared to other mathematical fields, such as numbers, algebra, measurement, data, and probability analysis, geometry is an important and inseparable component. Geometry has nearly the same proportions at every level among the five scopes of mathematics material in school (Figure 1).

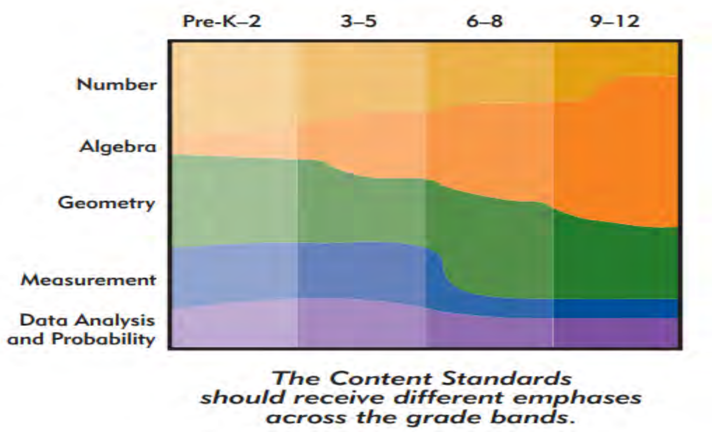

Figure 1. Proportion of Five Scopes of Mathematics Material in Schools (NCTM, 2000)

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Several factors influence learning geometry success, one of which is the ability to think geometrically. The following are some reasons why teachers recommend that their attention to geometric thinking in geometry learning can increase the level of geometric thinking. Van Hiele stated that geometry learning influences geometric thinking level, but biological maturity and grade level have less influence (Škrbec & Čadež, 2015). The lack of geometric learning leads to inadequate experiences (Armah et al., 2018). Van Hiele's research has resulted in the development of five levels of geometric thinking. Rigor, deduction, analysis, abstraction, and visualization are the five levels in order of importance (Škrbec & Čadež, 2015). It is critical for teachers to comprehend students who have reached a certain level of geometric thinking. The geometry learning in question must correspond to the level attained by students, so that students can experience an increase in order to achieve the next level (Mammarella et al., 2017).

Geometric thinking has five interrelated levels depicted as shown in Figure 2. It starts from level 0 which is marked by recognizing the geometry through the shape; level 1 is characterized by recognizing the properties and the relationship between geometric shapes through the properties; level 2 is identified by a meaningful definition of geometry due to the connection between geometric shapes perceived from the relationship between their properties and shape; level 3 is marked by a meaningful deduction, verification, understanding the role of definitions and axioms, understanding the sufficient and necessary conditions, and reasoning skill for each stage of the proof; level 4 is distinguished by understanding the formal deductive aspects, where symbols without a reference can be manipulated according to the laws of formal logic, understanding the role and needs of indirect and counter-positive evidence (Mayberry, 1983). Furthermore, Fuys et al. (1988) provide the following reasoning about the properties of geometric thinking levels. The first level is visual, recognizing the shape of its appearance while unable to see the components. The second level is descriptive reasoning, which involves reasoning about geometric concepts through informal analysis of parts and their properties. For example, a learner is aware of the characteristics of an equilateral triangle, which has three congruent sides and three equal angles. The third level is theoretical, which involves logically organizing properties and concepts, forming abstract definitions, and distinguishing between necessary conditions and a sufficient set of properties when defining a concept. The fourth level is formal logic, which involves thinking and organizing evidence logically. Students can prove the theorem as a construction process, so that the process is achieved after starting with an understanding of the role of the definition, the relevance of axioms, and an understanding of the meaning of adequate terms and conditions. The fifth level is the logic-law level, which indicates that a student can compare different geometries based on different axioms without using a concrete model. The ability to establish the consistency of the set of axioms, and the equivalence of different sets of axioms, and to generate an axiomatic system will be achieved at the highest level.

Moreover, Hohol (2020) elucidated the levels of geometric thinking in detail. The first stage is visual, as students understand geometric shapes as gestalt only on the visual level, without considering elements, parts, and geometric properties. The next stage is descriptive or analytical, in which students can identify not only the gestalts of their shape and name visually, but also blend with the nature and relationships between them. The third stage is abstract or relational, which is defined by students' ability to understand the hierarchical relationship between geometric properties and geometric concepts concerning necessary and sufficient conditions. The following level is formal deduction, where students obtain definitions, axioms, theorems, and proofs. The highest level is meta-mathematical. This designation is used to refer to geometers who have a geometric understanding of the relationship between Euclidean and non-Euclidean geometries.

Elementary school teachers should at least achieve deductive geometric thinking (Jupri, 2018). However, some previous research results showed that many teachers or pre-service teachers have not achieved deductive reasoning (Decano, 2017; Denizli & Erdoğan, 2018). Some research results on geometric thinking level with mathematics pre-service teachers as the subject revealed that the majority of them are at level 3 (Bulut & Bulut, 2012; Fitriyani et al., 2018), with level 4 being extremely rare and difficult to achieve.

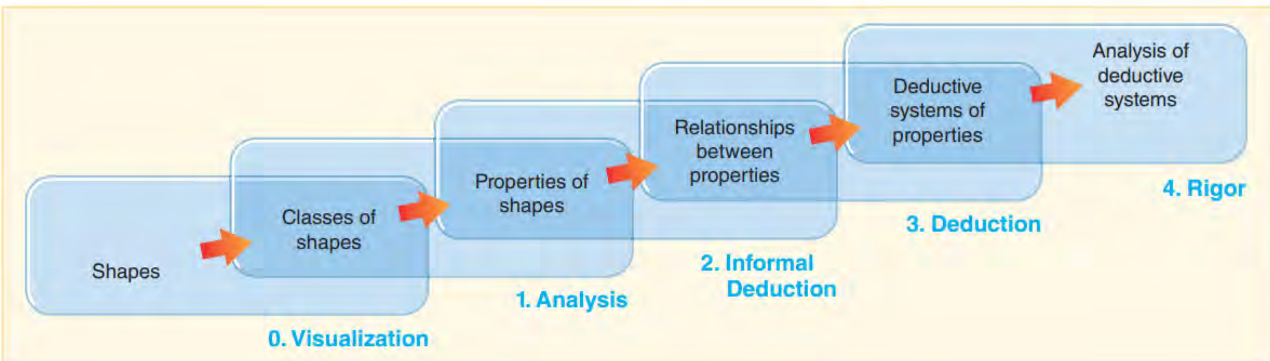

Figure 2. The Level of Geometric Thinking by Van Hiele (Van de Walle, 2018)

Research on geometry skills of students of Pendidikan Guru Sekolah Dasar (PGSD)/Elementary School Teacher Education Degree Program, Faculty of Education, Universitas Negeri Semarang, showed that the students' characteristics on the types of spatial ability and geometric thinking level are essential to induce the geometry lecture strategy. The results of the research data analysis indicated that the students were still in the sufficient category for the two abilities that had an effect on learning geometry (Trimurtini et al., 2021). When presented with application questions involving multiple geometric concepts, students continue to struggle. If students depart from a different level of geometric thinking, material delivery must be adjusted accordingly. As a result, even though the studied geometry concept is the same, the delivery strategy can be tailored to the students' geometric thinking level in a variety of ways. It is intended that students can master all levels of geometric thinking sequentially. Literature review on van Hiele's geometric thinking has been carried out, but has not focused on research participants and only calculated from the resulting effect size (Hassan et al., 2020).

Several previous studies on geometric thinking have been carried out in many countries. Most of the research conducted is very specific and focuses on one particular topic in geometry, for example, proving theorems in circles (Frank & Ablordeppey, 2020), rectangles (Syamsuddin, 2019), and also three-dimensional geometry (İbili et al., 2020). Hence, the forms of intervention in the research sample also vary greatly from the learning approach model to the variation of the learning media used ranging from computer-based media (Chang et al., 2007; İbili et al., 2020; Mdyunus & Hock, 2019) or manipulative media (Pathuddin et al., 2021; Trimurtini et al., 2020). The diversity and specifications of each research on geometric thinking open opportunities as well as challenges for researchers who will investigate geometric thinking. Thus, they do not only repeat existing research but find new elements that contribute to the development of geometry learning.

In addition, the effect size can be calculated from quantitative data obtained from quantitative research. According to Ilie et al. (2020) , the benefit of calculating this effect size is for detecting statistically significant results in research studies so that the findings can be synthesized across studies more accurately by conducting a meta-analysis.

The importance of this study for teachers is to know the stages of thinking from the simplest to the most complex farther they can overcome the difficulties of learning geometry and make it a guide to provide learning assistance in learning geometry that is in accordance with the level of students' thinking. Various forms of intervention to improve the ability to think geometrically the results of the last five years of research are presented and discussed, so that teachers can determine which learning aid is most appropriate to the condition of their students. The benefit of this study for researchers in the field of geometry education is that they can find links between previously researched topics about geometric thinking to determine the novelty of the research to be carried out.

The objective of this systematic review is to synthesize the discoveries from prevailing empirical research to present wider descriptions of geometric thinking and things related to it for future advancement. There are several steps in the systematic process. First, gather various empirical studies based on the criteria. Then, examine both qualitative and quantitative data. Finally, synthesize all relevant information from previous studies and explain the current status of the study as well as the effect sizes of the approaches used. Trends and theoretical frameworks can be identified as a result of this research.

Methodology

Research Goal

The goals of this systematic review are to (1) analyze the effect size of the various approaches used in geometric thinking research published in the last 5 years, (2) identify the research trend on geometric thinking in the last 5 years, and (3) reveal the geometric thinking theoretical framework used in the gathered studies.

Research Design

This systematic review involves several processes. Prior to interpreting all the studies that met the criteria, they were identified and evaluated according to the type of research and the empirical data presented. PRISMA (Primary Reporting Items for Systematic Reviews and Meta-Analysis) was employed, thus, all information sources (databases with coverage), the date of the last search, and searching limitation were explained including the limits used (Cooper et al., 2019).

The data were taken from the database of Science Direct, Scopus, Crossref, and Google Scholar using the keyword 'geometric thinking Van Hiele' which ranges from 2017 to 2021. The steps in this study are as described in Figure 3. The protocol of systematic review began with the preparation of research questions, then searching for articles on the database (Science Direct, Scopus, Crossref, and Google Scholar) using the keyword 'geometric thinking Van Hiele' published in 2017- 2021. A selection was made from the 905 titles obtained based on inclusion and exclusion criteria that began with the title, abstract, and duplication of titles from the four databases. Following the acquisition of the required articles, an assessment of data quality and data extraction was carried out. The final stage is the synthesis process, which is used to determine the results.

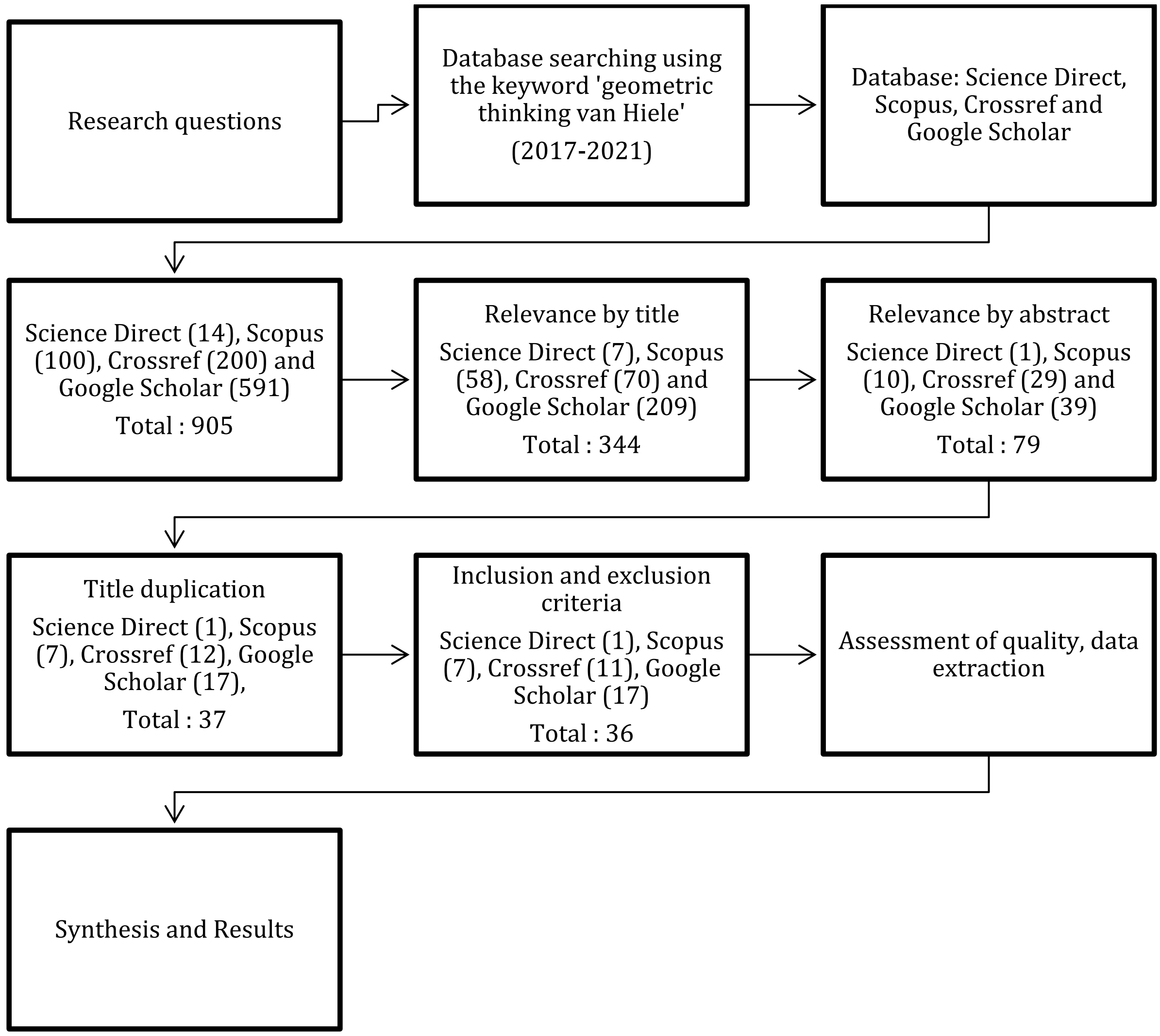

Figure 3. The Protocol of Systematic Review

The use of the keyword "geometric thinking van Hiele" aims to answer the research objectives on the theoretical framework of geometric thinking, whom the originator of the idea of geometric thinking is Van Hiele. For that reason, the development of the level of geometric thinking from the trend of research conducted in the last five years is observable.

Table 1 describes the terms of inclusion and exclusion. The objective of applying this requirement is to come up with an overview of the selection of research on geometric thinking that is used to consider research articles published in journals and conferences. The four large databases used are Science Direct, Scopus, Crossref, and Google Scholar. Some of these criteria are used for empirical research published from 2017 to 2021. In the beginning, 905 titles were obtained and reduced to 36 titles after undergoing the 5 stages of the systematic review protocol process. The other 869 titles were declined.

Table 1. The Criteria of Inclusion and Exclusion

Inclusion	Exclusion
Articles published from 2017 to 2021	Articles published outside of the specified time frame
Articles that have been published and entered into the database	Non-English articles
Articles having the participants coming from pre-service teachers or elementary school students	Unpublished articles that are not in the database
There are quantitative data and or qualitative data on geometric thinking	Consisting of less than 4 pages

Data Analysis

After a systematic review protocol (Figure 3) was carried out, 36 article titles were obtained from three data sources, namely Science direct (1), Scopus (7), Crossref (11), and Google scholar (17). The data analysis process began by grouping the data from 36 article titles based on the type of research (quantitative and qualitative), the country where the research is located, and the research subject (elementary school students and future teachers). The size effect was analyzed and summarized in this type of quantitative research to describe the effect of geometric thinking on the research results. Meanwhile, qualitative research and mixed qualitative-quantitative research were classified separately.

Furthermore, the data presented in the bibliometrics were analyzed using the VOSviewer software. The data input in Excel format was analyzed using VOSviewer software to examine the research trends over the last 5 years on the topic of geometric thinking. The analysis started from the relationship between keywords, and groups of keywords that are directly related to research trends in each year.

The theoretical framework on the topic of geometric thinking is investigated using various theories and research findings presented in these 36 articles. It began with the theories of several experts about the understanding of geometric thinking, then the level of geometric thinking in general and its achievement on the research subject of elementary school students and pre-service teachers, as well as the type of intervention carried out by researchers to improve geometric thinking skills.

Findings

The Effect Sizes of Various Approaches Used in Research on Geometric Thinking for the Last 5 Years (2017-2021)

After completing all protocol stages, 36 articles were obtained using three research approaches: quantitative, qualitative, and mixed (qualitative and quantitative). The gathered quantitative studies were analyzed quantitatively using the effect size (Cohen et al., 2007). There are 36 articles that have been processed and classified into three research approaches: quantitative (18 titles), qualitative (17 titles), and mixed (1 title). The quantitative group (table 2) was conducted in a number of countries, including Indonesia (6), Malaysia (2), the Philippines (1), Hong Kong (1), Ghana (1), Nigeria (1), Turkey (3), U.S.A (1), Jordan (1), and Israel (1).

Table 2. The Calculation of Quantitative Data's Effect Size

No	Author, year	Journal	Participant	Country	Domain Knowledge	Intervention	Effect Size
1	(Yi et al., 2020)	Teaching and Teacher Education	111 pre-service teachers	USA	geometry content knowledge, knowledge of students' van Hiele levels, and knowledge of geometry instructional activities	van Hiele's theory-based instructional	Geometry cognitive knowledge=1.177 (large). Geometric thinking knowledge= 1.421 (large). Geometry learning knowledge= 0.995 (large)
2	(Armah et al., 2018)	IJRES	75 pre-service teachers	Ghana	Geometric thinking level	van Hiele's Phase-based instruction	Enhancement in the experimental class= 3.097 (very large)
3	(Mdyunus & Hock, 2019)	International Journal of Instruction	96 fifth-grade elementary school students	Malaysia	The level of geometric thinking proposed by van Hiele's	Learning phase (VH-PL) module, Google SketchUp software	0.121 (medium)
4	(Pasani, 2019)	Journal of Southwest Jiaotong University	150 elementary school students	Indonesia	students' comprehension of geometric concepts	van-Hiele's Theory-Based Geometry Learning	0.10 (small)
5	(Tieng & Eu,	Pertanika Journal of	74 fourth-grade	Malaysia	van Hiele's levels of	media Geometer's Sketchpad	-0.24 (small)

No	Author, year	Journal	Participant	Country	Domain Knowledge	Intervention	Effect Size
	2018)	Social Sciences and Humanities	elementary school students		geometric thinking		
6	(Kristanti et al., 2018)	Journal of Physics	33 students of mathematics education	Indonesia	Geometric thinking	Creative problem-solving, book-based Al-Qur'an	0.420 (medium)
7	(Özçakır et al., 2020)	International Journal of Contemporary Educational Research	53 fifth-grade elementary school students	Turkey	Mathematically gifted	Dynamic geometry software	1.167 (very large)
8	(Usman et al., 2020)	African Journal of Educational Studies in Mathematics and Sciences	149 pre-service teachers	Nigeria	attitude towards geometry, gender	van Hiele's phase-based teaching strategy	van Hiele's teaching strategy = 0.43 (very large). Gender, attitude towards geometry = 0.00 (small). Intervention, gender, attitude towards geometry= 0.01 (small).
9	(Klemer & Rapoport, 2020)	EURASIA Journal of Mathematics, Science and Technology Education	88 second-grade elementary school students	Israel	Hebrew and Arabic-speakers	Computerized Origametria program, the GeoGebra environment	0.340 (very large)
10	(Çaylan et al., 2017)	Journal of Multidisciplinary Studies in Education	64 prospective elementary mathematics teachers	Turkey	Beliefs Towards Using Origami	Origami Course	0.366 (medium)
11	(Ng et al., 2020)	International Journal of STEM Education	174 students and 7 elementary school teachers	Hong Kong	Embodied cognition, Gestures	Dynamic geometry environment, 3D printing	For the DGE group = 1.612 (large) For the 3D printing group = 1.193 (large)
12	(Altakhyneh, 2018)	Journal of Education and Learning (EduLearn)	104 pre-service teachers	Jordan	Geometric thinking	Blended learning	0.075 (large)
13	(Decano, 2017)	American Journal of Applied Sciences	105 undergraduate students	Philippines	Cognitive development based on age, gender, and year level	van Hiele's Theory and Piaget Theory	Gender 0.1522 (very large) Deduction and rigor = 0.565 (very large)
14	(Sudihartinih, 2019)	Journal of Engineering Science and Technology	29 students of mathematics education	Indonesia	Self-efficacy, gender	GeoGebra software	Geometric thinking and Self efficacy = 0.127 (medium) Geometric thinking and

No	Author, year	Journal	Participant	Country	Domain Knowledge	Intervention	Effect Size
							Gender = 0.245 (medium)
15	(Primasatya & Jatmiko, 2019)	International Journal of Trends in Mathematics Education Research	67 fifth-grade elementary school students	Indonesia	students' critical thinking abilities.	geometry multimedia based on van Hiele's thinking theory	0.3416 (very large)
16	(Andini et al., 2018)	Journal on Mathematics Education	125 sixth-grade elementary school students	Indonesia	Informal deductive thinking, geometry basic skills	Flipbook multimedia	0.0156 (medium)
17	(Denizli & Erdoğan, 2018)	Journal on Mathematics Education	384 elementary students grade 1 to 4 (stage 1), 120 students (stage 2), 268 students (stage 3)	Turkey	Three-dimensional geometric thinking	Three-dimensional geometric thinking test	-
18	(Risnawati et al., 2019)	International Journal of Instruction	97 students of mathematics education	Indonesia	Mathematical Reasoning Ability	Plane Geometry Module based on van Hiele's level	0.8296 (very large)

The study's participants were divided into two groups: elementary school students (9 titles) and pre-service teachers (9 titles). This type of quantitative research included 4 research developments, 2 regression correlations, and 12 experiments. As shown in Table 3, the effect sizes of the 17 quantitative research titles were classified as small, medium, large, and very large. The strength of the relationship between the variables studied was measured by the effect size. These can be r, r-squared and eta-squared, which can describe the independent comparison of effect scales from one study to another (Cohen et al., 2007). The advantages of calculating effect size are threefold: (1) it can estimate sufficient sample size to detect statistically significant results in future research studies, (2) it can assess the practical significance of research studies, and (3) it can more accurately synthesize findings across studies (e.g., conducting a meta-analysis) (Ilie et al., 2020).

Table 3. Effect Size Grouping Types Based on Intervention and Domain Knowledge

Small effect	Medium effect	Large effect	Very large effect
Gender, attitude toward geometry. Intervention, attitude toward geometry, gender*)	Flipbook**)	Blended learning*)	Modul*)
media geometer's sketchpad**)	Self-efficacy, gender*)	Dynamic geometric environment, 3D printing**)	Multimedia**)
van Hiele's theory-based geometry learning, students' comprehension of geometric concept**)	Origami course*)	Geometry content knowledge, geometric thinking knowledge, geometry learning knowledge*)	Geometric thinking and gender. Deductive and rigor*)
	Creative problem-solving, book-based Al-Qur'an *)		Computerized origami program, GeoGebra environment**)
	van Hiele's phase of learning, module, google SketchUp software**)		van Hiele's teaching strategy*)
			Dynamic geometric software**)
			van Hiele's phase-based instruction*)

Information: *) = pre-service teachers as participant, **) = elementary school students as participant

Table 4 displays the studies that were collected using the qualitative (17 titles) and mixed (1 title) approaches. These studies were carried out in a number of countries, including Indonesia (11 titles), the Philippines (1), Ghana (2), Turkey (2), U.S.A (1), and Jordan (1). Geometric thinking research has been conducted in a variety of countries located on the world's continents. The distribution of researchers' countries of origin opens the possibility for collaborative research involving several countries. There are several studies, including qualitative descriptive research, case studies, design research, and classroom action research. The participants involved were elementary school students (5) and pre-service teachers (13).

Table 4. List of Qualitative Research on Geometric Thinking

No	Author, Year	Journal Name	Participant	Country	Focus
1	(Erdogan, 2020)	International Online Journal of Educational Sciences	65 Prospective junior high school mathematics teachers	Turkey	Problem-posing skills
2	(Fitriyani et al., 2018)	Infinity Journal	129 students of mathematics education	Indonesia	Geometric thinking development
3	(Rofii et al., 2018)	IJEME	66 elementary school students	Indonesia	Metacognition, geometry problem
4	(Ramlan & Hali, 2018)	JME	28 college students	Indonesia	Geometric reasoning, geometry transformation
5	(Armah & Kissi, 2019)	EURASIA Journal of Mathematics, Science and Technology Education	11 Mathematics education practitioners	Ghana	teaching strategies
6	(Salifu et al., 2018)	Journal of Education and Practice	351 Pre-service teachers	Ghana	geometric thinking level
7	(E Sudihartinih & Wahyudin, 2019)	Journal of Physics	90 Pre-service teachers	Indonesia	Geometry ability based on gender (sex: male and female)
8	(Nugraheni et al., 2018)	Journal of Physics	30 Sixth-grade elementary school students	Indonesia	Visual-spatial, geometry basic skills
9	(Crompton, 2017)	Journal of Educational Technology & Society	60 Fourth-grade elementary school students and 2 teachers	U.S.A	Mobile learning, IPads, angle
10	(Gür & Kobak-Demir, 2017)	Journal of Education and Practice	18 students of mathematics education	Turkey	Origami, geometry teaching, 2D object, 3D object
11	(Yudianto et al., 2018)	Journal of Physics	78 students of mathematics education	Indonesia	Space analytic geometry, van Hiele's levels
12	(Kusmayadi & Fitriana, 2021)	Journal of Physics	4 samples of students from 63 population of mathematics education students	Indonesia	Ethnomathematics problem
13	(Rahman et al., 2020)	Journal of Physics	Students	Indonesia	Scaffolding profile, solving geometry problem
14	(Hamzeh, 2017)	European Journal of Research and Reflection in Educational Sciences	55 students of mathematics education	Jordan	van Hiele's Model, Geometric thinking level

No	Author, Year	Journal Name	Participant	Country	Focus
15	(Mahfut et al., 2020)	Journal of Physics	1 male college student and 1 female college student	Indonesia	Imagination in solving a geometry problem, creative imagination, reproductive imagination
16	(Dimla, 2018)	Journal of Education in Black Sea Region	35 prospective mathematics teachers	Philippine s	Space function, Plane and Solid Geometry
17	(Pathuddin et al., 2021)	IJRISS	26 Sixth-grade elementary school students	Indonesia	Manipulative media, circle material
18	(Hamidah & Kusuma, 2020)	Journal of Physics	38 students of mathematics education	Indonesia	Learning style

The Trend of Geometric Thinking Research in the Last 5 Year-Publication

The last five year's research trend was obtained from the processed bibliometric data using a VOSviewer as shown in Figure 4. The same color indicates the same cluster, and the circle size shows the keyword's popularity. The larger the circle size, the more popular the topic is discussed in the 36 articles. Connecting lines between the circles mark the direct relationship between keywords.

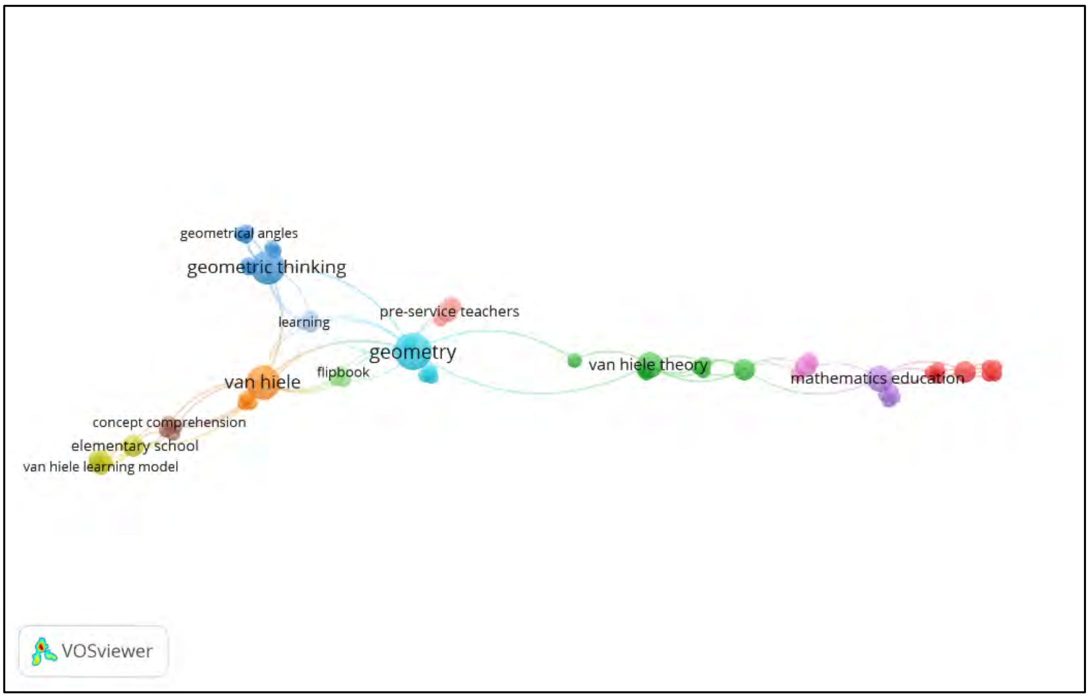

Figure 4. The Keyword Network Visualization of the 36 Processed Articles

Categories of Frequently-Occurred Keywords or Variables and Their Relationship

Observation of the most frequently studied topics in a period of time can reveal research trends on geometric thinking in the last 5 years. Comparisons are also made to see the trend of research on geometric thinking from time to time, so that the stages of development can be known. This is reflected in the topics that have just emerged in a period of time and the most popular topics. As shown in Table 5, there were 5 clusters of keywords that revealed the closeness of the keywords in the processed articles. The existing keywords discuss geometry material, intervention in geometry learning, cognitive aspects, affective aspects, and other aspects that may be related.

Table 5. Keyword Clusters in Research on Geometric Thinking

Cluster	List of Keywords
1	2d objects, 3d objects, dynamic computer activities, GeoGebra, geometric knowledge, geometric teaching, origami, speaking
2	elementary pre-service teacher, geometric thinking development, geometric thinking level, problem

Cluster	List of Keywords
	posing, pre-service teachers, teacher education, thinking level, van Hiele's theory
3	early graders, geometric thinking, geometrical angles, phase-based instruction, plane and solid geometry, three-dimensional geometry, van Hiele's model
4	basic competencies learning, circles, culture, elementary school, learning outcome, van Hiele's learning model
5	dynamic geometry, embodied cognition, gestures, mathematics education, technology-enhanced learning
6	geometry, mathematical reasoning, mobile learning, module development, plane geometry
7	cognitive development, geometry achievement, Piaget, van Hiele
8	concept comprehension, learning geometry, mathematics communication, learning theory
9	gender, Hiele's phase-based teaching, self-efficacy, pre-service mathematics,
10	pre-service teachers, teaching strategies, van Hiele's geometry test, van Hiele's levels
11	curriculum 2013, flipbook, informal deductive
12	learning, geometry test, thinking

The circle size of each keyword (Figure 4) shows the level of popularity of the 36 research titles processed. The larger the circle size indicates the greater the keyword usage in the research. This indicates that the variable has previously been extensively researched. The direct relationship between 'geometric thinking' keywords and other keywords is presented in Figure 5.

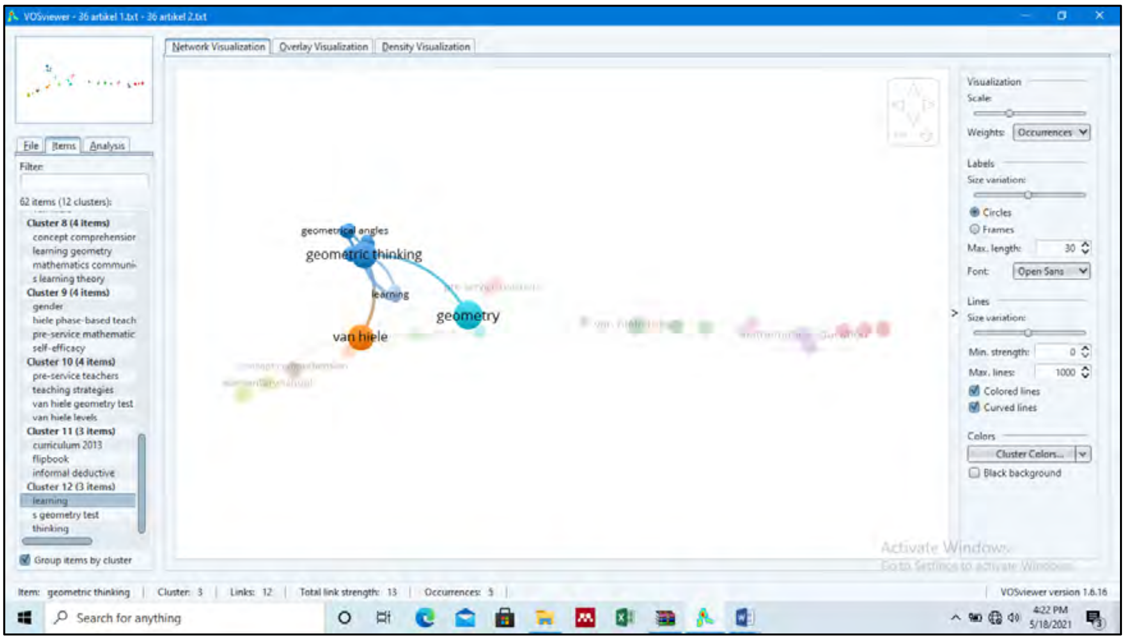

Figure 5. The Relation of 'Geometric Thinking' Keyword with Other Keywords

Figure 5 informs that the 'geometric thinking' keyword was included in cluster 3 with 12 links and 5 events. Keywords that were strongly related to 'geometric thinking' are geometry, van Hiele, learning, plane, and solid geometry. This keyword is directly related to other keywords that are in three different clusters, namely those in clusters 3 (dark blue), 6 (light blue) and 7 (orange). This can imply that, out of the 36 studies conducted, keywords in the three clusters are most likely to have become topics in a single research title.

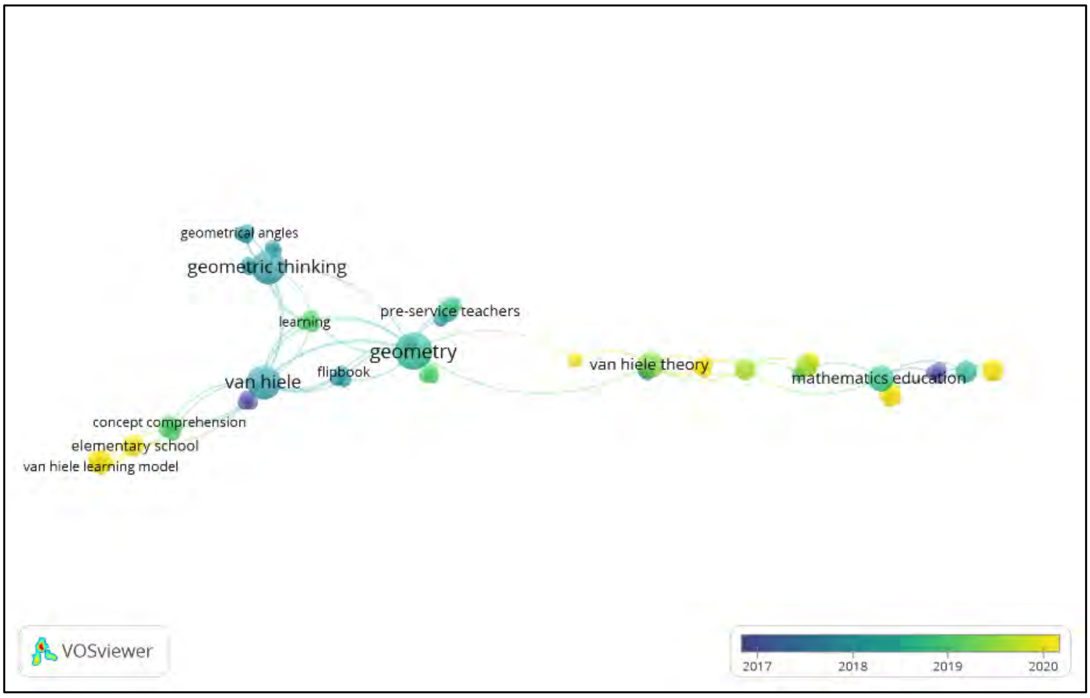

Figure 6. Publication Trend from 2017 to 2020

The trend, as shown in Figure 6, is indicated by colors, with lighter marks indicating newer publications. The most recent publications, denoted by a yellow color, demonstrate that the trend included keywords such as elementary pre-service teachers, elementary school, dynamic geometry, dynamic computer activities, and van Hiele's learning model. For researchers, information about the novelty of the topic is important to show the current state of the research carried out in accordance with the times.

The Theoretical Framework on Geometric Thinking Used in the Last 5 Year-Publication

The defined articles were coded and analyzed in terms of geometric thinking theory, model, or framework. A total of 36 articles were classified based on the definition, level, participants, types of intervention, and domain knowledge of the research. An overview of the theory, model or framework of geometric thinking was obtained from the 36 articles and is presented in Figure 7. As illustrated in Figure 7, there were several major topics discussed in the theoretical framework of 36 articles on geometric thinking. First, there is the concept of geometric thinking, which is a type of thought process or mental activity in a person about geometry. Furthermore, geometric thinking level is a mental process and individual skill to develop ideas related to mathematical situations and experiences in geometry. The level of geometric thinking is the level of thinking development that needs to be mastered hierarchically. The rationale lies in activities at every level of thinking, which consists of pre-introduction (level 0), visualization (level 1), analysis (level 2), informal deductive (level 3), deductive (level 4), and accuracy/rigor (level 5). Even so, it is difficult for pre-service teachers to reach the rigor level. The third is about basic geometry skills. After mastering the informal deductive stage, learners are expected to have basic geometry skills such as visual, verbal, drawing, logic, and application.

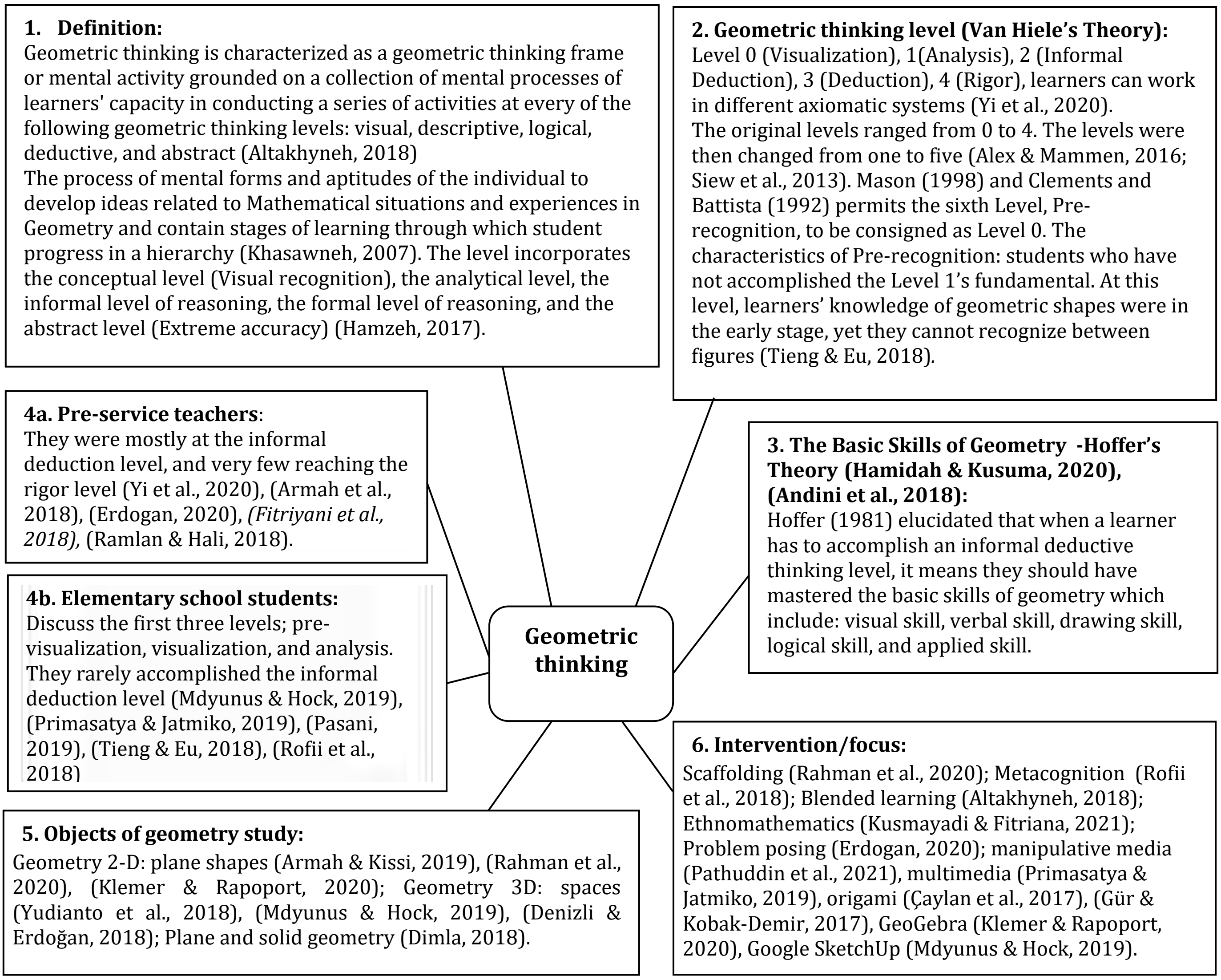

Figure 7. Theoretical Framework on Geometric Thinking

Studies on geometric thinking in the last five years are described in a theoretical framework as shown in Figure 7. The development does not only show theoretical definitions and characteristics of each level of geometric thinking, but it is also a very influential variable in the achievement of geometric thinking. The distribution of the geometric thinking level of elementary school students reached the informal deductive level, although there were students who remained to face difficulties in accomplishing the rigor level as the highest stage. Various previous researchers' efforts to increase the level of geometric thinking have been done starting from the intervention of learning approaches (ethnomathematics, blended learning), learning models (problem-posing), to learning media in geometry (manipulative, multimedia, origami, GeoGebra, Google SketchUp). Geometry topics are also scattered and definite, such as in Geometry 2D: plane shapes and Geometry 3D: spaces, planes, and solid geometry. Almost all topics fall into Euclid's system of geometry.

Discussion

The effect size shows a variable's influence on another and is affected by the number of samples used. An interesting fact was found in the summary table (Table 5), in which the 'very large' effect on geometric thinking was obtained in studies applying van Hiele's learning phase and various media, for instance, module, multimedia, origami computer program, GeoGebra, and dynamic geometry. This profound effect was observed in the participants, who were either elementary school students or pre-service teachers. Geometric thinking, which is frequently associated with van Hiele's theory, is composed of three major components: the description of theory assumption, geometric thinking level, and learning phase (Hohol, 2020). These three are interrelated and influence each other. On the other hand, manipulative media in geometry learning is widely suggested by researchers, either technology-based (Gecu-parmaksiz & Delialioglu, 2019) or concrete manipulative media. The latter enables learners to interact physically with abstract content, which is usually untouchable and cannot be visualized (Carbonneau et al., 2020).

Van Hiele's learning phases include information, orientation, explanation, free orientation, and integration. In the study conducted by Pathuddin et al. (2021), the information phase was applied by the teacher to ensure the students' ability to understand geometric shapes. Furthermore, in the orientation phase, abstract geometric shapes are introduced using learning media. They employ manipulative media to create a concept, followed by the teacher ensuring the truth of the concept in the explanation phase. The similarity of research results on manipulative media is appropriate for use in elementary school students, as the same assumption that the cognitive development of elementary school students is in the concrete operational development, and learning activities involving psychomotor will be more effective (Trimurtini et al., 2020). The difference is when students use manipulative media. While Trimurtini et al. (2020) suggested that manipulative media be used at the beginning of learning, Pathuddin et al.(2021) recommended the use of manipulative media in the second stage, at which the teacher has given a general explanation of the geometry materials. This discrepancy may occur due to the learning approach utilized, where constructivism is chosen when students are expected to build their knowledge at the beginning without much intervention from the teacher.

The literature review of geometric thinking already conducted by Hassan et al. (2020) shows that Van Hiele's learning phase intervention is divided into two, namely using manipulative (3 studies) and technology (12 studies). The influence of technological interventions produces effect sizes that vary from lowest to highest. While the influence of manipulative interventions is spread across the 3 lowest levels of effect size. Nonetheless Hassan et al. (2020) concluded that technological interventions are no better than manipulative. It is depending on the type of technology, when, and where the technology is used. In contrast to the findings of this study, that in the largest effect size (Table 3) there is an element of technology as a form of intervention to improve geometric thinking. The form of technology is in the form of computerized origami programs, GeoGebra environment, and dynamic geometric software.

Contrastingly, the technology-based manipulative media, for example, dynamic geometric software, is believed to expand working memory capacity, which is very efficacious for more complex learning assignments requiring bigger working memory resources (Bokosmaty et al., 2017). Technology as a medium in geometry learning is found in various forms ranging from multimedia, computerized origami, GeoGebra, mobile learning, and also dynamic geometric software. On the other hand, the existence of this technology-based media opens up opportunities for digital geometry learning, particularly during the COVID-19 pandemic. Digital learning will enable a pedagogical shift in mathematics to more entertaining and engaging teaching methods (Mulenga, 2020).

Several keywords appear in the studies published around 2020, including elementary pre-service teachers, elementary school, dynamic geometry, dynamic computer activities, and van Hiele's learning models. The word dynamic appears in two of the five keywords. Dynamic means advanced, full of energy, and enthusiastic. Dynamic geometry is realized in software that can provide different activities. The use of dynamic geometry software allows for the execution of several activities, including observing parts of objects through manipulation, recording, and constructing conjectures and theorems (Özçakır et al., 2020). In addition, dynamic geometry also provides unprecedented abilities for students to visualize and experiment (Luz & Yerushalmy, 2018).

The next part was about the participants, which consisted of elementary school students (13 studies) and pre-service teachers (23 studies). The geometry level investigated on elementary school students remained pre-introduction, visual, to analysis, with a few elementary school students reaching the informal deductive level. Similarly, pre-service teachers produced similar results, with the majority performing at the visual, analytical, and informal deductive levels. Few of them reach the level of deductive and very rarely achieve accuracy. This is in response to Burger and Shaughnessy's (1986) study, which was conducted with participants ranging from elementary school students to university students, and the results did not show information on attainment of the highest geometric thinking level. This is not surprising given that this level requires extensive meta-mathematical considerations. It is difficult to achieve because it is beyond the scope of the school geometry curriculum (Hohol, 2020).

The 36 articles (Part 5 Figure 7) were intriguing to investigate because they cover a wide range of geometry topics such as 2-D and 3-D geometry, plane and solid geometry, transformational geometry, and analytic geometry. There are even specific geometry materials, for example, points, lines, and angles, triangles, rectangles, or circles. The challenge is determining how to present it to students in order for them to achieve the rigor level with this Euclidean geometry. At the previous level, the deductive, students have already understood the formal aspects of deduction. Meanwhile, at the rigor level, they are expected to manipulate symbols without reference under the laws of formal logic (Mayberry, 1983). This highest level is also known as metamathematical, and students are hoped to achieve formal reasoning on geometrical associations in the Euclidean system and begin paying attention to many things such as non-Euclidean systems (Hohol, 2020). The need to distinguish various geometric systems at the elementary level is not great, but students learn the geometry of four different systems including topological geometry, Euclidean geometry, coordinate geometry, and transformation geometry (Kennedy et al., 2008). Although these geometric systems are related, each system has slightly different rules and vocabulary.

Data from ten studies at the college level (Table 2) showed that initially prospective teacher students reached the lowest level of visualization (level 0) and few reached deductive levels (Armah et al., 2018; Çaylan et al., 2017; Kristanti et al., 2018). Then after treatment that suits the needs and characteristics of students and geometric materials, students showed an increase in the level of thinking, but only one study showed that students were able to reach the highest level (rigor) (Çaylan et al., 2017). But from the research of Çaylan et al. (2017) found no detailed explanation of geometric materials and how students can achieve the highest level of geometric thinking. In the study, it was conveyed that the treatment given was in the form of learning using origami. While in Decano's research (2017), only a fourth-year student can reach the rigor level. At this level student can appreciate the investigation of various systems and be able to reason in the most appropriate way in various systems. This shows that age does not affect the achievement of geometric thinking levels (Škrbec & Čadež, 2015) and it is difficult to reach the highest level in geometric thinking (Hohol, 2020).

It is difficult but not impossible to achieve the highest level of geometric thinking (Yi et al., 2020). (Yi et al., 2020). The cause of this difficulty is that they are only able to recognize the physical aspects of phenomena and they lack logical and hypothetical reasoning (Decano, 2017). Departing from the resulting theoretical framework (Figure 7), there are fundamental theoretical differences between the highest level of metamathematical and formal deductive. The latter is mastered when the students can arrange evidence logically, understand the role of axioms and definitions, and provide reasons for each stage in proof. Furthermore, the highest stage is distinguished by students' ability to learn geometry without the use of a concrete model, achieving abstract deductive reasoning using Euclid's geometric system.

The literature on learning geometry needs to consider several studies derived from different approaches: the development of psychology, cognitive psychology, educational psychology, and education. The topic of geometry is interesting because all these approaches can contribute to our understanding of the teaching and learning of this complex subject. To understand the best way to solve the problem of learning geometry, we need to consider not only the cognitive processes involved in geometry, not only how geometric knowledge is developed, and not just how geometry is taught. But all these approaches are observed simultaneously (Mammarella et al., 2017). The theoretical framework (Figure 7) which is the result of this systematic review shows factors that affect directly or indirectly to geometric thinking. When compared with the results of Mammarella et al. (2017) thinking about learning geometry, the similarity with the theoretical framework (Figure 7) compiled tries to look at various aspects simultaneously. Although the difference in cognitive processes in geometric thinking has not been discussed in the theoretical framework.

Various forms of intervention in research and domain knowledge become the focus of research on elementary school students. The examples include scaffolding (Rahman et al., 2020) or learning assistance that is suitable for students' needs in solving geometry and metacognition problems (Rofii et al., 2018). There are various kinds of scaffolding that can be provided, including learning assistance to individual students, learning assistance by involving other students, or computer-based learning assistance (Kusmaryono et al., 2021). The two studies were conducted on elementary school students, taking into account their level of cognitive development which remained to be at the concrete operational stage. It is said that metacognition skills play a role in student learning success (Karatas & Arpaci, 2021; Surati et al., 2021).

Conclusions

Several conclusions have been drawn based on the findings and discussion of this literature review:

The effect sizes of various interventions and domain knowledge that have been the focus of geometric thinking research in the last 5 years ranged from small, medium, large, to very large, with everything influenced by the number of samples and data obtained. The 'very large' effect was obtained in van Hiele's learning phase intervention, which is closely related to geometric thinking and various concrete manipulative media and technology.

The last five year's research trends on geometric thinking can be grouped into interrelated 12 keyword clusters. In addition, the research trend around 2020 obtained several keywords such as pre-service teachers, elementary school, dynamic geometry, dynamic computer activities, and van Hiele's learning model. The theoretical framework of geometric thinking was formed from the definition of geometric thinking, the development of naming at the van Hiele level of geometric thinking, basic skills in geometry according to Hoffer, research participants consisting of pre-service teachers and elementary school students, various interventions, domain knowledge, research focus, and the object of geometry research. The fundamental difference in the characteristics of deductive and rigor thinking levels is to find appropriate scaffolding for the students to reach the highest level.

Recommendations

Referring to the findings of this systematic review, it is critical to conduct specific research on how to achieve the highest level of geometric thinking. Van Hiele first introduced the level of geometric thinking, which later evolved in line with Burger, Mayberry, and Masson's research, who found that the highest level of geometric thinking was difficult for students to achieve. Further study may provide a more realistic description of geometric thinking indicators following the conditions and needs. The widely used interlevel transformations of geometric thinking, known as van Hiele's learning phase, can be studied further to lead to more detailed scaffolding for a specific geometric level. Diverse concrete manipulative media and technology-based media should be thoroughly investigated in terms of their suitability for the participants' specific level of geometric thinking, whether elementary school students or pre-service teachers.

Further researchers can develop effective learning aids for the students to achieve the highest level of geometric thinking according to their respective fields of study.

For elementary school teachers, mastering geometric thinking is highly essential in achieving the goal of geometric learning. The understanding of geometric thinking characteristics shall better be taught from the lowest to the highest level, yet the teachers must be aware of the students' condition at each level so that the created scaffoldings suit them well. The scaffolding form included appropriate teaching aids in the form of manipulative media and the involvement of other students as peer tutors.

Limitations

There are some limitations to the findings in this systematic literature review. First, the size effect measurement can only be done in quantitative research, while in qualitative research only information is given about the focus of the research. Both restrictions on participants in the inclusion criteria can eliminate some information that affects the results of research trend analysis.

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Authorship Contribution Statement

Trimurtini: Conceptualization, design, analysis, writing. Waluya: Editing/reviewing, supervision. Sukestiyarno: Editing/reviewing, supervision. Kharisudin: Editing/reviewing, supervision.

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