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Students' Mental Addition Strategies and the Effects of Strategy Training: A Longitudinal Study*

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Abstract

This longitudinal study was conducted in order to determine and improve the mental addition strategies of students in their 2nd and 3rd years at elementary school. The first part of the study used the qualitative research method of case study, while the second part adopted the single group pretest-posttest quasi-experimental design. The study group consisted of 16 students who were attending grade 2 at an elementary school in Ankara, and who moved on to grade 3 the following year. In the first part, data were collected from the same students at the end of grades 2 and 3. In the first part of the study, no significant change was observed in the mental addition strategies of students who had passed from grade 2 to 3. In the second part, the same students were trained in "Mental Addition Strategies" for 5 class hours per week for a total of 6 weeks, and the final data were collected. Findings from the first part of the study showed that at both grades they used similar mental addition strategies. The second part of the study, on the other hand, showed that mental addition strategy training increased the variety of the mental addition strategies used by 3rd graders.

Keywords:

Mathematics Education, Elementary Mathematics Course, Mental Calculation, Mental Addition Strategies

Introduction

Aritmetic skills form the basis of mathematics. Mastering operations is an essential skill for children to quickly and competently retrieve and apply this information when necessary. In addition, knowing the basic operational skills paves the way for more advanced mathematical operations. In our daily life, we constantly face situations requiring the four operational skills. Transferring the knowledge gained in mathematics classes into skills and using them to solve daily problems start in earlier years of school life (Baykul, 2009).

In daily life, the four operations (addition, subtraction, division and multiplication) are generally done through four types of calculation methods: calculation via technological tools, written calculation, estimation calculation and mental calculation. In everyday life, estimation and mental calculation are preferred over others (Van De Walle et al., 2016). In practical solutions to an everyday problem, mental calculation or estimation can be done quickly without the need for paper, pen or a technological tool. There are examples showing that a series of algorithms should be used in teaching basic mathematical operations to elementary students (Baykul, 2009). Children usually learn these basic rules through memorization. However, this method is extremely ineffective as it ignores the principle that numbers are changeable. For example, teaching children to write 24 and place two zeros after it to multiply 24 by 100 is not useful. If students discover this method themselves, they will have sensed a structural feature of the number system (Busbridge & Özçelik, 1997). Also, this method does not offer a framework to help students organize the information that needs to be learned (Tertemiz, 2017). Children will be able to solve everyday problems quickly, accurately and easily by mastering the ability to calculate mentally.

Mental Calculation and Number Sense

The main purpose of mathematics instruction is to equip students with certain essential skills which enable them to learn rather than simply become loaded with information (Olkun & Toluk Uçar, 2018). Number sense plays an important role in the development of mental calculation skills in children. Students arrive at primary school and even kindergarten with various feelings, thoughts and knowledge about numbers. Over time, these feelings, thoughts and knowledge develop and become enriched through formal education or experiences (Barrera-Mora & Reyes-Rodriguez, 2019; Whitacre & Nickerson, 2008). People's knowledge of numbers, operations and their relationships with each other, and their ability to use this information easily in mathematical and everyday problems are directly related to their number sense (Yang et al., 2009; Yılmaz, 2017). According to Hope (1989), number sense is the feeling of being able to make logical predictions with numbers, to perform mathematical operations mentally, to notice number patterns and to choose the most effective solution to a problem involving numbers.

Even though it is difficult to define number sense due to its structure, the components that are effective in the four operations may be listed as follows: knowledge and skills about numbers (noticing the order among numbers), understanding the multiple representations of numbers and feeling their relative size, knowledge and skills about operations (understanding the effects, properties and relationships of operations), using knowledge and skills about numbers and operations in calculations (realizing that the solution may be found in different ways), understanding the meaning and effect of operations (understanding how different operations dictate the results), flexible calculation and counting strategies (mental calculation using knowledge of numbers and operations), being able to judge whether a calculation result is reasonable (judging by calculating by developing mental strategies), numerical estimation (finding the estimated result by rounding the number or using numbers flexibly) (Greeno, 1991; Hope & Sherrill, 1987; Reys et al., 1999; Reys et al., 1982; Şengül and Gülbağcı Dede, 2014; Yang, 2003; Yang and Tsai, 2010). Based on these, it may be stated that the skill of successful mental calculation both indicates and necessitates the presence of number sense so that students can develop their own strategies instead of learning standard rules of mathematics (Henry & Brown, 2008). In sum, children should be encouraged to discover various alternative strategies, such as guessing, mental rounding, knowing the meaning and size of numbers, doing operations, thinking flexibly and using different strategies, instead of depending at all times on rule-based algorithms or a pen and paper. Therefore, developing number sense will help a student to predict the answers to everyday mathematical problems and calculate mentally.

Mental Calculation and Mathematical Reasoning

A number of principles play an important role in the acquisition of the four operational skills. The most important is that these skills are based on advanced counting skills. In addition, structuring numbers (pattern clusters, 1 less/more, 2 less/more considering 5 and 10), which is included in the acquisition of the number concept, is particularly effective in the development of both number sense and mental operation strategies in children. The important thing is to perform multiple operations and see the strategy that suits the child. The teacher should therefore introduce many strategies and focus on the ones that their students find better. An individual's accumulation of knowledge is important in using strategies. The teacher should patiently allow the children to try out different strategies in the four operations and encourage them to discover still different ones, thus enabling meaningful learning. In-class practice should be used as an opportunity to find patterns and relationships (Tertemiz, 2017).

Other principles that are considered fundamental to understanding mathematical operations can be summarized as follows: Harniss et al., (2002. Cited in: Cockburn, 2005) list the principles that form the basis for understanding operations: (1) Place value of a number: The position of a number gives information about the value of the number in question. (2) Expanded notation (representing with signs): Reduction of numbers to digits (for example, the number 437 consists of four 100's, three 10's, and seven 1's), (3) Commutative property: The order of the numbers in an equation does not affect the result (for example, 8 + 7 = 7 + 8).

While this is valid in addition and multiplication, it is not so in subtraction and division. (4) Associative property: The grouping of numbers in an equation may be changed without affecting the result. For example, (8 + 7) + 4 = 8 + (7 + 4). On the other hand, such features are only valid in addition and multiplication. (5) Distribution properties: The distribution of numbers in an equation can be changed. For example, 7 x (8 + 4) = (7 x 8) + (7 X 4), (6) Equality: The number on one side of the "=" sign is equal to the number on the other side. According to Cockborn (2005), these principles affect all basic mathematics topics and are essential to their understanding.

In addition, practicing basic combinations when starting four operations instruction allows students to make sense of these combinations without memorizing them. For example, it is more important to understand that "7+6=13" and see the relationships between the numbers (such as 6+6+1=13 or 7+3+3=13) than to complete the operation accurately. However, when the child goes through this basic process mechanically, conceptual understanding does not occur. For another example, focusing on memorizing rules (such as 7x0=0 or 7x1=7) deprives children of mathematical competence. It also prevents children from contemplating their answers (Baroody, 2006). All these principles play an important role in mentally estimating results and checking the results in written operations and the problem solving process.

Mental calculation of the four operations needs to be considered together with mathematical thinking and reasoning skills. Reasoning is an important building block in learning mathematics, making sense of concepts and relationships, applying mathematics and defending ideas (Köse, 2016; Viseu, et al., 2021). Reasoning is the process of arriving at a rational conclusion by thinking through all aspects of a situation. Those who can reason on a subject have sufficient knowledge on that subject. They can compare their existing knowledge with new situations, examine all their dimensions, discover, make logical predictions, explain the reasons for their thoughts, reach conclusions, and explain and defend them (Ulu & Özedemir, 2018; Umay, 2003). To examine the student's reasoning process, it is necessary to ask questions such as "How did you reach this conclusion?" or "What did you focus on to reach this conclusion?" rather than to look at the accuracy or inaccuracy of the answer to a question. Or it is necessary to ask questions regarding the process incomplete or incorrect word choice. With these kinds of questions, adults gain insight into children's reasoning processes (Piaget, 2011). Mathematical skills support one another and are learned and developed as they are used. It is therefore not possible to evaluate reasoning independently from prediction, or mental calculation skills independently from number sense. Each is a skill used to improve and strengthen the other (Olkun, 2005; Ozsoy, 2012).

It is very difficult to acquire mental calculation skills in a few class sessions. Several techniques may be suggested to acquire this skill, but there is no standard teaching method or algorithm (Baykul, 2009; Van De Walle et al., 2016). Students may develop their own strategies after studying various examples. In order to develop these strategies, the first step is to know what they are. There are operations-specific strategies for mental addition, subtraction, multiplication and division. The present study is limited to mental addition operations. This is because mental addition is considered to be the foundation for the learning of other mental calculation skills (subtraction, multiplication and division) and to affect future processes.

Mental Addition Strategies

Among the four operations, the first one to be tackled is "addition" as it serves to teach and supports the other three operations (Liu et al., 2019). For example, multiplication is taught through repeated addition, and subtraction is taught through finding the unknown addend. Acting as a foundation for other operations, addition is affected by students' problem solving experiences, working memory capacities, age or schema automation (Imbo et al., 2007; Arnaud et al, 2008). This suggests that the mental addition strategies of individuals who are negatively affected by these variables remain weaker, or that individuals who are positively affected can develop a number of different strategies.

Baykul (2009) categorizes mental addition strategies under three main headings: "adding by taking advantage of the change and merge feature of addition", "adding by breaking the numbers down", and "adding the same number to one addend and subtracting it from the other". In general, since it is easier to add small numbers to large numbers than vice versa, whenever there is a larger number after a small number in an addition problem, the operation may be made easier by changing the places of numbers by using the change feature. Such addition problems may be facilitated by using the strategy of "adding by using the change and merge feature". In some cases, especially when adding one and two-digit natural numbers, breaking down the numbers in appropriate ways facilitates addition. This is known as the "adding by breaking the numbers down" strategy. Based on the characteristics of numbers, the breaking down may be done by using one of the following strategies: Completing to 10 or 100, making use of subtraction, breaking numbers down by multiples of 10, adding the digit values of numbers, adding by counting, and adding by rounding one of the numbers to 10. When the same number is added to one of the addends and subtracted from the other, the result of the addition does not change. This feature of the addition operation is known as the strategy of "adding the same number to one addend and subtracting it from the other". This strategy requires determining a suitable number to add to and subtract from the given numbers.

Strategies for mental addition have been categorized differently by different researchers. In the present study, the classification of Reys et al. (1995; p.310) was used for its comprehensiveness. The categorization is shown in Table 1 below.

As can be understood from Table 1, there are various strategies to be used in developing students' mental addition skills in mathematics classes. Also, mental addition has a two-way relationship with number sense, mathematical reasoning, and prediction skills.

Related Studies

When studies on mental calculation are examined, it can be seen that the topic has mostly been addressed in studies on number sense. Previous studies have concluded that there is a relationship between mental calculation skill and number sense in children (Heirdsfield, 2001; Heirdsfield, 2002), and that education can improve children's estimation and mental addition skills. Yazgan et al., (2002) and Siegler and Booth (2005) have stated that learning environments that encourage free exploration enable students to discover the relationships between numbers and operations, while McGuire et al., (2012) and Yang and Li (2013) noted that flexible mental calculation, estimation and decision-making skills about numerical values are dimensions that strengthen number sense. Regarding the topic of the present study, Aydın Güç and Hacısalihoğlu Karadeniz (2016) found that the most common strategy used by 5th graders during mental addition is the strategy of adding by using the change and merge feature of addition, and the least commonly used one is to break the numbers based on multiples of 10. Similarly, Duran et al., (2016) reported that the most common strategy among middle school students was "adding by dividing into tens and ones" while the least used one was "breaking down the numbers by taking 10 as reference".

On the other hand, certain studies examining children's knowledge and skills on numbers and operations revealed situations where children faced difficulty. These included children tending to use procedural algorithm in operations due to poor number sense perception, having a lack of understanding in numbers and operations, being better at memorized rules, regressing in intuitive number comprehension due to excessive dependence on pen and paper calculations, making mistakes related to "0" in operations (such as 4x0x3=12), having problems with digit value, and mostly doing operations from left to right (Mastrothanasis, Geladari et al., 2018; Rogers, 2014; Singh et al., 2019; Yang & Sianturi, 2019). As shown by previous studies, the development of mental calculation skills cannot be considered separately from children's number sense, their skills and the instruction offered. The knowledge and skills involved in this process directly affect each other as well. For example, while advanced number sense affects how operations are done, operation skills affect understanding the relationships among numbers. Therefore, this study aims to determine students' mental addition strategies on the one hand, and to contribute to the development of their strategies, save them from finger calculations, and help them make an easy transition to mental calculation on the other hand. Another rationale for the study is the belief that "Starting from grade 2, strategies for calculating with whole numbers should be the focus so that children's attention, flexibility and calculation fluency can develop by understanding natural numbers, addition and subtraction" (NCTM, 2000: 35, cited in Buchholz,

Table 1.
The Strategies Expected in Mental Addition (The example of 79+26)

A. Breaking into tens and ones	
A1. From left to right (Tens first)	(70+20=90; 9+6=15; 90+15=105)
A2. From right to left (Ones first)	(9+6=15; 70+20=90; 15+90=105)
A3. Cumulative addition	(70+20=90; 90+9=99; 99+6=105)
B. Keeping one addend constant	
B1. First addend	(79+20=99; 99+6=105)
B2. Second addend	(26+70=96; 96+9=105)
C. Rounding one or both addends to multiples of 10 and adjusting	
C1. First addend	(80+26=106; 106-1=105)
C2. Second addend	(79+30=109; 109-4=105)
C3. Both addends	(80+30=110; 110-1-4=105)
D. Rounding both addends to multiples of 5	(75+25=100; 100+4+1=105)
E. Using paper-pencil strategy mentally	
F. Saroban's mental image	

2004). In this way, students will produce a number of interesting and useful strategies to solve problems (Buchholz, 2004). Determining the strategies used by second graders when adding mentally is also believed to be important as applying the same procedure on the same students the following year in grade 3 can show their longitudinal development.

Purpose

The first part of this study aimed to determine the mental addition strategies of students at the end of grades 2 and 3, while the second part aimed to determine the effects of mental addition strategy training on the diversity of students' strategies.

In line with this general purpose, the following sub problems were studied:

- What strategies do students use when adding mentally at the end of grade 2?
- What strategies do the same students use when adding mentally at the end of grade 3?
- Which strategies do 3rd graders use when adding mentally after mental addition strategy training?
- Do mental addition strategies of 2nd and 3rd graders change from one year to the next, and before and after training?

Method

Study Design

The mixed-methods sequential explanatory design was used in this longitudinal study. In this design, the qualitative aspect of the study is applied first, and the quantitative research process is undertaken according to the results obtained. As it is sequential, the design does not allow the favoring of the quantitative or qualitative method over the other (Bowen et al., 2017; Ivankova et al., 2006). Cresswell and Plano Clark (2011) state that the results obtained from the qualitative data in this design determine the design of the subsequent quantitative dimension. The reason for conducting the study longitudinally is that it provides the opportunity to collect in-depth information about the same person or group by examining the change in the study group or variables over time, starting from a prespecified starting point until the study is complete, continuously or at certain intervals. Although the units followed in longitudinal studies are generally small in number, they yield deeper and more comprehensive information compared to other studies (Karasar, 2014).

The qualitative method was used when determining the mental addition strategies of the same 2nd and 3rd graders in the first part of the study as this method allows to explain the meanings and relationships from part to whole by questioning human problems via techniques such as observation and interview (Neuman, 2014, p. 634). The qualitative research method of case study was preferred as it provides the opportunity to examine a situation or a specific event at depth (Merriam, 1998).

In the second part of the research, after it was concluded that "no significant change occurred in the mental addition strategies of children who moved from grade 2 to 3", a training program was implemented to develop the mental addition strategies of 3rd graders, and the single group pretest-posttest experimental design was used to determine its effectiveness. In this design, the effect of the experimental process is tested via studies performed on a single group. The measurements of the subjects related to the dependent variable are obtained via a pretest prior to the study and a posttest afterwards, by using the same subjects and the same measurement tools (Büyüköztürk et al., 2015). The symbolic representation of the pattern is given in Figure 1.

Figure 1.
Single Group Pretest-Posttest Design

Group	Pretest	Process	Posttest
G	O_1	X	O_2

Study Group

The study group comprised a total of 16 second graders from an elementary school located in the Keçiören district of Ankara, who then moved on to the 3rd grade and remained in the same section. The study group was determined by using the non-random sampling method known as "Convenience Sampling". As the researcher in this method chooses a situation that is close and convenient to access, it adds speed and practicality to the study (Yıldırım ve Şimşek, 2016). Of the participants, 8 were male (S-1, S-2, S-3, S-4, S-5, S-6, S-7, S-8) and 8 were female (S-9, S-10, S-11, S-12, S-13, S-14, S-15, S-16).

Data Collection Tool

In order to determine students' mental addition strategies, 8 questions which would allow the use of different strategies were prepared. The questions were at an appropriate level for grades 2 and 3, and in line with the objectives stated in the Elementary Mathematics Curriculum (MEB, 2018). The objectives, their scope and sample questions are given below. The mental addition strategies offered by Reys et al., (1995) were taken into consideration as the questions were prepared, and care was taken to stay within the scope of the objectives. At every stage of the research, students were asked eight additions. In the preparation of the questions, the learning outcomes given in Table 2 were taken into consideration. In the

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study limited to learning outcomes, eight questions in three different forms each, which were similar to sample questions, were prepared. While preparing the questions, 4 questions for each learning outcome and eight questions in total for 2nd grade were asked. As for 3rd grade, at least two questions and eight parallel questions for each in total were prepared for each learning outcome.

Data Collection

Data in the first part of the study was collected at the end of grade 2 and the end of grade 3 from the same students, while data in the second part was obtained in the same way at the end of grade 3 after the experimental process.

In the evaluation of cognitive and upper cognitive strategies, mostly verbal reporting techniques are used. Since the application of think aloud protocols are realized within a learning activity (as an example, text reading or solving a mathematics problem), the fact that participants are informed in detail about mental strategies and that information is not lost is pointed out as the most significant advantage. Think aloud learning process is realized with student in three stages. In the first stage, the operator first explains the purpose of the study, informing about why think aloud techniques is a good way to understand how students solve a mathematical problem (Özkubat & Özmen, 2018). In the current study, the operator explained the purpose of study to the students and told them how to do think aloud protocol as follows: "I am interested in how you do additions mentally (without using pen and paper); therefore, I will ask you processes respectively so that you can do additions and listen to you how you do these processes. How you do is of importance for me, so I will use this recording device in order to be sure that you do not forget anything you say". In addition, the students were asked the question of "How did you find it?" after each answer in order to determine the first strategy coming to their mind while doing the processes in a clear and correct way. Following the introduction given, the operator becomes a model in the think aloud process about a mathematical operation in the second stage. In this process, operator thinks aloud over a mathematical operation by exhibiting such behaviours as self -questioning, self-instruction and self-monitoring (Özkubat & Özmen, 2018). In this study, the operator similarly became a model to the students once for each. At the last stage, the operator completed the training part of think aloud protocol by supporting the suitability and understandability of the tone of voice of the student during the process of mental aloud operation.

Experimental Process

During the experimental process, several "Mental Addition Strategies" were taught to 3rd graders 5 hours weekly for 5 weeks. These strategies were selected by taking into account the "Strategies Expected to be Used in Mental Addition" by Reys et al., (1995) and the 3rd grade addition objectives listed in the Elementary Mathematics Curriculum. The strategies included in the program were as follows:

Table 2.
Objectives of the Elementary Mathematics Curriculum and Sample Questions

Objectives and Scope	Sample Question
M.2.1.2.4. Can do mental addition.	
Mental additions are done with the number 10 and natural numbers that are multiples of 10 whose maximum sum is 100.	40+50=?
After this, two natural numbers whose sum does not exceed 50 are added mentally.	24+24=?
M.3.1.2.4. Can do mental addition.	
Mental additions are done with two two-digit numbers whose sum does not exceed 100;	76+21=?
A three-digit number and a single digit number;	283+7=?
A two-digit multiple of 10 and a three-digit multiple of 100.	800+70=?

Table 3.
Mental Addition Strategies Used in the Study and Sample Questions

Mental addition strategies	Sample Questions
Using paper-pencil strategy mentally	24+24=? 4+4=8; 2+2 =4; 48
The strategy of rounding both addends to multiples of 5	34+47=? 35+45= 80; 80-1+2= 81
The strategy of rounding one addend or both to tens	39+43=? 40+40= 80; 80-1+3= 82
The strategy of breaking into tens and units	62+24=? 60+20=80; 2+4= 6; 80+6= 86
The strategy of keeping one addend constant	76+21=? 76+20=86; 86+1= 87

For the instruction of each strategy within the scope of the study, prerequisite learning (skip counting and addition skills, etc.) was first ensured, lacks were amended and mistakes were corrected (Koç, 2018). After the preliminary work, each strategy was taught with the following learning-teaching cycle designed by the researcher teacher and a field expert academic.

Present the problem: The students were presented with a problem in line with the targeted strategy. Get the student to discover: This stage involved the discovery of the targeted strategy by the students. To ensure this, clues and guidance were offered. Recognizing the clues that students need in this stage and guiding them with the right questions require experiences teachers who know their students well. The clues sometimes involved asking questions with smaller numbers, which might be the simplest representation of the strategy, and at other times they involved the transfer of a previously learned topic to the learning environment.

Name the discovered strategy: Students named the strategy as a whole class. They were asked to engage in instructional strategies such as brainstorming, discussions, group work, etc. to express their views. What is important here is to guide the students in choosing a name that matches the nature of the strategy. For instance, the strategy of separating into tens and units may be given similar names such as the strategy of separating into digits, units first-tens later strategy, or tens first-units later strategy and so on.

Reinforce: This aimed to enable the students to fluently use the learned strategy in different questions and problems. By this stage, the strategy is already discoveredi and named commonly by the whole class. The aim of this stage may also be achieved in different ways according to the physical characteristics of the classroom or different student traits. To illustrate, according to the strategy where discovery happens through asking a question to the class, students were asked to solve the problem in small groups and then explain it to other students. This continued until each student was able to ask a question. The students were asked to use this strategy, add mentally, and check from a calculator. In order to explain the use of this strategy to their classmates, the students designed games and competitions and made videos, or had to teach the newly learned strategy to two people from their families. The cycle was repeated until all strategies were reinforced. After the experimental process (in the second phase of the study) think aloud protocols were held once again with 16 students, and these interviews were audio-recorded to avoid loss of data. The recordings were played to identify the strategies that students chose to use.

Data Analysis

In data analysis, interview recordings were played and the strategies preferred by students were coded in line with the table of expected strategies for mental addition developed by Reys et al. (1995). The findings were supported with direct quotations.

The following measures were taken to ensure validity and reliability, as suggested by Erlandson et al., (1993, cited in Yıldırım and Şimşek, 2016, p. 277-283).

Table 4.
Measures for Validity and Reliability

Internal Validity (Credibility)	Long term interaction In-depth data collection Expert examination Participant confirmation	While presenting the data, sample quotations for the answers of the students were given place. During these quotations, it was paid attention that the samples were comprised of the views reflecting the general answers.
External Validity (Transferability):	Using direct quotations Detailed description of the data collection tools, environment and stages of the study	In order to increase the possibility of being transferability, detailed descriptions were given place. In other words, data collection process, selection of participants and how the obtained data was analysed was explained in detail.
Internal Reliability (Consistency):	Checking validity (Examination of all stages of the study by an outsider) Two researchers acting together in all stages of the study	To obtain the consistency, the answers given by the students were firstly coded independently by two researchers and then, different codes were determined by coming together, and the discussion was carried on until getting a consensus. The codes obtained were presented by enumerating with repetition frequency.
External Reliability (Confirmability):	Offering explanations about data collection and analysis methods Checking confirmation (comparison of raw data with researcher's interpretations)	The data set was recorded in different times and controlled, so the confirmability of the research was tried to be obtained.

Findings and Interpretations

In this section, the methods that students use while adding mentally are presented under subtitles that consist of the research questions.

The First Stage of the Study: The Strategies Used by Students When Adding Mentally At the End of Grade 2

In order to determine second graders' mental addition strategies, an answer to the following question was sought: "What strategies do 2nd graders use when adding mentally?" Eight questions were asked to 16 students each. All students were observed to answer correctly after being given enough time. The findings are presented in Table 5.

The voice recordings of the students regarding their mental additions were analyzed with respect to the five strategies. Based on how frequently they were preferred in the present study, these strategies can be listed as follows from the most commonly preferred to the least:

"Using paper-pencil strategy mentally" was preferred 82 times and became the most preferred strategy. The second most preferred strategy, "rounding one or both addends to a multiple of 10" was used 19 times. This was followed by the strategies of "keeping an addend constant and breaking it into tens and ones", both of which were preferred 13 times. Among those who preferred to keep an additive constant, almost all preferred to keep the first addend. In the strategy of breaking up into tens and ones, almost all students started from the right that is by adding the ones first.

The least preferred strategy was "Rounding both addends to a multiple of 5", preferred by only one. Also, a general look at the table shows that while half of the students preferred to use 3-4 different types of strategies, the other half preferred to use only one.

The Strategies Used by Students When Adding Mentally at the End of Grade 3

In order to determine the same students' mental addition strategies a year later in grade 3, an answer to the following question was sought: "What strategies do 3rd graders use when adding mentally?" Eight questions were asked to 16 students each. All students were observed to answer correctly when given enough time. The findings are presented in Table 6.

Table 5.

Mental Addition Strategy Preferences of 2nd Graders

Grade: 2	Breaking into Tens and Ones			Keeping One Addend Constant		Rounding One or Both Addends to Multiples of 10			Rounding Both Addends to Multiples of 5	Using Paper-Pencil Strategy Mentally	Total Number of Questions Answered
Participant	From left to right (tens first) f	From right to left (ones first) f	Cumulative addition f	First addend f	Second addend f	First addend f	Second addend f	Both addends f	f	f	f
S1				5		1	2				8
S2				2	1	1	1	2	1		8
S3	1	2						1		4	8
S4						1	1	2		4	8
S5										8	8
S6				3						5	8
S7		4								4	8
S8		4								4	8
S9										8	8
S10										8	8
S11				2		1				5	8
S12	1	1				1	1	4			8
S13										8	8
S14										8	8
S15										8	8
S16										8	8
Total	2	11	0	12	1	5	5	9	1	82	128
Overall Sum			13		13			19	1	82	128

Table 6.
Mental Addition Strategy Preferences of 3rd Graders Prior to Training

Grade: 3	Breaking into Tens and Ones			Keeping One Addend Constant		Rounding One or Both Addends to Multiples of 10			Rounding Both Addends to Multiples of 5	Using Paper-Pencil Strategy Mentally	Total Number of Questions Answered
Participant	From left to right (tens first) f	From right to left (ones-first) f	Cumulative addition f	First addend f	Second addend f	First addend f	Second addend f	Both addends f	f	f	f
S1	1	1		1		1	1			3	8
S2	1	1								6	8
S3		1		2						5	8
S4	1	1							1	5	8
S5	1			3						4	8
S6	1			1		1	1			4	8
S7	2	1								5	8
S8	1	1		3				1		2	8
S9	2	1		1						4	8
S10	2	1		1	1		1			2	8
S11						1		4		3	8
S12			1				1			6	8
S13		2				1		2		3	8
S14		2						1		5	8
S15						1		1		6	8
S16					2			1		5	8
Total	12	12	1	12	3	5	4	10	1	68	128
Overall Sum			25		15			19	1	68	128

As can be understood from Table 6, it was found that the most commonly used mental addition strategy among 3rd graders was "using paper-pencil strategy mentally". This strategy was preferred 68 times. In other words, the students verbally stated the sequence of procedures that they applied on paper with the help of a pen. The second most preferred strategy, "breaking it into tens and ones", was preferred 25 times. The third most commonly preferred strategy was "Rounding one or both addends to a multiple of 10", used 19 times. The fourth most frequently used strategy was that of "keeping an addend constant", used 15 times. Among the sub-strategies of this basic strategy, "keeping the first addend constant" was preferred 4 times more than "keeping the second addend constant". The least preferred strategy was "rounding both addends to multiples of 5". It was chosen by one student only once.

When Tables 5 and 6 are examined together, it can be seen that the most preferred mental addition strategy in both grade levels when the students passed from grade 2 to 3 was "using paper-pencil strategy mentally". The strategies of "rounding both addends to a multiple of 5" and "rounding one or both addends to a multiple of 10" were used the same number of times in the two grade levels. This was also the case with the strategy of "keeping an addend constant". On the other hand, the strategy of "breaking into tens and ones" was preferred more in the 3rd grade.

The Second Stage of the Study: The Strategies Used by 3rd Graders When Adding Mentally After Mental Addition Strategy Training

The third sub-aim of the study was "Which strategies do 3rd graders use when adding mentally after mental addition strategy training?" Eight questions were asked to 16 students each. All students were able to answer correctly after being given enough time. Table 7 presents the findings.

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Table 7.

Mental Addition Strategy Preferences of 3rd Graders after Training

Grade: 3	Breaking into Tens and Ones			Keeping One Addend Constant		Rounding One or Both Addends to Multiples of 10			Rounding Both Addends to Multiples of 5	Using Paper-Pencil Strategy Mentally	Total Number of Questions Answered
	From left to right (tens first)	From right to left (ones first)	Cumulative addition	First addend	Second addend	First addend	Second addend	Both addends			
Participant	f	f	f	f	f	f	f	f	f	f	f
S1	1			1	1		1	1	2	1	8
S2	1		1		1	1	1		1	2	8
S3	1	1		1		1	1		2	1	8
S4		2		1		1		2		2	8
S5				1	1	1		2	1	2	8
S6		1	1	1		1		2		2	8
S7	1					2	1	2	1	1	8
S8	1			1	1		1	2	1	1	8
S9	1		1		1	1	1		1	2	8
S10	1					2	1	1	2	1	8
S11	1		1	1	1	2	1			1	8
S12	1			1	1	1	1	1	1	1	8
S13	1			1	1	2		2		1	8
S14	2	1		2		1		1		1	8
S15	1	1		1		1		1	1	2	8
S16	2			1	1	2		1		1	8
Total	15	6	4	13	9	19	9	18	13	22	128
Overall Sum			25		22			46	13	22	128

From a general perspective, Table 7 shows that the diversity of strategies that students were using increased after mental addition strategy training. In other words, it can be said that students could use various strategies. It was seen that the most commonly preferred mental addition strategy (46) was "rounding one or both addends to multiples of 10". The second most popular strategy (25) was "breaking into tens and ones". The two strategies that were preferred in medium frequency were "keeping an addend constant" and "using paper-pencil strategy mentally". A decrease may be seen in the use of the latter strategy. The strategy of rounding both addends to a multiple of 5 was used 13 times. Although this strategy seems to be used relatively less than others, it must be noted that this strategy, which was preferred only once by one student in the two think aloud protocols before the experiment, was preferred 13 times after it.

Comparison of Students' Mental Addition Strategies from Grade 2 to 3 With Respect to Grade and Training

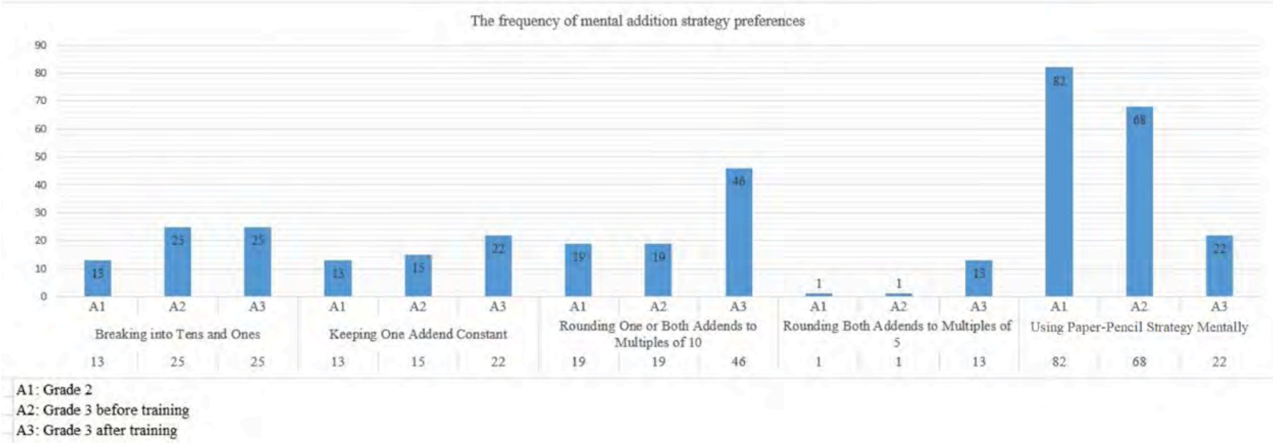
The 4th sub-problem of the study was "Do mental addition strategies of the same 2nd and 3rd graders change from one year to the next, and before and after training?" Chart 1 presents the frequency of mental addition strategy preferences by the same students in grades 2 and 3, based on grade level, and before and after training.

The chart above shows the frequency of mental addition strategies preferred by the same students in grades 2 and 3 before and after strategy training.

The most striking change that can be observed in the chart is the fall in the strategy of "using paper-pencil strategy mentally" from 82 to 68 and to 22. While the strategy of "rounding the addends to multiples of 5" was preferred only once by one student before the experiment, it was preferred 13 times afterwards. While the strategy of "rounding the addends to multiples of 10" was preferred 19 times before the experiment, it increased significantly to 46 after the application. The strategy of "breaking into tens and ones" was chosen the same number of times, 25 before and 25 after the mental addition training. While the strategy of

Chart 1.

The mental addition strategies of the same students in grades 2 and 3, and their strategies with respect to grade level and training



"keeping an addend constant" was preferred 15 times before the experiment, it was preferred 22 times after it. The overall facts shown in the chart; namely, the significant decrease in the pen and paper strategy, the 13-fold increase in rounding to multiples of 5, and the significant increase in rounding to multiples of 10, may point to the effectiveness of the mental addition strategy training.

Sample Student Answers

This section gives examples of how 2nd graders and 3rd graders before and after the experimental procedure solved the mental addition questions asked to them.

Sample 2nd grader strategies

A student who mentally computed 24+30 (S2), stated that the sum was 54. After this answer, the following dialogue took place between the researcher and the student:

Researcher: How did you reach this sum?

S2: I saved 30. I added 20 to it to reach 50. Then I added 4 and reached 54.

Researcher: Why didn't you save 24 but 30?

S2: As 24 is a smaller number, it's easier to add it to 30.

(The strategy used: Keeping one addend constant)

As can be seen, the student chose to keep one of the addends constant and added the other one. He explained his reason for keeping the second addend constant instead of the first one as the former being larger, and the smaller number being easier to add.

Another student who mentally computed 32+12 (S7) found the sum as 44. Following this answer, the dialogue below took place between the researcher and the student:

Researcher: How did you find this result?

S7: I added 30 and 10 and reached 40. Then I added 2 and 2 and reached 4. Finally, I added 4 to 40 and reached 44. (The strategy used: Breaking into tens and ones)

Sample 3rd grader strategies before mental addition strategy training

The following dialogue took place between the researcher and a student who mentally added 67+16 (S12) and answered 83:

Researcher: How did you reach the answer 83? Will you tell me step by step?

S12: I first rounded 67 to 70. Then I added 70 and 16.

Researcher: OK fine, how did you add 70+16?

S12: As 70 is larger, I kept it and added 10 to it to reach 80. Then I added 6 and the result was 86.

Researcher: You said it was 83. Then what did you do?

S12: At the beginning, I had increased the number by 3 when I rounded 67 to 70. Therefore, I need to subtract 3 from the sum. Three minus 86 is 83.

(The strategy used: Rounding one or both addends to a multiple of 10)

Sample 3rd grader strategies after mental addition strategy training

A student who mentally computed 283+8 (S5) answered 291, and the following dialogue occurred between this student and the researcher:

Researcher: How did you reach 291? Will you explain step by step?

S5: I accepted 283 as 285. So I increased the number by 2. I must remember this. I rounded 8 back to 5. Here, I decreased the number by 3. I'll remember this, too.

Researcher: Why did you do these?

S5: Because it's easier when the numbers are rounded to multiples of 5.

Researcher: OK, what did you do afterwards?

S5: Now I have 285 and 5. If I add them up, the result is 290.

Researcher: Fine. Is the operation finished now?

S5: No. As I had increased the number by 2 at the beginning, I need to first subtract 2. But I also took off 3 from the other addend, so I'll add 3.

Researcher: Well done! Go ahead.

S5: First I will add 3. If I add 3 to 290, I get 293. When I subtract 2 from 293, then the answer is 291. (The strategy used: Rounding both addends to a multiple of 5)

Results Discussion and Recommendations

The aim of this study was to determine the mental addition strategies used by the same students while attending grades 2 and 3, and to develop strategies they can access with mental addition strategy training implemented at the end of grade 3. In this section, the results derived from the findings of the sub-problems are discussed and several recommendations are made.

In the first and second sub-problems of the study, the mental addition strategies used by 2nd and 3rd graders were determined. The results showed that almost half of the students in grade 2, which was discussed in the first part of the study, and half of those in grade 3 could only use the strategy of "using paper-pencil strategy mentally" before the training. In other words, they could verbally express the algorithmic sequence of operations they applied with pen and paper. This result reveals that, when adding mentally, students often employed the algorithmic strategies they learned at school rather than alternative strategies. Corroborating these findings, Mastrothanasis et al., (2018) studied 80 students at grades 3, 4, 5 and 6 to determine their mental addition and subtraction strategies, and found that most could only mentally perform algorithmic operations with the help of pen and paper, and could not use more complicated strategies effectively. Yang and Sianturi's (2019) finding that children tend to do mostly operations from left to right was similar to the result of a study on secondary school students conducted by Singh et al., (2019). Huang and Yang (2018) studied the strategies used by 4th graders when solving questions necessitating number sense, and found that children used rule-based strategies, but middle and low-level children were more inclined to use these strategies. However, in a noteworthy study by Duran et al., (2016) focusing on the mental calculation strategies of middle school fifth graders when adding and subtracting, it was found that students did not use any strategy correctly and could not answer the questions.

Another result of the present study is that, when the same students at grades 2 and 3 performed mental addition prior to the training, they used the strategy of "rounding both addends to multiples of 10" only at a limited level, and they did not know or use the strategy of "rounding to multiples of 5". This may be attributed to the fact that the course curriculum outcomes do not adequately emphasize rounding to multiples of 5 (MEB, 2018). Likewise, Aydın Güç and Hacısalihoğlu Karadeniz (2016) found via think aloud protocols with 25 fifth-graders that most of them did not use mental addition strategies, preferred conventional strategies when computing mentally, and tried to solve most questions they encountered by using the same strategies.

In the present study, third graders were observed to have improved their addition strategies by breaking into tens and ones. This result may be a sign that the concept of place value is better understood by 3rd graders and that children can transfer this knowledge to the process of mental addition. Kamii (1986) views the concept of tens as a hierarchical system built on the already existing concept of ones. However, it is stated that early teaching of the concept of digits may cause tens and ones to be constructed separately in children's minds. Even though it may be considered appropriate to teach the concept of digit in early grades (Fuson & Briars, 1990), it is important to consider children's readiness levels. Contrary to the findings obtained, Yang and Sianturi (2019) reported difficulties by children with place value in their study about number sense. Children have difficulty understanding that a number increases ten times when it is multiplied by 10, or decreases ten times when divided by 10 (Rogers, 2014). Another result obtained in the study is that the diversity of the mental addition strategies used by 2nd and 3rd graders did not vary significantly. Similarly, Rezat (2011) studied 8th graders and concluded that they could use a single type of strategy when mentally adding rational numbers. This result parallels previous findings regarding mental calculation strategy use with natural numbers.

When the mental addition strategies of 3rd graders before and after instruction were compared, a significant increase was observed in the number and variety of strategies used after the instruction. Similarly, Yazgan et al., (2002), in their 8-week study aiming at developing elementary 5th grade students' mental calculation and estimating skills, concluded that these skills in children could be improved through education. By the same token, Heirdsfield and Lamb (2005) concluded that instruction on mental addition and subtraction led to a decrease in the use of useless strategies and an increase in more complicated ones among some 2nd graders, and to an increase in the variety of strategies used by those who could already use complicated strategies at the beginning. In other words, the results obtained from previous studies suggest that the teaching of mental addition strategies can contribute to students. Also, considering "calculation by using flexible calculation and counting strategies", which is an important component of number sense, children are expected to be able to use the various operation strategies mentioned in the study. This also hints at the development of number sense in children (Reys et al., 1999; Yang, 2003).

When the results of the present study and those of the previous ones are considered together, it is obvious that opting to use standard methods alone in mental calculation skills training may cause students to use conventional algorithmic rules when performing mental operations. This raises questions about whether or not classwork focuses on developing mathematical reasoning and thinking skills or student generated strategies. Also worth noting is the belief that an operative level of mathematical knowledge on the teacher's side may influence teaching style. When listing the action principles of mathematics education, the NCTM (2020) emphasizes the complexity of the process and the importance of teacher quality for deep learning. For these reasons, a deeper examination of the process may be recommended.

It would be useful to mention about the two limitations of the current study. The first one is that the questions asked to the students were prepared within the content of the learning outcomes regarding mental additions taking place in the Elementary Mathematics Curriculum for the 2nd and 3rd graders. The other limitation is that it is only limited to "Strategies Expected to be Used in Mental Addition" by Reys et al., (1995). For that reason, it is likely to see which group the children will choose and why they choose this group by given flexible number groups (e.g., 37+4, 50+30, 45+27). In other words, whether the given numbers have an effect on the strategy selection can be investigated in detail. Whether mental strategy preferences of the students will change depending on numbers or different strategy teaching is a case to be considered. In addition, only addition was studied in the current research. Similarly, some other studies could be carried out into other mental processes (subtraction, multiplication, division) and and at different grade levels.

In conclusion, the classroom environment should be one where children can share their thoughts and discuss their own mathematical ideas so that their mental calculation strategies may be enriched (Van De Walle et al., 2016). Having a classroom environment where children are not afraid to make mistakes, develop new ideas, receive encouragement for different mental calculation ideas, and share and discuss their opinions would increase participation in class. It would therefore be beneficial for teachers to use the mental addition strategies mentioned in this study or others with similar characteristics in their classrooms. Finally, future researchers may be recommended to hold in-depth examinations of classroom practices and textbooks by considering to what extent they include mental calculation strategies.

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