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# Investigating the Promise of a Tier 2 Sixth-Grade Fractions Intervention

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## Abstract

This pilot study examined the promise of a Tier 2 Grade 6 intervention program for students at risk for difficulties in mathematics. The study utilized a quasi-experimental design. The final sample included 112 students in treatment (Promoting Algebra Readiness) and 86 students in control (standard district practice) conditions. The Promoting Algebra Readiness program consisted of 93 lessons across four strands focused on key concepts and applications of fractions. Measures of mathematics achievement were collected at pretest and posttest. Feasibility and usability data indicated favorable impressions by users and strong levels of implementation fidelity. Gain scores of treatment students were significantly greater than those of control peers on two of four proximal measures of mathematics achievement. Positive nonsignificant effects were found on additional proximal and distal measures. Implications for educators delivering instruction for at-risk students in multitier service delivery models are discussed.

## Keywords

fractions, intervention, mathematics, learning disabilities

Work with rational number systems, and specifically fractions, forms the cornerstone of mathematics instruction in the middle grades for all learners (Common Core State Standards Initiative [CCSS], 2010; National Council of Teachers of Mathematics [NCTM], 2006; National Mathematics Advisory Panel [NMAP], 2008) including those at risk for or with mathematics learning disabilities (MLDs; Gersten et al., 2009). Increasingly standards have extended work with fraction concepts to the early elementary grades by leveraging students' initial interest in sharing and proportionality (Siegler et al., 2010) through understanding fractions as numbers, equivalence of fractions, and conceptual understanding of and procedural fluency with fraction operations (CCSS, 2010). This initial conceptual understanding and later work with fractions is associated with future performance in mathematics (Siegler et al., 2012), and students who fail to gain a firm understanding of fractions are at risk for later failure and potential access to more complex mathematics (Jordan et al., 2013). As such, a central argument for the importance of developing and building fraction understanding is the link to algebra with fluency with fractions serving as one of the three foundational areas for algebra success (NMAP, 2008). Fluency of fractions was noted as the most critical of the three foundational areas and that an increased focus on the teaching of fractions "must be acknowledged as critically important and improved before an increase in student achievement in Algebra can be expected" (NMAP, 2008, p. 18).

Given the increased emphasis on fraction understanding and the foundational role fractions play in facilitating understanding of more advanced mathematical content, current national achievement levels are concerning. National data show generally low levels of overall achievement. Recent results from the National Assessment of Educational Progress (NAEP, 2019) indicate that only 41% of Grade 4 students were performing at a level classified as at or above proficient. Results were even more disconcerting for students from low socioeconomic backgrounds (26%), minorities (20%–28%), English language learners (16%), and students with disabilities (17%). For students

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with disabilities, half were classified at or below basic. Perhaps more striking than overall results are individual item responses which indicate that students lack a basic conceptual understanding of fractions and are unable to solve simple fraction problems. For example, students were asked to solve a set of problems requiring them to compare the magnitudes of a given fraction (1/3, 2/3, 2/6, 4/6, 2/8, 4/8) to a benchmark fraction (1/2) and indicate whether the given fraction was greater than, less than, or equal to the benchmark fraction. Only 32% of students correctly answered all six comparisons, whereas 47% correctly compared three or fewer items (NAEP, 2019).

Akin to the focus on early literacy to improve long-term reading outcomes (National Reading Panel, 2001), there has been a strong focus on the role of early numeracy and whole number understanding (Frye et al., 2013) as foundational to later mathematics performance. The result has been the validation of numerous assessments to determine risk and monitor growth in whole number understanding (Fuchs et al., 2010; Gersten et al., 2012; Methe et al., 2011) and corresponding focus on developing and validating whole number interventions (Clarke et al., 2016; Dyson et al., 2013; Fuchs et al., 2005; Gersten et al., 2015). Collectively these efforts provide the basis for schools to implement research-based approaches to multitier systems of support (MTSS) in early mathematics (Witzel & Clarke, 2015). However, despite the critical role of fractions in supporting later mathematics including algebra, the depth of work in this area falls woefully short of levels needed to enable systematic efforts to build and implement comprehensive response to intervention (RTI) or MTSS models of service delivery in upper elementary and middle school mathematics (Gersten et al., 2009).

Research reported in this manuscript seeks to address this void. We report findings from a pilot study investigating the feasibility and promise of an intervention focused on improving students' understanding of fraction concepts and fluency with fraction procedures. We illustrate the dearth of interventions focused on supporting students' development of fraction concepts, identify potential themes that emerge from the existing intervention research, and describe the evidentiary basis on which the intervention—Promoting Algebra Readiness (PAR)—was designed.

## *What We Know About Teaching Fractions to Students With MLD*

Recent systematic reviews of the literature on fraction interventions for middle school students emphasize the relative dearth of research in the area. In the past 5 years, several syntheses have been conducted to examine outcomes of fraction interventions for students with MLD (Hwang et al., 2019; Roesslein & Codding, 2019; Shin & Bryant, 2015). Across these studies, Hwang et al. (2019) identified 13 studies that focused on fraction interventions for middle grade students (Grades 5–8). Shin and Bryant (2015) identified 11, and Roesslein and Codding (2019) included five that focused on Grades 5 and 6 only. When combined, there appears to be a large and growing body of evidence on the outcomes of fraction interventions for students in middle grades; however, considerable overlap in investigators and studies exists across these syntheses, thereby narrowing the range of research from which generalizations can be made. Pooling the literature specifically on mathematics interventions for middle grade students, Powell and colleagues (under review) identified five out of 51 studies focused on fractions. Most of these interventions emphasized problem-solving or fluency and computation, as opposed to conceptual understanding and advanced application. These findings point to a critical need to design fraction-focused interventions and evaluate their effectiveness for improving outcomes for middle grade students with MLD.

Although conclusions from research on the effectiveness of fraction interventions should be interpreted with caution, several themes emerge. First, interventions that use explicit and systematic instruction show improvements in the fraction outcomes for students with MLD. Second, instruction that uses visual and concrete representations is particularly beneficial for students with MLD, and the number line representation may be a particularly powerful representation that can be used to build, develop, and extend students understanding of fractions. Third, it is critical that content builds a robust understanding of fractions beyond equal parts. The PAR intervention was designed to incorporate elements of these evidence-based principles to support positive outcomes for middle grade students with MLD.

*Explicit and systematic instruction to support fraction understanding.* Shin and Bryant (2015) used previous research on evidence-based mathematics interventions (cf. Gersten et al., 2009) to examine 17 studies focused specifically on fractions across elementary, middle, and high school grades. Findings from these studies provide evidence that incorporating explicit and systematic instructional practices, using multiple representations, using heuristic strategies, and providing students with experience solving real-world problems significantly improved students with MLD fractions knowledge. Not all fraction outcomes (e.g., procedural fluency, problem-solving) were equally enhanced by these intervention approaches.

Additional research syntheses have extended these initial findings. In elementary contexts (Grades K–6), Roesslein and Codding (2019) reviewed 12 articles to better understand the intervention components and outcomes on students' fraction understanding. Of the 12 studies reviewed, all incorporated multicomponent interventions that included a range and sequence of examples, and most included explicit and systematic instruction, multiple representations, and student verbalizations. About half used contextual problems and strategy instruction. When examining the outcomes on students' fraction understanding, large effect sizes were observed for word problem-solving and computation, medium effect sizes for students understanding of equivalence and magnitude, and negligible effect sizes for distal measures. Variability was observed across studies when examining generalized fraction outcomes. Similarly, Hwang et al. (2019) examined 22 studies and noted that interventions with multiple representations produced positive outcomes for students with MLD in conceptual knowledge, procedural fluency, and contextually based problems.

*Multiple representation benefit students with MLD.* Recent research has provided more evidence on the role of using multiple representations, including concrete and visual representations in teaching fractions to students with MLD. Area models, number lines, and other visual representations can be used to build students' conceptual knowledge and procedural fluency (Siegler et al., 2010). In a small-scale study with Grade 5 students, Flores et al. (2018) integrated area and length models of fractions and abstract notation into students' Tier 2 intervention. The intervention was delivered via small groups and used explicit and systematic instructions—positive growth was observed.

Focusing more specifically on number lines, using number lines associated with measurement activities may be particularly useful to support students' understanding of fractions as quantities that can be ordered and have magnitude (Siegler et al., 2010). The number line serves as a critical link in transferring knowledge about part–whole relationships, the primary way in which students first encounter and think about fractions, to a measurement interpretation of fractions in which students begin to see fractions as distances on a number line (initially from 0; Schumacher et al., 2018). The robustness of the number line as an instructional tool is not only linked to developing an understanding of fraction magnitude and related concepts such as equivalence but also in transferring to distal outcomes including work in fraction operations (Fuchs et al., 2017; Tian & Siegler, 2016). Dyson et al. (2018) found significantly better performance on measures of fraction magnitude, fraction concepts, and to a lesser extent, fraction arithmetic for students with MLD who received an intervention that emphasized number lines to teach fractions when compared with an equivalent control group. The intervention was delivered using explicit and systematic instruction. Similarly, Hamdan and Gunderson (2017) trained students to represent fractions on either a number line or area model in two brief training sessions. They also used a nonnumerical control group. Although both groups improved at representing fractions in their respective conditions, only the group who received training using number lines improved their performance on an untrained fraction magnitude task. This study provides evidence that even brief training on using a number line to represent fractions may improve students' understanding of fraction magnitude.

*Content of fraction interventions should build robust understanding.* Building students' understanding of fractions should extend beyond part–whole relations and fair sharing to emphasizing equivalence and magnitude. This extends an initial additive perspective of fractions to a multiplicative perspective that is needed to understand proportions and ratios. Siegler et al. (2011) found that students' accuracy of fraction magnitude representations was closely related to both their proficiency in fraction arithmetic and their overall mathematics achievement. Moreover, fraction magnitude representations accounted for substantial variance in mathematics achievement scores, beyond fraction arithmetic proficiency. Fuchs and colleagues (2013) contrasted interventions focused on the part–whole interpretation with the measurement interpretation of fractions, which emphasizes fraction equivalence and magnitude. The intervention focusing on the measurement interpretation had significant positive effects on Grade 4 students' ability to conceptual knowledge and procedural fluency.

Despite these advances in the understanding of the field, it is important to interpret with caution and recognize the need for additional research in this area. Findings from the Hwang et al. (2019) meta-analyses caution against drawing simple conclusions from the fraction intervention research. In their article, studies were classified by the initial skill level of the sample, intervention type, and the focus of the dependent variable (or student outcome). Results found interventions were more effective for students with mathematics difficulties; and, within this group, impacts were greater for students with low achievement in contrast to those with MLDs. However, when collapsing across intervention type, results were varied when contrasted the impact of intervention with standard practice with interventions showing greater effect sizes for certain areas (e.g., word problems) but not for others (e.g., procedural skill). The mixed findings and the nascent nature of research on fraction interventions using group designs highlight the need for expanded research in this critical area of mathematics development for students at risk for or with MLD.

The purpose of this research study was to build on and expand current work in the field on teaching fractions to at-risk by investigating the feasibility and promise of a Tier 2 fraction intervention, PAR for use in an RTI or MTSS service delivery model. The PAR intervention was designed to increase student understanding of fraction concepts and fluency with fraction procedures through an explicit and systematic instructional design framework (Archer & Hughes, 2011) and the use of multiple representations including the number line. Three associated research questions were asked related to the purpose of the study:

**Research Question 1 (RQ1):** What were student perceptions of the PAR intervention?
**Research Question 2 (RQ2):** To what extent was the PAR intervention implemented as intended?
**Research Question 3 (RQ3):** What was the impact of the PAR intervention on student mathematics outcomes?

We hypothesized that, given the extensive development and iterative testing of the PAR intervention prior to the pilot study documented here, PAR would be considered usable, feasible, and impactful by teachers and students. In addition, due to the structure of the PAR intervention and professional development, we hypothesized that the PAR intervention would be implemented with fidelity. Finally, we hypothesized that because the PAR intervention integrated multiple critical components including the use of systematic and explicit instruction, multiple representations, and breadth of content, we would have a positive impact on a range of proximal and distal student mathematics outcomes.

## Method

This quasi-experimental study evaluated the effects of the PAR intervention on Grade 6 students' mathematics learning in four school districts in the Pacific Northwest during the 2014–2015 school year. In addition to measuring student mathematics learning in conjunction with delivery of the PAR intervention, we assessed and reported on teacher and student perceptions of the PAR curriculum, and fidelity of implementation of the PAR intervention.

### *Participants*

*Schools.* Middle schools in four school districts in Oregon were recruited to participate in the study. Districts were located throughout the state, in urban and rural areas. Seven middle schools in these four districts agreed to participate. All seven of these schools indicated they provided mathematics interventions to students who were struggling to meet state benchmarks for mathematics. Schools were assigned to the treatment ($n = 4$) or control condition ($n = 3$) based on their willingness to commit to staffing and delivering a daily, 45-min rational numbers intervention. Condition assignment occurred at the school level to reduce the likelihood of treatment diffusion (i.e., to prevent teachers from sharing intervention materials within the same school building). See Table 1 for condition assignment information by district, school, teacher, and student.

*Teachers.* Students in treatment schools were taught by six teachers. One of these teachers taught two treatment classes and another taught one treatment class and co-taught a second treatment class with another treatment teacher. Students in control schools were taught by three control teachers. One of these three control group teachers taught three control classes. In one school, control group students were taught at various times throughout the day by a single teacher (i.e., these students were not taught together in one or more cohesive groups). On average, teachers in the treatment condition had 9 years of teaching experience, including 7.5 years teaching math in the middle grades and 1.5 years providing intervention to students with or at risk for MLD. Teachers in the control condition, on average, had 10.67 years of teaching experience, including 7.67 years teaching math in the middle grades and 5 years teaching interventions to students with or at risk for MLD. All teachers in both conditions had taken graduate-level coursework in mathematics and mathematics teaching methods.

**Table 1.** Student Assignment to Classes, Teachers, Condition, and Schools in Participating Districts.

| | District | | | | | | |
|---|---|---|---|---|---|---|---|
| | A | | B | | C | | D |
| School | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| Condition[a] | 1 | 1 | 1 | 0 | 1 | 0 | 0 |
| Teachers | 1 | 1 | 2 | 1 | 2 | 1 | 1 |
| Classes[b] | 1 | 2 | 2 | 1 | 2 | 3 | NA |
| Students | 24 | 21 | 34 | 19 | 33 | 53 | 14 |

*Note.* NA = not applicable.
[a]Condition variable 1 = treatment, 0 = control. [b]In District D, students were distributed across all the math class periods taught by the teacher.

*Students.* Within each school, Grade 6 students who had scored below the 40th percentile on the state mathematics assessment (the Oregon Assessment of Knowledge and Skills [OAKS]; see section "Measures") at the end of Grade 5 were eligible to participate in the study. The 40th percentile was selected as the cutoff point to ensure students included in the analytic sample were students with or at risk for MLD, who may benefit from supplemental intervention in Tier 2 in preparation for algebra. In total, 112 students attended treatment schools and were assigned to classrooms that utilized the PAR intervention, whereas 86 students attended control schools and comprised the control group. One student in each condition did not provide consent to participate and was excluded from the study. Over the course of the year-long intervention, eight control students and 11 treatment students moved to other classes or schools not participating in the study (see section "Results" for a description of attrition). Over the course of the year, one student moved into each condition, but students did not change conditions at any point during the year. See demographic characteristics for participating students by condition in Table 2.

**Table 2.** Descriptive Statistics for Student and Teacher Characteristics by Study Condition.

| Student characteristics | PAR | Comparison |
|---|---|---|
| Male | 41% | 52% |
| Race | | |
| White | 85% | 92% |
| Black | 5% | 2% |
| Hawaiian/Other Pacific Islander | 1% | 0% |
| American Indian/Alaska Native | 1% | 1% |
| Asian | 3% | 1% |
| Multiracial | 4% | 4% |
| Hispanic ethnicity | 30% | 31% |
| LEP | 8% | 12% |
| SPED | 8% | 27% |
| OAKS score: *M* (*SD*) | 218.9 (6.2) | 218.4 (5.3) |

*Note.* The complete sample included 110 students and six teachers in the PAR condition and 84 students and three teachers in the comparison condition. PAR = Promoting Algebra Readiness; LEP = limited English proficiency; SPED = eligible for special education services; OAKS = Oregon Assessment of Knowledge and Skills Mathematics.

### Intervention

*Treatment.* The PAR intervention was designed to be delivered as a supplementary intervention in a multitiered system of support for students in upper elementary and early middle school. PAR focuses on teaching rational number concepts and skills identified in the Common Core State Standards for Mathematics (CCSS-M, 2010) and comprised 93 lessons across four intervention strands: (a) Strand 1: multiplication and division of whole numbers, (b) Strand 2: fractions as numbers, (c) Strand 3: addition and subtraction of fractions, and (d) Strand 4: multiplication and division of fractions. To develop the intervention, sixth-grade CCSS-M standards emphasizing rational number concepts were identified (i.e., sixth-grade standards in the number system domain of the CCSS-M), and a series of prerequisite skills were articulated for each standard to support the accessibility and appropriateness of the intervention for students with or at risk for MLDs in early middle school. These prerequisite skills were intentionally aligned with earlier grade-level CCSS-M standards and clustered into several categories: (a) *foundational* knowledge and skills that support the sixth-grade CCSS-M standards and provide an entry point for the particular intervention strand, (b) *bridging* knowledge and skills that involve pre-formal mathematical knowledge necessary to bridge foundational with abstract mathematical reasoning, and (c) *target* knowledge and skills that summarize the formal knowledge required to demonstrate understanding of the sixth-grade CCSS-M standards.

Across intervention strands, lessons were designed to increase student understanding of fractions and abstract concepts of rational number to prepare students experiencing difficulties in mathematics for coursework in algebra using an explicit, systematic approach to teach concepts and skills aligned with research evidence regarding best practices for instructing students struggling in mathematics (Gersten et al., 2009). For example, each lesson included an explicit statement of the instructional objective(s) for the lesson, an entry task to anchor learning to prerequisite knowledge and activate student interest, a teacher demonstration of concepts and skills to be taught in the day's lesson, guided practice to scaffold student learning, opportunities for independent practice with built-in differentiation strategies, and a lesson closure activity to summarize instruction and reinforce student learning. In addition, lessons were designed to incorporate varied visual representations (Siegler et al., 2010) and a concrete-representational-abstract (CRA) sequence (Miller & Hudson, 2007) to build students' conceptual understanding. For example, lessons focused on building students' conceptual understanding of rational number concepts introduced area models to represent fractions, which were further used in lessons focused on adding, subtracting, multiplying, and dividing fractions. Lessons began with the use of concrete materials (i.e., manipulatives), moved to visual representations of area models, and ultimately required students to complete algorithms involving fractions without concrete or representational aids. Finally, lessons embedded frequent practice opportunities to build students' procedural fluency and faded scaffolding over time to ensure mastery of foundational, bridging, and target knowledge and skills.

Students in the treatment condition received the PAR intervention in addition to core mathematics instruction, in a separate, contained, pull-out intervention session that corresponded to one class period of the middle school day. Certified teachers who were trained to deliver PAR provided the intervention in these pull-out sessions. The average class size for PAR intervention groups at the time of assignment was 16 students (range = 10–24). Students in two treatment classes ($n$ = 28) completed only the first three strands of the intervention due to scheduling and interruptions to planned intervention time, whereas all other treatment classes completed all four PAR strands. Core mathematics programs delivered in treatment schools were Common Core/Eureka Mathematics, College Preparatory Mathematics (CPM), and teacher created materials.

*Control.* Students in the control condition received core mathematics instruction and business-as-usual mathematics intervention supports. Intervention programs used in the control condition included Core Focus, Engage New York to pre-teach students struggling during core instruction, and teacher-created materials. Core mathematics programs delivered in control schools were CPM, Envisions, and Engage New York. One teacher delivered whole class instruction less than half of the time (21%–40%) and spent a higher percentage of time (41%–60%) in small group instruction. The remaining teachers spent a majority of time in whole class instruction (61%–80% and more than 80% of the time) and less than 20% of time in small group instruction. All three teachers spent less than 20% of core mathematics instructional time delivering individual instruction. Control group teachers reported frequent use of various practices to provide mathematics instruction to all students, including those struggling in math. Teachers used visual demonstrations of mathematics concepts (e.g., graphic organizers, word webs), opportunities for students to verbalize their mathematics thinking, guided practice, representations of mathematical concepts (e.g., number lines, place value models), peer-to-peer interactions, and teacher verbalizations of mathematics thinking (teacher think-alouds) to facilitate student learning on a daily basis. Two of these three teachers also provided students independent written practice daily, while one teacher did that on a weekly basis.

## Measures

Student outcome assessments and surveys were administered, and classroom observations were conducted to answer study research questions and document sample characteristics. In addition to these measures, student demographic data were collected from school partners through district-maintained student databases.

*Student assessments and surveys.* State mathematics assessment data were obtained from participating districts to screen students for eligibility for the intervention and assess baseline equivalence between the treatment and control groups. Several standardized, norm-referenced student assessments were administered to treatment and control students at pretest and posttest to assess the promise of the PAR intervention for improving distal mathematics learning. In addition, PAR mastery tests were administered to treatment and control students at the beginning and end of each strand of the intervention to assess proximal mathematics learning, and a survey of students' perceptions of the intervention was administered at the conclusion of the study.

*Oregon Assessment of Knowledge and Skills.* The OAKS (Oregon Department of Education, n.d.) is a criterion referenced test aligned with grade-level content standards. It is an untimed, computer-based, multiple-choice test that assesses calculation and estimation, measurement, statistics and probability, algebraic relations, and geometry concepts aligned with state standards. A score of 225 is considered passing or meeting state standards. School personnel oversaw administration of the OAKS in Grade 5 (2013–2014) and reported scores to the research team as part of screening in 2014–2015. Students were allowed to take the test up to 3 times during their Grade 5 year: The highest score was reported and used for analysis. OAKS scores were used to screen students into the study: Those who scored below the 40th percentile were eligible to participate.

*easyCBM.* easyCBM (Alonzo et al., 2006) Mathematics is a standardized, individualized, computer-administered assessment for students in kindergarten through Grade 8. In each grade, the measures include 15 items to assess each of three focal points, based on the NCTM Focal Point Standards. Items have also been mapped to CCSS-M domains. For the study, we used the easyCBM CCSS version. Measures are untimed, but the estimated administration time is 18 to 30 min. The internal consistency of the mathematics measures in middle school ranges from .92 to .95 and the concurrent validity correlation between the winter benchmark form and the SAT-10 is .82 (Anderson & Donchik, 2014).

*AIMSweb Mathematics Computation.* AIMSweb Mathematics Computation (M-COMP) is an 8-min, timed measure of student fluency with basic facts, computation, and conversion of whole numbers, fractions, percentages, integers, and exponents for students in Grades 1 to 8. In total, 30 alternate forms of equivalent difficulty are available at each grade level. Items are scored as either incorrect or correct (i.e., there is no partial scoring), and assigned a predetermined point value (1–3 points) based on the difficulty of the item. Fisher's $z$ transformed reliability coefficients for the M-COMP range from .82 to .90, split-half reliability ranges from .85 to .93, and alpha ranges from .82 to .91 (Pearson Education Inc., 2010). Correlations between M-COMP scores and the Group Mathematics Assessment and Diagnostic Evaluation are .76 at Grade 8 and .73 at Grade 3. In the study, students were administered the Grade 5 fall and spring benchmarks to assess student knowledge of prerequisite skills. In addition, we administered the Grade 6 fall and spring benchmark measures to assess grade-level mathematics knowledge.

*Algebra Readiness Progress Monitoring (ARPM).* The ARPM (Ketterlin-Geller et al., 2015) is a series of three measures designed to measure students' ability to manipulate whole numbers, rational numbers, and integers in relation to key algebra readiness knowledge and skills. Number Properties (NP) measures students' skills in recognizing and using number properties appropriately to solve mathematics problems efficiently by reasoning about the magnitude of two expressions. For example, when presented with the item 58 + 1.7 ___ 1.7 + 5.8, students are prompted to use their knowledge of properties of operations to reason about the magnitude of each expression and insert the correct symbol (>, <, =). Quantity Discrimination (QD) measures students' skills in recognizing differences in quantity within and across number systems (e.g., 25% ___ 1/5, where students select the correct symbol to make the comparison true). Proportional Reasoning (PR) measures students' skills making comparisons between entities in multiplicative terms. For example, students compare the magnitude of two proportions such as 9 to 12 compared with 3 to 4, and select the correct symbol (>, <, =) to render a correct statement. Each measure includes between 20 and 25 multi-select items that are dichotomously scored. Measures are group administered using paper-based tests, and students have 3 min to respond to as many items as possible. Scores are reported as total number correct. Internal consistency of the ARPM across measures ranges from .92 to .97, which serves as a measure of reliability. Rasch model fit statistics and item-total correlations meet expectations for use in measuring individual student progress (Ketterlin-Geller et al., 2015).

*PAR mastery tests.* Four researcher-developed proximal measures of student mathematics learning were developed to assess student mastery of CCSS-M content taught in each strand of the curriculum. Strand mastery tests were administered prior to introduction of strand content in the treatment group and again following completion of strand content prior to introduction of the subsequent intervention strand. Mastery tests were designed to assess conceptual understanding and procedural fluency, using a combination of multiple-choice and open-ended problem-solving tasks. Mastery tests were administered to students in the control group within the same assessment window as the measures were administered in the treatment group across schools. Measures are untimed, and scores are reported as raw scores representing the number of items answered correctly. To support feasibility of use in conjunction with the curriculum, mastery tests were intended to take no longer than 15 to 20 min to administer. Items on each of the four PAR mastery tests corresponding to each strand of the intervention were piloted during a previous feasibility study. Outlier items that were too difficult (e.g., no students in the feasibility answered them correctly), too easy (e.g., no students in the feasibility study answered them incorrectly), or that did not demonstrate reasonable variability in responding were thrown out. Internal consistency for the revised PAR mastery tests administered at pretest were .64 (Strand 1), .25 (Strand 2), .47 (Strand 3), and .30 (Strand 4) and administered at posttest were .76 (Strand 1), .40 (Strand 2), .66 (Strand 3), and .52 (Strand 4).

*Student perception survey.* A researcher-developed student perception survey was administered at the end of the study to students in the treatment group to assess their beliefs and preferences about the strategies and design features of the intervention and measure their perceptions of intervention effectiveness and their confidence learning mathematics as a result of participation in the PAR intervention. The survey included 17 items—10 items that asked students about specific features of the program and 7 items that asked about general features of the intervention. For example, students were asked to rate how much the story problem template helped them learn, and how much they liked it (scale = 1–4, where 1 = *low* and 4 = *high*). Other items asked how much knowledge students believed they gained from PAR, and what their confidence was solving and explaining various problems (scale = 1–10, where 1 = *low* and 10 = *high*).

*Ratings of Classroom Management and Supports (RCMIS).* The RCMIS (Doabler & Nelson-Walker, 2009) was used to measure the quality of implementation of PAR according to best practices in mathematics instruction. The RCMIS contains 11 items that are rated on a 4-point Likert-type scale, in three broad categories: (a) delivery of instruction, (b) classroom management, and (c) the learning environment. The RCMIS demonstrates high internal consistency ($\alpha$ = .92), high interobserver agreement (intraclass correlation coefficients [ICCs] = .79–.83), and moderate stability in ratings over time (ICCs = .30–.72). Research also indicates the RCMIS is predictive of student performance in early mathematics (Doabler et al., 2015).

*Fidelity of implementation.* Fidelity of implementation was measured using a researcher-developed rating instrument to assess the level of completion and quality of implementation of program-specific lesson components (i.e., entry task, activate knowledge, teacher demonstration, guided practice, independent practice, lesson closure, and exit ticket) twice per PAR strand, approximately once per month during the study. Level of completion was evaluated on a 3-point scale (1 = *not performed* to 3 = *complete*). Quality of implementation was evaluated on a 4-point scale (1 = *low quality* to 4 = *high quality*). Fidelity observations were conducted by project staff who had participated in the development of the intervention and had strong knowledge of program design features and intervention components. The fidelity instrument is available from the first author upon request.

## Procedures

*Timeline.* In fall 2014, following study recruitment, we obtained consent from teacher and student participants. In October, treatment teachers were trained to deliver the PAR intervention, students in treatment and control groups were administered five measures of mathematics knowledge: (a) easyCBM (Grade 6 fall benchmark), (b) AIMSweb M-COMP Grade 5 fall benchmark, (c) AIMSweb M-COMP Grade 6 fall benchmark, (d) ARPM NP, Quantity Discrimination, and PR, and (e) PAR Strand 1 pretest. Measures administered were counterbalanced to reduce fatigue and potential practice effects. Make-up testing was offered to support assessment of students absent on the original day of testing. Intervention delivery began the first week of November 2014.

At the conclusion of Strand 1, research staff administered the PAR Strand 1 mastery test and the PAR Strand 2 pretest. The same procedure was followed at the end of Strand 2 and at the end of Strand 3 (i.e., the completed strand mastery test and the pretest for the next strand were administered). At the end of Strand 4 (and the end of Strand 3 for the two treatment classrooms that did not complete Strand 4), the Strand 4 mastery test was administered, along with the rest of the pretest assessment battery, including (a) easyCBM (Grade 6 spring benchmark), (b) AIMSweb M-COMP Grade 5 spring benchmark, (c) AIMSweb M-COMP Grade 6 spring benchmark, and (d) ARPM NP, Quantity Discrimination, and PR. As was the case at pretest, measures administered were counterbalanced to reduce fatigue and potential practice effects and make-up testing was offered to support assessment of students absent on the original day of testing.

At posttest, students were also administered a survey of their perceptions of the PAR intervention. Individual student demographic information was obtained from each participating school district's central office.

*Professional development and coaching.* Treatment teachers who delivered the PAR intervention received 6 hr of training at the beginning of the study in fall 2014 to deliver Strands 1 and 2 of the curriculum. Intervention teachers also received 6 hr of training focused on Strands 3 and 4 of the PAR intervention in winter 2015, prior to their delivery. Training was provided by developers of the curriculum who were licensed educators with decades of experience designing and delivering mathematics interventions for students with or at risk for learning disabilities. All initial intervention strand training focused on the design of the curriculum and the strategies employed during delivery. Trainers modeled lessons and strategies for teachers and provided opportunities for teachers to practice implementing lesson features with feedback and support from the curriculum team.

Throughout the study, the same trainers and curriculum developers provided ongoing coaching to treatment participants. Trainers were available to respond to interventionist questions via email and made two visits to each participating treatment class during every intervention strand to support the fidelity and quality of implementation of the PAR intervention. In the control group, all professional development and coaching received was business-as-usual support.

*Training for data collection.* Trained research staff administered all student measures, collected surveys from teachers, observed PAR intervention using the RCMIS, and collected extant student demographic data from school districts. In fall 2014, prior to pretesting, research staff received 1.5 days of training related to observation measures and several additional multi-hour training sessions focused on the administration of student assessments. Observers were required to demonstrate inter-rater reliability (.85 or higher) with the project's lead observation coordinator prior to observing independently in classrooms. Research staff were also required to complete fidelity checklists as part of student assessment administration to ensure standardization was followed. An additional day of refresher data collection training was provided for observations and student assessments in late winter in preparation for data collection in the second half of the year, including spring posttesting.

### Statistical Analysis

We assessed effects of the PAR curriculum on mathematics outcomes using a mixed-model (multilevel) Time × Condition analysis (Murray, 1998). The analysis tested for differences between conditions on change in outcomes from pretest ($T_1$) to posttest ($T_2$), with gains for individual students clustered within schools. The model included effects of time, $T$ (coded 0 at pretest and 1 at posttest), condition, $C$ (coded 0 for control and 1 for *PAR*), and the Time × Condition interaction. The statistical model accounted for the clustering of students within schools, the level of assignment to study condition, with the following composite model:

$$Y_{tjk} = \left(\gamma_{000} + \gamma_{001}C_k + \gamma_{100}T_{tjk} + \gamma_{101}T_{tjk}C_k\right) + \left(u_{00k} + u_{10k}T_{tjk} + r_{0jk} + e_{tjk}\right),$$

$Y_{tjk}$ represents a score for assessment occasion $t$ on student $j$ in school $k$. The model includes three predictors: time, $T_{tjk}$, condition, $C_k$, and their interaction. Given the coding of $C$ and $T$, the model included the pretest intercept for the control condition, $\gamma_{000}$, the difference between conditions at pretest, $\gamma_{001}$, the estimate of gains for the control condition, $\gamma_{100}$, and the difference in gains between conditions, $\gamma_{101}$, the primary estimate of intervention efficacy. The model also includes four error variances: the school-level intercept, $u_{00k}$, the school-level gains, $u_{10k}T_{tjk}$, the student-level intercept, $r_{0jk}$, and the residual, $e_{ij}$. With only two assessments, the variances $r_{1jk}T_{tjk}$ and $\boldsymbol{e}_{tjk}$ are redundant and cannot be simultaneously estimated; so, the model excludes the $r_{1jk}T_{tjk}$ term (Murray, 1998). The student-level intercept, $\boldsymbol{r}_{0jk}$, is also equivalent to the within-student covariation between pretest and posttest assessments.

*Model estimation.* We fit models to our data with SAS PROC MIXED version 9.4 (SAS Institute, 2016) with maximum likelihood (ML) estimation. ML estimation for the Time × Condition analysis uses all available data to reduce the potential for biased results even in the face of substantial attrition provided the missing data were missing at random (Schafer & Graham, 2002). Collins et al. (2001) demonstrated that sophisticated missing-data approaches, such as ML, do not introduce bias; the assumptions of the approach are relatively benign compared with complete case analysis (Allison, 2009; Schafer & Graham, 2002). In the present study, we did not believe that attrition or other missing data represented a meaningful departure from the missing at random assumption, meaning that missing data did not likely depend on unobserved determinants of the outcomes of interest (Little & Rubin, 2002). All students with pretest or posttest mathematics achievement scores were included in the analyses ($n \geq 192$ for primary outcomes and $n \geq 170$ for strand measures). The statistical models assume independent and normally distributed observations. We addressed the first assumption by modeling the multilevel nature of the data. The data in the present study also did not markedly deviate from univariate normality; skewness and kurtosis fell within ±2.0 for all measures.

*Effect sizes and interpretation of results.* Hedges' $g$ effect sizes were calculated at the school level to characterize the magnitude of intervention effect sizes (What Works Clearinghouse, 2020). We designed the pilot study based on a priori power analyses that assumed ICCs from .10 to .20, approximately 3.5 schools per condition, an average of 28 students per school, and pre–post outcome correlations of .50 to .71. With these assumptions, the minimum detectable intervention effects ranged from 0.71 to 1.21 standard deviations. Because this is a relatively underpowered intervention development study, we emphasize Hedges' $g$ effect sizes and their 95% confidence intervals (CI).

## Results

Tables 2 and 3 summarize participant characteristics and student mathematics outcomes by assessment time and condition, respectively. Below, we present results from tests of baseline equivalency and differential attrition across conditions as well as treatment effect estimates, fidelity of implementation, and students' and teachers' perceptions of the PAR curriculum.

### Baseline Equivalency and Attrition

Preliminary analyses revealed baseline equivalency across conditions on student demographic characteristics and outcome measures with two exceptions: The PAR condition included fewer special education eligible students (8% vs. 27%; $\chi^2(1) = 10.59$, $p = .001$, odds ratio = 0.25), and students in the PAR condition had higher mean Strand 1 scores at pretest, 4.1 versus 3.3, $t(191) = 2.41$, $p = .017$, $g = 0.34$. The study conditions were also equivalent with respect to teacher mathematics knowledge ($p = .269$).

Examination of student-level attrition between pretest and posttest revealed 10% ($n = 19$) of the sample did not complete a posttest assessment. Rates of attrition did not significantly differ between PAR and comparison conditions, 7% versus 13%, $\chi^2(1) = 1.83$, $p = .176$. We evaluated the extent to which student attrition threatened the internal validity of this study using a two-way analysis of variance. These analyses examined the effects of condition, attrition status, and their interaction on pretest outcomes. We found no statistically significant interactions between attrition and condition predicting baseline outcomes ($p$s $> .352$), suggesting that the effect of attrition on outcomes did not vary by condition.

### Treatment Effect Estimates for the PAR Curriculum

We tested the hypothesis that students in PAR schools experienced greater gains in mathematics outcomes during Grade 6 than students in comparison schools. Complete results are summarized in Table 4. Consistent with our hypotheses, students in PAR schools significantly outperformed students in comparison schools on the Strand 2 proximal assessment ($g = 0.84$, 95% CI = [0.12, 1.56], $p = .030$) and Strand 3 proximal assessment ($g = 0.68$, 95% CI = [0.14, 1.22], $p = .023$). Nonsignificant positive effects of PAR were observed on all other outcome measures ($g$s $\geq 0.15$), including easyCBM ($g = 0.44$, 95% CI = [−0.14, 1.02]).

### Fidelity of Implementation

Project staff conducted three observations of each classroom in the PAR condition. Observers completed three rating scales during each observation: RCMIS (11 items rated on scale of 1 = *not present* to 4 = *highly present*), level of completion (seven items rated on scale of 1 = *not performed* to 3 = *complete*), and quality of completion (seven items rated on scale of 1 = *low* to 4 = *high*). Total scores for each rating scale were calculated as the mean across item ratings. Inter-observer reliability ICCs for the total scores were greater than .80, indicating substantial agreement between observers. RCMIS scores ranged from 2.1 to 3.7 ($M = 3.0$, $SD = 0.7$), level of completion total scores ranged from 2.1 to 3.0 ($M = 2.5$, $SD = 0.3$), and quality of completion total scores ranged from 1.3 to 3.8 ($M = 2.8$, $SD = 1.0$).

**Table 3.** Descriptive Statistics for Student Math Outcomes by Assessment Time and Condition.

| | $T_1$ | | | | $T_2$ | | | |
|---|---|---|---|---|---|---|---|---|
| | PAR | | Comparison | | PAR | | Comparison | |
| Measure | *M* (*SD*) | *n* | *M* (*SD*) | *N* | *M* (*SD*) | *n* | *M* (*SD*) | *n* |
| ECBM | 25.6 (6.3) | 110 | 24.3 (5.5) | 82 | 28.7 (7.6) | 93 | 23.8 (6.9) | 72 |
| ARPM | 33.3 (13.5) | 110 | 33.6 (12.5) | 84 | 37.3 (15.4) | 102 | 36.2 (14.6) | 73 |
| AW Grade 5 | 7.7 (3.3) | 107 | 8.0 (3.6) | 82 | 7.4 (3.6) | 97 | 7.6 (3.7) | 70 |
| AW Grade 6 | 11.3 (5.7) | 107 | 11.2 (5.0) | 84 | 12.9 (6.1) | 98 | 12.0 (6.7) | 70 |
| Strand 1 | 4.1 (2.5) | 109 | 3.3 (2.1) | 84 | 5.6 (3.1) | 104 | 4.2 (2.4) | 77 |
| Strand 2 | 4.5 (1.6) | 103 | 4.2 (1.8) | 76 | 6.5 (2.3) | 101 | 4.6 (1.6) | 74 |
| Strand 3 | 5.6 (2.3) | 100 | 4.9 (2.0) | 74 | 7.7 (2.5) | 99 | 5.3 (2.1) | 68 |
| Strand 4 | 5.1 (2.0) | 73 | 3.8 (1.6) | 69 | 5.4 (2.3) | 96 | 4.1 (2.0) | 72 |

*Note*. $T_1$ and $T_2$ correspond to pre- and post-intervention for all outcome measures except the strand proximal assessments, which were sequentially delivered across the study period. The sample sizes (*n*) represent students with a particular measure at each assessment period. Strands 1 to 4 = PAR mastery tests. PAR = Promoting Algebra Readiness; ECBM = easyCBM; ARPM = Algebra Readiness Progress Monitoring; AW = AIMSweb Math Computation.

### *Student Perceptions of the PAR Curriculum*

Students in the PAR condition completed a survey of their perceptions of the curriculum. On scales from 1 = *low* to 10 = *high*, students rated how the PAR class compared with their other mathematics class ($M$ = 7.1, $SD$ = 2.4), the amount of knowledge they gained in the PAR class ($M$ = 7.6, $SD$ = 2.1), how strategies learned in the PAR class helped them monitor their learning ($M$ = 7.4, $SD$ = 2.2), and how the PAR class improved their confidence in mathematics ($M$ = 7.9, $SD$ = 2.1). One scales from 1 = *low* to 4 = *high*, students rated how much they liked specific components of the PAR curriculum ($M$ = 2.9, $SD$ = 0.6 across components) as well as the extent to which components helped them learn ($M$ = 3.2, $SD$ = 0.6 across components).

## Discussion

The pilot study of the PAR intervention showed initial evidence of usability, feasibility, and promise. Student perceptions of the intervention, RQ1, showed generally favorable impressions of the program. For RQ2, fidelity of implementation, results were more varied but they generally indicated that teachers delivered the intervention as intended. Finally, for RQ3, promise of impact, positive significant results were found on two of four proximal measures with positive nonsignificant scores on the other two proximal measures and all four of the distal measures. Across research questions, the PAR intervention exhibited promise that would warrant a further investigation in a large scale randomized control trial. While noting general promise, the nascent nature of the pilot study along with specific limitations of the research warrants tempering interpretation of results while providing guidance for future research. Limitations and future directions are discussed below.

Within a small-scale pilot study, interpretation of results should be done with caution as the goal is to provide evidence of promise, and pilot studies are not fully powered to detect effects. Effect sizes reported in syntheses of fraction interventions demonstrate a wide range of impact on student outcomes generally and when reported out by intervention type, instructional components, and student skill (Hwang et al., 2019; Shin & Bryant, 2015). Results from this study demonstrated a similar pattern with slightly lower effect sizes ranging from .15 to .84. When designing the PAR intervention, we not only focused on work with fractions but also ensured background knowledge on key concepts of multiplication and division that lay the groundwork for understanding fractions and then focused on key concepts and applications of fractions across the four operations. The resulting intervention was notable for its breadth and depth of coverage in comparison to the current research base in which interventions are more limited in focus to one or two particular aspects of fraction understanding. Of the 17 studies reviewed by Shin and Bryant (2015), 16 consisted of 30 lessons or fewer with a limited focus on more advanced work including multiplication and division of fractions. However, that choice and relative depth and breadth of the PAR intervention also reflects the complexity of providing mathematics interventions to students who are in upper elementary or middle school and who may have a range of skills and varying needs for teaching or priming relevant background knowledge on whole numbers. The breadth of content covered may in part explain the varying findings on the proximal assessments linked to the strands of the PAR intervention. Namkung and colleagues (2018) found that Grade 4 students with severe deficits in whole numbers skills were 32 times more likely than students with adequate skills to experience difficulty with fractions and were 7 times more likely to experience difficulty than

**Table 4.** Results of a Nested Time × Condition Analyses of $T_1$ to $T_2$ Gains in Math Outcomes.

| Effects | ECBM | ARPM | AW Grade 5 | AW Grade 6 | Strand 1 | Strand 2 | Strand 3 | Strand 4 |
|---|---|---|---|---|---|---|---|---|
| Fixed effects | | | | | | | | |
| Intercept | 23.82*** | 33.22*** | 7.76*** | 10.26** | 3.30** | 4.10*** | 4.92*** | 3.77*** |
| | (2.35) | (3.18) | (0.86) | (1.78) | (0.76) | (0.46) | (0.69) | (0.52) |
| Time | −0.51 | 1.78 | −0.95 | 0.65 | 0.80 | 0.39 | 0.42 | 0.27 |
| | (1.27) | (2.62) | (0.88) | (0.72) | (0.43) | (0.43) | (0.38) | (0.22) |
| Condition | 2.24 | 0.57 | 0.05 | 1.37 | 0.86 | 0.42 | 0.80 | 1.00 |
| | (3.09) | (4.16) | (1.13) | (2.34) | (1.00) | (0.60) | (0.90) | (0.69) |
| Time × Condition | 3.23 | 2.21 | 0.53 | 0.93 | 0.70 | 1.68* | 1.59* | 0.43 |
| | (1.65) | (3.38) | (1.14) | (0.94) | (0.56) | (0.56) | (0.49) | (0.31) |
| Variances | | | | | | | | |
| School intercept | 13.18~ | 17.37 | 0.86 | 8.06~ | 1.33~ | 0.29 | 1.10~ | 0.60 |
| | (9.49) | (15.79) | (1.08) | (5.67) | (0.99) | (0.31) | (0.81) | (0.50) |
| School gains | 1.61 | 4.50 | 0.81 | 0.09 | 0.14 | 0.17 | 0.08 | <0.01 |
| | (1.50) | (6.31) | (0.70) | (0.69) | (0.18) | (0.17) | (0.12) | (NA) |
| Student | 19.52*** | 65.80*** | 5.55*** | 13.27*** | 2.72*** | 1.10*** | 1.82*** | 2.10*** |
| | (2.96) | (14.71) | (0.98) | (2.50) | (0.46) | (0.25) | (0.38) | (0.35) |
| Residual | 13.95*** | 116.80*** | 5.69*** | 15.84*** | 2.82*** | 2.01*** | 2.52*** | 1.70*** |
| | (1.57) | (12.62) | (0.65) | (1.79) | (0.30) | (0.22) | (0.28) | (0.20) |
| Time × Condition effects | | | | | | | | |
| Hedges' *g* | 0.44 | 0.15 | 0.15 | 0.15 | 0.25 | 0.84 | 0.68 | 0.20 |
| 95% CI | [−0.14, 1.02] | [−0.43, 0.72] | [−0.67, 0.96] | [−0.23, 0.53] | [−0.26, 0.76] | [0.12, 1.56] | [0.14, 1.22] | [−0.20, 0.60] |
| *p* values | .1078 | .5424 | .6632 | .3684 | .2679 | .0301 | .0227 | .2386 |

*Note.* Condition coded 0 for control and 1 for PAR. $T_1$ and $T_2$ correspond to pre- and post-intervention for all outcome measures except the strand proximal assessments which were sequentially delivered across the study period. Table entries show parameter estimates with standard errors in parentheses except for Hedges' *g* values and *p* values. Tests of fixed effects accounted for schools as the unit of analysis within condition and used five degrees of freedom. Strands 1 to 4 = PAR mastery tests. ECBM = easyCBM; ARPM = Algebra Readiness Progress Monitoring; AW = AIMSweb Math Computation; CI = confidence interval.
~$p < .10$. *$p < .05$. **$p < .01$. ***$p < .00$.

students with less severe difficulty. Such findings indicate the critical role foundational skills may play and the importance for some but not all students. The lower rates of growth and nonsignificant results on Strand 1 (multiplication and division of whole numbers) in comparison to Strands 2 and 3 suggests that the Grade 6 students in our study may have not needed instruction in these foundational skills. Although critical to understanding of fractions, the content of Strand 1 is expected to be mastered in earlier grades level. Nonsignificant findings on Strand 1 point to the need to more closely attend to student initial skill at intervention onset. Researchers have begun to explore the role of initial skill in mathematics (e.g., Clarke et al., 2019) and fractions specifically (Fuchs et al., 2017). Fuchs and colleagues (2017) found that intervention effects of a Grade 4 fractions intervention were not moderated by initial skill but that only the performance of students with less severe initial skill deficits was normalized compared with a normative group of non-at-risk peers. Results of this nature demonstrate the complexity of considering initial skill, intervention response, and the importance of aligning interventions to student skills. One logical step would be to study the PAR intervention strands in isolation or in some combination more closely aligned to student's skill when entering the intervention and examining student response to the intervention.

Differential response due to initial skill also points directly to the use of adaptive designs in both research and practice (e.g., Coyne et al., 2016). Coyne and colleagues (2016) deployed an intervention model in which student RTI content dictated the pace of lessons within the intervention. Student groups who mastered content were accelerated through the intervention and conversely groups who failed to meet criteria were retaught critical intervention content. Given the extensive scope and sequence within fractions, a similar model seems logical. In this particular case, moving students quickly through Strand 1 foundational content directly into more challenging content may be a better use of limited instructional time, and from a research standpoint provide a better index of the intervention's capacity to more finely address student learning needs. Although beyond the scope of a small-scale pilot study, adaptive or smart research designs (Almirall et al., 2018) are becoming more prominent in intervention research and could offer greater insight into how the PAR intervention functions and interacts with student response.

Corresponding to the focus on specific strands is the need to examine the proximal assessments linked to each strand. Proximal assessments in the study had lower internal consistency than desired. The lower internal consistency scores at pretest and in general may reflect the novel content of the assessments for students, the breadth of the content taught and assessed within each strand, and the reduced number of items included in each strand assessment to support feasibility of administration. For example, Strand 2 of the curriculum focuses on building students' conceptual knowledge of fractions, including fraction identification, representations, and equivalence. The revised Strand 2 assessment includes 11 total assessment items—nine multiple-choice and two open-ended—to capture the range of content taught in Strand 2. Regardless of the reason, continued improvement of the proximal assessments used to document student understanding and outcomes warrants further development and exploration along with general refinement of the measurement net as a whole. It is also critical to note that we had positive but nonsignificant results on the distal measures used in the study. The use of distal measures is relatively unique when examining the impact of fraction interventions. In their review of fraction interventions, Shin and Bryant (2015) found that only 4 of 17 studies included non-research development outcomes measures. While nonsignificant findings on distal measures are somewhat expected in an underpowered pilot study, future research should continue to focus on the capacity of an intervention's effects to generalize across a range of measures to ensure that the intervention helps students develop a robust understanding of fractions that can be applied in a variety of contexts.

The PAR intervention was also delivered in a whole class format rather than in small groups as is typical in the research literature (Hwang et al., 2019) and called for by experts in the field (Gersten et al., 2009). The whole class format is also in contrast to service delivery in elementary school, where mathematics intervention work can be considered as supplementary or in addition to the core, as middle school interventions are often linked to class periods. The study of PAR in middle school dictated our focus on whole class PAR groups because schools in our study had a dedicated mathematics period in which students were grouped together based on skill. Future research should focus on investigating PAR delivered in small groups or other arrangements that capture the range of how interventions are delivered in school settings. In addition, the study utilized a business as usual control condition. A significant limitation of the current study was a lack of documenting the counterfactual including the content covered in the control condition, documenting the frequency and duration of any intervention experiences, and the instructional approaches used within those experiences. Observations focused on key instructional design elements such as opportunities to respond, use of models, and teaching sequences such as CRA would help unpack how PAR was similar to or different from other intervention approaches. Recent findings indicate that standard or typical instruction includes many of the same elements found in fraction interventions and that student outcomes on different domains of fraction performance are similar or stronger with standard instruction (Hwang et al., 2019). More detailed documentation of the control condition is needed to ease out shared and unique instructional elements across intervention and control settings. A stronger design for future research would either more systematically document the business as usual condition to provide greater context for interpreting the effects of PAR versus current practice or employ the use of an active control group to better control for confounding variables such as time.

Future research should also consider how we conceptualize interventions and in particular interventions classified as Tier 2. Largely, this classification is dictated by when an intervention is delivered (i.e., after nonresponse to core instruction) but with interventions targeting more advanced content, greater flexibility is needed when considering how to study and link to school practice. For example, situations in which student performance is low enough at the group level may warrant thinking of and delivering interventions as part of core or supplemental instruction provided to all students. Work by Jitendra and colleagues (2015, 2017) highlights this approach where instruction targeting mathematics content, PR, is taught at the whole class level based on the significant number of students who struggle in the area. Conversely, an intervention that includes foundational content, such as PAR, could be implemented in a Tier 3 setting combined with specific Tier 3 features such as data-based individualization (Schumacher et al., 2017). Research should investigate the deployment of interventions at varying grade levels and at different time points along the spectrum of RTI and MTSS models. Finally, our work focused broadly on investigating whether PAR impacted student outcomes, but specific calls have noted the need to delve more deeply into the components of fraction intervention programs to determine the active ingredients critical to impact to help unpack the black box of intervention effectiveness (Doabler et al., 2016). A key component of the PAR intervention was the use of multiple representations to model fraction concepts and procedures. Hwang et al. (2019) documented a strong impact ($g$ = .49–.88) for the use of multiple representations. Our impact results generally fell within this range. As called for by leading experts (NMAP, 2008), the primary representation utilized by the PAR intervention was the number line. As few as 5 years ago (Shin & Bryant, 2015), with some notable exceptions (Fuchs et al., 2017), most fraction interventions did not use the number line as a core intervention feature. Future research that explores hypothesized high leverage representations, such as the number line, and other active intervention components is needed to gain a better understanding of how and why interventions impact student understanding.

Besides the limitations noted above, specific limitations also include the use of a quasi-experimental design in which schools willing to implement the PAR intervention constituted the treatment condition. For example, the treatment group had significantly more students with disabilities. Based on findings from a synthesis on fraction interventions indicating that students with low achievement had generally stronger outcomes than students with MLD (Hwang et al., 2019), results in this study may have been impacted by the differences between conditions on this key student variable. Although there were limited differences on pretest measures of student achievement, a stronger design would utilize random assignment to mitigate threats to internal validity. In addition, our study was limited in terms of geographic region and the demographics of the participating sample. Future research should be systematically conducted to investigate the impact of PAR in a range of communities, teachers, and learners. Such endeavors will add to the growing knowledge base on how best to teach fractions and prepare students for continued growth in their mathematical knowledge.

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