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Analysis of a teaching learning process of the derivative with the use of ICT oriented to engineering students in Chile

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Abstract
This work aims to analyze the responses of a group of engineering students related to problems about tangents in a teaching learning process of derivative in a differential calculus course. The methodological design, oriented to a group of 161 students from two Chilean universities, considers different onto-semiotic configurations in problem-situations about tangents. The methodology implemented integrates information and communication technologies in differentiated activities, favoring the use of languages and progressive approach to the meaning of the derivative. The exploratory-type analysis was carried out applying some tools of the onto-semiotic Approach of mathematical knowledge and instruction. Difficulties were found in the concept of function and the Euclidean conception of the tangent line, which brings with it a weak interpretation of the derivative function and its geometric representation. It is concluded that the implementation of the geometric interpretation through information and communication technologies makes it possible to improve the teaching of the derivative.

Keywords: teaching of the derivative, problems about tangents, information and communication technologies, engineering education

INTRODUCTION

The subject of differential and integrated calculus, oriented to engineering, is known for its high levels of failing and for a teaching based on the transmission of knowledge with a very marked emphasis in the development of algebraic abilities (Zuñiga, 2007). This has caused an interest in researching the processes carried out in the teaching of this subject.

With the aim of establishing teaching standards (Martínez et al., 2013) engineering schools of different countries are currently promoting educational models based on competencies, emphasizing, amongst other aspects, the active role of the student, the use of information technology resources and virtual learning platforms in teaching, and the preparation of teachers in updated teaching methodologies (Alvarado et al., 2018; Esnaola & Ansó, 2019; Hernández et al., 2018; Marqués, 2012; Martín et al., 2017). Information and communication technologies (ICTs) are an essential tool in the incorporation of knowledge both face to face and virtually, as they, amongst other things:

a. facilitate autonomous and collaborative work among students (Ahumada, 2013; López & Hernández, 2016);
b. strengthen metacognitive competencies (Tourón & Santiago, 2015); and
c. promote social interaction and collaborative problem solving (Aguilera et al., 2017; González et al., 2017; Hernández & Tecpan, 2017; Martín & Tourón, 2017; Núñez & Gutiérrez, 2016).

Along these lines, a curriculum is being implemented that is less technical and more and more practical in the context of conceive-design-implement-operate (CDIO) (Gustafsson et al., 2002). However, there exists a difficulty in the implementation of these new curricular processes from the teachers as, on one hand, they seldom manage teaching methodologies (Hitt & Dufour, 2014)

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Contribution to the literature

- The results of this research confirm that the Euclidean conception hinders the construction of the Cartesian and Leibnizian conceptions of the tangent line to a curve.
- The integration of information and communication has allowed the construction of the meaning of the derivative considering the various configurations of the primary objects and has expanded the language in the design of activities.
- The design of the proposed activities and the applets used by the researchers can serve as a model for teachers who are interested in teaching the derivative.

and, on the other hand, the structure of the subject program is extensive and inflexible (Letelier et al., 2005). As a consequence, teachers of calculus carry out classroom interventions that consider mainly algebraic activities, giving priority to symbolic language and not benefitting the movement between languages, and this has caused scarce learning of the fundamental concepts of calculus, as has been shown through diverse research (Hitt, 2005). For this reason, there are many studies that have dealt with these issues, providing theoretical arguments and teaching proposals so that students can be successful in learning calculus (Antonio et al., 2019; Balcaza et al., 2017; García & Flores, 2016; Pino-Fan et al., 2011).

One of the mathematical objectives taught in the calculus subject in engineering courses is the derivative due to its applicability. The derivative is a complex calculus topic that blends many associated meanings: real function, Cartesian plane, slope, equation of a line, secant line, tangent line, real limit of a function, etc. On the other hand, it possesses diverse representations, depending on the situation problems, for example: the graph, as a slope of a tangent line; the analytical, as the limit of the incremental quotient and a global or point aspect depending on the needs of the task to be carried out (Sánchez-Matamoros et al., 2006). Additionally, different properties, procedures and arguments that move between descriptive, geometric, graphic, tabular, and symbolic languages make its comprehension even more complex. The articulation of the components in which this complexity breaks out is present in almost all the emerging theoretical frameworks in the area of mathematical education. This article takes as its theoretical referent the onto-semiotic approach (OSA) of mathematical knowledge and instruction (Godino et al., 2007, 2019). Working with the different meanings of a mathematical object is an aspect proposed in the OSA, which argues for analyzing the complexity of mathematical objects through their pluri-meanings (partial meanings).

In this theoretical approach, we can say that, in competence terms, the consequences of the future engineer not understanding the different meanings of the derivative and its different forms of representation is that it will be difficult to establish logical relationships between the mathematical elements necessary for and during the resolution of a problem or a specific situation (Sánchez-Matamoros, 2004). For example, from understanding the geometric meaning of the derivative, the future engineer is able to understand and reflect on the optimization process. That is, to apply the geometric meaning of the derivative in a cost optimization situation, understanding the result for better decision making.

Habitually, teaching processes present a dogmatic version of the derivative, favoring their algebraic representation and infrequently favoring their graphic or tabular representation. This aspect has generated research from different theoretical approaches about the difficulties in comprehension and the processes that facilitate its learning (Artigue, 1998; Robles et al., 2010). The difficulties are related to situation problems about tangents, maximums and minimums, and variations and velocity (Pino-Fan et al., 2011), in the graphic comprehension of the tangent as a limit in a set of secant lines, and in the notion of the rate of change in a point in a curve (stationary points, points of inflexion, points of maximums and minimums, etc.) (Orton, 1983). Difficulties have also been observed related to the different ways of expressing f', (tabular, algebraic, and graphic) (Font, 2000a, 2000b, 2005) and in using the analytical expression of the derivative (limit of the incremental quotient) or the geometric interpretation (as a slope of a tangent line) (Artigue, 1998). In concrete terms, the difficulties in comprehending the derivative are due to the lack of argumentation using diverse languages by the students, shown in situation problems about tangents and, to a lesser extent, in situation problems of maximums and minimums, and variations and velocity (Artigue, 1995, 1998; Asiala et al., 1997; Azcárate, 1990; Badillo et al., 2011; Balcaza et al., 2017; Berry & Nyman, 2003; Borgen & Manu, 2002; Flores, 2014; Font, 2008, 2009; Gutiérrez et al., 2017; Habre & Abboud, 2006; Hitt, 2005; Inglada & Font, 2003; Londoño et al., 2013; Orton, 1983; Pino-Fan et al., 2015; Sánchez-Matamoros, 2004; Sánchez-Matamoros et al., 2008; Zandieth, 2000).

In terms of the incorporation of ICTs in the learning of complex mathematical objects, along with its applications and interpretations, the results indicate that the use of these technological tools offers students an appropriation of the concepts, allowing for a verification of that done in algebraic form, an appropriate interpretation of the results, and the capability to apply what was learned (Pico et al., 2017; Pineda et al., 2020; Salas-Rueda & Lugo-García, 2019). Lagrange et al. (2001) present the results of a meta-analysis of more than 600 publications with research reports and experiences of innovation in the use of ICTs in mathematical education. The calculus line is that which has received greatest interest and investment in the use of technologies and, in particular, the derivative mathematical object as it allows students to approach the concept considering different representations (Tall, 2001).

In agreement with López (2008), we argue that the use of new technologies will allow us to approach the concept of the derivative through several configurations such as the manipulative, the computational, and the algebraic, until we arrive progressively at the conceptualization of an abstract definition, going through activities that permit the exploration and proving of conjectures with the use of these concepts.

With the support of some tools from OSA, the objective of this work is to carry out an exploratory analysis of the resolutions of engineering students in the field of tangent problems, done in the context of a proposal of teaching and learning of the derivative that considers manipulative, computational, and algebraic configurations. This is considered in the definition of concepts, propositions, procedures, and arguments in a movement between geometric, descriptive, graphic, and symbolic languages through the integration of ICTs.

In particular, in this study, the proposal for teaching the derivative is centered on the student and integrates ICTs into the design of the intended activities in order to favor a gradual approach to the derivative. More specifically, this study takes into consideration diverse representations of the derivative through the field of the problem about tangents (geometric interpretation of the derivative), protecting the movement between different languages: written, numeric, graphic, and symbolic, through the integration of ICTs.

THEORETICAL FRAMEWORK

Onto-Semiotic Focus of Mathematical Instruction and Knowledge

The theoretical developments proposed by OSA, recently explained in Godino et al. (2019), aim to respond to some problems generated in the field of mathematical education. In OSA, it is assumed that mathematical activity is a human activity centered on problem resolution, in a determined time-space, through a sequence of practices that often consider processes (of signification, conjecture, argument, etc.). Thus, OSA proposes the notion of situation-problem in mathematical practice (sequence of practices) that takes place during the resolution of these problem situations.

These sequences take place in time and tend to be considered, in many cases, as processes. In particular, the use and/or emergence of the primary objects of the configuration (problems, definitions, propositions, procedures, and arguments) takes place through their respective mathematical processes of communication, problematization, definition, enunciation, elaboration of procedures (creation of algorithms and routines), and argumentation (applying the process-product duality). On the other hand, the aforementioned dualities give way to the following processes: institutionalization–personalization, generalization–particularization, analysis or decomposition–synthesis or reification, materialization or concretion–idealization or abstraction, expression/representation–signification.

OSA also assumes the principle that the knowledge of an object by a subject (be it an individual or an institution) is a set of semiotic functions that this subject can establish as those in which the object intervenes as expression or content.

In addition, the correspondence between an object and the system of practices where the object intervenes is interpreted as the “meaning of this object” (institutional or personal). For example, when a subject carries out and evaluates a sequence of mathematical practices, she or he activates a conglomerate formed by situation-problems, languages, definitions, propositions, procedures, and arguments articulated in what OSA calls a configuration of primary objects (Font et al., 2013). To delimit the meanings of a mathematical object, OSA proposes the tool called “analysis of systems of practice” (personal and institutional) and the onto-semiotic configurations involved with them (Godino, 2014; Godino & Batanero, 1994).

Representativity of the complexity of mathematical objects in OSA: The case of the derivative

Font et al. (2013) explains that the notion of the complexity of a mathematical object and the articulation of the components of that complexity play an essential role. Understanding the complexity, in terms of a plurality of meanings, is a result of the pragmatic vision of the meaning that is assumed in OSA. From a pragmatic point of view, the meaning of a mathematical object is understood as a set of practices in which the object intervenes in a determining fashion (or not). A mathematical object, that has originated as an emergent of the system of practices that allows the resolution of a determined field of problems, becomes over time framed in different programs of research. Each new research program allows us to resolve new types of problems, apply new procedures, relate the object (and thus, define) in a different way, use new representations, etc. In this way, as time passes, new subsets of practices (directions) appear that broaden the meaning of the object.

For the derivative mathematical object, Pino-Fan et al. (2011) characterize its complexity through nine configurations of primary objects:

1. tangent in Greek mathematics;
2. variation through the Middle Ages;
3. algebraic methods to find tangents;
4. cinematic conceptions for the tracing of tangents;
5. intuitive ideas of limits for the calculation of maximums and minimums;
6. infinitesimal methods in the calculation of tangents;
7. calculation of fluxions;
8. calculation of differences; and
9. the derivative as limit.

In Pino-Fan et al. (2013), these nine configurations are used for the reconstruction of the global meaning of the derivative which is used to value the representativity of the intended meaning in the curriculum of the bachelor's degree in Mexico (from the configurations of the primary objects activated in the mathematical practices proposed both in the study plan and the textbooks of that level).

The characterization of the complexity of the derivative done in Pino-Fan et al. (2011) facilitates having elements to design questionnaires that allow us to characterize the comprehension of the students, future teachers, and in-service teachers about the derivative. For example, in Pino-Fan et al. (2015), a questionnaire was designed to determine the comprehension of future teachers of the derivative, in which tasks were included to activate the different partial meanings of the derivative characterized in Pino-Fan et al. (2011).

In accordance with OSA, "problems about tangents" are understood as the practices done by the student to resolve problems in which the slope of the tangent line (geometric meaning of the derivative) has a relevant role in its resolution, which implies conceiving it also as "knowledge and application of norms" that regulate the practice and the primary objects that intervene in it (problems, procedures, propositions, and arguments) (Galindo & Breda, 2020; Godino et al., 2019).

In the same way, we will consider, in this work, three epistemic configurations: manipulative (the student works with manipulative devices without using algebraic notation or calculation), algebraic (characterized by symbolic language and deductive demonstration, as well as resources such as algebra and analysis), and computational (characterized by iconic language, the simulation is incorporated as a procedure and the preferable argument is inductive), adapted from Alvarado et al. (2018), in the context of the derivative.

METHODS

In this section, the context of the study, the instruments for data collection and the analysis of this data are explained.

Context of the Study

Participants

161 students participated in the research, of which 91 belong to the business engineering program of the Faculty of Economy and Business and 70 belong to the civil computer engineering program of the Faculty of Engineering, with ages between 18 and 19 years old, and from two Chilean universities. The study plan of civil computer engineering places the subject of calculus in the first academic semester and the study plan of Business Engineering places the calculus subject in the third academic semester and has the algebra course as a prerequisite.

Bases for a didactic proposal

The intervention plan considered, for the development of the teaching the derivative, the following elements:

a. **The problem fields:** The proposal considers one of the main problems that gives rise to the derivative object, PT: problems of tangents (Pino-Fan et al., 2011).
b. **Epistemic configurations:** The proposal considers three configurations used in teaching to facilitate students' comprehension: manipulative, computational, and algebraic, which are differentiated especially by the tools (procedures and languages) and the set of resources the student has to resolve the problem.

   i. In the manipulative configuration, the student works with paper, ruler, and pen. The language used in this configuration is that which is characteristic of descriptive procedures and analytical geometry.
   ii. In the computational configuration, the student has a notebook, cell phone or tablet, internet, *GeoGebra* (free version), QR code, educational videos, and applets. The language and procedures are of a graphic, geometric, and descriptive type.
   iii. In the algebraic configuration, the student has a notebook, cell phone or tablet, internet, educational software such as *Symbolab* (free version) and *Wolfram Alpha* (free version). The language and procedures are of a symbolic and tabular type.

Table 1. Week 1: Timing and planning of problems about tangents

Session	Didactic action	Objective of the didactic action	Languages	Configurations
1 (Asynchronic)	Visualization video 1, Study theoretical manuscript Task 1	Introduction to the line tangent to a curve. Obtaining the slope of the tangent line through approximations by the slope of secant lines.	Geometric Graphic Descriptive	Computational
2 (Synchronic)	Task 2	Obtaining the slope of the tangent line through approximations for the slope of secant lines. Physical application of slope of a line tangent to a curve.	Tabular Geometric Graphic Descriptive	Manipulative algebraic
3 (Synchronic)	Task 3	Geometric interpretation of the derivative in a particular point	Symbolic Graphic	Computational algebraic
4 (Synchronic)	Task 4	Determination of tangent and normal lines.	Symbolic	Algebraic

Table 2. Week 2: Temporalization and planning of the problems about tangents

Session	Didactic action	Objective of the didactic action	Languages	Configurations
5 (Asynchronic)	Visualization video 2, Study theoretical manuscript	Introduction to the derivative function	Graphic	Computational
6 (Synchronic)	Task 5	Articulation of the derivative of a function in a point and its derivative function	Tabular Graphic Symbolic Descriptive	Computational Algebraic
7 (Synchronic)	Task 6	Generalization of the derivative of a function in a point and its derivative function	Graphic Symbolic Descriptive	Computational
8 (Synchronic)	Task 7	Applications of the derivative function	Tabular Graphic Symbolic	Computational Algebraic

c. **Synchronic and asynchronic work:** The didactic trajectory considers synchronic and asynchronic sessions directed by the teachers. The synchronic sessions are carried out in the established timetable and require interaction from the student. However, the asynchronic sessions do not have an established timetable and consider the student's autonomous work.

Development of the teaching

The program of the calculus subject is developed in 18 weeks, and each week has four chronological hours of theory classes called *lectures* and two chronological hours of classes of cooperative work and exercises called *tutorials*.

For the development of the teaching in the field of problems of tangent, two weeks were considered, each of them with three synchronic and one asynchronic session. In the synchronous sessions, both professors were present in the classrooms of the two universities, in the schedules established for the lectures and tutorials. These activities led by the teachers allowed the activation of the knowledge acquired by the students during the asynchronous sessions and collaborative group activities that favored dialogue, feedback and consolidation of the knowledge acquired by the students of both universities. The asynchronous sessions included activities such as watching educational videos, reading theoretical and practical notes and carrying out guided activities, available on the Moodle virtual platform of both institutions. This material was prepared by the teachers, in order to guide the students' autonomous work.

The planning of the study of the derivative included problems about tangents (PT). In this stage we considered six synchronic and two asynchronic sessions, over a period of two weeks (**Table 1** and **Table 2**).

To begin the study of the line tangent to a curve, it was proposed in session 1 that students examine an educational video and a theoretical manuscript and then do task 1, which allows them to construct the concept of the line tangent to a curve through its genetic decomposition (Orts et al., 2016). The problems of task 1 consider broadening the Euclidian conception to the Cartesian through the construction of the tangent line as a limit of the secant lines using an applet of *GeoGebra*.

In session 2, task 2 was carried out. This considered problems such as, for example, the construction of a tangent line as a limit of secant lines using manipulative configurations and geometric language; physical

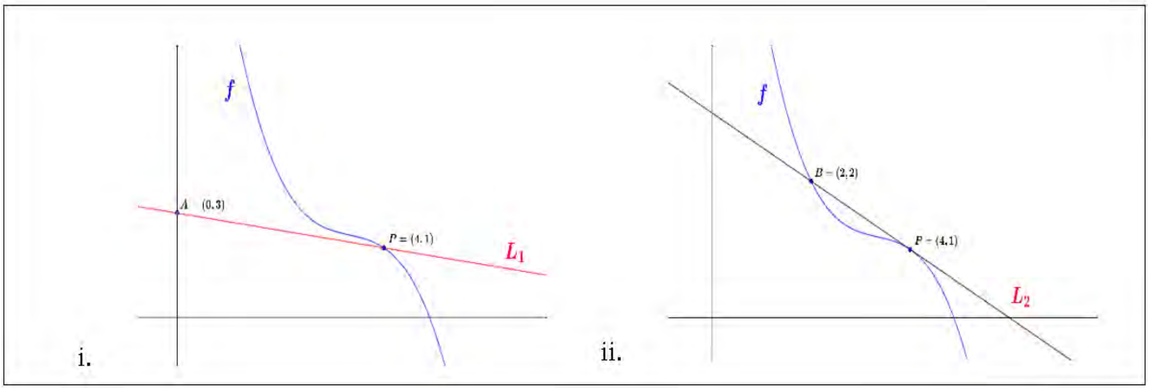


Figure 1. Task applied to students at the end of the first week (Task 1: Determine which of the following lines allows you to calculate $f'(4)$ and obtain its value. Justify your answer)

applications relating the concept of slope of the secant line with that of mean velocity; and the concept of instant velocity as the approximation of the calculated mean velocities, highlighting their equivalence with the slope of the tangent line. The purpose of this task is the thematization of the diagram of the tangent line through the conscious use of its properties and subsequent de-capsulation that allows the return from the object to the process that generated it (Orts et al., 2016).

In session 3 problems were considered in which the slope of the line tangent to a curve was related with the derivative of the function as a point of tangency. The purpose of the task is to know, as a process, the identification of the tendency of the slope of the secant lines with the derivative of the function in the point of tangency (Orts et al., 2016). Keeping infinitesimal methods in mind, the calculation leads students to the highest levels of generality of the tangent (Santi, 2011). Session 4 considered the problems of equations of tangent and normal lines, consolidating the concept of the derivative at a point as limit of the slopes of the secant lines. The purpose of this activity is the thematization of the diagram of the line.

In session 5 the student analyzed the second video and the theoretical notes which considered different types of function, singular points in which the existence of tangent lines is evaluated, and the concept of the derivative function is introduced. The aim of session 6 was to move from the derivative of a function in a point to the derivative function. If we consider that the students have practiced the calculation of the slope of a line and the geometric meaning of the derivative in a point in previous sessions, we can thus suppose that they can obtain the symbolic expression $f'(x)$ without much difficulty (Font et al., 2007). For this, tasks are considered in which the symbolic expression of the function $f(x)$ is known and the function that all the slopes of the tangent lines fulfil is constructed, and this corresponds to the symbolic function of $f'(x)$ (Font, 2005).

Among the exercises developed, an adaptation of the problem proposed by Font (2009) is considered and the movement between graphic, tabular and analytical expressions of f' is encouraged. Session 7 allowed students to generalize, through the *GeoGebra* application, the derivative function as a function which, at each value, corresponds to the slope of the tangent line of the graph of f in the point $(x, f(x))$. It is expected that the student will interpret the derivative function $f'(x)$ as a function whose images, $y_0 = f'(x_0)$, correspond to the slopes of the tangent lines of the function f in x_0, thus causing the decapsulation of the object of the tangent line. Session 8 considered problems of equation in tangent and normal lines using the derivative function. The purpose of this activity is the thematization of the diagram of the tangent line and the consolidation of the geometric representation of the derivative.

Data Collection Instruments

For the exploratory analysis of the students' learning in the field of problems about tangents, an instrument was applied at the end of each week. To continue, for reasons of space, two representative tasks of each instrument we applied are presented. The instrument of week 1 provides us with information about the student's specialized knowledge of tangents, as it considers in its development the steps from its application, or implicit use, to a conscious use of the properties of the tangent line. In other words, the student carries out the thematization of the diagram of the tangent line and concludes the geometric meaning of the derivative in a point to answer the question (**Figure 1**).

In this task, the student must first identify the line that allows him or her to determine $f'(4)$, and then must adequately justify that the line L_2 allows him or her to calculate $f'(4)$ through its slope m_{L_2}, and for this, he or she must consider, in the argument, the geometric interpretation of the derivative, mentioning that the line L_2 corresponds to the tangent line of the curve in P utilising the cartesian concept (limit of secant lines) or the Leibnizian concept (better linear approximation to a curve) for that. To calculate m_{L_2}, the student must know, as action, the slope of a line from two points contained in it, and for this must use the points $P = (4, 1)$ and $B =$

Task 2. Let $f(x) = x^2$ be a real function of a real variable.

a) With the support of *Symbolab*, obtain the slope of the tangent line of the graph of f in the point of abscissa $x_0 = 1$, i.e., $m_T = \lim_{x \to x_0} \frac{f(x)-f(x_0)}{x-x_0}$

b) What is the value of the derivative of the function f in the point of abscissa $x_0 = 1$?

c) Complete the following table.

x_0	-5	-4	-3	-2	-1	0	1	2	3	4	5
$f'(x_0)$											

d) Based on the previous table, represent the points $(x_0, f'(x_0))$ in a cartesian plane.

e) Trace the graph of the curve that contains the points $(x_0, f'(x_0))$.

f) Model the curve previously drawn for any point and explain in your own words what it represents.

Figure 2. Task applied to students at the end of the second week

$(2,2)$ contained in the line L_2, and calculate, $m_{L_2} = \frac{1-2}{4-2} = -\frac{1}{2}$. Then student must obtain the value of the derivative requested by thematizing the diagram of the line tangent to the curve to conclude that $f'(4) = m_{L_2} = -\frac{1}{2}$.

The instrument of week 2 corresponds to an adaptation of the problem proposed by Font (2009, p. 17-18). This allows us to analyze the movement of the student through the graphic, tabular and symbolic representations of f' (**Figure 2**).

It is expected that, in this task, the student will apply the concepts and results of the geometric interpretation of the derivative. In section a) the student, supported by mathematical software, obtains the slope of the line tangent to the graph of the quadratic function in a point $(1,1)$, i.e., $m_T = 2$. Following this, in section b) the student must link the previous result with the concept of the derivative in that point, i.e., $f'(1) = 2$. In section c) the procedure from sections a) and b) is repeated with other points and then the data is summarized in a table. In section d) and e) the information tabulated is used to trace a graph that contains the points $(x_0, f'(x_0))$. Finally, section f) models the traced curve, concludes that it is the function $f'(x) = 2x$ and explains that the curve represents the graph of a function that, at each value, makes the slope of the tangent line correspond to the graph of f in the point $(x, f(x))$.

Analysis Methods

Mixed-type analysis (Johnson & Onwuegbuzie, 2004), by means of the notion of configurations of primary objects of OSA, permitted an exploratory-type analysis, in the sense of quantifying the correct actions (frequency) related to the students' resolutions of the tasks proposed for each of the primary objects. From a qualitative point of view, it was possible to identify correct and incorrect responses and arguments of the students on resolving the proposed tasks.

RESULTS AND DISCUSSION

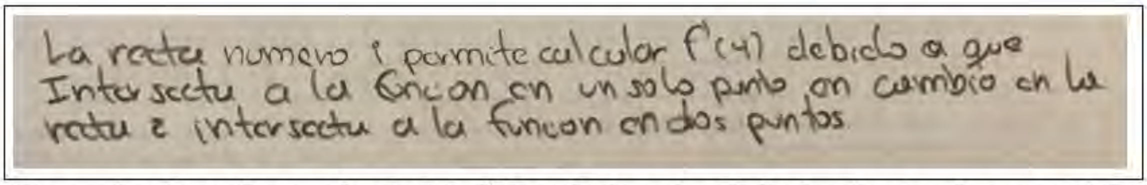
In the following the results obtained by the 91 students of business engineering and the 70 students of civil computer engineering in items 1 and 2 are presented. In addition, an exploratory analysis was done, showing with images the procedures and arguments of the responses of some students.

Table 3 presents the results of task 1 of students of business engineering and civil computer engineering. It is observed that 59% and 46%, respectively, identify that the line L_2 allows them to obtain $f'(4)$, but only 41% and 31% adequately justify that the line is tangent to the graph f in the point P, i.e., carrying out the correct process of thematization of the tangent line in the geometric interpretation of the derivative. We must observe that 68% and 69% identify that the points offered allow them to calculate the slope of the lines and do this calculation correctly, determining the slope m_T, but only 33% and 23% give a symbolic or descriptive answer that concludes the activity. These results show that there exists a relationship between the learning of the tangent of a curve and the learning of the geometric interpretation of derivative.

Table 3. Frequency of correct answers in task 1 (n1=91, n2=70)

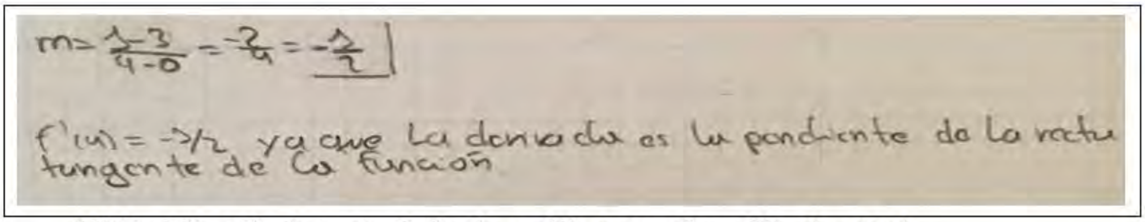
Actions for the problems about tangents	PO	n1-AF	n1 (%)-RF	n2-AF	n2 (%)-RF
Identifies that line L_2 is line allowing them to compute $f'(4)$	Representation	54	59	32	46
Justifies that the line L_2 is a tangent line to the curve	Argument	37	41	22	31
Correctly identifies the points belonging to the line that permit the calculation of the slope	Representation	62	68	48	69
Correctly uses the formula for slope	Procedure	55	60	48	69
Correctly relates the slope of the tangent line to the curve with the derivative of the function in the point of tangency	Definition	30	33	16	23

Note. PO: Primary objects; AF: Absolute frequency; RF: Relative frequency


La recta numero 1 permite calcular f'(4) debido a que Intersecta a la funcion en un solo punto en cambio en la recta 2 intersecta a la funcion en dos puntos

[The number 1 line allows us to calculate $f'(4)$ as it intersects with the function in only one point. On the other hand, line 2 intersects with the function in two points].

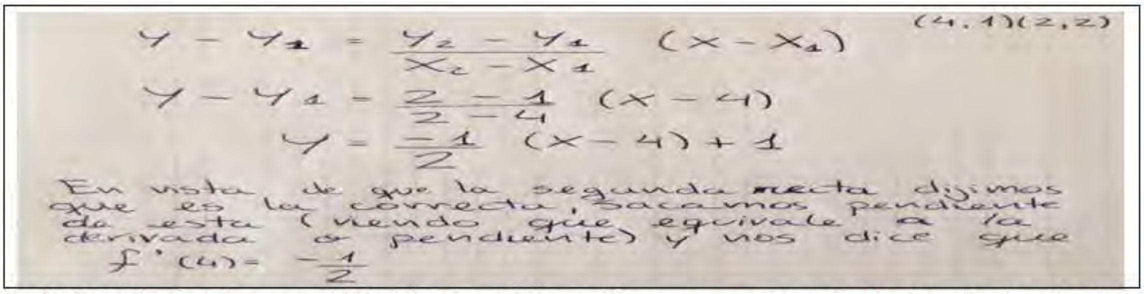
Figure 3. Argument after incorrectly identifying the line tangent to a curve


$m = \frac{1-3}{4-0} = -\frac{2}{4} = -\frac{1}{2}$

f'(4) = -1/2 ya que la derivada es la pendiente de la recta tangente de la funcion

[$f'(4)$=-1/2 as the derivative is the slope of the tangent line of the function].

Figure 4. Identification of the derivative of the function in the point of tangency with the slope of the tangent line


(4,1)(2,2)

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - y_1 = \frac{2-1}{2-4}(x-4)$

$y = \frac{-1}{2}(x-4) + 1$

En vista de que la segunda recta dijimos que es la correcta, sacamos pendiente de esta (viendo que equivale a la derivada o pendiente) y nos dice que $f'(4) = -\frac{1}{2}$

[In light of the fact that we said that the second line is the correct one, we take its slope (seeing that it is equivalent to the derivative or slope) and this tells us $f'(4)$ = -1/2].

Figure 5. Correct process of the derivative of the function in the point of tangency

In **Figure 3**, the justification used by a student to wrongly identify line L_1 as the tangent of the curve f, where we can show the influence of the Euclidean conception of the tangent line (it only has one point in common with the curve). This result does not differ from prior research that reveals the obstacle to construct the Cartesian conception and later Leibnizian product of the Euclidean conception (Biza & Zachariades, 2010; Orts et al., 2016; Santi, 2011).

In **Figure 4**, it is observed that while the student does not correctly identify the line tangent to the curve, he or she does know, as a process, the identification of the derivative of the function in the point of tangency with the slope of the tangent line.

In **Figure 5**, it is observed that a student determines the equation of the line that contains the points and then concludes the relationship between the slope of this and the derivative in the point of abscissa.

Finally, errors of an arithmetic type (procedural) are observed in the calculation of the slope of the tangent line as are errors of a conceptual type in the moment of defining the slope of the line that contains two points, which impedes the student from obtaining the expected result.

Table 4 presents the results of item 1 answered by students of business engineering and civil computer engineering respectively. It is observed that 80% and 92% of the students carried out stage 1 of the problem correctly, i.e., they determined algebraically the slope of the line tangent to the graph of the quadratic function in the point $(1, 1)$. 78% and 90% of the students correctly link the result obtained in stage 1 with the geometric interpretation of the derivative in said point. 78% and 90% trace the points $(x_0, f'(x_0))$ in the Cartesian plane. 60% and 69% correctly model the derivative function. Finally, only 30% and 24% correctly explain that the curve represents the graph of a function that, at each value, makes the slope of the tangent line correspond to the graph of f in the point $(x, f(x))$.

It is important to note that a group of students affirmed that the derivative function was $f'(x) = 2$, which shows the problem existing in the comprehension of the functions in concordance with the results obtained by Cuesta et al. (2010), Gómez et al. (2015), and Mercado et al. (2010).

Table 4. Frequency of correct answers of task 2 (n1=91, n2=70)

Actions for the problems that allow movement from the graphic representation of the derivative in one point towards the derivative function	PO	n1-AF	n1 (%)-RF	n2-AF	n2 (%)-RF
Correctly uses the definition of the slope of the tangent line as the limit of the slopes of the secant lines to complete the table	Procedure	73	80	64	92
Correctly relates the slope of the line tangent to the curve with the derivative of the function in the point of tangency	Definition	71	78	63	90
Traces the curve that contains the points $(x_0, f'(x_0))$ in the Cartesian plane	Procedure	71	78	63	90
Models the traced function	Procedure	55	60	48	69
Explains that the curve represents the graph of the function that, at each value, makes the slope of the tangent line correspond to the graph of f in the point $(x, f(x))$	Argument	27	30	17	24

Note. PO: Primary objects; AF: Absolute frequency; RF: Relative frequency

In the following, the conclusions about the teaching proposal are presented, considering the main results and implications, which will allow for a reflection around the need to readjust, modify or reorganize the teaching proposal.

CONCLUSIONS

The objective of this work was to carry out an exploratory analysis of engineering students' resolutions in relation to a field of problems about tangents done in the context of a proposal of teaching and learning of the derivative that contemplated manipulative, computational and algebraic configurations, considering in the definitions, propositions, procedures, and arguments a movement between geometric, descriptive, graphic, and symbolic language through the integration of ICTs.

In terms of the field of problems about tangents, in a relatively large group of students, it was obtained that the Euclidean conception of the line tangent to a curve makes the construction of the geometric interpretation of the derivative difficult. 41% and 31% of the students of business engineering and civil computer engineering respectively, identified the line tangent to a curve arguing correctly, and only 33% and 23%, respectively associated the slope of the line tangent to a curve with the derivative of the function in the point of tangency (**Table 3**).

One possible explanation for this result is that the Euclidean conception is an obstacle for the construction of the Cartesian and Leibnizian conceptions of the line tangent to a curve, that are essential to understanding the relationship that exists between the slope of the line tangent to a curve and the derivative of the function in the point of tangency. This relationship is fundamental for the construction of the geometric interpretation of the derivative. This result is no different to research about the line tangent to a curve of Biza and Zachariades (2010), Orts et al. (2016), and Santi (2011).

One of the limitations of the study is that, although even though in the teaching initial activities focused on the construction of the meaning of the line tangent to a curve were considered, these were not sufficient. In this way, in a future version of the teaching process it is considered necessary to add activities focused on overcoming the obstacle produced by the Euclidean conception of the tangent line of a curve. In this sense, in a redesign of the teaching process, it is considered necessary to add activities focused on overcoming the obstacle produced by the Euclidean conception of the tangent line to a curve.

On the other hand, in accordance with Cuesta et al. (2010), Gómez et al. (2015), and Mercado et al. (2010), it became clear that there was a consequent influence of the misinterpretation of mathematical concepts that depend, to a great extent, on the concept of function, as only 30% and 24% of the students of business engineering and civil computer engineering, respectively, modelled and explained the derivative function (**Table 4**). This result explains a second limitation of the study. Specifically, for a redesign of the instructional process, it is necessary to dedicate one or a few sessions aimed at recovering the previous knowledge of the students in relation to the basic concepts of the function object.

Finally, while the integration of ICTs has allowed for the construction of the meaning of the derivative, considering the different configurations of the primary objects, and has broadened the language of the design of the activities, it is observed that they have implied a cognitive cost for the student in that the students must assimilate distinct forms of communication in a limited time of their academic load of the semester. The greatest limitation was the time available that, while it may have been more than enough for some students, it was insufficient for all of them to take onboard all of the programmed objectives. This result contributes to a third limitation of the study, because, although the time was long for some of the students, it was insufficient for others to assimilate all the programmed objectives. In this sense, for a redesign of the teaching process, it is considered ideal to rethink the workload destined to the development of the activities proposed to the students.

Although the study has some limitations, an important contribution of this work is that the activities designed and implemented and the applets used are useful material both for researchers of this mathematical object and for teachers who are interested in teaching the derivative. One of the reasons is that the student's understanding in relation to the derivative from its geometric meaning (slope of the tangent line) and with the use of ICT, is a fundamental aspect for the development of the skills of future engineers to solve problems on tangents, be they problems within the mathematical field or problems related to situations experienced in the engineering context.

Considering that all teaching processes are perfectible, a future line of research is to value the didactic suitability (Breda et al., 2018; Moreno, 2017) of the instruction process to identify its limitations and strengths in detail in order to suggest modifications, reorganizations, and improvements in the activities and evaluation instruments. Following this, it is of interest to add other practice systems to thus contribute to a better comprehension of the derivative in the context of higher education.

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