Epistemological Obstacle in Learning Trigonometry

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Abstract: *Epistemological obstacle is emphasized in mathematics education. Students often have limited students' context of knowledge in understanding trigonometry. Knowing the epistemology obstacle can help a teacher understand the student's misconception. Therefore, this study aimed to identify students' epistemological obstacles of trigonometry and trigonometric function. This research used two subjects of 11th grade high learners of high school. The research data were collected in audio, photograph, and trigonometry test. Data were analyzed by using in and between conditions students interview. This study suggests that students tend to associate the trigonometric value into a particular angle, such as having difficulty using angle in radian and could not recognize the value of π. They also tend to follow the procedural steps in converting angle from radian to degree and vice versa without knowing how the formula is constructed. They have difficulty figuring out the trigonometric function's value, especially for angle in the quadrant and the coordinate of a point trigonometry graph, primarily related to the radian unit.*

INTRODUCTION

Trigonometry, a mathematical concept, especially within the framework of a trigonometric function, has become a part of mathematics curricula in high school. As of the essential concepts of mathematics, trigonometry is both a unifier and a hypernym for many mathematical subjects such as geometry, function, and calculus (Weber, 2008). Trigonometric functions, formed by the definition of trigonometry as ratios of right-triangle side in a unit circle, are a part of calculus courses. Several studies have common conclusions that the traditional methods in teaching trigonometry are inadequate to introduce the students into a trigonometric function's concept (Kamber & Takaci, 2017; Orhun, 2001; Weber, 2005). The traditional methods mentioned here involve procedurally teaching the courses within the framework of definitions, theorems, proofs, and problem-solving of trigonometry as a ratio of right-triangle only.

Understanding mathematical concept is quite challenging for students. However, if a student is not asking herself questions and solving problems, she is not doing mathematics. While doing a mathematics task, a student may manifest error. Errors are not only the effect of ignorance, of uncertainty, of chance but also the effect of a previous piece of knowledge that was interesting and successful but now is revealed as false or irrelevant. Errors of this type are not erratic and unexpected. They constitute obstacles (Brousseau, 1989). An error can be identified by seeing the result of students' work. For instance, in the research of Kamber & Takaci (2017) research, a student implements one of the properties $\sin(mx) = m\sin(x)$ in determining the value of $\sin 270^o$. The student gets the result that:

$$\sin 270^o = \sin(3.90^o) = 3.\sin 90^o \quad (1)$$

The student knows that the property is correct in algebra, so the student applies the equation to determine the other sine values of angles. While analyzing this error, it raises questions (Do the students not understand how to find out trigonometric values? Do the students use the wrong concepts? Do the students mix up the understanding of algebra and trigonometry? Do the students understand the meaning of sine? Do the errors occur because of prior knowledge of the students or occur during the learning process). For sure, each student has reasons or arguments that support the answer. The student uses a concept in a particular context and applies them to another context (Brousseau, 2002). The students might not realize that what they are doing is incorrect, for it makes sense to them. The error made by students indicates learning obstacles experienced by students.

The obstacles arose as a result of learning where situations experienced by students are insufficient in facilitating students to obtain correct and complete knowledge (Begg et al, 2003). Analysis of students' difficulty in learning trigonometry was studied by several researchers (Kamber & Takaci, 2017; Orhun, 2001; Weber, 2005). However, there was no analysis of students' understanding of trigonometry based on the epistemological obstacle. Classification of the students' difficulty through the classification of students' learning obstacles will facilitate the teacher, educators in solving the misconception and error. Hence, this article is concerned with identifying epistemological obstacles. This study aims to examine the epistemological obstacle of trigonometry and trigonometric functions and addresses the following research questions:

- What is the kind of epistemological obstacles of high school students in mathematics who have learned trigonometry within the context of the Theory of Didactical Situation?

This article is organized as follows: The following section presents the theoretical framework, the methodology adopted by this research, followed by a section that presents the results and provides the discussion of the results; and the last section concludes the paper with a summary of the discussion.

THEORETICAL FRAMEWORK

Obstacle in Theory of Didactical Situation

The theory of didactical situation, a learning theory in mathematics education, works in a framework that the knowledge gained by students comes from students' adaptation to the didactic situation given to the students (Brousseau, 2002). Thus, there will be situations that shape the students' knowledge. An appropriate learning situation can give correct concepts and then construct them into new knowledge. However, if a student experiences a learning situation that could not provide both the concept and knowledge construction, then the learning situation is insufficient to facilitate students to obtain the correct and complete knowledge (Artigue et al, 2014). In this case, there is a possibility that students' knowledge becomes an obstacle to further learning (Skordoulis et al, 2009).

Brousseau (2002) revealed that learning obstacles could be seen or indicated by students' mathematics errors. Errors can be explicitly identified in the students' work. However, errors indicated as an obstacle are not accidentally happened (e.g., careless in calculating), but those are resistant to changes and will be repeated by students. These errors occur consciously and are re-used and challenging to disappear. We illustrate obtaining new knowledge in the theory frame of a didactical situation as Figure 1.

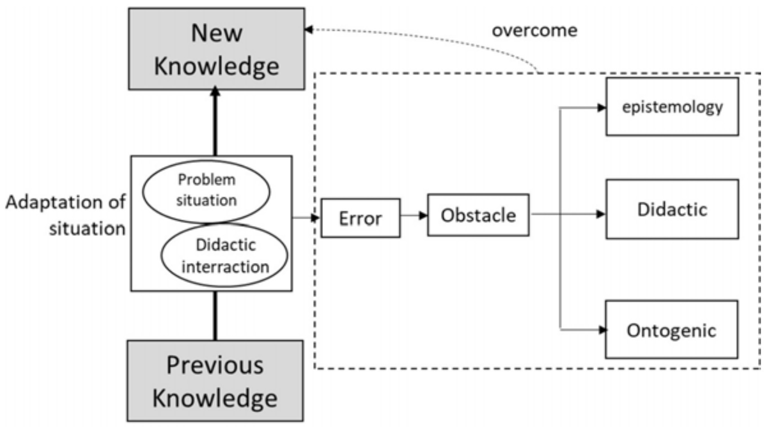


Figure 1 Scheme of the Obtaining New Knowledge

An obstacle is an integral part of the learning process. A reflection on obstacles becomes essential in changing learning models as important content in the learning process. It offers a thought-provoking opportunity to develop mathematical abilities because it can explicitly determine the direction of how the learning process takes place (Sztajn et al, 2012).

Brousseau (2002) revealed that the emergence of learning obstacles did not merely occur because of a single system of interaction. He determines that the learning obstacles can be derived from three origins (learning obstacles originating from ontogenic, didactical, and epistemological sources). The Ontogenic obstacles are developmental related to the stages of a child's mental growth. The Didactical obstacles arise as a result of teaching decisions. These obstacles can be avoided by developing alternative instructional approaches. Brousseau defines epistemological obstacles as forms of knowledge that have been relevant and successful in specific contexts, including school contexts, but that has become false or simply inadequate at some point, and whose traces can be found in the domain's historical development (Artigue et al., 2014). It emerges regardless of the instructional strategy since the concept itself is the source of the problem. Thus, identifying and characterizing an obstacle is essential to analyzing and constructing didactical situations.

Understanding and overcoming epistemological obstacles are essentially the same thing in many circumstances. Epistemological obstacles look backward, focusing on what was wrong, inadequate in the ways of knowing while understanding anticipates new ways of knowing (Sierpinska, 1990). Epistemological and cognitive considerations are, of course, not independent: The aim is to identify the conditions for a planned process of learning through which students construct and use those features of trigonometry that the epistemological analysis has identified as constituting the concept. This result of the study functions as an instrument to guide theoretically informed local decisions about teaching in trigonometry for teachers (Brousseau, 1989).

Understanding the concept of trigonometry and trigonometric function

Of the many sentences that can be formulated about trigonometry and trigonometric functions, let us choose this one:

- Trigonometry: ratios of sides of a right-angled triangle.
- Trigonometric functions: Real Functions defined for the domain of real numbers which relate an angle and for the range of real numbers

The logical sentence of this sentence can be written as Figure 2:

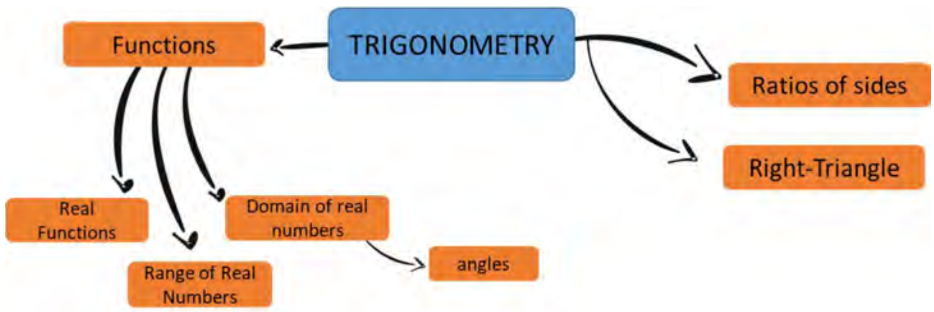


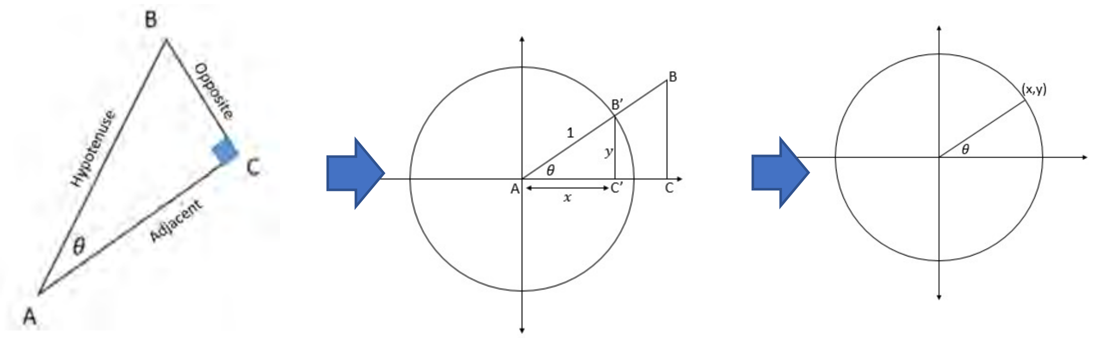
Figure 2 Logical structure of the trigonometry and trigonometric definition

This structure defines the sense of the definition. It shows the difference between trigonometry and trigonometric functions. In trigonometry, the objects point out the ratios of two ideas: side and right triangle. It can be inferred that the object of trigonometry refers to the angle in the right triangle. In the case of trigonometric functions, an image from $\mathbb{R}$ to $\mathbb{R}$ generally points to an angle. It can be inferred that the object of trigonometric functions refers to any angle. Understanding both definitions will lead to the perception that there is no difference between trigonometry and trigonometric functions. However, in saying this, it defines a type of relation.

The trigonometric function definition is developed from the trigonometry concept in the right triangle. As a historical approach to trigonometry, the trigonometric function definition is related to the trigonometry in the right triangle (Moore, 2012). To explore this relation, let a right triangle ABC has right angle θ. The θ is in standart position, so that $B'(x, y)$ is in unit circle and point $C'(x, 0)$ is in x-axis (see Figure 3). The triangle ABC is congruent with triangle $A'B'C'$. Thus, these properties are followed:

$$\frac{\text{opposite}}{\text{hypotenuse}} = \frac{y}{1} = y \quad (2)$$

$$\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{x}{1} = x \quad (3)$$



Right triangle | θ in standar position | The point of (x, y) in unit circle

Figure 3. Consistency in Trigonometric Definition

The equation (2) and (3) are known as the definition of sine and cosine. Thus the value of $\sin(\theta)$ is y and the value of $\cos(\theta)$ is x. Thus, the coordinate of $(x, y) = (\cos x, \sin x)$. The process of drawing trigonometric function can be drawn from a unit circle (see: Figure 4)

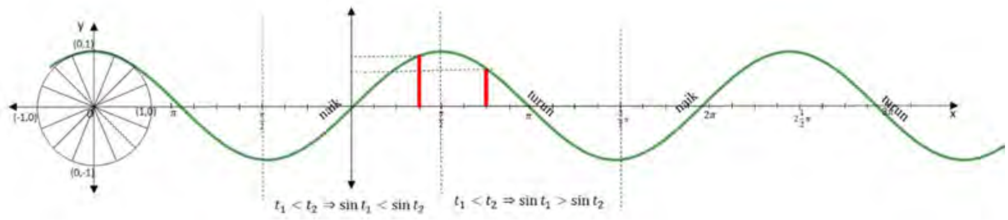


Figure 4 Sine Function Graph from A Unit Circle

METHOD

This study is qualitative which was conducted in a case-study pattern. According to Yin (2003), a case study is an empirical research method used in cases where more than one source of proof or data is present. This study focuses on high school students' obstacles in trigonometric functions. For this purpose, the Theory of Didactical Situations was used, and then a coding table was formed. Based on this table, students' mental structures and mechanisms were coded within the context of the theory of the didactical situation. In line with this coding, the students were interviewed to reveal their trigonometric functions' mental structures and thinking mechanisms.

Participant

This study was conducted in Indonesia with two high school students as participants. A purposive sampling method revealed the learning obstacle regarding their conceptual understanding of trigonometric function. The participants were selected according to the following criteria:

(1) Participants are expected to have completed the trigonometry course in the curriculum. Trigonometry courses include the topics of ratios of right-triangle sides, radian, trigonometric graph, the value of trigonometry, period, maximum, and minimum of a trigonometric function. The reason behind this criterion is that the genetic decomposition steps for understanding the trigonometric function concept are among the learning outcomes of these courses. In line with the instructors' views teaching these courses, it is necessary for the students to freely express their ideas about a problem and provide a basis for them.

(2) Based on the students' performance in mathematics. The categories were created for the students who have high performance in mathematics based on information from the teacher.

Tools for data collection

In this study, the data were retrieved from two sources. The first one is the written responses that the students gave to the Trigonometric Function-Understanding test, and the second one is the tape recording of clinical interviews concerning these questions. A measurement tool was developed to analyze the learning obstacle of the trigonometric functions. The questions were taken from Kamber & Takaci (2017) and Weber (2005). Based on expert opinion, the first question regarding the mathematical argumentation was added from Maknun et al. (2018), and the second question regarding converting angle from radian to degree and vice versa were added. Further, the sixth question regarding the graphical representation of trigonometric function was adapted from Brousseau (2002). The previously constructed schemas, which the students are presumed to know to understand the trigonometric function, are as follows:

(1) The intuitive notion of ratios of trigonometric such as formulas of sine, cosine, and tangent in right-triangle,
(2) The Cartesian plane (that is, the concept of points as objects; functions, curves, and areas as processes resulting from the generalization of the action of representing the components of points),
(3) Real numbers (that is, the concept of number as an object; arithmetic and algebraic transformations as processes),
(4) Sets,
(5) Real functions with real values (that is, the function as a process, operations with functions, and the coordination of the algebraic and geometric representations of functions)

The questions corresponding to the trigonometric functions used in this study to figure out students learning obstacles are presented in Table 1.

Questions		Questions corresponding to trigonometric understanding
1		The action of invoke the concept image of equation of trigonometric function and validate the statement
2	2a	The action of converting angle from degree to radian
	2b	The action of converting angle from radian to degree
3	3a	The action of sorting the sine value through estimating their value from the drawing angles
	3b	The action of sorting the cosine value through estimating their value from the drawing angles
4		The action of analysing about maximum and minimum values of sine function,
5		The action of determining the coordinate of particular point in graphical representation of trigonometric functions.
		The action of determining the domain and range of the trigonometric function through graphical representation.

Table 1 Questions corresponding to trigonometry test

Desired Responses

Question 1

It is not true that sin x = $\frac{1}{2}$ is 30°, because the domain of the x is $90^o \leq x \leq 360^o$. Thus the 30° is not the single answer. There is another solution, i.e., 150°.

Question 2

In this question, the students were not expected only on the question's correctness, but they could also explain how the formula is constructed. The construction of radian and degree equality is figure

$$\text{Full angle (in radian)} = \frac{\text{circumference of circle}}{\text{radius}} \quad (4)$$

$$360^o = \frac{2\pi r}{r} \quad (5)$$

$$360^o = 2\pi \text{ radian} \quad (6)$$

$$180^o = \pi \text{ radian} \quad (7)$$

$$\frac{180^o}{\pi} = 57{,}3^o = 1 \text{ rad} \quad (8)$$

$$1^o = \frac{\pi}{180} = 0{,}0174 \quad (9)$$

Question 3

Knowledge about the definition of trigonometric functions is needed to answer this question. In the cartesian plane, the angle coordinate represents the value of sine and cosine. Thus, if $P(x, y)$ is a point in the unit circle, the sine value is y, and the cosine value is x. Hence, to arrange the value of sine and cosine to the correct order, the students should compare each coordinate. Since the coordinates were not available in this question, students needed to estimate each angle. The correct order is $\cos\beta < \cos\alpha < \cos\theta$ and $\sin\theta < \sin\alpha < \sin\beta$.

Question 4

n answering this question, the students could use any trigonometry context. One of the contexts they can use is the unit circle. The expected answer from students is that they can explain from the unit circle that there is no possible value of function $y = \sin x$ such that smaller than -1. Otherwise, they can illustrate through trigonometry graph.

Question 5

This question has a connection with the coordinate system that they have ever learned before. The cartesian plane was used to pair numbers from ℝ to ℝ. We used this question to see if the students used radian or degree to determine the coordinate. Using radians in answering this question is the response we expect to appear. Thus the correct answer to this question is $(\pi, -1)$.

Clinical interview

The students were interviewed to analyze their understanding and views of the mathematical concepts in the questions and obtain in-depth data by revealing their mental mathematical structures. The interviews were tape-recorded with consent from the students. There were no time limits during the application or the interview. Following the test analysis, the students have interviewed the questions coded as 0 and 1, according to Table 2. During the interviews, discussions were held concerning the answers given by the students to the questions chosen from the test. In the clinical interviews, the students were asked questions such as "What did you think in this question?" and "Can you explain what you have done in this question?" Furthermore, the students were asked some follow-up questions such as "How would you think if I give you these angles?"

Data Analysis

The content analysis method was used to analyze the data obtained in the study. The answers given by the students were analyzed under the themes of learning obstacles (epistemological, didactical and ontological obstacle) proposed by (Artigue et al., 2014; Brousseau, 2002). In this study, we are re-examining the interpretation of students' errors and how they are produced; we found recurrent errors, and showed that they are grouped around conceptions.

The written documents about the questions in the application were examined, and the answers given by the students were coded as 0, 1, and 2 to determine the incorrect or incomplete answers. The coding was performed according to Table 3. Then, the mathematical reasons behind appearing the epistemological obstacle for the questions coded as 0 or 1 were investigated through clinical interviews with the students. The interview recordings were transferred to a computer and transcribed. For example, the transcribed documents obtained from the test of S-1 and s-2 were examined; Questions 3 was coded as "0," and S-1 and S-2 were interviewed about these questions. The interview showed that both S-1 and S-2 had difficulty in the geometric representation of the trigonometric value, particularly in a unit circle, although both the students had approached it through the positive and negative values in each quadrant. This situation may be attributed to the fact that the student could not find trigonometric value as a coordinate of points in a unit circle.

0 (incorrrect)	1 (partially correct)	2 (correct)
An incorrect answer is given to the question, or the question is not answered (lack of schema that should have previously been known to understand the concept of trigonometry).	Some steps required to solve the question are performed, but the explanation about why perform those answers is not well explained.	A correct answer is given to the question and well explained.

Table 2 Code and explanation

The questions given to the students in the interview strengthened the finding in students' work. Throughout the interview session, students could provide arguments for their answers related to trigonometry on why a mathematical statement is true or why the answers to math problems are obtained (Sriraman & Umland, 2014). The forms of arguments given by students can vary, for example, informal evidence, explanation, and working steps. Thus, the interview can determine students' understanding of trigonometry and students' mistakes when working on a given problem. The theory of learning obstacles proposed by Brousseau (2002) informed this study's theoretical backgrounds and goals. In the analysis process, the researcher identified common themes found in students' work, such as error and misunderstanding in trigonometric function, to discover the learning obstacles. After identifying these themes, the researcher searched the data for additional instances that could support and contradict these themes.

RESULTS

In this section, the findings obtained from the data analysis are discussed. The findings include some of the answers given by the students to each question. The data was obtained from the interview analysis. The students learning obstacle of trigonometric was evaluated within the theory of the didactical situation. In the clinical interview dialogues, the researcher was denoted with the letter "R" whereas the students were denoted with the letter "S." The answers given by the students during the interview were represented as S-1 and S-2, as shown in Table 3.

	1	2a	2b	3a	3b	4	5
S-1	0	1	1	0	1	2	0
S-2	0	1	1	0	1	1	2

Table 3 Codings of the answers given to the questions by the students

Question 1

In this question, the aim is to invoke the concept image of the equation of trigonometric function and validate the statement. When Table 3 is examined, it is seen that both the students answered the first question incorrectly. As indicated in Table 2, both students could not invoke the concept image of the equation of trigonometric function and validate the statement. The excerpts from the clinical interview are given below.
R: Can you explain why you think this statement is true?

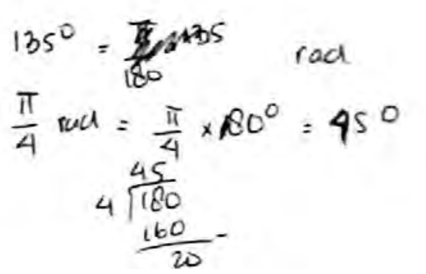
S-1: "I remember it from the [trigonometric] table that $\sin 30° = \frac{1}{2}$"

S-2: "Because $\sin 30° = \frac{1}{2}$"

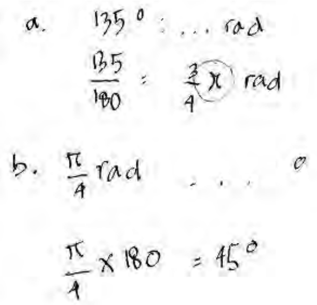
Both students expressed the concept of trigonometric values correctly. However, they ignored the parts where the value of x not only 30° since the domain of x is $90^o \leq x \leq 360^o$. As a result, it can be suggested that their intuitive notion of space schemas is incomplete. In this case, the students tend to associate the sine values with the table, which lists the values of sines, cosines, and tangents at a particular angle given by the teacher. So the sine and cosine values that students know are $0, \frac{1}{2}, \frac{1}{2}\sqrt{2}, \frac{1}{2}\sqrt{3}$ and 1. It can also be seen in students' answers to other questions, limiting the sine value to $0, \frac{1}{2}, \frac{1}{2}\sqrt{2}, \frac{1}{2}\sqrt{3}$ and 1. Limiting sine values on that values will lead to obstacles. The sine value is a real number in the interval [-1, 1].

Question 2

In this question, the students were expected to convert an angle from degree to radian and vice versa. Both the students had the same accurate answers and procedure in answering question 2 as presented in Figure 1.



S-1 solution in question 2a and 2b



S-2 solution in question 2a and 2b

Figure 4 Students solution in question 2

Figure 4 demonstrates that S-1 and S-2 correctly answered Question 2b. S-1 gave the unfinished answer, while S-2 correctly answered the same question. Thus, it may be concluded that S-1 was unable to convert degree to radian. Below are the student's explanation and an excerpt from the interview.

R: Can you explain Why did in question 2a you divided the angle with 180, while in question 2b, you multiplied it with 180?

S-2: From the formula I learned from the teacher this formula, the degree to radian is divided by degree while radian to degree multiplied by 180

R: So, did you know how the formula was constructed? I mean, why does the formula provide that way?

S-1: I do not know

S-2: Perhaps there is a reason, but I do not know why. I did not learn that.

As can be seen, both the student could answer the question in a procedural action through a fixed formula. When asked why they should divide or multiply them by 180, they did not know the reasons and followed what had been taught. In converting angles from degrees to radians and vice versa, there was a relationship between π and 180°. To further understand the students' understanding of π, we asked them to determine the value of $\sin\left(\frac{\pi}{4}\right)$, students answered. "Change it to degree first".

R: What is the value of π?

S-1: In a circle, as far as I know $\pi=\frac{22}{7}$, or 3.14. However, the value of π can be different. It is not always $\frac{22}{7}$, especially if there is a word "sin" in front of it. In that case, π is 180"

S-2: It is 180

R: What about if we find out the circle area, what is the value of π?

S-2: "... Hmm, it means π is not $\frac{22}{7}$, isn't it? (doubt). Yes, perhaps it is a provision. In the circle is $\frac{22}{7}$, but in degree becomes 180?"

The interview above found that the relationship between π and 180° influenced students to see the value of π. It can be seen that S-2 doubts why there are two different values of π. It can be seen that S-2 doubts why there are two different values of π, namely $\frac{22}{7}$ and 180°. Whereas S-1 believes that the value of π will be different if the domain area is trigonometric.

Question 3

In this question, the students were asked to sort the value of the sine of the angle from the smallest to the largest through the representation of the given angle without knowing how big the angle was in each picture. An excerpt from the interview with both students is presented below:

R: What is your answer?

S-1 : The order is $\sin\alpha, \sin\beta, \sin\theta$

S-2 : The first order is $\sin\theta$, because it is negative, while $\sin\alpha$ and $\sin\beta$ are positive because they are in quadrant I and II. However, I do not know which one between $\sin\alpha$ and $\sin\beta$ is smaller since they are positive.

R: Can you explain the answer?

S-1: I predict the value. I assume there is an angle such as 60^o. Then the one in quadrant II is likely 120^o because it is more than 90^o and this one is more that 270^o. Let us say it is 300^o, it is approximately about $\frac{1}{2}\sqrt{3}$. So, I predict the angle which is not too far from a special angle. So, th order is $\sin\alpha, \sin\beta, \sin\theta$. For cosine is the opposite [of sine] and will be like this: $\cos\theta, \cos\beta, \cos\alpha$. So evaluating from this [table of trigonometry], the cosine value is the opposite of sine

R: Why does a particular quadrant have positive and negative values in trigonometry?

S-1: I do not know the reason. That is what I learned

R: Can you explain your answer?

S-2: The $\sin\theta$ is the smallest because it is negative [value], while $\sin\alpha$ and $\sin\beta$ are positive because [they are] in quadrants I and II. I do not know which the smallest one [between] $\sin\alpha$ and $\sin\beta$ because they are both positive. Quadrant I positive for sine, cosine, secan, cosec, tan, cotan; sine and cosec [positive in] quadrant I; tangent and cotangen [positive in] quadrant III; cosine equal secan [positive in] quadrant IV"

A further question was proposed for S-2 because he said that he could know exactly the order if he knew what the degree was.

R: What about these? Can you sort these from the smallest to the greatest order ($\sin 10^o$, $\sin 110^o$, $\sin 250^o$, and $\sin 335^o$.)

S-2: "[the order is] $\sin 250^o$, $\sin 335^o$, $\sin 10^o$, $\sin 110^o$ because this angle [pointed out to the angle 250^o and 335^o] are negative, then to determine the smallest of them. Look at $\sin 30^o$ and $\sin 60^o$. $\sin 30^o$ [is] $\frac{1}{2}$ and $\sin 60^o$ is $\frac{1}{2}\sqrt{3}$, so if you have a bigger number [read: angles], then the sine value is also bigger. So this $\sin 335^o$ is the smallest because it is negative."

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Question 4

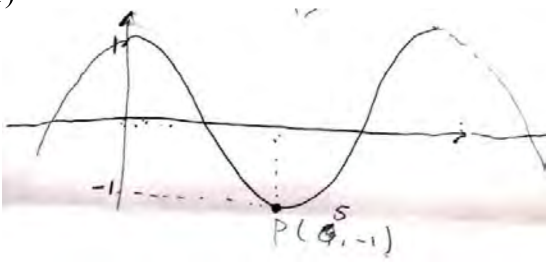
In this question, the students were asked to analyze maximum and minimum values of a sine function, in contrast to Kamber and Takaci (2017), where most students can answer correctly and explain well the reasons why it is an impossible $\sin x = 2$. An excerpt from the interview is presented below.

R: What is your answer? Could you explain your answer?
S-1: "It is impossible because no matter how big the angles are, for example, more than 360, you will find the remaining angles and return to a special angle again",
S-2: It is impossible, because the maximum sine value is 1.
R: Why cannot it be more than 1?
S-2: I do not know the reason.

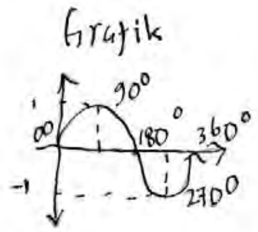
Although S-2 gave the correct answer, he could not explain why the sine value could not be more than one. In comparison, S-2 provided an argument by estimating for angles greater than 360° by using the concept of correlated angles. By analyzing that, any angle can undoubtedly be related to the angle in the first quadrant, wherein the first quadrant was found a particular angle with no value of more than two. S-1 concluded that the statement $\sin x = 2$ is impossible.

Question 5

This question asked students to determine the coordinate of a particular point in a graphical representation of a trigonometric function. When Figure 5 is examined, it is seen that student S-1 answered the sixth question incorrectly, while student S-2 answered correctly. Student S-2 had no difficulty deciding the P coordinate with the cosine function graph he already knew. However, student S-1 provided a different perspective in finding the coordinate of P that her answer was (5,-1)



S-1 solution in question 5



S-2 solution on 5

Figure 5 Students' answer in question 5

The excerpts from the clinical interview are given below

R: Can you explain your answer?

S-1: This one [pointing at y-axis] is -1. Because the cos $0° = 1$ [pointing the graph], the cosine starts from here (pointing the peak of cosine graph) and about the x, it is 5 perhaps [thinking again for a moment] as I remembered, this one (pointing at x-axis) is (in) degree. That is the problem, I am not sure about it.

S-2: I draw the trigonometric function like this (Figure 5)

From question 5, the epistemological obstacle in deciding the point coordinate in a trigonometric graph can be known. S-1 could not recognize the accurate coordinate of a point. Instead of answering $(\pi, -1)$, she answered $(5, -1)$ in which 5 represented her assumption of the length from the point to central coordinate in the x-axist. These epistemological obstacles may result from false intuition in determining prior knowledge to answer the questions or make less straightforward generalizations (Subroto & Suryadi, 2018). The students need to get more concrete activities in trigonometry (Maknun et al, 2020)

DISCUSSION AND SUGGESTION

Students' errors in understanding trigonometry and trigonometric functions reveal students' obstacles related to these concepts. In this study, the students were asked five questions prepared to figure out students' errors of these concepts within the Theory of Didactical Situations framework.

In this study, no students could invoke the concept image of the equation of trigonometric function and validate the statement. The students were unable to invoke the concept image of the equation of trigonometric function is they did not have the intuitive notions of trigonometric function in all angles. For example, S-1 could not assign the all value of the trigonometry equation because she only views the value for angle $0° \leq x \leq 90°$. In general, it was observed that the students who could not analyze trigonometric values in all angles were unable to evaluate a particular angle larger than 90°. Thus, it can be said that errors made regarding trigonometry values are consistent and lead to learning obstacles. Further, the students also had difficulty understanding the concept of radian measurement. Below we explain the learning obstacle related to the epistemological origin.

Obstacle 1 : Finding the value of trigonometry in the term of trigonometric functions

Understanding trigonometric functions usually result in understanding trigonometry in the right triangle. However, this development is not possible without a concept shift of attention. In particular, focusing on the form of the angles or on the rule for generating terms of a function. Demir & Heck (2013) notice this matter and design the bridging concept through a unit circle. Below we shall make some comments on these obstacles:

1. Domain of trigonometric functions in solving trigonometric equations

This obstacle focuses on a trigonometric value on the triangle domain. However, it had been clear from the question that the domain of an angle is $0° \leq x \leq 360°$, both the students could not recognize the possibility of another value of x. This obstacle is not because they did not know that sin $(150^o) = \frac{1}{2}$, but because it is unclear enough for students the term of $0° \leq x \leq 360°$ is for, and because they focus only on the question itself. So that they directly answered that the value of x must be 30° only. Overcoming this obstacle amounts to defining trigonometry as a function, in a similar way to how the students learn functions.

2. The bigger the angle the bigger the value of sine function

The above conception quickly develops if the angles are introduced through excessive practice and stressing of certain angles (30°,45°,60°,90°). The student S-2 had difficulty in answering this problem, where she assumed that the bigger the angle was, the greater the trigonometric value would be. She analyzed from the sine values at special angels that she had memorized. In the particular angle that she memorized, from the selected angle, i.e., 0º, 30º, 45º, 60º, and 90º, indicating an increasingly more prominent sine value. She must assume the sequence of angles (according to the smallest to the biggest trigonometric value) on unclear angles on a unit circle. This case may tend to make unjustified inductive jumps and believe that if they observe on the sine value is getting more prominent as the angles bigger, and this means that the sequence of the trigonometric value depends on how big the angles are.

3. The smaller the angle the smaller the value of cosine function

Like the finding in the sine case above, S-2 was asked to sort cosine values at the same angle. She chose to reverse the order of sine values immediately. It was widely known that the trigonometric value at that angle was that the sine value was getting greater along with the enhancement of the angle. On the contrary, the cosine value was getting smaller with the angle enhancement. The result above suggests that the students sort the sine and cosine values through the trigonometric value at a particular angle, most probably having in mind the table of trigonometric values.

Maknun et al. (2018) found that the particular angle studied by the students limited them to understand the trigonometric value, especially when the students were asked to memorize the table of trigonometric values in a particular angle.
Let us now consider the obstacle:

Obstacle 2: Distinguish the value of π

1. π has two different value (3,14... or 180)

π is the ratio of the circumference of any circle to the diameter of that circle. π has a decimal value of approximately 3,14. However, it is an irrational number, which cannot be expressed as the quotient of two integers. The students in this study were placed in a situation where they must answer the value of π and identify it in circle and trigonometry. The students were interviewed after they finished the task (converting radian to degree and vice versa). This situation raises their question concerning its value (why does it have a different value for the same mathematics symbol?). Before this interview, the students did not see any contradicts in the value of π. The feeling of paradox only when the students were remembered the π in a circle. No one can deliberately choose which value of π but agree that 3,14 and 180 are the values. The students said that both values are correct because the use of π depends on the context (is it trigonometry context or circle context?).

2. Constructing The formula in converting degree to radian and vice versa

Although, the students were able to convert angels from radians to degrees and vice versa well with the formula of multiplying and dividing by 180. However, when asked how the formula was originally from, the students found difficulty. This process revealed that procedural skill was the obstacle for students in understanding the whole mathematics concept. This difficulty is also found in the research of Kansanen and Meri (1999). This finding is similar to what Prihandhika et al. (2020) found that students still focus on procedural understanding in answering mathematical questions. Unlike the previous epistemological obstacles, the students generalized an outcome in a particular context when talking about the value of π. Students assume different values of π in circle and trigonometry context. This understanding was inseparable from the procedure by students in converting angels in radians to degrees and vice versa. Perform

Some Remarks on Obstacles in Trigonometry

It can be seen that students had understood the concept of trigonometric values, specifically at a particular angle. Students were also right in analyzing the trigonometric values at that particular angle. The sine value got more prominent when the angle got bigger and vice versa for the cosine. However, this fact did not apply to all circumstances. When discussing angels in general, a different perspective was needed. This perspective was a prerequisite for the emergence of an epistemological obstacle. In other words, students' knowledge of trigonometric values at certain angles becomes an epistemological obstacle in understanding trigonometric values at all angles (Brousseau, 2002). It means the students' knowledge was limited only to a particular context. This condition results in difficulties when students are given problems in a different context. This may occur due to the teachers of the students. First, if it occurred due to the teacher, the teacher tended to provide a single way or knowledge to the students when teaching a concept. To anticipate this matter, the teacher may collaborate with other teachers to enhance their creativity in designing the concept (Asari et al. 2018; Fauziyah et al., 2021). Second, if it occurred due to students' problems, students were incapable of keeping their pace with the teachers' explanation (Cesaria & Herman, 2019). .

Didactical handling of obstacle

The angle measurement forms an obstacle to the conception of trigonometric functions. For obvious reasons of the proximity of the depth analyses, this obstacle is more difficult to overcome than the one they pose to the conception of trigonometry as ratios of right-triangle. The learning obstacles are primarily about the conceptual problem, especially in trigonometry. This result supports the study of (Kurniasih & Rochmad, 2020), which stated that students with high mathematical beliefs experience conceptual problems in integrating their abilities. Thus, the lesson should be designed to fulfill the students' obstacles as follows:

(1) Students tend to associate the trigonometric value into a particular angle
(2) The students could not distinguish the value of π
(3) The students tend to follow the procedural steps in converting angle from radian to degree and vice versa without knowing how the formula constructed
(4) The students challenging to figure out the value of a trigonometric function, especially for all angles
(5) The students have difficulty figuring out the coordinate of a point in a trigonometric graph, primarily related to the radian unit.

Let us take the example of errors in point (4). To solve a problem, students intuitively guess required trigonometric values by finding the coordinate of sine and cosine in the unit circle. They fail when the angle is not available if placed in the quadrant with the same sign (positive and negative) trigonometric value, confusion with an angle bigger than 90°, or inability to understand the problem. Further, having previously succeeded in converting the degree to radian and vice versa does not entirely prevent the phenomenon from occurring obstacle in constructing the formulae.

Lastly, S-1 and S-2 are students with a high GPA among others; however. It was observed that their errors indicate the obstacles which might lead to the misconception. Thus, future studies may investigate a relationship between the learning obstacle of trigonometry and trigonometric functions and GPA. The prerequisite when choosing students was that they had already taken trigonometry courses and had completed them. When the learning objectives of these courses are examined, it is expected that the students who completed this course will be able to comprehend trigonometry and trigonometric functions. However, the results obtained from this study concerning the learning obstacle regarding trigonometry and trigonometric functions contradict these learning objectives. Thus, the sufficiency of curricula of trigonometry courses might be questioned. We can conclude that the traditional approach to trigonometry education is not adequate for conceptual understanding. This study suggests that the teaching of trigonometry courses should be evaluated and organized within the Theory of Didactical Situations framework to ensure that students can better understand the concepts in the trigonometry courses.

CONCLUSION

The study revealed that epistemological obstacles affected students' understanding of trigonometry and trigonometric functions. It was also found that students'understanding of the trigonometry and trigonometric functions was related to the procedural skill in how to solve the questions. Drawing on the results of the study, it can be said that the errors appearing in students' answers come from the epistemology obstacle such as understanding the angle, understanding of the value of π. In this case, the lack of understanding of the concept of radians becomes an epistemological obstacle in understanding the value of π.

Obstacles can also cause more fundamental educational issues. Many didactical practices justified by the simply additive classical model must be reviewed and perhaps rejected. But this model affects both internal (within and between classes) and external (between teachers and society) negotiations with the educational system, in terms of the teaching curriculum. However didactical issue is not only the diagnosis of errors, their explanation, and the description that follows changed, but also the teacher's and students' roles and obligations have been reassigned. Teachers' epistemologies must be changed in order to integrate this new paradigm of didactical communication.

These findings contribute in several ways to our understanding of didactics obstacles encountered by the students in trigonometry and provide a basis for knowing what material should mostly lead to the students' misconception. Some pratical recommendations for teachers in order to follow up these findings are:

- Teacher should not neglect the appearance of obstacle by students since the fact that various items of knowledge, even incorrect ones, may be required to enable the establishment of definitive knowledge (Brousseau, 2002).
- Design the learning sequences from what the students have known
- Unit circle can fill the gap between the definition of trigonometry as ratios of right-triangle and trigonometric functions
- Radian measurement should be stressed to make better understanding on trigonometric function especially on trigonometric graph.

DATA AVAILABILITY STATEMENT

The authors confirm that the data supporting the findings of this study are available within the article [and/or] its supplementary materials.

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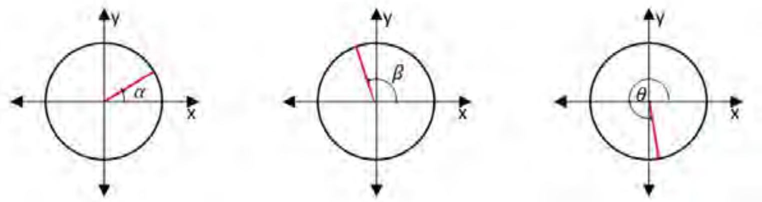
APPENDIX: TRIGONOMETRY TEST

1 For $90^o \leq x \leq 360^o$, Is it true that the value of x in sin x = $\frac{1}{2}$ is 30°?

2 a) $135^{\mathrm{o}} = \cdots.$ radian

b) $\frac{\pi}{4}$ radian $= \cdots$(degree)

3

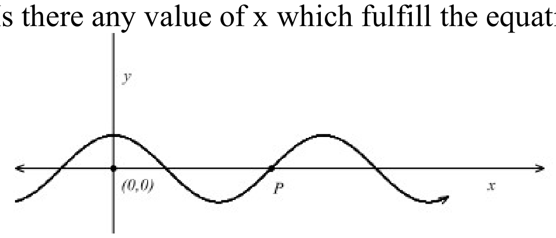


a) Sort the sine value from the smallest to the largest value

b) Sort the cosine value from the smallest to the largest value *) problem adapted from (Weber, 2005)

4 Is there any value of x which fulfill the equation of $\sin x = -2$?

5



What is the coordinate of the point P?

*) problem adapted from (Brousseau, 2002)