Acta Didactica Napocensia

Volume 14, Number 2, 2021 - DOI: 10.24193/adn.14.2.25

Secondary/High School Mathematics Teachers' and Mathematics Teacher Candidates' Opinions about HTTM (History/Theory/Technology/Modeling) Learning Process

Çağlar Naci HIDIROĞLU

Abstract: The aim of this study is to investigate the opinions of the secondary and high school mathematics teachers and mathematics teacher candidates who have been educated about HTTM learning process and experienced HTTM activities in the classroom environment about HTTM learning process. The study is a basic qualitative research study conducted with a qualitative understanding. The sample of the study consisted of three secondary school maths teachers working in different secondary schools and three high school math teachers working in different high schools in Turkey during 2016-2017 academic year, five senior class students who are secondary school mathematics teacher candidates taking teaching practice course and applying teaching practices at different secondary schools and five high school mathematics teacher candidates applying teaching practices at different high schools. The data of the study is comprised of video analysis of the interviews conducted through semi-structured interview form with the secondary and high school mathematics teachers and mathematics teacher candidates. Content analysis technique was used in the research. As a result of the data analysis, a two-dimensional structure related to HTTM learning process was obtained. In the first dimension of the findings 15 themes were obtained. These are preparation, time, flexibility, interdisciplinary nature, contemporary skills, technology integration, history of science integration, working with groups, metacognitive skills, conceptual understanding, projects, problem posing, motivation, evaluation and teacher competency. In the second dimension of the findings; benefits, difficulties and suggestions related to 15 themes were put forward.

Key words: Mathematics teacher education, learning theories, mathematical modeling, educational technology, history of mathematics.

1. Introduction

Throughout life, learning processes of an individual take place in a variety of different environments. However, educators focus more on schools and classrooms as learning environments. There are many studies indicating that schools and classrooms are directly effective in students' learning process, developing mathematical skills and succeeding in life. Therefore, the design of effective learning environments in schools is an important goal for researchers and educators.

To ensure the outcome of the purpose of mathematics education and to produce critical and reflective thinkers who are prepared to solve problems in their lives, attention must be given to important and relevant mathematics (Brenner, 1998). Mathematical modeling, which is an important tool to achieve this, is the process of expressing a situation/event in real life mathematically and explaining it with the help of mathematical models (Berry & Houston, 1995; Blum & Niss, 1991). Mathematical modeling is an effective instrument in the process of assessing students (Borromeo Ferri, 2013; Hıdıroğlu & Özkan Hıdıroğlu, 2016; Peter Koop, 2009), and by being used as an activity/problem in the learning process, it provides students with a rich learning environment that includes a rich cognitive and metacognitive process (Borromeo Ferri, 2006; 2010; Hıdıroğlu, 2012; 2015; Hıdıroğlu & Bukova Güzel, 2013; 2014;

Received February 2019.

Cite as: Hidiroğlu, C. N. (2021). Secondary/High School Mathematics Teachers' and Mathematics Teacher Candidates' Opinions about HTTM (History/Theory/Technology/Modeling) Learning Process. *Acta Didactica Napocensia,* 14(2), 346-375, https://doi.org/10.24193/adn.14.2.25

2015; 2016; 2017; Lesh & Doerr, 2003; Maaβ, 2006; Stillman, Galbraith, Brown & Edwards, 2007) in order to develop the necessary knowledge and skills. Mathematical modeling ensures a richer learning experience as it embraces the aspect of doing mathematics (Burkhardt, 2006). Average learners are capable of forming powerful mathematical models and constructs: These conceptual systems that form the basis of these models are often more sophisticated than those they are currently taught in schools (Kaiser & Schwarz, 2006; Lesh & English, 2005).

1. 1. The Modeling Perspectives and the Place of HTTM in Modeling Perspectives

Due to some differences in researchers' philosophies, goals, subject areas and educational understandings, studies on mathematical modeling are handled in six categories: Educational (didactical and conceptual), theoretical [epistemological], realistic [applied], contextual, sosyo-critical ve cognitive-metacognitive modeling (Blomhøj, 2009; Kaiser, 2005; Kaiser & Sriraman, 2006). Although these six perspectives differ from each other in some issues, they play important roles in supporting each other in various points and thoughts (Hıdıroğlu & Özkan Hıdıroğlu, 2016).

(1) Realistic/applied modeling basically aims to develop problem solving and modeling skills. It draws attention to the use of mathematical knowledge in different contexts by making use of problem situations in engineering and other fields. (2) Theoretical (epistemological) modeling tries to reveal the mathematical concepts and theories in the solution of modeling problems and the relationship between these by giving secondary importance to real life context. (3) Socio-critical modeling focuses on the socio-cultural and ethno-math dimensions of mathematics. According to this approach, through maths teaching, students should be adorned with critical thinking skills that can be specifically used for their own society and cultural structure. (4) Contextual modeling deals with meaningful and complicated real-life situations that are far from artificiality. (5) Educational (didactical and conceptual) modeling can be considered as a kind of combination of realistic and contextual modeling approaches. In order to better serve the basic purpose in educational modeling, limitations can be made in learning environments and mathematical modeling problems (closed-ended problems). (6) Cognitive-metacognitive modeling focuses on students' mental actions in modeling process. In this process, it is crucial to analyze cognitive and metacognitive thinking processes and to determine their roles and relationships.

All these modeling approaches basically aim to ensure learning and to educate more successful individuals in real life. On the other hand, different points prioritised by each of them change their focus and cause them to give less importance to some essential issues. Effective learning environments can be designed by taking into account both these prioritised important points and the ones that are ignored, and by blending them effectively in the learning process (Hıdıroğlu & Özkan Hıdıroğlu, 2016). HTTM ***(H**istory**T**heory**T**echnology**M**odeling)* learning approach, which was developed with this consideration and with a holistic-pragmatist understanding, includes a rich and novel learning process with its four-dimensional and multi-stage structure.

1.1.1. HTTM Learning Perspective and the Dimensions of HTTM (History/ Theory/ Technology/ Modeling). HTTM learning model aims to develop concepts and modeling skills in a technology- and modeling-aided medium by utilizing the history of science and the theories in mathematics, physics, chemistry and biology during learning process. The most fundamental dimension of HTTM learning model is mathematical modeling (Hıdıroğlu & Özkan Hıdıroğlu, 2016; see Figure 1).

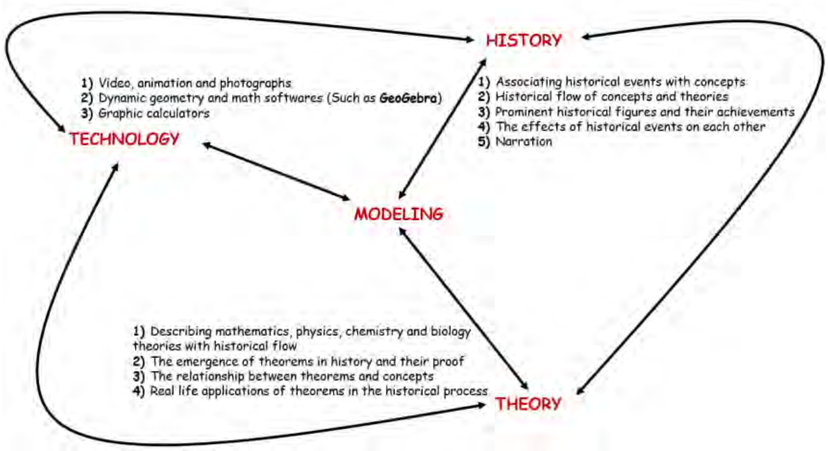

Figure 1. Dimensions of HTTM learning model

In *modeling* dimension of HTTM (1), different models such as physical, mental, simulation, mathematical, emergent and hypothetico-deductive models that emerge during the mathematical modeling process are of great importance in the simplification and elaboration of a theory, in the elucidation of a concept, in the association of concepts, in the discovery of deficiencies or misconceptions in conceptual development, and in the description of the relationships between real life and disciplines. (Hıdıroğlu & Özkan Hıdıroğlu, 2016). In the *history of science* dimension of HTTM (2), discovering the relationships between the scientific progress in the history of science and theories and concepts will provide a rich mental process both in the development of concepts and in the discovery of theories (Hıdıroğlu & Özkan Hıdıroğlu, 2016; Swetz, 1994). Piaget (1978) also proposed that historical evolutionary processes of concepts should be known in order to assure conceptual understanding. According to *technology* dimension of HTTM learning process (3); current technology should be actively used with the integration of dynamic math and geometry softwares, video, animation and photographs (Hıdıroğlu & Özkan Hıdıroğlu, 2016). In *theory* dimension of HTTM (4), the emergence of theories in the historical process and their interactions with the concepts are important. In this way, the importance of discoveries and inventions in history is taught and it is revealed how concepts and theorems are developed.

1.1.2. The Learning Cycle Model of HTTM (Components and Basic Steps). HTTM learning model consists of nine key components and eight key steps that connect these components to each other (Hıdıroğlu and Özkan Hıdıroğlu, 2016; see Figure 2).

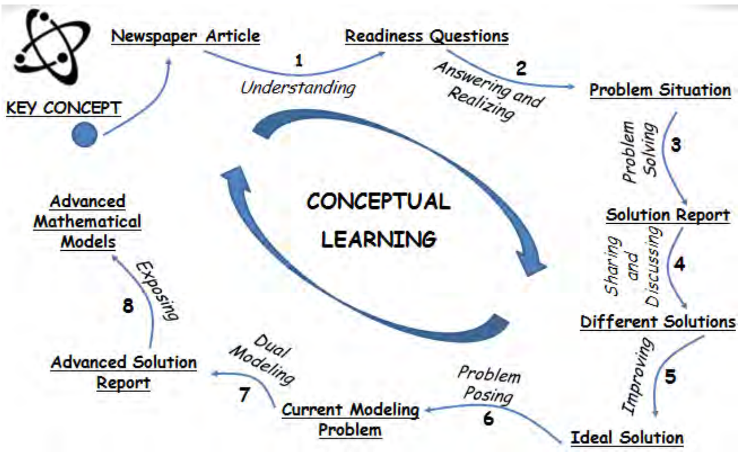

Figure 2. The learning process of HTTM model (Components and basic steps)

In HTTM learning process, *newspaper article* consists of the written texts including real life situations/events supported by the history of science. In newspaper article, prominent figures in the history of science, concepts, theories, theorems, events are presented through narrating in story form. *Readiness questions* in HTTM are the questions that enable students to understand and internalize the scenario in the newspaper article and reveal the problem situation. Problem situation in HTTM is a mathematical modeling problem. The problem handled should be open-ended and it should be structured according to the level of students. The problem situation can be adjusted so that it can be suitable for the level of each student. The problem situation can be of two different types: Holistic and atomistic problem situation.

In step 3 (see Figure 2), the technology-supported mathematical modeling process (individual or collaborative working) emerges (see Figure 3). The technology-supported mathematical modeling cycle consists of nine basic components (*complex real world situation, real world problem situation, model of real world problem situation, sub-mathematical models, main mathematical model/s, mathematical solution, real world solution, solution decision and solution report*) and nine basic steps (*problem analysis, constructing a systematic structure, mathematization, meta-mathematization, mathematical analysis, interpretation, validation, revising and reporting*). In addition, auxiliary components such as mathematical results, real world results, computer model/s and mental representation of the situation model play an important role in the technology supported mathematical modeling.

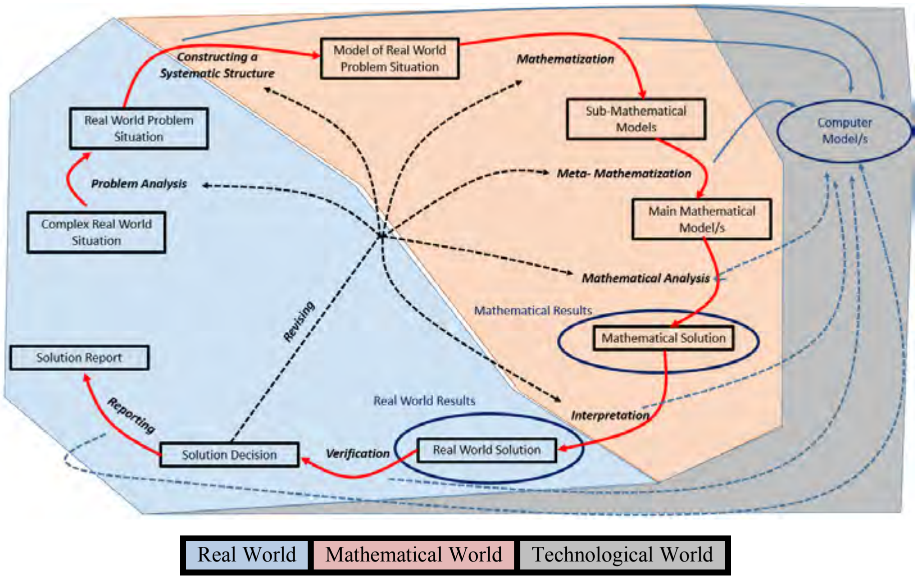

Figure 3. Technology-supported mathematical modelling cycle-Worlds, components and steps (Hıdıroğlu, 2015)

Solution reports in HTTM are the technology-aided written or visual reports that reflect the solutions of the student/student groups regarding the problem situation. In HTTM learning process, students generate many different models in the solution report (see Figure 4).

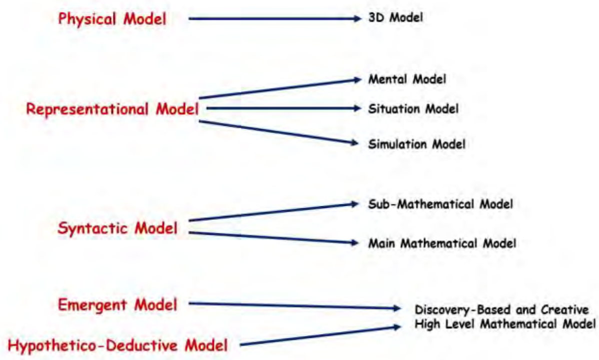

Figure 4. The different models revealed in HTTM learning process

In HTTM learning process, students reach their ideal solutions after discussing different solutions in the classroom environment. In the next stage, they design an updated modeling problem (i.e. that is important currently) with the help of what they have used and discovered in the solution. In HTTM learning process, by establishing associations between their solutions and the solutions of the problem they have designed, students give rise to dual modeling process (Saeki & Matsuzaki, 2013) that has taken it place in the literature. The most important features of problem posing and dual modeling process in HTTM learning process are involving students in high level conceptual learning process, developing their creative thinking skills and creating opportunities for them to develop innovative high level mathematical models. When the literature is reviewed, it is seen that two different expressions emerge for these high level mathematical models: Emergent or hypothetico-deductive mathematical models.

1. 2. Importance of the Study

Two of the most important goals of mathematics education are to make students aware of the importance of overcoming real-life problems and to enable them to learn how to use their mathematical and technological knowledge and skills effectively in this process (Clayton, 1999). The rapid development of technology and its increasing importance and effect on education (Hohenwarter, Hohenwarter, Kreis & Lavicza, 2008) make it important to consider the need for more quality dynamic math / geometry software and technological tools that can be used in education and the ways to use these tools more effectively in the learning process (Hıdıroğlu, 2015; Lingefjärd, 2000). According to Hıdıroğlu and Bukova Güzel (2014), thanks to technological software and tools students have the opportunity to compare the relationships between geometric and algebraic representations; examine mathematical or physical concepts through multiple representations; study interactively between the table, the algebra and the geometry; and make mental transitions between different fields thanks to these tools' acting as a bridge between geometry, algebra and analysis. In HTTM learning process, mathematical modeling problems arising from the history of science are dealt with the help of dynamic software (GeoGebra, Maple, Derive, Cabri, etc.). In addition to software, videos and animations that include high level of interaction provide opportunities for students to design problems, solve problems, observe the results by redefining conditions, create models, discover new relationships and features (Baki, 2002). National Council of Teachers of Mathematics [NCTM] (1998) also emphasizes not only how technology can best support mathematical learning, but also how the existence of technology will affect students' mathematical ability, different thinking, and conceptual skills. Accordingly, HTTM integrates different representations such as video (documentaries / films covering the history of science), photography (suitable for the problem case), 3D model / learning material (related to historical architectural structures or problem situation) into the learning process.

The history of science and mathematics is a critical resource for students to use cognitive, affective and psychomotor skills, to follow the emergence and development of concepts, to realize the solutions of problems by experiencing them and to discover different models enriched with visuals that enable them to learn the concepts inside (Berlinghoff & Gouvea, 2004; Mitchell, 1995; Reimer & Reimer, 1992; Swetz, 1994). According to NCTM (2000), mathematics education should be supported with learning environments enriched by the history of mathematics. According to Jankvist (2009) and Fried (2001), the problems selected from the history of science, various models used by mathematicians in history, the special situations they experienced, the life stories of the scientists, specşal periods from their lives and the effective pictures about them should be included in the curriculum. Jankvist (2009) emphasized that the history of mathematics and science can be handled as goals (whys) and tools (hows). In the learning process of HTTM, history of sciences and theories, as a tool, basically maintain the enrichment of the learning environments necessary for conceptual learning while they provide a broad perspective to the students on the content of history of science, its functioning and development of concepts as a goal. History of science and mathematics can focus on the problems and solutions of ancient mathematics scholars, reveal different possible solutions under limited conditions and present an important map of how to create a more effective cognitive-constructivist learning environment by considering the development processes of concepts (Siu, 2003; Swetz, 1994; Veljan, 2000). In this respect, the use of the history of mathematics and science in mathematics learning process is supported by many researchers (Bidwell, 1993; Ernest, 1998; Gulikers & Blom, 2001; Leng, 2006; Liu, 2003; Mcride & Rollins, 1977). At the same time, the history of mathematics presents the reasons behind the definitions, rules and formulas of mathematics. Historical problems and the responses given to these in history enhance mathematical thinking skills and indicate the development process of mathematical concepts (Baki, 2014). According to Piaget (1973); the concepts are more easily understood by children when they are ranked from concrete to abstract, from easy to difficult or from known to unknown. Similarly, Bettelheim (1976) and Egan (1991) argued that if the connections between real events and subject matters are more firmly established, the concepts can be understood more quickly and will not be forgotten for a long time. HTTM learning process includes important scientists in the history of science, civilizations, problem situations, the emergence and development process of theories, theorems and concepts in the learning process.

Gardner (2008) mentions five essential minds types for the future including the Disciplined mind, Synthesising mind, Creating mind, Respectful mind and Ethical Mind. The history of science and the intellectual development of science play a significant role for students to develop thoughts special to a certain discipline. However, the HTTM's potential to reveal different solutions, the process of group work and the stage of sharing the solutions are of great importance in increasing the respectful mind levels of students. HTTM highlights the importance of students' obtaining complicated and essential information from completely different sources (such as history of science, different student solutions / thoughts), their making sense of this information, using an objective perspective while evaluating their thoughts and integrating them in a meaningful way for themselves or their group.

In this sense, HTTM creates environments that will enhance the Synthesising mind skills. The problem posing and dual modeling process in HTTM enables students to present original questions / solutions and go beyond their existing knowledge and synthesis to develop new things. This will improve their creating mind levels. In addition, Gardner (2008) and Anderson & Krathwohl (2001) emphasized the process of “discovery" in the learning process and mentioned that the rich learning environments containing this process couldn't be created. HTTM, on the other hand, enables the creation of such rich mental environments in the development process of high-level mathematical models (Hıdıroğlu, 2018b).

An important criterion for evaluating mathematics achievement of countries is international exams such as PISA and TIMSS. Turner (2007) states that the problems in PISA are related to daily life problems, that they are prepared by taking into account the skills needed today, that many of the problems in PISA include some phases of the modeling process, although not a complete mathematical modeling problem, that students with high modeling skills are more successful in such exams and that the difficulty of the problems is closely related to their including modeling stages of the related problems. Since HTTM exposes students with open-ended modeling problems in the learning process, it is an essential tool for the development of their modeling skills and thus the success of countries in international exams.

Considering all these aspects, HTTM has the potential to create a rich learning process. In order for HTTM learning process to be implemented effectively, mathematics teacher candidates and mathematics teachers should have a positive perception about HTTM learning process and they should be trained in this direction. In addition, their views, as practitioners, on how to better implement HTTM are important. For this reason, the study was carried out with teachers and teacher candidates who had been trained for a while on HTTM and who had the opportunity to apply HTTM as a new learning approach in their secondary and high school level courses. In this way, it was aimed to get effective feedback about HTTM.

Considering all these aspects, HTTM is thought to bring a different perspective to the literature, enrich the learning process with different dimensions and provide opportunities for rich mental processes in mathematics learning process. In this sense, the aim of this study is to investigate the opinions of the secondary and high school mathematics teachers and mathematics teacher candidates who have been educated about HTTM learning process and experienced HTTM activities in the classroom environment about HTTM learning process. In this way, it will be possible to reveal how a learning approach that takes into account the understanding of today's curriculums and which can be effective in the process of learning mathematics is perceived by its practitioners. The research problem is expressed as follows: What are the opinions of secondary and high school mathematics teachers and mathematics teacher candidates about HTTM learning process?

The sub-problems of the study were determined as follows:

1) On what aspects/characteristics of HTTM did secondary and high school mathematics teachers and mathematics teacher candidates present their opinions?

2) How did secondary and high school mathematics teachers and mathematics teacher candidates express their opinions about these aspects/characteristics related to HTTM learning process (advantage, disadvantage and suggestion)?

2. Method

2. 1. The Design of the Study

The study is a basic qualitative research study conducted with a qualitative understanding. The reason why this kind of research is adopted in this study is because of the fact that basic qualitative research studies, by describing any situation and process, aim to display how a process takes place and what kind of process is followed in the realization of truth (Merriam, 2009).

2. 2. Participants

The sample of the study consisted of three secondary school maths teachers working in different secondary schools and three high school math teachers working in different high schools in Turkey during 2016-2017 academic year, five senior class students who are secondary school mathematics teacher candidates taking teaching practice course and applying teaching practices at different secondary schools and five high school mathematics teacher candidates applying teaching practices at different high schools. The study was conducted with a total of 16 participants (6 teachers and 10 teacher candidates). In the selection of the participants, criterion sampling method, one of the purposeful sampling methods, was used. Prior to the study, a five-week training on HTTM learning process (see Figure 4) was conducted under the leadership of the researcher with the secondary school, high school mathematics teachers, mathematics teacher candidates and researchers. Then, the teachers and teacher candidates carried out a HTTM practice with the students in classroom environment. In this way, a rich source of data related to HTTM learning process were tried to be obtained at both secondary and high school level. The findings were presented by giving the participants codes (see Table 1).

Table 1.

Participant Codes

Participant Codes	*Teacher / Teacher Candidates*
T1	School A / Secondary School Mathematics Teacher
T2	School B / Secondary School Mathematics Teacher
T3	School C / Secondary School Mathematics Teacher
T4	School D / High School Mathematics Teacher
T5	School E / High School Mathematics Teacher
T6	School F / High School Mathematics Teacher
T7	G University / Secondary School Maths Teacher Candidate 1
T8	G University / Secondary School Maths Teacher Candidate 2
T9	G University / Secondary School Maths Teacher Candidate 3
T10	G University / Secondary School Maths Teacher Candidate 4
T11	G University / Secondary School Maths Teacher Candidate 5
T12	H University / High School Maths Teacher Candidate 1
T13	H University / High School Maths Teacher Candidate 2
T14	H University / High School Maths Teacher Candidate 3
T15	H University / High School Maths Teacher Candidate 4
T16	H University / High School Maths Teacher Candidate 5

2. 3. Data Collection Instruments

The data of the study is comprised of video analysis of the interviews conducted through semi-structured interview form with the secondary and high school mathematics teachers and mathematics teacher candidates. The semi-structured interview technique that existed in the classification of Aiken (1997) and Punch (2005) was made use of. While the interview questions were being prepared, HTTM learning process of Hıdıroğlu and Özkan Hıdıroğlu (2016) was taken into consideration. In the interview, the opinions related to the structure of HTTM learning process, its applicability, effectiveness, its effect on conceptual learning, its suitability with the curriculum, the assessment and evaluation process and the application process were tried to be revealed.

In the process of developing of the interview form, two experts in the field other than the researcher reviewed the form and the necessary adjustments were performed based on the expert opinions. Before the interview form was presented to the participants, a pilot study was carried out with two people having the same characteristics as the participants, and the general and probing questions were given their final forms.

2. 4. Data Collection Process

The data collection process of the study is as follows:

1) Selecting the mathematics teachers and mathematics teacher candidates: Firstly, with the aim of identifying the opinions about HTTM learning process, it was planned to conduct studies with both the secondary and high school mathematics teachers/teacher candidates. In this respect, the candidates determined to be involved in the study were informed through a form explaining the purpose and process of the study. The participants were selected in accordance with the feedback and willingness of the mathematics teachers and mathematics teacher candidates.

2) Training of the teachers and teacher candidates on modeling and HTTM learning approach: A contact was maintained with the teachers and teacher candidates and according to their availability, they were trained on modeling and HTTM learning process through sessions and modeling practices for a total of 25 hours during a five-week period (see Figure 5).

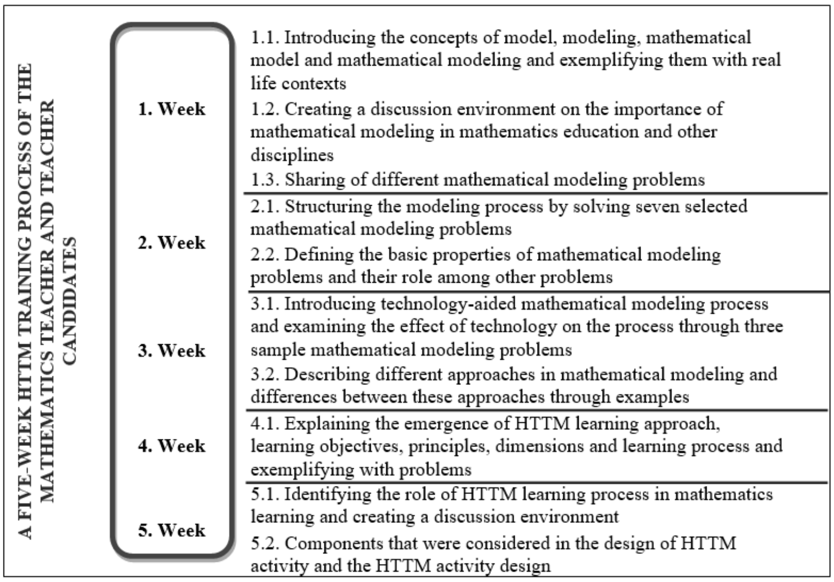

Figure 5. A Five-Week HTTM Training Process of the Participants

3) Conducting in-class practices that constitute HTTM learning process: The math teachers (in the schools where they work) and the mathematics teacher candidates (within the scope of their teaching practices course) firstly carried out different practices that include HTTM-supported mathematical modeling problems with their students in the relevant schools (secondary and high schools) individually or in group in the classes they chose. In this process, the researcher guided the teachers

about what they could do before the lessons. The teachers and teacher candidates, on the other hand, selected whatever HTTM activities they liked and then conducted these in-class HTTM-supported learning practices for about four weeks including a total of approximately 15 hours.

4) Conducting the semi-structured interviews at the end of the training and practices provided: Individual interviews were conducted with 16 participants and these interviews were video-recorded. 78 pages of video analyzes obtained at the end of the interviews formed the data of the study.

2. 4. Analysis of the Data

Content analysis technique was used in the research. In this way, deeper and systematic explanations about the participants' opinions on HTTM learning approach were presented. The process of data analysis in the study is as follows:

1) Coding the data

2) Identifying the temporary categories

3) Making the codes suitable for the categories

4) Finalization of the temporary categories

5) Defining the general characteristics of the categories

6) Determining differences between the categories

7) Organizing the findings in accordance with the aim of the study, the codes and categories obtained

In the analysis of the data, interrater reliability was calculated as 87%. Miles and Huberman (1994) signified that an interrater reliability with 80% at least is a good level for qualitative reliability. Accordingly, it was seen that in the study the interrater reliability was high. While the findings of the study are presented, the categories emerging are described with the relevant data in detail (Patton, 2002). For the reliability issue, while collecting the data, peer review and control strategies (Merriam, 2013) were taken into consideration and the procedure in the study was explained in detail (Charmaz, 2006).

3. Findings and Discussion

As a result of the individual interviews conducted with the secondary/high school mathematics teachers and mathematics teacher candidates, it was seen that the participants generally highlighted how HTTM learning process contributed to them, what difficulties they experienced in HTTM learning process and what could be done for these problems. The two-dimensional categories that are discussed in this regard are presented in Figure 6.

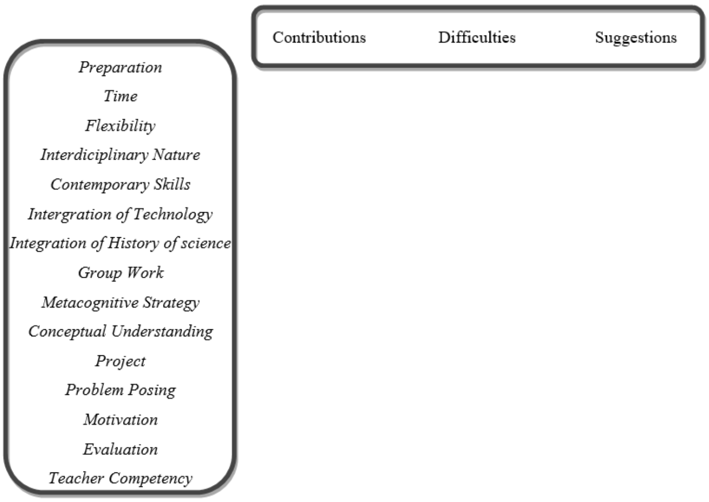

Figure 6. Two-dimensional categories obtained as a result of data analysis

3. 1. Preparation

The participants stated that they were able to create an effective learning environment with the students when they made the necessary preliminary studies in HTTM learning process *(such as identifying possible interesting and different solutions/opinions/ strategies by the students, defining the important concepts and their roles for the solution in the required disciplines, taking into consideration students' prior knowledge in various disciplines)* and supplied the appropriate materials. T7 noted that before he had implemented HTTM activities, he had thought about the solution in detail and had taken notes. T2 stated that he showed his own solution to the teachers from different disciplines before the implementation, a situation which, he said, provided advantages for him while applying the practice. T5 reported that he had discussed the solution with the physics teacher before the application and talked to him about possible student attitudes. T5 also emphasized that it is of great importance for the teacher to be prepared in order to implement such activities effectively. T15 stated that he had solved the problems beforehand and examined whether the prior knowledge of the class in which he was going to make the implementation was sufficient for the solution.

T7: Before applying The Ancient Theater (referring to HTTM activity), I thought a lot about the solution of the problem and took notes to remember in class.

T2: I tried to solve the Alexandria Lighthouse (referring to HTTM activity) problem myself first. Later, I showed the parts that I am not sure about and the solution that I found to the science, geography and computer teachers. This provided me advantage. I became more helpful to the students and was less stressful.

T5: We discussed with the physics teacher about the motions of the planets and thought about the different solutions that the students could present. After all, the teacher must be prepared for the applicability of such activities, otherwise the teacher will have difficulty during the application.

T15: I had solved HTTM problems that I would apply beforehand. Upon thinking about the class of implementation, I decided to implement them after making sure that their prior knowledge was enough for the solution.

3. 2. Time

The participants mentioned that the preparation, implementation and evaluation of HTTM activities were very time consuming. T2 stated that in order to practice one HTTM activity, at least 6-7 lesson hours are required. T2 also highlighted that the teachers who have already reported that they do not have sufficient time in general might not devote time to HTTM activities. T11 said that such activities required a long preparation and this would cause difficulties for the teacher in the first practices. T11 stated that the teachers can get prepared for HTTM-supported course process in a shorter time than the time they spend to gain experience. T5, on the other hand, suggested that newspaper articles and readiness questions of HTTM activities should be given to the students before the lesson and the students should come to the class by getting prepared. In this way, he added, there would be less problems related to time.

T2: I think; the teacher should allocate 6-7 lesson hours in the learning process of a HTTM activity. Currently, teachers complain that there is not enough time. No matter how different and effective these activities are, teachers may not be able to use them because they take a lot of time. I think I'll use it but everyone may not think like me.

T11: Such activities require a long preparation and this is hard for the teacher. Maybe the teacher can gain experience as she practices it. In this way, these can be prepared in a shorter time. But I think it will be difficult in the beginning. This will also be time consuming.

T5: HTTM learning process takes a lot of time both before and after the practice. I thought it might be something to save time. We can in advance give the students the history-based text that we first provided, even later with the readiness questions. Children can answer them beforehand and then come to class. so time is saved. I did the applications in class, but I suppose I will do it like that later. In this way, children get prepared for the class in advance.

3. 3. Flexibility

The participants emphasized that it is very easy to use a HTTM activity at different class levels and that such activities can be handled at all levels. For example, T1 stated that he implemented HTTM activities at different levels in secondary school and HTTM activities created effective learning environments for all levels.

> *T1: I applied the same HTTM practice in the 6,7 and 8th classes and in all of them the students tried to solve the problem enjoyably. This is also a great convenience for the teacher. Even though the students' levels were different, they created a process that was suitable for their level. Even in the 6th grade, different ideas emerged and they were better in some subjects than in the other upper classes.*

A high school mathematics teacher T5 emphasized that technology entered in the process in accordance with the mental structure of the students and that it was not a must for the student to use technology in HTTM as long as it was not necessary. T5 said that students were more comfortable in the solution when they did not feel obliged to use the technology, and could create more creative ideas with technology when they wanted.

> *T5: In HTTM, the student who determines the integration of technology into the learning process according to his/her own mental structure makes use of technology if he likes to. This makes them more comfortable. When they don't feel obligated to use it, they are more comfortable in the solution and can create more creative ideas with technology.*

Differently, T7 stated that the structure of HTTM problems can be adjusted according to the level of the students. T7 added that when he wanted to apply the problem situation given for the 8th grade to the lower classes, he decreased the number of the variables and simplified some sentences in the newspaper article.

> *T7: We can make changes in the structure of HTTM problems according to student levels. For example, I wanted to make practice in the 5th grade, and unlike the 8th grade, I included numerical values for some variables in the problem situation. In this way, the problem situation became more appropriate to the learning level of 5th grade. I also split some sentences up in the newspaper article. I have had two sentences and I have made the sentences more understandable. I can say the lesson was as enjoyable as in the 8th grade.*

3. 4. Interdisciplinary Nature

The participants suggested that the interdisciplinary nature of HTTM learning process constitutes a rich mental process and that they relate mathematics to different disciplines through the real life applications of history of science, theory and mathematical world. In addition, some teachers emphasized that the interdisciplinary structure of HTTM was felt more in some problems and therefore it was required to get help from teachers from different disciplines. T4 said that the HTTM activity of Kepler and the Motions of the Planets contained an interdisciplinary learning process in which maths and physics are intertwined, and that the interdisciplinary structure supported making associations and so provided conceptual learning.

> *T4: The practice of Kepler and Motions of the Planets actually requires an absolute collaboration of mathematics and physics and covers many issues. In this sense, we were able to introduce the students into a rich process. It was different for them as well, but I'm sure they understood the topics better by associating them with each other.*

> *T6: With its basic understanding, learning process and theoretical background, HTTM learning model contributes considerably to different understandings and ideas by combining the factors that should not be ignored in mathematics, physics, chemistry and biology education such as technology, modeling, theory and history of science. The lighthouse activity I applied shows how intertwined physics and mathematics are.*

The participants emphasized that the interdisciplinary structure of HTTM learning process created difficult situations, such as in terms of allocating some common time by the teachers in different disciplines and determining how and in which course to apply the practice. T3 pointed out that he needed science teachers in problems involving science or geography issues and belived that this makes the process difficult for the teacher.

T3: I have obviously had problems with the lighthouse activity and the problems that require science knowledge. In those cases, I needed science teachers who could help me. Even it could be a geography teacher that could help me. It would have been easier if there was a process I could only handle on my own. Nevertheless, it's important that the practice is interdisciplinary.

3. 5. Contemporary Skills

The participants maintained the idea that HTTM learning process develops the main 21st century skills that are highlighted in the curriculum including creative thinking, questioning, mathematical modeling, digital skills, working with groups, conceptual association, mathematical communication. T9 emphasized that HTTM learning process develops the skills required for PISA and TIMSS and supports the contemporary skills by revealing all the levels in Revised Bloom's Taxonomy. T13 asserted that HTTM learning environments improve students' cognitive and affective skills and provide opportunities for creative thinking and conceptual learning.

T9: HTTM activities provide an appropriate learning environment for all levels of Revised Bloom's Taxonomy, as well as the development of basic knowledge and skills measured in international exams such as TIMSS and PISA.

T13: The fact that students are continously exposed to HTTM activities in the learning process allows students to develop their creative thinking and conceptual understanding by minimizing cognitive and affective deficiencies.

T4 emphasized that high-level mathematical models that were developed at the end of problem posing and dual modeling in HTTM learning process enhanced students' creative thinking skills while T2 suggested that students engage in high-level conceptual learning processes with such practices and that they create original and high-level mathematical models.

T4: The exploratory models introduced in HTTM learning process allow students to think creatively.

T2: By involving problem posing and multiple modeling process into the high level conceptual learning process through HTTM activities, students push their boundaries both in terms of concept and skills. Thanks to the advanced mathematical thinking skills that appear, creative thinking skills are developed and students are provided with the opportunity to create original and high-level mathematical models.

3. 6. Integration of Technology

The participants revealed that a learning environment intertwined with technology (video, animation, photography, graphic calculator, GeoGebra, calculator, etc.) was created in HTTM learning process. T5 stated that his students did not want to use technology at the beginning of HTTM learning process, but they used technology since they believed it was necessary even though they did not have to. T5 maintained that with the use of technology in the HTTM learning process, numerous important opportunities (such as discovery of knowledge, indicating the relationship between variables, making the analysis of models easier, eliminating operational difficulties, concretising abstract concepts, visualizing formulas, demonstrating the relationship between concepts) were utilized. T13 stated that digital competence is essential in 21st century curricula and HTTM supported learning environments both improve students' digital competencies and support their learning.

T5: Although HTTM does not oblige students to use technology, it somehow incorporates technlogy into the process. Once the student realizes the benefit of technology, he/she starts to regard it as necessary. The ones who declared that they would not use it in the first HTTM practices used it in the later stages. Products of technlogy such as softwares, pictures, animation and videos provide students with an environment in which they can contribute to the development of appropriate strategies for the discovery of knowledge, to interact with each other and to make discussions. It facilitates the analysis of mathematical models by providing an understanding of the relationship between variables. With the use of technology in the problem solving process, operational difficulties are eliminated, abstract concepts become concrete, mathematical relations and formulas are visualized and relations between concepts are concretised.

T13: Digital skills are now an essential competence area in the learning process. Digital environments are vital for students to acquire the necessary skills and learn the necessary mathematical concepts. HTTM also highlights the importance of technology for mathematics and learning.

3. 7. Integration of History of Science

The participants stated that dealing with the important figures, problems, theories and events in the history of science through narration during HTTM learning process increased the students' motivation and enhanced the memorability of the concepts. In addition, the participants alleged that they hadn't included the problems in the history of mathematics in their courses before and that they lacked the relevant knowledge in this direction. T1 emphasized that it was the first time that he included the history of science in his course and that the motivation of the students was high. T1 added that the students who normally had less attention in the course also tried to participate actively in this process. T6 suggested that the history of science made the lesson enjoyable. According to T6, the students learned mathematical concepts better in the history of science. T4 emphasized that the fact that HTTM activities are intertwined with the history of science created a very effective learning environment, but when he does not have any HTTM activities in hand, he cannot create such activities himself and he does not have much idea about how to create such activities.

T1: Obviously, I have never mentioned the history of mathematics in my lessons before. In this sense, I felt uneasy; but I realized that the students' motivation was quite high. Even the students who didn't listen to my lectures often took the floor. That was nice.

T6: The lesson was very enjoyable. The students attentively listened to the scientific developments in the old city of Alexandria. They made different inferences. I wouldn't have attracted much attention if I had given the lighthouse problem of Alexandria with a direct lighthouse photograph without storifying. Children felt history.

T4: Thanks to the history of science, HTTM learning process creates an effective lesson. If I were a parent of a student who watched the course from outside, I would find the course and the teacher qualified. But there's a problem here. I can't design them when my HTTM activities are finished. I don't have much idea on how to design it (laughs).

3. 8. Group Work

The participants affirmed that their students did not use to make group works; however together with HTTM learning process, they exhibited a more effective mental process and they improved both verbal and mathematical communications by increasing their intra-group and inter-group interactions. According to T16, students were more successful in group work than individual work in HTTM learning process and were able to create synergy within the group. This enabled them to have a positive attitude towards working with group. T1 indicated that his students were not used to group work; however, they reported that together with HTTM process they would like to have more group works. T1 also stated that the students were more active through group work in HTTM learning process and that the students got help from each other.

T16: The fact that the students continuously listen to and communicate with their friends creates an environment that increases their social interaction and mathematical communication skills... students working in group complement each other. In this way, they learn from each other and they are more successful than when they work individually. When they realise this, they want to work together more.

T1: Our students are not accustomed to group work but I saw that they wanted to work together during my HTTM practice. So if I do this way, the students will be more active. In this respect, I think, such a thing is easier for the teacher as he/she will not have to constantly explain something. There is a lot of noise in group work, but the students get a lot of help from each other.

3. 9. Metacognitive Skill

In HTTM learning process, the students constantly organized their own thoughts by listening to different thoughts in the group. The students tried to contribute to the group with the ideas they organized. T11 emphasized that the students who participated in the group discussions during HTTM learning process made self-evaluations and this improved their metacognitive skills (such as realizing their deficiencies, developing themselves, improving their incompetencies).

T11: It enables the students to participate in the solution and realize their deficiencies. Effective environments in which they can develop themselves and improve their incompetencies are created.

T4 emphasized that after seeing different solutions, students questioned the effectiveness of their own solutions and thoughts in the learning process. According to T4, the students revised or changed their thoughts while reaching the ideal solution during HTTM learning process (HTTM activities at Aspendos Antique Theater and at Alexandria Lighthouse).

T4: Students saw different solutions in HTTM practice and questioned their own solutions. Some revised their thoughts, others completely changed them. I think this process is effective. They thought about how to make a better solution. This was the case for the activity of the Ancient Theater and the Lighthouse.

3. 10. Conceptual Understanding

The participants stated that HTTM activities support conceptual learning through presenting the real life applications (giving context) of mathematics in a technology supported environment, introducing the historical development process of the related mathematical or other concepts (physics, chemistry, biology) and revealing relationships with other concepts (association). T6 stated that the practices in HTTM prevented students from memorizing; because he stated that such practices cannot be done via memorization. According to T6, problems used in textbooks are not sufficient for students' conceptual learning. T6 stated that in the HTTM activity he applied in his class, his students associated many important concepts (motion, function, equation, distance, velocity, acceleration, integral, gravitation, the planets and their positions) and discussed these concepts with their context in real life. T6 and T14 emphasized that HTTM learning process provided effective learning. T14 added that since there were open-ended problems in HTTM and real-life situations involving more than one concept, the students were trying to solve the new situation that they just encoured with what they already knew.

T6: With these activities, students will be away from memorizing. Because it's not something to memorize. It requires different and flexible thinking. There are many questions that are based on the formula in the textbooks and there is information in the problem only enough to apply the formula, and those questions do not even measure whether students can distinguish the necessary information. ... That's why students fail in the international PISA and TIMSS exams. In the application of motions of the planets, many concepts such as motion, function, equation, distance, velocity, acceleration, integral, gravitation, the planets and their positions are associated. I think the child perceives how this can happen in real life and understands it more effectively.

T14: Conceptual learning is important to us. In this process, the student first understands what mathematical concepts mean in real life and how they intertwine with mathematics. Solutions are also associated with too many issues. Since there are no questions that direct the student, the student tries to go to the conclusion by associating new knowledge with what he knows.

3. 11. Project

The participants stated that the rich mental processes that emerged in HTTM learning process provided them creative and good ideas for project work. For example, T4 maintained that they would implement a project with some students from the HTTM activity namely Kepler and Motions of the Planets and they would get help from geography, physics and chemistry teachers in this process. T10 also stated that HTTM activities are an essential resource for projects due to its rich content.

T4: HTTM gave me very good ideas. For example, I was inspired by the problem of Motion of the Planets. With some of my students, we will visualize the Earth's motion relative to all planets and the Sun in the GeoGebra environment. For this, we will get some information about all the planets and the Sun from our geography, physics and chemistry teachers. This will be our project.

T10: I think HTTM activities are a suitable resource for TUBITAK projects. We can create beautiful products by designing such activities with students. There's both technology and math history. The content is quite rich and different.

3. 12. Problem Posing

Generally, the participants stated that they had not done any problem posing activities with their students in the classroom before. However, they noted that the students were very willing in problem posing stage during HTTM learning process. In addition, T1 emphasized that during problem posing in this process, the students' work with the group was more productive due to the emergence of more thoughts and more confident students. T6 said that with the problem posing and multiple modeling process, there were environments for students to think at high level. According to T6, the students performed more skillfully at this stage of HTTM learning process than the first stages.

T1: I had never designed problems before. However, this process was very enjoyable. It may be because it was something new for the students, but working with the group rather than the individual work was more efficient in this process. More thoughts appeared and the students felt safer.

T6: Students are experiencing the problem posing and multiple modeling process in the learning process with HTTM activities. In this way, the HTTM learning process puts them in the process of high-level thinking and pushes their limits both in terms of concept and skill. At the last stage, I would say that the students performed more skillfully than the first stages of the activity. The process made them better towards the end.

T16 stated that in the designing of HTTM learning process, environments that improve students' creative thinking skills were created. He also highlighted that the students produced many different thoughts and put forward ideas that would improve both themselves and the other students. T16 stated that some students had problems in transferring their thoughts to mathematics at this stage.

T16: I think HTTM process created an environment to reveal their creativity and to help them break their habits. The students were free at this stage and raised many different ideas. I just saw that sometimes they found it hard to express what they thought mathematically. But the students surprised me and each other with their ideas.

T10 asserted that HTTM learning process supported the students in reaching a richer mental process in the design process. In this way, the students were able to have better and higher level thoughts. T10 also suggested that working with group was more effective in HTTM designing.

T10: At this stage, I think that before they are asked to design a problem directly, creating an environment where they can use their experiences through the historical HTTM problem given to them creates a very effective designing process. The student has better thinking and I think he is designing more qualified problems. ...During this phase, I got more efficiency in group work.

3. 13. Motivation

The participants emphasized that when HTTM learning process is supported by the history of science, technology and group work, a learning environment that keeps student motivation at a high level can be created. T1 stated that technology-aided mathematics learning made the lesson colorful and fun for the students, and added that HTTM learning process can also create such an environment. T13 proposed that the more materials and technology are used in the lesson as in HTTM, the more students can participate in the lesson and the more effective and fun environment for the students can be created.

T1: By integrating technology into the lesson, we make the lesson more colorful and fun. HTTM activities created such a learning environment. The students enjoyed the lesson very much.

T13: I think that we eliminate or minimize any negative thoughts or prejudice against mathematics by using technology and physical materials as the lesson becomes more meaningful. In other words, the more materials and technology we use, the more active our students are. Thus, a permanent, meaningful, productive and entertaining lesson will be carried out and mathematics will no longer be an incomprehensible course for students. HTTM activities provided us with such an environment.

3. 14. Evaluation

The participants stated that HTTM learning process provided suitable environments for multiple evaluation techniques. According to the participants, in HTTM learning process, while students were performing in-group evaluation, self-evaluation and inter-group evaluation, the teacher developed a rich environment in which he could evaluate his students both individually and in groups. According to T3, in HTTM, appropriate environments were provided for the students to see their own deficiencies and develop their thoughts by participating in the evaluation. T3 also added that assessment environments in HTTM support students' metacognitive, empathy, social and communication skills. T6 reported that during HTTM process, the student actively questioned his/her own thoughts, compared the thoughts in the group with his/her own thoughts and evaluated the perspective of different groups. T6 emphasized that the groups wanted to evaluate each other and the students produced very quality ideas.

T3: HTTM is very suitable for the student to participate in the evaluation process. Active participation of students in assessment will improve their metacognitive skills and mental processes in terms of seeing and improving their own deficiencies. In addition, evaluating students' group-mates will support their empathy skills, social interactions and communication.

T6: In HTTM, the student constantly questions his / her own thoughts and compares the thoughts in the group with his / her own thoughts. Groups also share their solutions with other groups. In this way, there is evaluation between the groups. I wanted the groups to evaluate each other by presenting their grounds, of course. I got very nice expressions. Some of them gave me ideas. Such environments greatly increase their awareness.

T12 emphasized that as the students were constantly active in groups or individually in HTTM learning process, the teachers could find enough opportunities to monitor them, realize the shortcomings and eliminate these. T12 also remarked that there were many things that students learned and this enabled them to develop their own thoughts and give them better feedback.

T12: Students are constantly talking and staying in the learning process while dealing with HTTM activities. Continuous progress does not create rupture in them. I was very comfortable as a teacher. One reason was that I was well prepared in advance and another one was that the lesson was student-centered so I didn't have to talk constantly. It also gave me the opportunity to listen to the students more efficiently. I had the opportunity to eliminate their missing sides. While listening to the students and the groups, I improved myself a lot. I think I gave the students more quality feedback.

3. 15. Teacher Competency

The participants believed that the teacher should have sufficient knowledge and skills in order for the HTTM learning process to proceed effectively and to improve teachers' attitudes towards using HTTM activities. T1 revealed that the fact that he had used GeoGebra with his students before was an advantage for him in HTTM learning process. T1 also added that HTTM implementation process will be difficult for those who do not use technological software such as GeoGebra. T6 suggested that teachers should reflect their knowledge and skills in technology-assisted field teaching in order to design effective HTTM activities and build HTTM learning process. In addition, according to T6, the teacher can develop the necessary skills in the application process; therefore, he said that he should be willing but not be afraid of the implementation. T8 emphasized that HTTM learning process provides an environment for the teacher to develop their in technology use, mathematical thinking, reasoning, synthesis, digital skills, creativity, and history-supported mathematics teaching.

T1: I used GeoGebra with my students in my classes. This was an advantage for me and I had no difficulty in terms of technology. But those who have not used it will have some difficulty in the application process at first.

T6: For HTTM activities, the teacher should be willing to improve himself in many ways. It is difficult to design effective HTTM activities. I think the implementation process is also difficult. Not every teacher can do that. At this stage, the teacher should have teaching skills in fields including technology and history. I think there will be environments where he can develop such skills if he is willing to implement the process. The teacher should not be afraid but willing.

T8: HTTM learning process also develops teachers. It supports the use of technology and the development of digital competence. In addition, the open-ended structure, creativity of the problem posing process, the ability to synthesize, reasoning, mathematical thinking, the ability to use history in lessons always increases. In my opinion, if the teacher continually makes use of such problems (HTTM activities), he/she becomes much better than the previous states.

3. Conclusion

In this study, individual interviews were conducted with secondary school / high school mathematics teachers and mathematics teacher candidates who received a five-week training on mathematical modeling and HTTM learning process and then used HTTM activities in the classroom environment, and their opinions on HTTM learning process were identified. As a result of the data analysis, a two-dimensional structure related to HTTM learning process was obtained. In the first dimension of the findings 15 themes were obtained. These are preparation, time, flexibility, interdisciplinary nature, contemporary skills, technology integration, history of science integration, working with groups, metacognitive skills, conceptual understanding, projects, problem posing, motivation, evaluation and teacher competency. In the second dimension of the findings; benefits, difficulties and suggestions related to 15 themes were put forward. In this section, the results of the study are compared with the literature.

Under the theme of *Preparation*, it was concluded that a strict preliminary preparation was required for HTTM learning process and that it was difficult for the teacher to deal with open-ended problems. Zawojevski, Lesh & English (2003), Lesh & Doerr (2003), Şahin & Eraslan (2018) and Pilten, Serin & Işık (2016) also emphasized the difficulty and importance of the preparation stage prior to modeling activities. It can be said that the multi-dimensional structure in HTTM makes pre-preparation even more necessary. Tekin Dede and Bukova Güzel (2014) stated that "preparation" stage is the first of the four basic stages in model building activities. Producing resource books containing HTTM activities and possible solutions to prevent difficulties in pre-preparation and supporting teachers by experts in the preparation process can provide a more effective learning process.

Under the theme of *Time*, it was concluded that it was difficult to allocate sufficient time in the lessons for HTTM activities and therefore this learning process could be applied at most two or three times during a semester. Blum (1991) and Şen-Zeytun (2013) mentioned that one of the difficulties in mathematical modeling applications is that they are time consuming. Lesh & Doerr (2003) argued that since model eliciting activities take a long time, they should be given as project assignments. It was suggested by the participants that some parts could be given to students as homework in order to make more applications. Teachers can associate Flipped classroom system with HTTM learning process in order to eliminate the time problem in HTTM applications. With the understanding of flipped classroom, newspaper article in HTTM learning process (first two basic steps) can be presented in a way that students can work outside of the classroom. The solutions of the problems designed can be discussed in the classroom. According to the teacher/teacher candidates, the learning process in the courses of other disciplines can be structured if necessary in line with the content of relevant HTTM activities.

As for *Flexibility* theme, it was concluded that the HTTM learning process and activities were flexible (can be changed depending on the situation or circumstances). Since the main purpose of this kind of learning environment is to provide learning and create rich mental environments, it is possible to make changes based on the level of the students in the weight of the dimensions, the use of technological software, the process and the structure of the problem in order to be effective in such environments. In this sense, HTTM learning process with its flexible structure takes into account the Educational modeling approach. According to Mousoulides, Christou & Sriraman (2006), mathematical modeling in secondary and high school mathematics curriculum enables students to produce more flexible solutions to events with the help of mathematics. Gürbüz, Çavuş Erdem, Şahin, Temurtaş, Doğan, Doğan, Çalık and Çelik (2018) also mentioned the importance of flexibility of interdisciplinary mathematical modeling activities which constitute an interdisciplinary learning environment like HTTM. Mathematical modeling is a cyclical process in which temporary models are developed and refined depending on the creativity of students (Blum & Leiß, 2007; Czocher, 2013; Lesh, Hoover, Hole, Kelly & Post, 2000). This view explains the flexibility in the construction of mathematical models and even in the evaluation of mathematical models. The modeling process first gives the modeler the opportunity to correct errors. It then provides opportunities for researchers who examine the modeling process to go beyond assessing the accuracy of mathematical operations or assessing the existence (or level) of mathematical concepts. Furthermore, in the mathematical modeling process, the process of designing the model is at the forefront rather than the model's appropriateness. In the solution of a problem, it is possible to encounter different but quality mathematical models. This flexibility promotes creativity and provides an environment that reassures the student.

In the *Interdisciplinary Nature* theme, it was concluded that HTTM learning process brought together different disciplines such as mathematics, physics and geography and supports the conceptual understanding by revealing the relationship between concepts in different disciplines in the context of real life. Blomhøj & Kjeldsen (2006), English (2009), Lingefjärd (2006), Blomhøj & Jensen (2007), Maaß (2006), English & Watters (2004), English (2015), English, Hudson & Dawes (2013), Doğan, Şahin, Çavuş Erdem & Gürbüz (2018) considers mathematical modeling, which is the basic dimension of HTTM learning approach, as an effective tool for interdisciplinary learning environments such as STEM. The training of individuals who can establish interdisciplinary relationships and have the skills of model building will provide individuals with the necessary basic knowledge throughout their lives (Thomas & Hart, 2010).

Under the theme of *contemporary skills*, it was concluded that in HTTM learning process (*especially dual modeling and problem posing, technology-supported mathematical modeling process*), the students used the historical modeling problem to design and solve the current mathematical modeling problems, and to associate them with the old problems, creating rich mental environments that reveal the skills needed today. According to Blomhøj & Kjeldsen (2006), Blum & Niss (1989), English (2009), Lingefjärd (2006), English & Watters (2004), Frejd (2012) and Maaβ (2011), the transfer of real-life environments into mathematics in the learning process is essential in the development of basic skills. In addition, in their study, Cinar, Pırasa & Sadoglu (2016), Egli (2012), Eroglu & Bektas (2016), Kızılay (2016), Lee, Park & Kim (2013), Thomas (2014), Wang (2012), Yıldırım (2016), Yıldırım & Türk (2018) stated that knowledge should be associated with daily life in learning process so that environments where students can improve their skills in current technology (such as technology literacy) and economy (financial literacy) can be created. HTTM activities can create environments that provide an appropriate learning process for all levels / stages of the development of basic knowledge and skills measured in exams such as TIMSS and PISA and revised Bloom's Taxonomy (Anderson & Krathwohl, 2001). For this reason, the difficulties to be experienced in the first phase of HTTM activities should be regarded as normal and the students' current skills that need to be revealed and developed in this direction should be focused. Teachers should apply various adjustments in the activities, if necessary, taking into account the student level when implementing HTTM activities. This is important for students to overcome cognitive barriers. In this way, in the first stage, students can overcome their cognitive barriers to some extent and reveal a more effective learning process in the next process.

Under the theme of *integration of technology*, it was determined that technology was not a must in the process of HTTM learning process; but when necessary, technology enabled the students to progress effectively in the process, increased their motivation and made them use their high level mental processes. Metin, Birişçi & Coşkun (2013) stated that technology improves the quality of learning. In addition, Hıdıroğlu & Bukova Güzel (2016; 2017) emphasized that although technology does not change the basic steps in the mathematical modeling process; it creates auxiliary components and enriches the sub-steps that shape the basic steps. Technology has also been important in the emergence of different mental processes and cognitive disabilities in HTTM learning process. The National Council of Teachers of Mathematics [NCTM] (1998) highlighted not only how technology can best support mathematical learning, but also how the existence of technology will affect students' mathematical strengths, different thinking and conceptual skills. However, in the light of basic computer sciences; computational thinking, an important field of study in HTTM learning process, which is defined as problem solving, system design and trying to understand human behavior (Wing, 2006), and the effect of HTTM learning process on students' computational thinking skills can be important research subjects.

Under the theme of *integration of history of science*, it was concluded that rich learning environments could be created by using problem situations in the history of science, emergence of theories, theorems and concepts, and plot. In addition, effective integration of technology and mathematical modeling with the history of science has been concluded to create suitable environments for conceptual learning and HTTM creates effective learning environments for contextual learning. The history of science discussed in the HTTM process, as Siu (2003), Swetz (1999) and Veljan (2000) suggested, presented an important map about the learning environment focusing on the problems and solutions of ancient mathematical scientists, both by revealing different possible solutions under limited conditions and by taking into consideration the development processes of concepts more effectively. With the integration of the history of science into HTTM learning process, as Baki (2014) pointed out, historical problems and responses to these problems in history have been discussed, and according to the participants, the context of history of science supported the learning process in which students can improve their mathematical thinking skills and conceptual learning. In the integration of the history of mathematics into mathematics classes, Jankvist (2009) mentioned the role of "whys" and "hows". Based on the "whys", the history of mathematics helps to learn and teach mathematics with motivational, cognitive and evolutionary ways. To illustrate, the history of mathematics shows learners that mathematics is a product and endeavor of human (Freudenthal, 1981); brings alternative perspectives and solutions to mathematics issues, concepts and problems (Katz, 2007); and provides an important set of teaching practices in line with the breakpoints of the evolutionary development of mathematical thought in the historical process (Farmaki & Paschos, 2007). While the use of the history of mathematics as an aim (whys) in the learning process deals with the sociological, epistemological and historical issues of mathematics, its use as a tool (hows) addresses the cognitive and affective skills such as learning mathematics, mathematical thinking, problem solving, attitude and motivation. Here HTTM learning process basically has a role of a tool; however, it does not ignore the role of an aim regarding the evolution of concepts and theories in the history of science. In this way, it is aimed to provide conceptual learning and develop the necessary skills in HTTM learning process. In this sense, it can be said that HTTM learning approach will be an effective theoretical framework for mathematics education research supported by the history of mathematics. HTTM learning process begins at first from a context and storyline supported by the history of science. Numerous researchers (Busse, 2011; Maaß, 2006; Busse & Kaiser, 2003; Galbraith & Stillman 2001; Şen-Zeytun, 2013; Hıdıroğlu, 2015; Hıdıroğlu & Bukova-Güzel, 2015; 2016) emphasize that context plays an important role in mathematical modeling studies. The participants mentioned the importance of the resources they would use in the design of HTTM activities. Alpaslan & Işıksal Bostan (2016) and Jahnkey (2000) also stated that there is a need for primary historical resources that teachers can access more easily. In addition, Alpaslan, Işıksal Bostan & Haser (2014), Fauvel (1991), Hıdıroğlu ve Can (2018), Hıdıroğlu & Özkan Hıdıroğlu (2016) also underlined the importance of developing the necessary skills for teachers to design mathematical history supported learning environments.

In the theme of *group work,* the teachers and teacher candidates stated that although the students they were practicing with did not have previous experience, they performed effective group activities in HTTM learning process. In addition, it was concluded that working with group in HTTM learning process was vital in the development of students' communication and mathematical thinking skills and in creating synergies and that HTTM created appropriate learning environments in crowded classrooms for teachers' guidance. Urhan & Dost (2016), Hıdıroğlu & Bukova Güzel (2015; 2016; 2017), Carreira & Baioa (2011), Fox (2006), Lesh & Doerr (2003) maintained that working with group creates effective learning environments. Maaß (2006) asserted that group work is crucial in the development of modeling competencies. Zawojewski, Lesh & English (2003) reported that synergy increases the interest and motivation of each group member in the modeling process and helps people to exceed their potential. According to Eraslan (2011), Zawojevski, Lesh & English (2003) and Watson & Chick (2001), by working with group during the modeling process, students are faced with higher levels of mental processes and able to cope with them. Zawojevski, Lesh & English (2003) proposed that in the modeling process, effective working environments can be created with groups of three or four people. In groups with more people, sub-groups within the group itself can appear and this affects the learning environment negatively (Zawojevski, Lesh & English, 2003). According to Hıdıroğlu & Özkan Hıdıroğlu (2016), it is of great importance to form groups of 3-5 people, to assign students with high level of technology skills to different groups and to create mixed groups considering the success level if HTTM learning process is to be carried out.

Under the theme of *metacognitive* skills, it was concluded that in the learning process of HTTM, rich mental environments were created in which students could constantly question their thoughts. In this sense, intra-group discussions, inter-group mental interaction, and the efforts of groups to optimize their solutions by considering other solutions allowed for metacognitive activities in HTTM. Zawojevski, Lesh & English (2003) asserted that students form continuous hypotheses during the model building activities, test and develop these hypotheses, and review the model and solution in the process. Yimer & Ellerton (2006) reported that the plan and the results are evaluated as metacognitive action in the solution process. According to Hıdıroğlu & Bukova Güzel (2015) and Magiera & Zawojevski (2011), students' metacognitive actions in the modeling process are fed from two sources: individual thoughts or experiences and group thoughts or experiences. Similarly, intergroup interaction plays an active role in HTTM learning process together with these two sources. Hurme & Järvelä (2001), Biryukov (2004), Schoenfeld (1992) and Blomhøj (1993) emphasised the existence of metacognitive actions by stating that students use previous experiences in problem solving and make connections between old and new ideas. In HTTM learning process, the current modeling problem design and dual modeling stages provide opportunities for similar environments.

Under the theme of *conceptual understanding*, it was concluded that both handling concepts with the history of science and the internalization of technology with real life context in HTTM learning process supported conceptual learning. In the literature, Aydoğan Yenmez (2012), Urhan & Dost (2016) and English & Watters (2004) revealed that the mathematical modeling process supports conceptual understanding. Göker (1997) also added that if a person who learns a branch of science also learns the history of that science, this provides a deeper and more comprehensive knowledge of the concepts. Mousoulides, Chrysostomou, Pittalis & Christou (2010) and Lingefjärd (2000) stated that a technology-aided environment contributes to students' conceptual understanding and mathematical development in mathematical modeling process in schools. According to Rivers (2001), the use of more cognitive and metacognitive strategies in organizing information and attempting to reach a solution with a deeper and more abstract conceptual structure and schema are the characteristics of experienced and metacognitive skills learners. HTTM learning process creates effective learning environments for obtaining, comparing and correlating different representations of mathematical concepts and expressions with the help of dynamic software, the history of science and real-life situations.

Under *project and problem posing* themes, it was concluded that HTTM learning process and activities led the teachers to identify project topics and design different problems. In addition, in HTTM learning process, the students were challenged to design skills and dual modeling. As Erduran (2013) and Siew, Amir & Chong (2015) stated, there is a need for students who are open to innovations, who have scientific curiosity and who are questioning. In HTTM learning process, it is essential to design and solve current modeling problems similar to the problem fed from the history of science (in accordance with socio-critical modeling understanding) both to reveal different solutions and to reach creative ideas based on what students know. Besides, the students were more confident and successful at designing, they were active and willing in the process. In this process, it was found out that some students had some problems in expressing their thoughts mathematically and the teachers helped them. According to Hıdıroğlu & Özkan Hıdıroğlu (2016), mathematical models created in HTTM learning process increases interaction and creativity and provides the environment for the emergence of high-level mathematical models (emergent or hypothetico-deductive models). Many researchers in the literature (Cotabish, Dailey, Robinson, & Hunghes, 2013; Kim & Choi, 2012; Rishowski, Todd, Wee, Dark & Harbor, 2009; Şahin, Ayar & Adıgüzel, 2014) reported that students should be in learning environments in which they will design and exhibit innovative ideas.

Under the theme of *motivation*, it was concluded that the history of science, video, animation, photography, dynamic software, the relationship between real life context and mathematics and group activities increased students' motivation during HTTM learning process. Eric (2010), Maaβ (2011) and Özbilen (2018) asserted that mathematical modeling problems increase students' motivation. Yenilmez (2011), Baki (2008), Baki & Bütüner (2010), Sullivan (2000), Baki & Yıldız (2010), Fraser & Koop (1978) maintained that mathematics learning environments supported by the history of science increase students' motivation. According to Maaβ (2011), with the transfer of real-life environments to mathematics in the learning process, students realize the use of mathematics in daily life, have a desire to learn mathematics and are better motivated for lesson. According to Türker Biber, Akkuş İspir & Sonay Ay (2015), contexts should be handled with creative drama, and accordingly newspaper articles and problem situations in HTTM learning process can also be carried out with creative drama and the motivation of students can be increased.

In the *evaluation* theme, it was concluded that HTTM learning process provides multiple assessment environments; HTTM activities are not only a tool for conceptual learning, but also an effective tool in assessing learning. HTTM learning process allows for multiple assessment environments, such as checklists, observation, group and individual reports, intergroup, intra-group and self-assessment, portfolio assesment (also in digital form). Similarly, mathematical modeling problems/activities are effective tools in evaluating students (Borromeo Ferri, 2013; Peter Koop, 2009) and in developing their necessary knowledge and skills (Borromeo Ferri, 2006; Lesh & Doerr, 2003; Stillman, Galbraith, Brown & Edwards, 2007). Tekin Dede & Bukova Güzel (2013), Eraslan (2011), Yu & Chang (2009) revealed that the modeling process is not suitable for national examination systems. In recent years, countries have been focusing on new approaches, reviewing exam systems and improving curriculum modeling skills. On the other hand, it has been reported that teachers have difficulty in evaluating students' work in complex processes involving modeling such as HTTM activities (Blum & Niss, 1991). As Asturias (1994), Coletti (1987), Sole (2013), Thompson & Senk (1998), Şahin & Eraslan (2018) pointed out, rubrics are important in the solution of open-ended problems while assesment and evaluation tools are essential in HTTM learning process as suggested by Hıdıroğlu and Özkan Hıdıroğlu (2016). In observing and evaluating the cognitive and metacognitive processes in HTTM learning process, teachers can use graded scoring keys, elaborate and re-create them by changing the main factors (Hıdıroğlu, 2018a). As Sezer (2005) reported, with the help of a rubric that enables teachers to observe various students of low levels, it is possible to determine what level the students have in any dimension.

In terms of *teacher competency*, the teachers/teacher candidates believed that it is of great importance to improve themselves to make HTTM learning process effective. One of the most important components in HTTM learning process is the teachers who carry out this process and reveal the thoughts of their students. In order to fully serve the purpose of HTTM learning process, first of all, together with field knowledge, technological and pedagogical knowledge and skills of teachers and teacher candidates should be enhanced to a good level (Hıdıroğlu & Özkan Hıdıroğlu, 2016). HTTM activities including mathematical modeling, technology and conceptual learning, the relationship between theory and practice in the learning process, and the history of science can be an important learning tool in undergraduate courses. Blum (2002), Blum & Ferri (2009) and Lingefjard (2007) stated that technological incompetencies of teachers' function as an important obstacle for teachers to create effective learning environments including mathematical modeling and emphasized that their relavant competencies should be developed. Blomhøj & Kjeldsen (2006), Yu & Chang (2009), Thomas & Hart (2010) and Eraslan (2011) suggested that the teachers who experienced mathematical modeling for the first time experienced uncertainties, difficulties and discomforts at the beginning of the learning process with mathematical modeling. Similarly, in the study, the teachers and teacher candidates maintained that HTTM-supported process increased their performance in the classroom. It is possible to reach similar results in the literature suggesting that teacher competency in modeling develops from experience (Bukova Güzel, 2011). In order to overcome inadequacies, it is vital that teachers and teacher candidates gain experience through frequent exposure to such environments. In-service trainings or projects should be supported to develop their skills in designing such activities so that teachers can become a good practitioner of HTTM activities in the classroom.

HTTM learning model structured through dealing the modeling approaches with a holistic and pragmatic point of view, aims to develop basic concepts and technology-supported modeling skills in technology-supported modeling process by making use of the history of science, mathematics, physics, chemistry and biology theories. It is thought that HTTM learning model will be an important tool for learning process by combining the factors that should not be ignored in education. Since it is the product of a postmodernist understanding, HTTM learning approach, which has a more flexible structure within itself, provides various opportunities for the teacher in structuring a process in accordance with the class levels, facilities and student competencies. There are ongoing practices at various levels about the effects of the HTTM learning process, which was designed by the author in the light of literature, on learning, and there exist studies conducted in progress to reflect the theoretical basis of HTTM.

References

Aiken, L. R. (1997). *Qustionnaires and inventories: Surveying opinions and assessing personality*. New York: John Wiley & Sons, Inc.

Alpaslan, M., & Işıksal Bostan, M. (2016). Ortaokul öğrencilerinin matematik tarihi bilgileri ile okul matematiğinde tarih kullanılmasına ilişkin tutum ve inanışları [Middle school students' knowledge of history of mathematics and their attitudes and beliefs towards using history in school mathematics]. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi, 31*(1), 142-162.

Alpaslan, M., Işıksal, M., & Haser, Ç. (2012, July). *Relationship between pre-service mathematics teachers' knowledge of history of mathematics and their attitudes and beliefs towards the use of history of mathematics in mathematics education*. Retrieved December 1, 2018 from https://www.researchgate.net/publication/268515404_Relationship_between_Pre-Service_Mathematics_Teachers'_Knowledge_of_History_of_Mathematics_and_Their_Attitudes_and_Beliefs_towards_the_Use_of_History_of_Mathematics_in_Mathematics_Education.

Anderson, L.W. (Ed.), Krathwohl, D.R. (Ed.), Airasian, P.W., Cruikshank, K.A., Mayer, R.E., Pintrich, P.R., Raths, J., & Wittrock, M. C. (2001). *A taxonomy for learning, teaching, and assessing: A revision of Bloom's taxonomy of educational objectives* (Complete edition). New York: Longman.

Asturias, H. (1994). Using students' portfolios to assess mathematical understanding. *The Mathematics Teacher, 87*(9), 698-701.

Aydoğan Yenmez, A. (2012). *Ortaöğretim matematik öğretmenlerinin pedagojik alan ve pedagojik bilgilerindeki gelişimin, modelleme yaklaşımına göre tasarlanmış bir mesleki gelişim ve eğitim etkinliği sürecinde incelenmesi [An investigation of in-service secondary mathematics teachers evolving knowledge through professional development activities based on modeling perspective]*. Unpublished doctoral thesis, Ortadoğu Teknik Üniversitesi Fen Bilimleri Enstitüsü, Ankara.

Baki, A., & Bütüner, S. Ö. (2010, September). Matematik tarihi etkinlikleriyle zenginleştirilmiş sınıf ortamından yansımalar [Reflections from classroom environments enriched with the mathematics historical activities]. Paper prensented at *IX. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi* (pp. 104), Dokuz Eylül Üniversitesi, İzmir.

Baki, A., & Yıldız, C. (2010, April-May). Matematik tarihinin kitaplardaki kullanımı ve öğretmen görüşleri [Use of the history of mathematıcs in textbooks and teacher views]. Retrieved November 21, 2018 from http://www.eab.org.tr/eab/media/kitap/EAB_Kongre_Kitap_2010.pdf.

Baki, A. (2002). *Öğrenen ve öğretenler için bilgisayar destekli matematik [Computer aided mathematics for learners and teachers]*. İstanbul: BİTAV-Ceren Yayın Dağıtım.

Baki, A. (2008). *Kuramdan uygulamaya matematik eğitimi [Mathematics education from theory to practice]* (Genişletilmiş 4. Basım). Ankara: Harf Eğitim.

Baki, A. (2014). *Matematik tarihi ve felsefesi [History and philosophy of mathematics]*. Ankara: Pegem Akademi.

Berlinghoff, P. W., & Gouvea, Q. F. (2004). *Math through the ages: A gentle history for teachers and others*. Farmington: Oxton Publishers.

Berry, J., & Houston, K. (1995). *Mathematical modelling.* Bristol: J. W. Arrowsmith Ltd.

Bettelheim, B. (1976). *The uses of enchantment: The meaning and importance of fairy tales*. New York: Random House.

Biryukov, P. (2004). *Metacognitive aspects of solving combinatorics problems*. Retrieved December 2, 2014 from http://www.cimt.org.uk/journal/biryukov.pdf.

Bidwell, J. (1993). Humanize your classroom with the history of mathematics. *Mathematics Teacher, 86*(6), 461-464.

Blomhøj, M., & Kjeldsen, T. H. (2006). Teaching mathematical modelling through project work- Experiences from an in-service course for upper secondary teachers. *ZDM-Zentralblatt für Didaktik der Mathematik, 38*(2), 163-177.

Blomhøj, M. (1993). Modelling of dynamical systems at O-Level. In J. de Lange, C. Keitel, I. Huntley, & M. Niss (Eds), *Innovation in mathematics education by modelling and applications* (pp. 257-268). Chichester: Ellis Horwood.

Blomhøj, M. (2009). *Different perspectives on mathematical modelling in educational research- Categorising the TSG21 papers*. Retrieved July 2, 2018 from https://rucforsk.ruc.dk/ws/portalfiles/portal/3820977/IMFUFA_461.pdf#page=6.

Blomhøj, M., & Jensen, T. H. (2007). What's all the fuss about competencies? Experiences with using a competence perspective on mathematics education to develop the teaching of mathematical modelling. In W. Blum, P. L. Galbraith, H. W. Henn, & M. Niss (Eds), *Modelling and applications in mathematics education* (pp. 45-56). New York: Springer.

Blum, W., & Borromeo Ferri, F. (2009). Mathematical modelling: Can it be taught and learnt? *Journal of Mathematical Modelling and Application, 1*(1), 45-58.

Blum, W., & Leiß, D. (2007). How do students and teachers deal with modelling problems?. In C. Haines, P. Galbraith, W. Blum & s. Khan (Eds), *Mathematical modelling: Education, engineering and economics* (pp. 222-231). Chichester: Horwood.

Blum, W., & Niss, M. (1989). Mathematical problem solving, modelling, applications, and links to other subjects – State, trends and issues in mathematics instruction. In M. Niss, W. Blum ve I. Huntley (Eds). *Modelling applications and applied problem solving* (pp. 1-19). England: Halsted Pres.

Blum, W., & Niss, M. (1991). Applied mathematical problem solving, modelling, application, and links to other subjects-state, trends and issues in mathematics instruction. *Educational Studies in Mathematics. 22*(1), 37-68.

Blum, W. (1991). Applications and modelling in mathematics teaching a review of arguments and instructional aspects. In M. Niss, W. Blum and I. Huntley (Eds), *Teaching of mathematical modelling and applications* (pp. 10-29). Chichester: Ellis Horwood.

Blum, W. (2002). ICMI Study 14: Applications and modelling in mathematics education- Discussion document. *Educational Studies in Mathematics, 51*(1), 149-171.

Borromeo Ferri, R. (2006). Theoretical and empirical differentiations of phases in the modelling process. *Zentralblatt für Didaktik der Mathematik, 38*(2), 86-95.

Borromeo Ferri, R. (2010). On the influence of mathematical thinking styles on learners' modeling behaviour. *Journal für Mathematik Didaktik, 31*(1), 99-118.

Borromeo Ferri, R. (2013). Mathematical modelling in European education. *Journal of Mathematics Education at Teachers College, 4*, 18-24.

Brenner, M. E. (1998). Adding cognition to the formula for culturally relevant instruction in mathematics. *Anthropology and Education Quarterly, 29*(2), 214-244.

Bukova Güzel, E. (2011). An examination of pre-service mathematics teachers' approaches to cunstruct and solve mathematical modelling problems. *Teaching Modelling and Its Applications, 39*, 19-36.

Burkhardt, H. (2006). Modelling in mathematics classrooms: Reflections on past developments and the future. *ZDM-Zentralblatt für Didaktik der Mathematik, 38*(2), 178-195.

Busse, A., & Kaiser, G. (2003). Context in application and modelling- An empirical approach. In Q. Ye, W. Blum, K. Houston, & Q. Jiang (Eds), *Mathematical modelling in education and culture* (pp.3-15). Chichester, England: Horwood.

Busse, A. (2011). Upper secondary students’ handling of real-world contexts. In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds), *Trends in teaching and learning of mathematical modelling* (pp.37-46). New York: Springer.

Carreira, S., & Baioa, A. M. (2011). Students’ modelling routes in the context of object manipulation and experimentation in mathematics. In G. Kaiser, W. Blum, R. Borromeo Ferri & G. Stillman (Eds), *Trends in teaching and learning of mathematical modelling: ICTMA 14* (pp. 211-220). Netherlands: Springer.

Çınar, S., Pırasa, N., & Sadoğlu, G. P. (2016). Views of Science and mathematics pre-service teachers regarding STEM. *Universal Journal of Educational Research 4*(6), 1479-1487.

Clayton, M. (1999). Industrial applied mathematics is changing as technology advances. C. Hoyles, C. Morgan, & G. Woodhouse (Eds), *Rethinking the Mathematics Curriculum* (pp. 22-28). London: Falmer.

Colletti, A. B. (1987). *Teaching methods and applied techniques*. USA: Keystone Publications, Inc.

Cotabish, A., Dailey, D. Robinson, A., & Hughes, G. (2013). The effects of a STEM intervention on elementary students' science knowledge and skills. *School Science and Mathematics, 113*(5), 215-226.

Czocher, J. (2013). *Toward a description of how engineering students think mathematically*. Unpublished doctor of philosophy dissertation, The Ohio State University, Columbus, Ohio.

Doğan, M.F., Şahin, S., Çavuş Erdem, Z. & Gürbüz, R. (2018, June). *Investigation of teachers’ awareness of interdisciplinary mathematical modeling problem*. Retrieved June 19, 2019 from http://theicmme.org/docs/Abstracts_Book/ICMME-2018_Book_of_Abstracts.pdf.

Egan, K. (1991). *Primary understanding: Education in early childhood*. New York: Routledge.

Egli, S. (2012). *Using STEM education to promote 21st century math skills*. Unpublished master thesis, Minot State University, North Dakota.

English, L. D., & Watters, J. J. (2004). Mathematical modeling in the early school years. *Mathematics Education Research Journal, 16*(3), 59-80.

English, L. D. (2006). Mathematical modeling in the primary school: children's construction of a consumer guide, *Educational Studies in Mathematics, 63*(3), 303-323.

English, L. D. (2009). Promoting interdisciplinarity through mathematical modelling. *ZDM Mathematics Education, 41*, 161-181.

English, L. D. (2015). STEM: Challenges and opportunities for mathematics education. In Beswick, Kim, Muir, Tracey, & Wells, Jill (Eds) *Proceedings of the 39th Conference of the International Group for the Psychology of Mathematics Education*, PME, Hobart, Tas, pp. 4-18.

English, L. D., Hudson, P., & Dawes, L. (2013). Engineering-based problem solving in the middle school: Design and construction with simple machines. *Journal of Pre-College Engineering Education Research (J-PEER), 3*(2), 43-55.

Eraslan, A. (2011). İlköğretim matematik öğretmen adaylarının model oluşturma etkinlikleri ve bunların matematik öğrenimine etkisi hakkındaki görüşleri [Prospective elementary mathematics teachers’ perceptions on model eliciting activities and their effects on mathematics learning]. *İlköğretim Online, 10*(1), 364-377.

Erduran, S. (2013). Fen bilimlerine alanlararası bakış ve eğitimde uygulamalar [Interdisciplinary view of science and applications in education]. *Fen Bilimleri Öğretimi Dergisi, 1*(1), 43-49.

Eric, C. C. M. (2010). Tracing Primary 6 students' model development within the mathematical modelling process. *Journal of Mathematical Modelling and Application, 1*(3), 40-57.

Ernest, P. (1998). The history of mathematics in the classroom. *Mathematics in School, 27*(4), 25-31.

Eroğlu, S., & Bektaş, O. (2016). STEM eğitimi almış fen bilimleri öğretmenlerinin STEM temelli ders etkinlikleri hakkındaki görüşleri [Ideas of science teachers took STEM education about STEM based activities]. *Eğitimde Nitel Araştırmalar Dergisi, 4*(3), 43-67.

Farmaki, V., & Paschos, T. (2007). Employing genetic 'moments' in the history of mathematics in classroom activities. *Educational Studies in Mathematics, 66*(1), 83-106.

Fauvel, J. (1991). Using history in mathematics education. *For the Learning of Mathematics, 11*(2), 3-6.

Fox, J. (2006). *A justification for mathematical modelling experiences in the preparatory classroom*. Retrieved November 14, 2017 from http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.491.8677&rep=rep1&type=pdf.

Fraser, B. J., & Koop. A. J. (1978). Teachers' opinions about some teaching material involving the history of mathematics. *International Journal of Mathematical Education in Science and Technology, 9*(2), 147-151.

Frejd, P. (2012). Teachers' conceptions of mathematical modelling at Swedish upper secondary school. *Journal of Mathematical Modelling and Application, 1*(5), 17-40.

Freudenthal, H. (1981). Should a mathematics teacher know something about the history of mathematics?. *For the Learning of Mathematics, 2*(1), 30-33.

Fried, N. M. (2001). Can mathematics education and history of mathematics coexist?. *Science and Education, 10*, 391-408.

Galbraith, P., & Stillman, G (2001). Assumptions and context: Pursuing their role in modelling activity. In J. Matos, S. Houston, W. Blum & S. Carreira (Eds), *Modelling and mathematics education* (pp. 317-327). Chichester: Horwood.

Göker, L. (1997). *Matematik tarihi ve Türk-İslam matematikçilerinin yeri [History of mathematics and the place of Turkish-Islamic mathematicians]*. İstanbul: MEB Yayınları.

Gulikers, I., & Blom, K. (2001). A historical angle, a survey of recent literature on the use and value of history in geometrical education. *Educational Studies in Mathematics, 47*, 223-258.

Gürbüz, R., Çavuş Erdem, Z., Şahin, S., Temurtaş, A., Doğan, C., Doğan, M. F., Çalık, M., & Çelik, D. (2018). Bir disiplinler arası matematiksel modelleme etkinliğinden yansımalar [Reflections from an interdisciplinary mathematical modeling activity]. *Adıyaman University Journal of Educational Sciences, Special Issue,* 1-22.

Hıdıroğlu, Ç. N., & Bukova Guzel, E. (2017). The conceptualization of the mathematical modelling process in technology-aided environment. *The International Journal for Technology in Mathematics Education, 24*(1), 17-36.

Hıdıroğlu, Ç. N., & Bukova Güzel, E. (2013). Teknoloji destekli ortamda matematiksel modellemede modelin doğrulanmasındaki yaklaşımların ve düşünme süreçlerinin kavramsallaştırılması [Conceptualization of approaches and thought processes emerging in validating of model in mathematical modeling in technology aided environment]. *Kuram ve Uygulamada Eğitim Bilimleri Dergisi (KUYEB), 13*(4), 2487-2508.

Hıdıroğlu, Ç. N., & Bukova Güzel, E. (2014). Matematiksel modellemede GeoGebra kullanımı: Boy-ayak uzunluğu problemi [Using GeoGebra in mathematical modeling: The Height-Foot Length problem]. *Pamukkale Üniversitesi, Eğitim Fakültesi Dergisi, 36*(2), 29-44.

Hıdıroğlu, Ç. N., & Bukova Güzel, E. (2015). Teknoloji destekli ortamda matematiksel modellemede ortaya çıkan üst bilişsel yapılar [Metacognitive structures occuring in mathematical modelling within a technology enhanced environment]. *Türk Bilgisayar ve Matematik Eğitimi Dergisi, 6*(2), 179-208.

Hıdıroğlu, Ç. N., & Bukova Güzel, E. (2016). Transitions between cognitive and metacognitive activities in mathematical modelling process within a technology enhanced environment. *Necatibey Faculty of Education Electronic Journal of Science and Mathematics Education, 10*(1), 313-350.

Hıdıroğlu, Ç. N., & Can, B. (2018, October). HTTM (History/Theory/Technology/Modeling) öğrenme ortamının fen bilgisi öğretmeni adaylarının matematiksel düşünmeye ilişkin algılarına ve matematiksel modelleme becerilerine etkisi [Effect of HTTM (History/ Theory/ Technology/ Modeling) learning environment on preservice science teachers' perceptions of mathematical thinking and mathematical modelling skills]. Paper presented at *14. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi*, Pamukkale Üniversitesi, Denizli, Türkiye, pp. 196-197.

Hıdıroğlu, Ç. N., & Özkan Hıdıroğlu, Y. (2016). Modelleme yaklaşımlarına bütüncül bir bakış ve yeni bir öğrenme modeli önerisi: HTTM (History/Theory/Technology/Modeling) modeli ve kuramsal temeli [A holistic view of modeling approaches and a new learning model proposal: HTTM (History/Theory/Technology/Modeling) model and its theoretical basis]. In Ö. Demirel & S. Dinçer (Eds), *Eğitim bilimlerinde yenilik ve nitelik arayışı*, (pp. 1109-1142). Ankara: Pegem Akademi.

Hıdıroğlu, Ç. N. (2012). *Teknoloji destekli ortamda matematiksel modelleme problemlerinin çözüm süreçlerinin analiz edilmesi: Yaklaşım ve düşünme süreçleri üzerine bir açıklama [Analysing mathematical modelling problems solving processes in the technology-aided environment: An explanation on approaches and thought processes]*. Unpublished master thesis, Dokuz Eylül University, İzmir, Turkey.

Hıdıroğlu, Ç. N. (2015). *Teknoloji destekli ortamda matematiksel modelleme problemlerinin çözüm süreçlerinin analizi: Bilişsel ve üstbilişsel yapılar üzerine bir açıklama [Analysing problem solving processes of mathematical modelling in the technology aided environment: An explanation on cognitive and metacognitive structures]*. Unpublished master thesis, Dokuz Eylül University, İzmir, Turkey.

Hıdıroğlu, Ç. N. (2018a). An overview of the HTTM (History / Theory / Technology / Modeling) learning process (Expectations, needs and flow of the process): Galileo and the Tower of Pisa experiment HTTM task. In A. B. Bostancı & S. Koçak (Eds), *2nd Education Research and Teacher Education [ERTE] Congress Proocedings Book* (pp. 814-828), Adnan Menderes University, Aydın.

Hıdıroğlu, Ç. N. (2018b). HTTM (History/Theory/Technology/Modeling) öğrenme sürecinde oluşturulan üst düzey matematiksel modeller [High-level mathematical models created in the HTTM (History/Theory/Technology/Modeling) learning process]. In Ş. Çınkır (Ed.), *Vth International Eurasian Educational Research Congress 2018 Conference Proceedings* (pp. 1043-1045). Ankara: Anı Yayıncılık.

Hohenwarter, M., Hohenwarter, J., Kreis, Y., & Lavicza, Z. (2008). *Teaching and learning calculus with free dynamic mathematics software GeoGebra*. Retrieved January 17, 2015 from https://archive.geogebra. org/static/publications/2008-ICME-TSG16-Calculus-GeoGebra-Paper.pdf.

Hurme, T. R., & Järvelä, S. (2001). Metacognitive processes in problem solving with CSCL in mathematics. In P. Dilenburg, A. Eurelings & K. Hakkarainen (Eds), *European perspectives on computer-supported collaborative learning* (pp. 301-307). Maasticht, The Netherlands: University of Maasticht.

Jahnke, H. N. (2000). The use of original sources in the mathematics classroom. In J. Fauvel & J. vanMaanen (Eds), *History in mathematics education, the ICMI study* (pp. 291-328). Dordrecht, The Netherlands: Kluwer.

Jankvist, T. U. (2009). A categorization of the whys and hows of using history in mathematics education. *Educational Studies in Mathematics, 71*, 235-261.

Kaiser, G., & Schwarz, B. (2006). Mathematical Modelling as bridge between school and university. *ZDM-Zentralblatt für Didaktik der Mathematik, 38*(2), 196-208.

Kaiser, G. (2005). Introduction to the working group "Applications and modelling". In M. Bosch (Ed.), *Proceedings of the Fourth Congress of the European Society for Research in Mathematics Education* (pp. 1613-1622). Universitat Ramon Llull: FUNDEMI IQS.

Kaiser, G., & Sriraman, B. (2006). A global survey of international perspectives on modelling in mathematics education. *ZDM-Zentralblatt für Didaktik der Mathematik*, *38*(3), 302-310.

Katz, J. V. (2007). Stages in the history of algebra with implications for teaching, *Educational Studies in Mathematics, 66*, 185-201.

Kızılay, E. (2016). Fen bilgisi öğretmen adaylarının FETEMM alanları ve eğitimi hakkındaki görüşleri [Pre-service science teachers' opinions about STEM disciplines and education]. *The Journal of Academic Social Science Studies, 47*, 403-417.

Kim, G. S., & Choi, S. Y. (2012). The effect of creative problem solving ability and scientific attitude through the science based steam program in the elementary gifted students. *Elementary Science Education, 31*(2), 216-226.

Lee, J. W, Park, H. J., & Kim, J. B. (2013). Primary teachers' perception analysis on development and application of STEAM education program. *Elementary Science Education, 31*(1), 47-59.

Leng, W. N. (2006). Effects of an ancient Chinese mathematics enrichment programme on secondary school students' achievement in mathematics. *International Journal of Science and Mathematical Education, 4*, 485-511.

Lesh R., & English L. D. (2005). Trends in the Evolution of models & modeling perspectives on mathematical learning and problem solving. *ZDM-Zentralblatt für Didaktik der Mathematik, 37*(6), 487-489.

Lesh, R., & Doerr, H. M. (2003). Foundations of a models and modeling perspective on 3 mathematics teaching, learning, and problem solving. In R. Lesh and H. M. Doerr (Eds), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning and teaching* (pp. 3-34). Mahwah, NJ: Lawrence Erlbaum.

Lesh, R., Hoover, M., Hole, B., Kelly, A., & Post, T. (2000). Principles for developing thought-revealing activities for students and teachers. In A. Kelly & R. Lesh (Eds), *Handbook of research in mathematics and science education* (pp. 113-149). Mahwah, NJ: Lawrence Erlbaum Associates.

Lingefjärd, T. (2000). *Mathematical modeling by prospective teachers using technology.* Doctoral dissertation, University of Georgia, USA.

Lingefjärd, T. (2006). Faces of mathematical modeling. *ZDM-Zentralblatt für Didaktik der Mathematik, 38*(2), 96-112.

Lingefjärd, T. (2007). Mathematical modelling in teacher education- necessity or unnecessarily, In W. Blum, P. L. Galbraith, H. W. Henn & M. Niss (Eds), *Modelling and applications in mathematics education: 14th ICMI Study* (pp. 333-340). New York: Springer.

Liu, H. P. (2003). Do teachers need to incorporate the history of mathematics in their teaching?. *Mathematics Teacher, 96*(6), 416-421.

Maaß, K. (2006). What are modelling competencies?. ZDM-*Zentralblatt für Didaktik der Mathematik. 38*(2), 113-142.

Maaß, K. (2011). How can teachers' beliefs affect their professional development?. *ZDM Mathematics Education, 43*(4), 573–586.

Magiera, M. T., & Zawojewski, J. (2011). Characterizations of social-based and self-based contexts associated with students' awareness, evaluation, and regulation of their thinking during small-group mathematical modeling. *Journal for Research in Mathematics Education, 42*(5), 486-520.

McBride, C. C., & Rollins, H. J. (1977). The effects of history for mathematics on attitudes toward mathematics of college algebra students. *Journal for Research in Mathematics Education, 8*(1), 57-61.

Merriam, S. B. (2009). *Qualitative research: A guide to design and implementation*. San Francisco, CA: Jossey-Bass.

Metin, M., Birişçi, S., & Coşkun, K. (2013). Öğretmen adaylarının öğretim teknolojilerine yönelik tutumlarının farklı değişkenler açısından incelenmesi [Investigation of prospective teachers' attitudes towards instructional technology in terms of different variables]. *Kastamonu Eğitim Dergisi, 21*(4), 1345-1364.

Mitchell, M. (1995). *Mathematical history, activities, puzzles, stories and games*. Virginia: National Council of Mathematics Teacher.

Mousoulides, N., Christou, C., & Sriraman, B. (2006). *From problem solving to modelling- A meta analysis*. Retrieved October 14, 2018 from https://www.researchgate.net/publication/254102378_From_Problem_Solving_to_Modelling_A_meta-analysis

Mousoulides, N., Chrysostomou, M., Pittalis, M. & Christou C. (2010). *Modeling With Technology In Elementary Classrooms*. Retrieved July 15, 2013 from https://www.researchgate.net/profile/Nicholas_Mousoulides/publication/228468165_Modeling_with_technology_in_elementary_classrooms/links/0deec521b3a8e181b0000000.pdf.

National Council of Teachers of Mathematics [NCTM], (1998). *Principles and standards for school mathematics: Discussion draft*. Reston, VA: Author.

National Council of Teachers of Mathematics [NCTM], (2000). *Curriculum and evaluation standards for school mathematics*. Reston, VA: NCTM Publications.

Özbilen, A. G. (2018). STEM eğitimine yönelik öğretmen görüşleri ve farkındalıkları [The opinions of elementary teachers about STEM education]. *Bilimsel Eğitim Araştırmaları, 2*(1), 1-21.

Peter Koop, A. (2009). Teaching and understanding mathematical modelling through Fermi problem. In B. Clarke, B. Grevholm & R. Millman (Eds), *Tasks in primary mathematics teacher education* (pp. 131-146). New York: Springer.

Piaget, J. (1973). *To understand is to invent*. New York: Grossman.

Piaget, J. (1978). *Success and understanding*. Cambridge, MA: Harvard University Press.

Pilten, P., Serin, M. K. & Işık, N. (2016). Sınıf öğretmenlerinin matematiksel modellemeye ilişkin algılarını belirlemeye yönelik bir olgubilim çalışması [A phenomenological study on classroom teachers' perceptions about mathematical modeling]. *Electronic Turkish Studies, 11*(3), 1919-1934.

Punch, K. F. (2005). *Introduction to social research–quantitative & qualitative approaches*. London: Sage

Reimer, W., & Reimer, L. (1992). *Historical connections in mathematics*. California: AIMS Educational Foundation.

Riskowski, J. L., Todd, C. D., Wee, B., Dark, M., & Harbor, J. (2009). Exploring the effectiveness of an interdisciplinary water resources engineering module in an eighth grade science course. *International Journal of Engineering Education, 25*(1), 181-195.

Rivers, W. P. (2001). Autonomy at all costs: An ethnography of metacognitive self-assessment and self-management among experienced language learners. *The Modern Language Journal, 85*(2), 279-290.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334-370). Macmillan: New York.

Sezer, S. (2005). Öğrencinin akademik başarısının belirlenmesinde tamamlayıcı değerlendirme aracı olarak rubrik kullanımı üzerinde bir araştırma [A research on the use of rubrics as a complementary assessment tool in determining the academic success of the student]. *Pamukkale Üniversitesi Eğitim Fakültesi Dergisi, 18*, 61-69.

Siew, N. M., Amir, N., & Chong, C. L. (2015). The perceptions of pre-service and inservice teachers regarding a project-based STEM approach to teaching science. *Springer Plus, 4*(8), 1-20

Siu, M. K. (2003). Proof and pedagogy in Ancient China: examples from Liu Hui's commentary on Jiu Zhang Suan Shu. *Educational Studies in Mathematics, 24*, 345-357.

Sole, M. (2013). A primer for mathematical modeling. *Journal of Mathematics Education at Teachers College, 4*, 44-49.

Stillman, G., Galbraith, P., Brown, J., & Edwards, I. (2007). A framework for success in implementing mathematical modelling in the secondary classroom. *Mathematics: Essential Research, Essential Practice*, *2*, 688- 697.

Sullivan, K. M. (2000). *Preservice secondary mathematics teachers' attitudes about the history of mathematics*. Unpublished master thesis, University of Nevada, Las Vegas.

Swetz, J. F. (1994). *Learning activities from the history of mathematics*. Portland: Walch Publishing.

Şahin, N., & Eraslan, A. (2018). İlkokulda model oluşturma etkinlikleri nasıl uygulanmalı? [How to apply model eliciting activities in primary school?]. *Eğitim Kuram ve Uygulama Araştırmaları Dergisi, 4*(1), 99-117.

Şen Zeytun, A. (2013). *An investigation of prospective teachers' mathematical modeling processes and their views about factors affecting these processes*. Unpublished doctoral dissertation, Middle East Technical University, Ankara, Turkey.

Tekin, A., & Bukova Güzel, E. (2014). Model oluşturma etkinlikleri: Kuramsal yapısı ve bir örneği [Model eliciting activities: The theoretical structure and its example]. *Ondokuz Mayıs Üniversitesi Eğitim Fakültesi Dergisi 33*(1), 95-111

Thomas, K., & Hart, J. (2010). Pre-service teacher perceptions of model eliciting activities. In R. Lesh, P. L. Galbraith, C. Haines, A, Hurford (Eds), *Modeling students' mathematical modeling competencies* (pp. 531-539). New York, NY: Springer Science & Business Media.

Thomas, T. A. (2014). *Elementary teachers' receptivity to integrated science, technology, engineering, and mathematics (STEM) education in the elementary grades*. Unpublished doctoral dissertation, University of Nevada, Reno.

Thompson, D. R., & Senk, S. L. (1998). Using rubrics in high school mathematics courses. *The Mathematics Teacher, 91*(9), 786-793.

Turner, R. (2007). Modelling and applications in PISA. In W. Blum, P. Galbraith, H.-W. Henn, & M. Niss (Eds), *Modelling and applications in mathematics education* (pp. 433-440). New York: Springer.

Türker Biber, B., Akkuş İspir, O., & Sonay Ay, Z. (2015). Matematik tarihinin öğretimi için alternatif bir öğretim yöntemi: Yaratıcı drama [An alternative teaching method for teaching history of mathematics: creative drama]. *İlköğretim Online, 14*(4), 1384-1405.

Urhan, S., & Dost, Ş. (2016). Matematiksel modelleme etkinliklerinin derslerde kullanımı: Öğretmen görüşleri [The use of mathematical modelling activities in courses: Teacher perspectives]. *Elektronik Sosyal Bilimler Dergisi, 15*(59), 1279-1295.

Veljan, D. (2000). The 2500-year old Pythagorean theorem. *Mathematics Magazine, 73*(4), 259-272.

Wang, H. H. (2012). *A new era of science education: science teachers' perceptions and classroom practices of science, technology, engineering, and mathematics (STEM) integration*. Unpublished doctoral dissertation, Minnesota University, Minnesota.

Watson, J. M., & Chick, H. L. (2001). Factors influencing the outcomes of collaborative mathematical problem solving: An introduction. *Mathematical Thinking and Learning, 3*(2-3), 125-173.

Wing, J. M. (2006) Computational thinking. *Communications of the ACM, 49*, 33-35.

Yenilmez, K. (2011). Matematik öğretmen adaylarının matematik tarihi dersine ilişkin düşünceleri [The relationship between prospective mathematics teachers' beliefs and attitudes towards history of mathematics and their creativeness level]. *Pamukkale Üniversitesi Eğitim Fakültesi Dergisi, 30*, 79-90.

Yıldırım, B., & Türk, C. (2018). Sınıf öğretmeni adaylarının stem eğitimine yönelik görüşleri: uygulamalı bir çalışma [Pre-Service primary school teachers' views about STEM education: An applied study]. *Trakya Üniversitesi Eğitim Fakültesi Dergisi, 8*(2), 195-213.

Yıldırım, B. (2016). *7. sınıf fen bilimleri dersine entegre edilmiş fen teknoloji mühendislik matematik (STEM) uygulamaları ve tam öğrenmenin etkilerinin incelenmesi [An examination of the effects of science, technology, engineering, mathematics (STEM) applications and mastery learning integrated into the 7th grade science course]*. Unpublished doctoral thesis, Gazi University, Ankara.

Yimer, A., & Ellerton, N. F. (2006). *Cognitive and metacognitive aspects of mathematical problem solving: an emerging model*. Retrieved July 15, 2017 from http://docplayer.net/44153621-Cognitive-and-metacognitive-aspects-of-mathematical-problem-solving-an-emerging-model.html

Yu, S., & Chang, C. (2009). *What did Taiwan Mathematics teachers think of model-eliciting activities and modeling?*. Retrieved June 21, 2018 from http://120.107.180.177/1832/9802/98-2-04pa.pdf.

Zawojewski, J. S., Lesh, R., & English, L. D. (2003). A models and modelling perspective on the role of small group learning. In R. A. Lesh & H. Doerr (Eds), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning and teaching* (pp. 337-358). Mahwah, NJ: Lawrence Erlbaum.

Authors

Çağlar Naci HIDIROĞLU, Assoc. Prof. Dr., Pamukkale University, Faculty of Education, Department of Mathematics and Science Education, Denizli (Turkey). E-mail: chidiroglu@pau.edu.tr. ORCID ID: https://orcid.org/0000-0002-3774-4957.