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AN INVESTIGATION ON PRIMARY SCHOOL STUDENTS' 3D GEOMETRIC THINKING

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Abstract

This study aims to investigate how primary school students' three-dimensional geometric thinking changes across grades. The survey model was used, and the study group was comprising of 520 primary school students in a large city of Turkey. In the study, the Three-Dimensional Geometric Thinking Test, which is a paper and pencil test, was used to collect data. The scores taken from the test were compared across the grades and the relationships between the components of three-dimensional geometric thinking were examined. The findings showed that as the grades increased, students' scores taken from the Three-Dimensional Thinking Test also increased significantly. Moreover, a medium and positive correlation was found between the components of three-dimensional geometric thinking. The results of the study revealed that grade level is a significant variable on three-dimensional geometric thinking, yet some important three-dimensional geometric thinking skills can be developed independent from the grade level. The current study intends to shed light on the development of three-dimensional geometric thinking starting from early grades, and to provide important information for organizing the three-dimensional geometric content in the curriculum and its implementation.

Keywords: Geometric thinking, three-dimensional geometric thinking (3DGT), geometry teaching, primary school students.

INTRODUCTION

3D geometry basically includes 3D geometric objects, their properties, and movements. 3D geometric thinking (3DGT) can be defined as the perception and interpretation of geometric objects, their movements, and properties using spatial ability (Gutiérrez, 1992; Yeh & Nason, 2004). When it is about teaching and learning 3D geometry, the skills of recognizing 3D objects, understanding their structures and elements, classifying, measuring, and solving problems related to these objects come to the forefront (Gutiérrez, 1992; Ministry of National Education [MoNE], 2018; National Council of Teachers of Mathematics [NCTM], 2000). These skills, which are necessary for students to understand mathematics and other disciplines in school, are also important for the students to have make sense of external world's natural objects such as tree, mountain, and seashell, etc. and ideal objects such as circle, triangle, prism, etc. (Yeh & Nason, 2004). In addition, these skills are critical for many professions such as engineering, architecture, carpentry, health, art, cinema, and television. While the mathematics curricula is being prepared, those skills are determined by considering the grades, and different skills are expected from students at each grade. However, it had been observed that the research on 3D geometry has yielded some results which require questioning this approach of curricula. Studies show that four years old children are able to use pictorial information in geometric object tests, eight year olds are able to draw geometric objects without looking at the picture (Murph & Wood, 1981), five years olds start drawing a house by taking its volume into account, six year olds are able to separate a circle from a sphere with the colors they use in their drawings (Wolf, 1988), six and seven year olds begin to notice the relationships between 3D objects and their nets, nine to eleven year olds can correctly form the net of a 3D object with all its parts (Piaget & Inhelder, 1956), five and a half year olds can fold the shapes drawn on paper in their minds (Harris, Newscombe, & Hirsh-Pasek, 2013), ten year olds begin considering the structures produced using identical cubes, both represented in drawing and concrete forms (Olkun, 1999); 8 to 9 year olds, as a result of their experience, begin to notice the components of the polyhedral (Ambrose & Kenehan, 2009). Furthermore, the first-grade kids can also reason about two-dimensional representations of three-dimensional objects (Hallowell, Okamoto, Romo, & La Joy, 2015) and as a result of collective discussions, they can begin to associate three-dimensional objects with their two-dimensional representations and make the transition from physical models to mental models (Conceição, & Rodrigues, 2020). On the other hand, there are studies showing that some between the age of 11 and 15 cannot fully perceive prism drawings (Ben-Chaim, Lappan, & Houang, 1985), some have significant difficulties in finding the number of identical cubes in structures before they are 11 (Battista & Clements, 1996), some 13 to 17 year olds confuse the concepts 'volume' and 'surface area' (Hirstein, 1981), some 14 to 16 year olds draw 3D objects without depth (Mitchelmore, 1980), 14 years olds have difficulties in counting the identical cubes forming a prism (Olkun, 2003a), 13 year olds are not successful in relating 3D objects to their nets (Wright & Smith, 2017), and in drawing two-dimensional views of 3D structures on paper from different directions (Yolcu & Kurtuluş; 2010). Moreover, eighth grades still have difficulties in associating the properties of a cube (e.g., understanding the properties of a triangle drawn inside a cube, comparing the length of a diagonal on an edge or on a face) (Fujita, Kondo, Kumakura, Kunimune, & Jones, 2020) and they prefer to use formulas without attempting spatial solutions while solving questions about 3D objects (Ibili, Çat, Resnyansky, Şahin, & Billinghurst, 2020). Correspondingly, those studies indicate that young pupils do not bear some 3DGT related essential skills, which may also not be developed properly even at older ages. Within that context, studies are needed to explain how 3DGT skills develop at primary school level. On the other hand, given that 3DGT includes many objects and properties perceived in early childhood, it can be argued that investigating 3DGT of young students is of critical importance. In particular, it is envisaged that examining 3DGT according to grades will provide important data both for the organizing of the curricula according to grades and for the studies to be conducted on different age groups regarding the issue.

Components of 3DGT

The geometric thinking model determined by Van Hiele (Van Hiele, 1986) provides powerful explanations on the levels of geometric thinking and had been an important resource for various researches in the geometry teaching. Questioning whether Van Hiele's study was sufficient or not to illuminate other areas of geometry such as 3D geometry, Gutiérrez (1992) examined the relationship between Van Hiele's geometric thinking levels and 3DGT; and determined the levels of 3DGT, independent from the age. Level 1 (Recognition) is the level where 3D objects can be compared visually by taking into account some particular elements such as vertices, edges, faces, etc. Level 2 (Analysis) is the level where 3D objects can be compared according to their features, such as angle size, edge length, parallelism, etc. Individuals at this level can recognize the properties of objects by observing them or based on their names; and can determine the results of the movements of the objects by looking at their positions before and after the movement. Level 3 (Informal deduction) is the level where 3D objects and their properties can be analyzed based on their representations or mathematical structures. Individuals at this level can relate the elements (faces, edges, vertices) of 3D objects compared with their initial and last positions. Level 4 (Deduction) is the level where high visualization skills are required, and the mathematical structures, properties, and movements of objects can only be analyzed based on formal definitions or some properties of objects. When the skills required for those four hierarchical levels, determined by Gutiérrez, are examined, it is evident that recognizing, comparing, distinguishing the properties of, analyzing, and understanding the movements of 3D objects improve as the level increases; and that such improvements take place through the visualization skills developed in time (Gutiérrez, 1992).

NCTM (2000) determines the skills expected from students according to their grades and explains the skills of 3DGT at four different levels; namely preschool-2nd grade, 3rd-5th grades, 6th-8th grades, and 9th-12th grades. The standards set by NCTM consist of skills such as recognizing, constructing drawing, comparing, and defining 3D objects between preschool and 3rd grade. At the 3rd to 5th grades, skills like classification of 3D objects according to their properties, dividing-integration, recognition of identical and similar objects, making assumptions and inferences according to the properties of geometric objects, constructing 3D objects, comparing objects with their drawings, and determining the surface areas and volumes of 3D objects in standard and non-standard units are included. It can be said that as the grade increases, the targeted skills are enriched and deepened, and these skills include more spatial visualization and mental operations. Therefore, it had been seen that 3DGT, which can improve independently from the grades according to Gutiérrez, is addressed in NCTM standards according to grades, and despite it is not explicitly stated, a relationship is established between the improvement of these skills and the student age or academic level.

In their study conducted considering the standards determined by NCTM, Pittalis and Christou (2010) defined the 3DGT skills of 5th to 9th grades under the following five categories; (i) the ability to manipulate different representational modes of 3D objects, (ii) the ability to recognize and construct nets, (iii) the ability to structure 3D arrays of cubes, (iv) the ability to recognize 3D shapes' properties and compare 3D shapes, and (v) the ability to calculate the volume and the area of solids.

In this study, the following path is pursued to determine the components of 3DGT: Before all the skills determined by Gutiérrez (1992) for the levels of 3DGT, the skills included in the NCTM standards (2000), and the skills determined by Pittalis and Christou (2010) were comparatively analyzed in detail. Then, the literature on the subject was reviewed extensively and associated with the above-mentioned skills. As a result, considering the target audience of this study, it had been seen that two new components could be added to the components determined by Pittalis and Christou (2010) (Determining the Positions of 3D Objects Relative to Each Other, Recognizing 3D Objects) and with some changes the components of 3DGT were defined as follows (Akkurt-Denizli, 2016).

1. Determining positions of 3D objects relative to each other

Young students learn to use concepts such as above, behind, beside, between, etc. to express the relations between the positions of objects, while determining the positions of the objects (NCTM, 2000). Children aged 3-4 can show 3D objects which are usually shown as targets and whose positions are marked; but the position of an object can often be precisely encoded in the period of mid-childhood (Piaget, İnhelder, & Szeminska, 1960). This skill, on the other hand, is related to spatial ability (Yeh, 2013). It is observed that the skill of determining the positions of the objects those are relative to each other, has not developed sufficiently in students with lack of experience; and such students cannot perceive the different views of the objects in the space and their positions such as right-left, front-back (Kol, 2010). Given that this skill is essential for other skills, such as understanding the appearance of 3D objects from different directions and recognizing the nets of 3D objects, it is evident that this is one of the basic skills of 3DGT.

2. Recognizing 3D objects

Students who distinguish 3D objects visually and by considering only their simple elements (including square, triangle, etc.) at first can analyze 3D objects and distinguish them with all their properties as their geometric thinking level improves (Gutiérrez, 1992). While students are expected to name 3D objects and recognize some of their basic properties during the period between pre-school and 2nd grade, from 3rd to 5th grade, they are expected to be able to analyze a 3D object, recognize all the elements that make up a 3D object, and consider these elements when naming these objects (NCTM, 2000). Recognizing a 3D object with all its properties, regardless of its shape, color and position, requires the use of advanced spatial relations (Gutiérrez, 1996). Therefore, it is concluded that recognizing 3D objects is a basic component of 3DGT, especially for young students.

3. Using different representations of 3D objects
Perception of mathematical objects is possible only through the use of their representations (Duval, 1993). 3D geometric objects are generally represented by actual physical objects, objects in the computer environment and drawings on paper (Gutiérrez, 1992). In order to understand the representations on paper, which are most commonly used in schools, the depth in the drawing and the elements of the object must be recognized, and the object must be visualized as a whole (Deregowski, & Bentley, 1987). Therefore, it can be said that the representation that requires the utmost mental processing is the representation through drawing. Parzyzs (1988), examining the representations of 2D and 3D geometric objects, stated that the representation through drawing is the distant representation of a 3D object. In addition, the representation of 3D objects in two dimensions requires the visualization of the objects through drawings and improves the spatial abilities of students (Olkun, 2003b). Although the representation through drawing, including different mental operations, is difficult to understand, particularly for young students, and thus it can be considered as a distinctive component of 3DGT skills.

4. Recognizing properties of 3D objects and comparing them
Elements of 3D objects, such as vertices, edges and faces make it possible to distinguish and analyze this object. For instance, a triangular pyramid is distinguished and named by means of a number of triangular faces that make it up. Understanding the category, in which a 3D object belongs, to is only possible by distinguishing its invariant properties (Pittalis & Christou, 2010). Comparison of 3D objects according to their properties is an important skill that requires analyzing the properties of objects and classifying them according to such properties (Gutiérrez, 1992). Since the relationship between a geometric object and its representation is generally more complex and ambiguous for 3D objects (Parzysz, 1988), it is more difficult for young students to distinguish 3D objects from two-dimensional ones by the means of representation through drawing and to distinguish the properties of 3D objects (Clements, 2004). For this reason, distinguishing the properties of 3D objects by the means of representation through drawing, which is commonly used in schools, and comparing 3D objects is seen as a basic component of 3DGT.

5. Establishing relationship of 2D - 3D
Transition between 2D and 3D involves the acts of unfolding and folding. Since these acts require an object to be transformed into a different object through visualization in mind, the ability to establish the 2D-3D relationship requires a more advanced visualization, apart from recognizing objects (Cohen, 2003; Potari & Spiliotopoulou, 2001). The ability to establish the 2D-3D relationship, which is also related to the processes of constructing and rotating 3D objects (Piaget & Inhelder, 1956), is an ability that can be improved through experience, awareness, and special activities (Cohen, 2003). Piaget's statement that *students who can perform the acts of unfolding and folding are two or three years ahead of others*, addresses the importance of this component in the development of 3DGT skills in children.

6. Recognizing 3D structures made up of identical objects
Recognizing structures made up of identical objects involves analyzing those objects, visualizing their views from different directions, and constructing their abstract components in the mind. Counting cubes in a structure made up of identical cubes requires important cognitive operations, such as coordination, construction, and combination (Battista & Clements, 1996). Such operations, which require the identification and organization of components in a structure (Battista, 2004), help primary school students to construct the volume formula (Battista & Clements, 1998). For example, rows or columns consisting of 16 cubes can be identified and used to count cubes in a rectangular prism-shaped structure consisting of 64 identical cubes. Students have difficulties especially in counting the identical objects in the structures presented to them as drawings (Ben-Chaim et al., 1985; Olkun, 1999) and in drawing the structures consisting of identical objects (Yolcu & Kurtuluş, 2010). In this context, recognizing 3D structures made up of identical objects is seen as a component of 3DGT for young students.

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7. Calculating area and volume of 3D objects

Calculating the areas and volumes of 3D objects by the means of units plays an important role in structuring the numerical operations in these measurements and in associating the formula with the structure. Understanding and visualizing the internal dynamics of the structure is necessary for this association (Battista & Clements, 1996). The size and shape of the surface, which the area of will be measured, and the shape of the units to be used in the measurement (triangle/square) are among the variables that determine the difficulty of area measurement (Owens & Outhred, 2006).

Volume measurement, which is more difficult for students compared with area measurement, requires more detailed construction of the object and more spatial visualization. Operations such as calculating the number of identical objects required to fill a box without leaving any space, finding the number of cubes in a structure consisted of identical cubes, and calculating the number of cubes required to complete a structure consisted of identical cubes, help configure the volume formula, as they allow the use of different strategies (Batista, 2004). It can be said that the area and volume measurements made with non-standard units form the basis for the development of this skill in young children.

The seven components described above do not mean that there will be no other components related to 3DGT or that the classification will only be done this way. For example, the ability to classify objects hierarchically, which develops, according to Van Hiele (1999), at the level of informal deduction, can be considered as another component of 3DGT. Since the components of geometric thinking are determined for younger students in the study group, this component is not included in our study.

When the components of 3DGT are examined, it is understood that each one of them require different tasks, and such tasks may contain particular difficulties for different age groups. Therefore, it is thought that determining students' 3DGT would be possible through tasks related to as many components of 3DGT as possible. The fact that many components of 3DGT are handled together, brings up the issue *which representations will be used to measure students' 3DGT skills*. When working on three-dimensional geometry, actual physical objects, objects in the computer environment, and objects on paper can be used. Concrete models, drawings, figures, diagrams, and representations in the computer environment are generally preferred in daily mathematical activities in the classroom. It is stated that concrete materials are the models that should be used in the first place for the development of students' abstract mathematical thinking skills (Erbaş, Kertil, Çetinkaya, Çakıroğlu, Alacalı, & Baş, 2014; Sarama & Clements, 2016). But as concrete models seem not to have an important effect on students' 3DGT after the age of 5, representations such as drawings and symbols should be preferred from this age onwards (Sarama & Clements, 2016). When curricula are examined, it is seen that teachers have been encouraged for the use of technology, and studies have emphasized that technology-supported activities have positive effects on students' learning (Altun, 2011; Olkun & Altun, 2003; Yeh & Nason, 2004; Erdoğan, Özdemir Erdoğan, Galan, & Güler, 2012; Ibili et al., 2020). However, it is also known that representations on paper, which require more mental processing than others, are being used quite frequently. It is only possible to include three-dimensional objects in books by using their two-dimensional representations on paper. Furthermore, representations on paper are generally included in assessment tools for three-dimensional geometry. Although it is not easy to understand the properties of three-dimensional objects from their representations on paper and to determine the rules of the transition from 3D to 2D, representations on paper are important for students to develop mental processes while learning objects (Sarama & Clements, 2016). For those reasons, it can be said that drawings, which are the most frequently used but also most complex representations for students as they require important mental operations, constitute an adequate tool to investigate students' three-dimensional geometric thinking.

In this context, the purpose of our study is to examine the evolution of primary school students' 3DGT geometric thinking according to grade and the relationships between the components of 3DGT. In the study, the Three-Dimensional Geometric Thinking Test (3DGTT), which includes five of the above-mentioned components of 3DGT and prepared by using the representation through drawing, was used (Akkurt-Denizli, 2016).

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Answers were sought for the following questions concerning these five components:

1. Does primary school students' state of 3DGT vary significantly depending on grade?

    a. Do primary school students' scores taken for the component of recognizing 3D geometric objects vary significantly depending on grade?

    b. Do primary school students' scores taken for the component of recognizing properties of 3D objects and comparing them vary significantly depending on grade?

    c. Do primary school students' scores taken for the component of establishing relationship of 2D - 3D vary significantly depending on grade?

    d. Do primary school students' scores taken for the component of recognizing 3D structures made up of identical objects vary significantly depending on grade?

    e. Do primary school students' scores taken for the component of calculating the area and volume of 3D objects vary significantly depending on grade?

2. Is there a significant correlation between the scores taken by primary school students for the sub-components of 3DGTT?

The literature review showed that a limited number of research has been done on 3DGT skills of primary school students and that students' 3DGT has not been examined according to their grades. It is seen that studies conducted on primary school students generally give information about students' thinking about only one component of 3DGT. The study conducted by Pittalis and Christou (2010) on 5th to 9th grade students by defining the components of 3DGT, and the study conducted by Ibili et al. (2020) on 8th grade students by creating a geometric thinking test on the basis of the components defined by Pittalis and Christou are considered to be important as they included several components of 3DGT. However, no study in which the components of 3DGT of primary school students are addressed as related to each other, is found. This study is important as it addresses the evolution of 3DGT of primary school students across the grade levels, and it is thought that the study can contribute to the improvement of curriculum, prepared by considering the characteristics of students in each grade level. In the current study, examining the relationships between the components of 3DGT requiring different skills may give information about which of these components develop in a related way, while also shedding light on how they should be handled in the teaching process. For this reason, it is thought that the study can provide important insights into the 3DGT of primary school students and 3D geometry teaching at primary school level and beyond.

METHOD

Research Model

The current study aims at determining primary school students' state of 3DGT as accurately as possible, without making any effort to change anything is a descriptive study designed in the survey model (Karasar, 2009; Büyüköztük, Çakmak Kılıç, Akgün, Karadeniz, & Demirel, 2010). This model allows forming generalizable results and making comparisons between different groups (Büyüköztürk et al., 2010; Creswell, 2002).

Study Group

Typical case sampling, one of the purposive sampling methods, was used in the current study (Büyüköztürk et al., 2010). The study had been conducted in six state schools (three primary schools and three middle schools) having students among medium socio-economic level, in a big city of Turkey. The schools selected in the sampling process were determined according to the information obtained directly from the Ministry of National Education.

The study group was comprised of 2nd to 5th graders in the fall term of 2015-2016 education year. The ages of these students varied between 7 to 11. The middle schools with 5th graders were the schools with the majority of the students graduating from the three selected primary schools. Thus, it was ensured that the students in the study group were at a similar socioeconomic level. The reason for the selection of 2nd to 5th graders, instead of 1st-4th grades, was that this group of students can best reflect the primary school level. First grade students are not at a level to read and answer any pencil and paper tests at the beginning or in the middle of the term. Furthermore, 3D geometry subjects are included in the curriculum of each class at different times during the academic year, and this might affect the results of a test to be applied during the year. By applying the test to students of 2nd to 5th grades simultaneously at the beginning of the semester, it was thought that the way 3DGT of primary school students varies across the grades could be best reflected. As a result, the study represents 3DGT of primary school students (1st- 4th grades) even if the study group consists of the students who were at the beginning of the grades from 2nd to 5th. The distribution of the students in the study group across the grade levels is given in Table 1.

Table 1. Distribution of the students in the study group across the grade levels

Grade Level	**Gender**		
	Female	**Male**	**Total**
1st grade	63	65	128
2nd grade	62	61	123
3rd grade	61	69	130
4th grade	56	83	139
Total	242	278	520

As can be seen in Table 1, of the 520 students making up the study group, 46.5% are females and 53.5% are males. The study was carried out in the classes of the teachers who accepted to participate in the study in the selected schools and with the participation of all the students who were in the classroom during the test application.

Data Collection Tool and Its Implementation

The Three-Dimensional Geometric Thinking Test (3DGTT), which was developed by Akkurt-Denizli (2016) to determine the 3DGT of primary school students, was used as the data collection tool in the study. The 3DGTT is a paper-pencil test that includes questions on the five components of 3DGT and has 45 questions in total. This test was prepared by seeking expert opinions according to the determined components of three-dimensional geometric thinking, without considering a hierarchical structure. The distribution of the questions in the test across the components of 3DGT is given in Table 2.

Table 2. Distribution of the questions in the 3DGTT across the components of 3DGT

Components of 3DGT	**The number of questions**	**The total number of questions (sub-items included)**
Recognizing 3D geometric objects	4	4
Recognizing properties of 3D objects and comparing them	7	18
Establishing relationship of the 2D and 3D	4	4
Recognizing 3D structures made up of identical objects	3	8
Calculating area and volume of 3D objects	4	11
Total	22	45

The 3DGTT consists of multiple choice and short answer open-ended questions (Examples in Figure 1 and Figure 2).The highest score to be taken from this mixed test, which is scored as true (1)-false (0), is 45. In order to establish the content validity, the test was submitted to the review of 13 experts, and the KR-20 reliability coefficient of the test was found to be 0.87 (Akkurt Denizli, 2016; Akkurt-Denizli & Erdoğan, 2018).

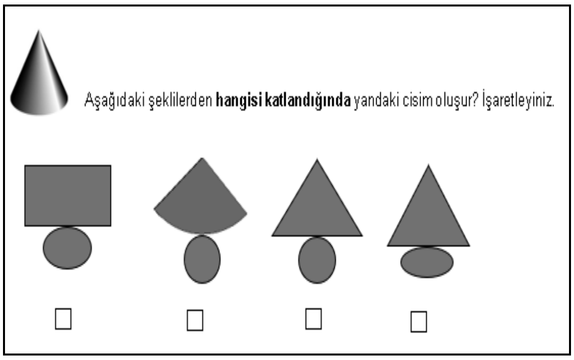

Figure 1. Establishing relationship of the two-dimensional and 3D

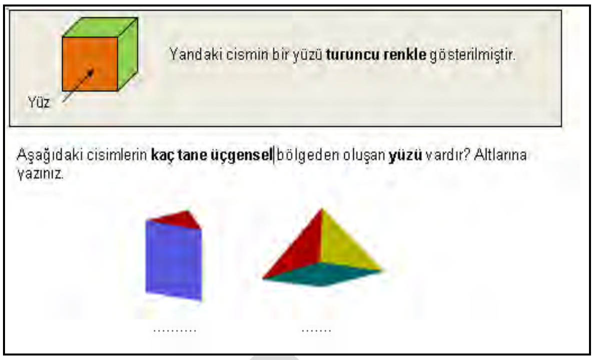

Figure 2. Recognizing properties of 3D objects and comparing them

The question in Figure 1 is that: "Which of the following shapes should be folded to obtain the next object? Mark it." The question in Figure 2 is that: "One face of the next object is shown in orange. How many faces of the objects below are made up of triangular areas? Write under them."

The application of the test was carried out by the researcher in the students' own classes and during the class hours. The students were told that this test would not affect their math scores at school and that they should take the test without any help. The students were informed that they would have a maximum of 50 minutes to complete the test, and they were given detailed information on the basis of the instruction of the test about how they would answer the test questions (Akkurt-Denizli, 2016). The sheets of the students who completed the test were checked quickly by the researcher, and the students were asked to read and answer the questions/ pages they had forgotten. After 50 minutes, all students' tests were collected.

Since this study was conducted in schools with similar socio-economic levels, and the data of the students with any diagnosed special needs were excluded, we can argue that the study group's features imposed no challenges against our study. In all classes, data were collected by the researcher. When collecting data, no events, which might had affect the students' answers, occurred. Since the data collection tool 3DGT is valid and reliable (Akkurt-Denizli, 2016), this test imposed no challenge to this study.

Data Analysis and Statistical Techniques

The answers given by each student to the questions in the test were coded as true (1)-false (0), and the scores for each component and for the test were calculated.

One-Way Variance Analysis (ANOVA) was used to compare the total test scores of students from different grades. MANOVA (Multivariate ANOVA) was used to examine how the scores of the students for the different components of 3DGT vary depending on grade level.

First of all, ANOVA and MANOVA analyses were performed to test whether the data satisfy the assumptions. Missing data were examined, and no missing data were found in the data set. In the examination of Q-Q graphs for the evaluation of univariate normality, it was found that the normal distribution was protected, and the skewness coefficients of the variables remained within ±1 limits. So, the scores did not show a significant deviation from the normal distribution (Çokluk, Şekercioğlu, & Büyüköztürk, 2010). In the process of examining multivariate normality and linearity, Mahalanobis distances were calculated for all dependent variables first. The values obtained were compared with the value of $X^2_{(p=.001,\ df=4)}=18.467$ specified in the X^2 table, and it was determined that there was no outlier since the critical value of 18.467 was not exceeded. Then, the Scatter Plot Matrix was examined, and it had been seen that the scatter plots of all binary relations of the dependent variables were close to an elliptical shape, that is, multivariate linearity was attained (Tabachnick & Fidell, 2007). As a result of the Levene Test, performed to test the homogeneity of the variance matrices between the groups, the assumption of homogeneity of the variance matrices was satisfied since the

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$p>.05$ condition was met for the difference scores of the variables. According to the results of the Box's M Test, performed to test the homogeneity of the covariance matrices, the $p>.05$ ($p=.559$) condition was met. The results of these analyses showed that the research data were suitable for ANOVA and MANOVA analyses and satisfied the assumptions of these analyses.

For the interpretation of the effect size in both analyses, the criteria determined by Cohen (1988) were used. Cohen (1988) interprets the η2 value calculated as follows: .01: small effect, .06: medium effect, .14: large effect.

Pearson Correlation Coefficient was calculated to examine the relationships between the components of 3DGT. The degree of correlation was calculated as follows: between ± .70 and ± 1: strong correlation, between ± .30 and ± .70: medium correlation, and ± .00 and ± .30: small correlation, .00: no correlation (Büyüköztürk, 2007). Data from this study were analyzed using SPSS 15.0.

RESULTS

First of all it had been examined how the 3DGT of students varies depending on their grades. Then, it was determined how the scores obtained for each component of 3DGT vary depending on grade level. Finally, the relationships between the components of 3DGT were examined.

How 3DGTT Scores Vary depending on Grade Level

The mean scores, the highest and lowest scores are presented in Table 3.

Table 3. Students' 3DGTT scores

Grade Level	The Number of Students	3DGTT Scores			
		Mean	Std. Deviation	Lowest score	Highest score
1st grade	128	21.04	7.56	5	40
2nd grade	123	23.64	7.96	5	42
3rd grade	130	24.17	8.27	6	43
4th grade	139	27.94	8.24	5	44
Total	520	24.28	8.38	5	44

Table 3 shows that the 1st grade students got the lowest mean score with 21.04 while the 4th grade students got the highest mean score with 27.94, followed by the 3rd graders with 24.17 and 2nd graders with 23.64. This finding shows that as the grades increase, the mean score taken from the test also increases. Regarding the highest and lowest scores, it had been seen that with increasing grade, only small increases occur in the highest score while the lowest score does not vary depending on grade. The lowest score is 6 among the 3rd graders and it is 5 among other grades.

One-way Variance Analysis (ANOVA) was run to determine whether there is a significant difference among the scores taken from the 3DGTT and grade level. The findings are presented in Table 4.

Table 4. ANOVA scores

Source of the Variance	Sum of Squares	Degree of Freedom	Mean Square	F	p	Significant Difference	η2
Between-Groups	3261.174	3	1087.058	16.885	.000*	1-3. 1-4. 2-4. 3-4	.089
Within-Groups	33219.557	516	64.379				
Total	36480.731	519					

* p<.01

Table 4 shows that the 3DGTT scores vary significantly depending on grades [$F_{(3-516)}=16.885$, $p<.01$]. In addition, η2 value, calculated for the effect size, was found to be .089. Thus, it was concluded that the grade variable has a medium effect on the scores taken from the 3DGTT (Cohen, 1988). The results of the Scheffe Test conducted for paired group comparisons showed that this significant

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difference between different grades from the significant differences between the 1st and 3rd graders, between the 1st and 4th graders, between the 2nd and 4th graders and between the 3rd and 4th graders. These significant differences seem to be in favor of upper grade levels in all the paired comparisons. This finding shows that the test score taken at the 4th grade level is significantly higher than the scores taken at all the other grade levels. Although mean scores increase as the grade increases, the score differences between the 1st and 2nd graders and between the 2nd and 3rd graders were not found to be significant.

How the Scores Taken for the Components of 3DGTT Vary Depending on Grades

The mean scores taken by the students for the components of 3DGTT were compared in a graph (Figure 3). As the numbers of questions regarding the components in the test are different (e.i. the highest scores taken for the components), the scores taken by the students for the components in the 3DGTT were converted into t scores.

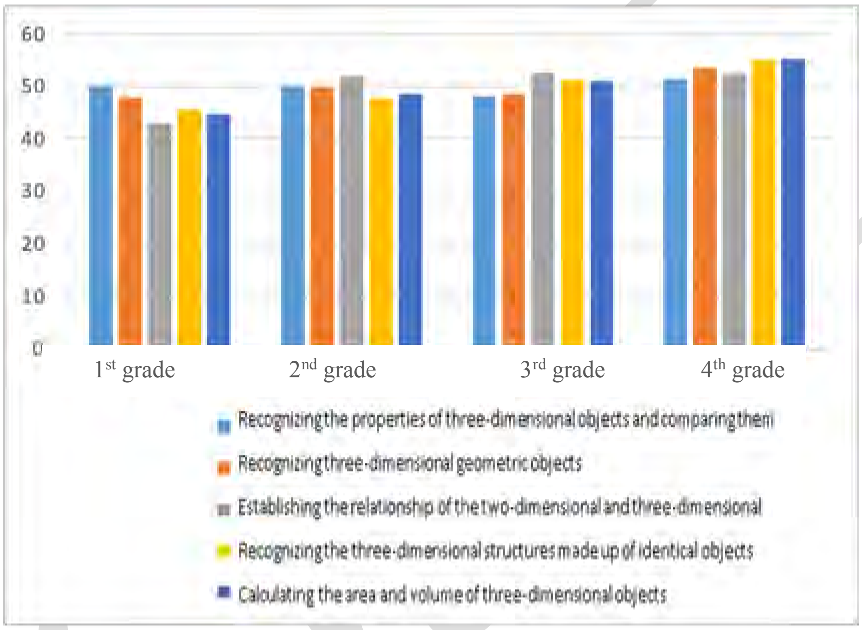

Figure 3. Students' t scores for the components of 3DGTT

Figure 3 shows that the scores taken by the students for all the components of 3DGTT increase, except for the component of recognizing the properties of 3D objects and comparing them between 2nd graders' scores and 1st graders' scores. On the other hand, while the lowest mean score was obtained for the component of establishing the relationship of the 2D and 3D by the 1st graders, the 2nd graders got the highest mean score for this component. It can be said that a small decrease occurred in the mean score taken for the component of recognizing the properties of 3D objects and comparing them (50.22 versus 50.14). At the 3rd grade, the mean score taken for the component of establishing relationship of the 2D and 3D increased, and it became the component with the highest mean score as at the 2nd grade. It can also be said that there are decreases in the mean scores taken for the components of recognizing 3D geometric objects and recognizing the properties of 3D objects and comparing them while there are increases in the mean scores taken for the other components at the 3rd grade. At the 4th grade level, all the mean scores except for the mean score taken for the component of establishing relationship of 2D and 3D are higher than the scores taken for these components in the other three grades.

The results showed that the highest mean score for the component of establishing relationship of 2D and 3D relationship was obtained in the 3rd grade (42.95- 51.89-52.61- 52.37 for the 1st, 2nd, 3rd, 4th grade levels, respectively) while the highest mean scores for all the other components were obtained in the 4th grade. With increasing grades, the mean scores taken for the components of recognizing the 3D structures made up of identical objects and calculating the area and volume of 3D objects increased. While the lowest mean score for the component of recognizing the properties of 3D objects and comparing them was taken in the 3rd grade, the lowest mean scores for the other components were taken in the 1st grade.

MANOVA was used to investigate how the scores taken by the students for the components of 3DGT in the 3DGTT vary across the grades**.** MANOVA assumptions are examined under the heading of "Data Analysis and Statistical Techniques". Statistics for this analysis are given in Table 5.

Table 5. Statistics of the MANOVA analyses

Source of the Variance	Wilks' λ	F	Hypothesis df	Error df	p	η2
Grade level	.624	17.57	15.00	1413.808	.000*	.146

* p<.01

As a result of the analysis, significant differences were found between the grades in terms of the components of 3DGT [Wilk's Lambda (λ)= .624, $F_{(3-516)}$=17.57, p<.01]. The calculated η2 value was found .146, which shows that the effect of the grade variable on the scores taken for the components is large (Cohen, 1988). In other words, the grade variable explains 14% of the variance in the dependent variable, and this variable has a great effect on the dependent variable. The results showing how the scores taken for each component vary across the grades are presented in Table 6.

Table 6. Means and standard deviations and ANOVA results

Component	Grade	n	Mean	Std.Dev.	Df	F	p	Significant Difference	Possible maximum scores
Recognizing 3D geometric objects	1	128	1.12	1.10	3-516	8.82	.000*	1-4, 2-4, 3-4	4
	2	123	1.38	1.18					
	3	130	1.22	1.16					
	4	139	1.81	1.28					
Recognizing properties of 3D objects and comparing them	1	128	10.39	4.01	3-516	2.38	.069		18
	2	123	10.35	4.08					
	3	130	9.54	4.33					
	4	139	10.87	3.99					
Establishing relationship of 2D and 3D	1	128	1.32	0.95	3-516	33.47	.000*	1-2, 1-3, 1-4	4
	2	123	2.34	1.16					
	3	130	2.42	1.06					
	4	139	2.39	.97					
Recognizing 3D structures made up of identical objects	1	128	4.06	2.10	3-516	26.20	.000*	1-3, 1-4, 2-3, 2-4, 3-4	8
	2	123	4.50	1.92					
	3	130	5.26	2.08					
	4	139	6.07	1.86					
Calculating area and volume of 3D objects	1	128	4.11	2.18	3-516	29.95	.000*	1-2, 1-3, 1-4, 2-4, 3-4	11
	2	123	5.05	2.39					
	3	130	5.72	2.29					
	4	139	6.79	2.58					

* p<.01

Table 6 shows that regarding the component of *recognizing 3D geometric objects* there are increases in the scores taken with increasing grades, except for 2nd and 3rd grades (1st grade:1.12; 2nd grade:1.38; 3rd grade:1.22; 4th grade:1.81). Thus, a significant correlation was found between the mean scores taken for this component and grade [$F_{(3-516)}$=8.82, p<.01]. The calculated η2 value was found .048, indicating that the effect size is medium (Cohen, 1988). The significant difference found as a result of Scheffe Test seems to stem from the significant differences between the 1st and 4th grades, between the 2nd and 4th grades and between the 3rd and 4th grades in favor of the 4th grade. In other words, the 4th grade students got a significantly higher mean score for the component of recognizing 3D objects than the

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students from the other grades. Moreover, the highest score (4) and the lowest score (0) to be taken for this component were taken in all the grades.

Regarding the component of *recognizing properties of 3D objects and comparing them* (1st grade: 10.39; 2nd grade: 10.35; 3rd grade: 9.54 and 4th grade: 10.87), the lowest mean score was taken in the 3rd grade. The highest mean score was taken in the 4th grade, followed by 1st and 2nd grades. It is seen that there is no significant correlation between the scores taken for this component and grade level [$F_{(3\text{-}516)}$=2.380, p>.01]. The highest score for this component (18) was taken in the 3rd grade. The highest score taken in the other grades is 17. The lowest score taken in the 2nd, 3rd and 4th grades is 1, and it is 2 in the 1st grade level.

For the component of *establishing relationship of 2D and 3D* (1st grade: 1.32; 2nd grade: 2.34; 3rd grade: 2.42 and 4th grade: 2.39), the highest mean score was taken in the 3rd grade and the lowest mean score was taken in the 1st grade. A significant correlation was found between the scores taken for this component and grade level [$F_{(3\text{-}516)}$=33.47, p<.01]. The results of Scheffe Test showed that this significant difference between the grades stemmed from the significant differences between the 1st and 2nd grades, between the 1st and 3rd grades and 1st and 4th grades in favor of the 4th grade. The highest score (4) and the lowest score (0) to be taken for this component had been taken in all the grades.

For the component of *recognizing 3D structures made up of identical objects*, the mean scores increase as the grade increased (1st grade: 4.06; 2nd grade: 4.50; 3rd grade: 5.26; 4th grade: 6.07). A significant correlation was found between the scores taken for this component and grade [$F_{(3\text{-}516)}$=26.20, p<.01]. The effect size of the grade variable on the scores taken for this component was found to be medium (η2=.13), but closer to large effect size (Cohen, 1988). The results of Scheffe Test showed that this significant difference stemmed from the significant differences between the 1st and 3rdgrades, between the 1st and 4th grades, between the 2nd and 3rd grades, between the 2nd and 4th grades, and between the 3rd and 4th grades in favor of the 4th grade. These results show that there are no significant differences only among the 1st and 2nd grades. The highest score to be taken for this component (8) had been taken in all the grades. The lowest score taken for this component is 1 in the 4th grade and 0 in all the other grades.

The mean scores taken for the component of *calculating the area and volume of 3D objects* increased as the grade increased. The mean scores taken for this component in the 1st, 2nd, 3rd and 4th grades are 4.11, 5.05, 5.72 and 6.79, respectively. A significant correlation was found between the scores taken for this component and grade [$F_{(3\text{-}516)}$=29.95, p<.01]. The calculated effect size was η2=.14, showing that the significant difference has a large effect (Cohen, 1988). As a result of Scheffe Test, it was found that this significant difference stemmed from the significant differences among 1st and 2nd grades, 1st and 3rd grades, 1st and 4th grades, 2nd and 4th grades, and 3rd and 4th grades in favor of higher grade levels. These results show that there are no significant differences only among the 2nd and 3rd grades. The highest score to be taken for this component (11) had been taken in all the grade levels and the lowest score (0) was taken in the 1st, 2nd and 3rd grade levels while the lowest score taken in the 4th grade level is 2.

Relationships between the Components of 3DGT

In order to investigate the relationships between the components of 3DGT, correlation coefficients had been calculated; and positive, medium and significant correlations had been found among all the components. Thus, it can be said that as an increasing score is taken for any of the components, the scores taken for the other components tend to increase. The correlation values among the components of 3DGT are given in Table 7.

The component of recognizing properties of 3D objects and comparing them has medium, positive and significant correlations with the component of recognizing 3D geometric objects (r=.36, p<.05), medium, positive and significant correlations with the component of establishing relationship of 2D and 3D (r=.45, p<.05), medium, positive, and significant correlations with the component of recognizing the 3D structures made up of identical objects (r=.42, p<.05), medium, positive, and significant correlations with the component of calculating the area and volume of 3D objects (r=.44, p<.05).

Table 7. Correlations between the components of 3DGT

Components		**Pearson Correlation Coefficient**				
		Recognizing properties of 3D objects and comparing them	Recognizing 3D geometric objects	Establishing relationship of 2D and 3D	Recognizing 3D structures made up of identical objects	Calculating the area and volume of 3D objects
Recognizing properties of 3D objects and comparing them	Pearson K.	1	.360*	.454*	.425*	.442*
	Sig. (2-tailed)		.000	.000	.000	.000
	N	520	520	520	520	520
Recognizing 3D geometric objects	Pearson K.		1	.353*	.307*	.314*
	Sig. (2-tailed)			.000	.000	.000
	N		520	520	520	520
Establishing relationship of 2D and 3D	Pearson K.			1	.418*	.523*
	Sig. (2-tailed)				.000	.000
	N			520	520	520
Recognizing the 3D structures made up of identical objects	Pearson K.				1	.465*
	Sig. (2-tailed)					.000
	N				520	520
Calculating the area and volume of 3D objects	Pearson K.					1
	Sig. (2-tailed)					
	N					520

* p<.01

The component of recognizing 3D geometric objects has medium, positive and significant correlations with the component of establishing relationship of 2D and 3D (r=.35, p<.05), medium, positive, and significant correlations with the component of recognizing 3D structures made up of identical objects (r=.30, p<.05), medium, positive, and significant correlations with the component of calculating the area and volume of 3D objects (r=.31, p<.05).

The component of establishing relationship of 2D and 3D has medium, positive, and significant correlations with the component of recognizing the 3D structures made up of identical objects (r=.41, p<.05) and medium, positive, and significant correlations with the component of calculating the area and volume of 3D objects (r=.52, p<.05).

The component of recognizing the 3D structures has medium, positive, and significant correlations with the component of calculating the area and volume of 3D objects (r=.46, p<.05).

DISCUSSION and CONCLUSIONS

The scores taken from the 3DGTT revealed that the primary school students' 3DGT improves as their grade increase. Understanding a 3D object requires the visualization of the object as a whole in mind as well as distinguishing the elements of the object (Deregowski & Bentley, 1987). The visualization of a 3D object, which the drawing of is given, can be accomplished through the spatial ability and this ability develops in hierarchical stages depending on experience (Guay & McDaniel, 1977). Accordingly, as the grade increases, the experiences gained may enable the improvement of this ability and facilitate the understanding of 3D objects. In addition, except for the 1st and the 2nd grades and the 2nd and the 3rd grades, significant differences were found among all the grades in favor of upper grades. Thus, it can be claimed that the most remarkable improvement occurs while the students pass from 3rd grade to the 4th grade.

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When the lowest and highest scores taken from the 3DGTT are examined, it had been seen that there are 1st grade students who obtained 40 points, 2nd grade students who obtained 42 points, 3rd grade students who obtained 43 points and 4th grade students who obtained 44 points. These results may indicate that 1st grade students, who are expected by the mathematics curriculum (MEB, 2013) to recognize the elements making up the objects, and to classify the objects according to their "being round or cornered", might have a higher ability of 3DGT than the anticipated. Moreover, there are 2nd grade students who got 42 points in the test, which includes questions requiring important visualization processes, such as determining the view of a structure made of cubes from different angles and comparing objects with different surface areas. Those results show that in younger students, 3D thinking skills can develop independently from the instruction. On the other hand, the presence of students who got 6 in the 3rd grade and 5 in the other grades indicate that some students performed well below their grade's average. These results support the view of Van Hiele (1999) that a child may have a higher level of geometric thinking, compared with an older child. In addition, increases had been observed in the scores depending on the grades, and the significant score differences found in favor of the upper grades may be a result of students' achieving more experience in 3D geometry each year, depending on the structure of mathematical course curriculum. These results show that students' 3DGT mainly depends on their grade but may also develop independently from the grade.

NCTM (2000) contends that the students from preschool to 2nd grade should be able to explain the properties of 3D objects and their constituent parts, to use their representations; and that 3rd to 5th grade students should be able to analyze the properties of such objects and classify them according to their properties. In the 1st to 4th grade mathematics curriculum, which was in effect in Turkey when this study was being conducted, the properties of 3D objects were first mentioned in the 2nd grade and, at this level, it was aimed to make students recognize the properties such as vertices, edges, and faces. At the 3rd grade, students were expected to visualize the parts of 3D objects in their minds when they are given the nets of these 3D objects (MEB, 2013). In that case, it is understood that an improvement depending on the grade was expected in terms of the recognition of objects' properties. In our study, the scores obtained by the students at the 1st to 4th grades on the component of recognizing the properties of 3D objects were compared and were found to be very close. It is an important result that there are no significant differences in terms of the grades among students' scores for this component. In that case, it became evident that students at different grades are at a similar level in determining the vertices, edges and faces of 3D objects and comparing different 3D objects according to their properties. However, these results should not be read as 'the students' skills of recognizing the properties of 3D objects do not improve at all as their grade levels increase'. It should be noted that the current study is based on the 3DGTT, limited with the representations through drawings. Our study reveals the fact that young students' noticing some features of three-dimensional objects, which are not included in the curriculum, is in line with the result that 8- and 9-years old students' noticing the components of polyhedras, as also observed by Ambrose & Kenehan (2009). The fact that an average of about 10 points /18 points was obtained at all grades for this component, indicate that students begin to notice the properties of three-dimensional objects earlier than expected by the curriculum.

The findings obtained for the component of recognizing 3D geometric objects show that the relevant skill develops depending on the grade. The fact that the significant difference found stemmed from the significant differences between 1st and 4th, 2nd and 4th and 3rd and 4th grades can be regarded as an indicator of a significant improvement in this skill at the 4th grade. This result may also indicate that the skills of distinguishing 2D and 3D objects and recognizing similar 3D objects develop only at the ages of 9 and 10 (Mitchelmore, 1980). The results related to this component show that students' related skills start to develop at the 1st grade or earlier, and in this respect, they support the study by Hallowell et al. (2015), who showed that the 1st grade students were able to envisage two-dimensional representations of three-dimensional objects. However, the absence of a significant difference among grades till the 4th grade addresses that it may be beneficial especially for the students at lower grades to be provided experience on three-dimensional objects on paper. The views that concrete objects are not as effective as it is thought to be after the age of 5, and that the use of representations, such as drawings and symbols, is necessary after that age (Sarama & Clements, 2016) are considered to be important in respect with that.

The significant differences found between 1st and 2nd, 1st and 3rd, and 1st and 4th grades in favor of upper grades for the component of establishing relationship of the 2D and 3D indicate that this skill shows a remarkable improvement after the 1st grade. This result may be due to the fact that the 1st grade students aged 6-7 in the study group had just begun to recognize the relationship between a 3D object and its net but yet not have an idea about the object that the net will create (Piaget & Inhelder, 1956). In addition, although the subject of nets is first mentioned in the 3rd grade according to the mathematical curriculum, which was in effect in the academic year during the current study was conducted (MEB, 2013), the improvement observed in the students when they passed to the 2nd grade, and the lack of significant difference between 2nd and 3rd grade students are important outcomes of that study. This outcome addresses that, students started to visualize the parts of 3D objects in their imagination, independent from the curriculum. In addition to these, the existence of students with skills related to this component in the first grade supports the studies of Harris et al. (2013), who argued that 5.5-year-old children can fold shapes on paper in their minds.

The facts that the students' mean scores taken for the component of recognizing 3D structures consisting of identical objects vary significantly depending on grades, and that, significant differences were found between all the grade levels, other than the 1st and 2nd grades, in favor of upper grades, proves that as their grade increase students use significantly more advanced strategies in visualizing and counting identical cubes (Battista & Clements, 1996; Olkun, 2003a). According to the mathematical curriculum, which was in effect in the academic year during the study was conducted, students encounter drawings of buildings consisting of identical cubes for the first time in the 4th grade (MEB, 2013). However, the significant score differences observed for this skill after the 2nd grade indicate that there is a remarkable leap after this grade and student start to recognize 3D structures consisting of identical objects independent from the curriculum .

Accordingly, it had been found that the mean scores obtained for the component of calculating the area and volume of 3D objects vary significantly depending on grade may indicate that students can use unit squares to calculate areas on the basis of the experience they gained over a few figures (Heraud, 1987). It had also been found that the skill of calculating volumes with unit cubes develop depending on the grade (Battista & Clements, 1996; Olkun, 2003a). Students' experience with geometric shapes and 3D geometric objects improves as their grade increase. In addition, in our study, which had been conducted during an academic term when area measurement with non-standard units was addressed in the 3rd grade and the volume measurement was addressed in the 5th grade according to the mathematical curriculum, it has been concluded that students showed improvement for this component from the 1st grade onward. This result addresses that important steps had been taken towards the development of the concepts of area and volume as of the first years of primary school, independent from the curriculum.

When the relationships between the scores taken by the students for the components of 3DGT were examined, it was observed that while the scores obtained for one component increased, the scores obtained for the others also increased and positive and medium significant correlations were found between all the components. This result points out the necessity to consider as many components as possible together in order to determine the 3DGT state of primary school students. This result is also believed to provide guidance for the teaching to be delivered on the relevant subject. For example, in the mathematics curriculum, which was in effect in the academic year during the current study was conducted, the properties of 3D objects were addressed in the 2nd grade and the identical structures in the 4th grade for the first time (MEB, 2013). Different skills can be developed at different ages and at different grades, and this also affects the teaching to be done according to grade. Moreover, the results obtained are considered to be remarkable in terms of understanding the importance of teaching the components of 3DGT in an integrated manner. In mathematical curriculums there sometimes may

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occur considerable changes on the objectives regarding the 3D objects from time to time. Also, the objectives addressed for each grade, or the content of the current objectives might change. For example, the properties of 3D objects were addressed in the 2nd grade when the current study was conducted (MEB, 2013), but this subject is included in the 3rd grade at the current curriculum (MEB, 2018). It is thought that much more effective results can be obtained from such changes, if the facts that the development of the components of 3DGT depends on grade level, and that the components of 3DGT are interrelated, are taken into consideration.

As a conclusion, the scores obtained for both the components of 3DGT and the whole test show that 3DGT starts to develop from the 1st grade onward, and that some students were able to correctly answer questions concerning objectives normally addressed in the upper grades according to the mathematical curriculum. This result reveals that some important 3DGT skills develop independently from the objectives set in the mathematics curriculum for each grade. However, as the grade increases, despite the scores for the components sometimes decrease, the mean scores at the test and the significant score increase too especially after the 3rd grade; and this shows the effects of grade and related objectives on the development of 3DGT. These results reveal the importance of determining the subjects and objectives of curriculum according to grades and the importance of considering the fact that, while planning the instructional process, some skills may be developed at an early age to a certain degree. In particular, inclusion of the basic skills related to the components of calculating the area and volume of 3D objects, establishing relationship of 2D and 3D, recognizing the 3D structures made up of identical objects and addressed in higher grades together with other components as of the first years of primary school seems to be necessary for the development of geometric thinking at an early age. On the other hand, the results of the study show that the components of 3DGT develop simultaneously. This result points out the necessity of handling the components of 3DGT with an integrated manner in the curriculum and in the instructional process as much as possible.

Limitation of the Study and Suggestions

The data obtained in the current study are limited to the students' responses to the three-dimensional geometric thinking test. Therefore, the study is not extensive enough to explain why and how students gave these responses. Examination of such issues is worth considering in future researches.

In the current study, three-dimensional geometric thinking of students at different grades had been compared. In addition, a longitudinal study is considered as necessary to see the development of students' three-dimensional geometric thinking as their grades change.

Administration of the test used in the current study to students in upper grades can provide more comprehensive information about the change occurring in 3DGT depending on grade. Such a study can also be useful in determining the subjects related to 3D geometry in primary and secondary school mathematical curriculums.

One-to-one interviews to examine how students are affected by the variables while examining 3D object drawings, and how they perceive 3D objects in different positions and colors can provide detailed information about young students' 3DGT.

It is thought that it would be beneficial to determine by what means the students, who got higher scores since they were in lower grades and who got lower scores since they were in upper grades, think in a different way from others. One-on-one interviews with students with very low and very high scores at each grade will provide information about the 3DGT state of those students and offer important insights into teaching, in which individual differences will be taken into account.

The fact that younger students have some important 3D thinking skills makes us curious about what those students have about higher order skills such as "classification", which is not included in the current study. Studies with young students, including such skills, can provide more comprehensive information about their 3DGT.

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Ethics and Conflict of Interest

This article is based on Zeynep Akkurt Denizli's PhD dissertation entitled "The Development, Application, and Evaluation of a Three-Dimensional Reasoning Test for Grades 1 to 4", conducted under the supervision of Abdulkadir Erdogan. It is hereby declared that the authors of the current study acted in accordance with the ethical rules in the course of the research process. There are no conflicts of interest by and between the authors. This research was conducted with the 26 November 2015 dated and 14588481-605.99-E.12164053 numbered permission granted by the Ministry of National Education. We hereby wish to present our gratitude to primary schools, middle schools, classroom teachers, and all students who participated in the research.

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