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Introducing conditional probability using the Monty Hall problem

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Introducing conditional probability using the Monty Hall problem

Abstract

This study developed a teaching module that incorporated the Monty Hall problem to introduce conditional probability in a college introductory statistics course. This teaching module integrated a guess – experiment – discussion approach with game-based instruction. The researchers piloted this module and made modifications. The work of 20 non-mathematics major undergraduate students was examined for changes in their perceptions about conditional probability through a pre-and-post survey design. The Structure of the Observed Learning Outcome (SOLO) taxonomy was used for data analysis to show the quality of the student work. Findings suggest that most students' perceptions were at higher levels after the teaching module.

Practitioner Notes

1. This article examines an exploration activity for introducing the concept of conditional probability to non-mathematics major undergraduate students.
2. Through a guess-experiment-discussion framework integrated with game-based instruction, 75% of the student demonstrated positive changes in perceiving conditional probability after this teaching module.
3. Because this is a one-time introductory teaching module, this is just the beginning.
4. Continuous efforts to implement effective teaching interventions for the learning of conditional probability is needed.

Keywords

introduction to probability, statistics education, simulation, game-based teaching model

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Introduction

Conditional probability plays an important role in making informed decisions. In undergraduate education, conditional probability is a common topic in required mathematics courses for majors in social sciences, natural sciences, business, and health fields. However, the idea of conditional probability is complex, and research has shown that learners struggle to understand the concept (e.g.; Díaz & Batanero, 2009; Falk, 1986). The purpose of this study was to develop, implement, and assess a teaching module for introducing the concept of conditional probability using a guess – experiment – discussion approach through game-based instruction.

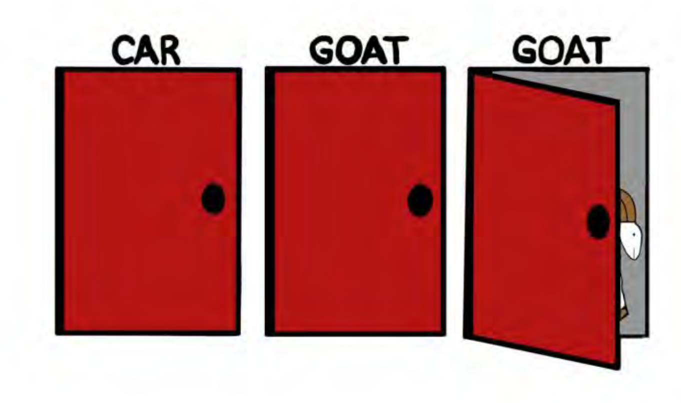
Throughout the years, mathematics educators have used games as a pedagogical tool to increase student learning (e.g., DeLegge & Ziliak, 2021; van Putten et al., 2020). DeLegge and Ziliak (2021) used games in a community of 35 undergraduate non-mathematics majors. Results showed that rich and deep discussions happen when students play games, which increases their appreciation and enjoyment of learning mathematics. Van Putten et al. (2020) found that working collaboratively on games through worksheets increased learners' understanding of mathematics. Several researchers have studied the use of games to facilitate discussions of strategies for conditional probability (e.g., Batanero et al., 2004, Butterworth & Coe, 2004; Mayberry, 1994). For example, Butterworth and Coe (2004) used several games from the television game show *The Price is Right*, including PLINKO and the Money Game as probability scenarios. Mayberry (1994) focused on different conditional probability questions through Bridge, a card game.

In addition to studies on the role of games for teaching mathematics and statistics concepts, researchers investigated the ways of assessing students' perceptions of and reasoning about conditional probability. One of the ways is by using the Structures of the Observed Learning Outcomes (SOLO) taxonomy (Biggs & Collis, 1982). This model has been adapted and used to assess students' probabilistic thinking structures (e.g., Tarr & Jones, 1997; Watson & Kelly, 2007). In Australia, Watson and Kelly (2007) used the SOLO taxonomy to inform the structural complexity of 69 students' responses to four conditional probability questions. Similarly, Tarr and Jones (1997) developed a framework, based on SOLO, for assessing conditional probability and independence. This framework focused on probabilistic situations involving "with" and "without" replacement scenarios and was validated through eight tasks by 15 students from fourth to eighth grades.

The teaching module developed in this study involved examining winning strategies using the Monty Hall problem, a version of Bertrand's paradox (Bertrand, 1889). Monty Hall was the host of the TV show *Let's Make a Deal,* and the following game became a popular probability problem bearing his name. In this game, there were three closed doors with a goat behind two of the doors and a car behind the third. The contestant was asked to pick one of the three doors, but before opening the chosen door, Monty Hall would reveal a goat behind one of the two remaining doors. After this reveal, Monty would give the contestant the option to either stay with their originally chosen door or switch to the remaining door before revealing whether the contestant won a car or a goat. Figure 1 illustrates the Monty Hall problem. While it seemed that winning was due entirely to chance, the results of a great number of trials showed that the probability of winning is higher when switching to the remaining door, $\frac{2}{3}$, than when staying with the original door, $\frac{1}{3}$. The same conclusion can also be made using theoretical conditional probability. This result has appeared counterintuitive to many people and stimulated valuable discussions in the mathematical community.

Figure 1:

Game scenario where the host showed the goat in the third door when the contestant had originally chosen the middle door



While the Monty Hall problem is not new, in the current study, the researchers developed a teaching module using a simulation of the Monty Hall game to *introduce* the concept of conditional probability to undergraduate students in a course for non-mathematics majors. Through the study, the researchers used the SOLO model to examine changes in students' perceptions of conditional probability after using the Monty Hall game teaching module.

Using the Monty Hall Problem to Teach Conditional Probability

The Monty Hall problem has been used in different learning and teaching scenarios and is well-known in mathematics. In this section, the researchers focused on previous research using this problem as it related to the teaching of conditional probability.

Batanero et al. (2009, 2014) analyzed the variety of errors learners made when working with the Monty Hall problem. Their work focused on using the Monty Hall problem in a teacher preparation program to emphasize common errors and help teachers become aware of the erroneous perceptions students may have. The first erroneous perception was assuming *independence* (Kelly & Zwiers, 1986) because the student may not see how switching a door in present time can change the probability of where the car was placed initially. The second perception is the *erroneous perception of the sample space*. This can happen when reducing the sample space to two doors that are equiprobable as a consequence of ignoring the host's knowledge when selecting to show a goat. The third perception is an *incorrect assignment to the initial probabilities*. This is a variation of the second challenge when incorrectly applying the addition rule for probability as a consequence of believing the probability of winning the game is 50% because the car should be in one of the two unopened doors. The fourth perception is the *wrong convergence interpretation* that occurs when overusing the empirical Law of Large Numbers (LLN) based on a small number of trials.

Another perception, the belief of probability always being 50% regardless of staying or switching doors was discussed by Borovcnik (2012). When working with the Monty Hall problem, potential mistakes are a mixture of private thinking and partially understood mathematical concepts. Some private thinking involves emotions and responsibility when losing. If one loses a game, the choice of staying is associated with bad luck, but switching is associated with the responsibility of the player. Thinking that the probability is 50% is a consequence of assuming that the two remaining doors were equiprobable from the beginning.

Existing literature has been focused on analyzing several approaches, strategies, and reasoning to solve the Monty Hall problem (e.g., Batanero et al., 2009; Saenen et al., 2018). While there is research on using the Monty Hall problem to motivate undergraduate students in active learning, no research has incorporated the Monty Hall problem for *introducing the concept of conditional probability* at the postsecondary level and examining subsequent changes in student perceptions about conditional probability. This research intends to fill in this gap in the scientific literature.

Research Question

The research question that guided the study is: What changes in students' perceptions of conditional probability were present after their participation in the Monty Hall teaching module?

Methods and Data Source

A pre-and-post survey single group design was used to answer the research question. This design allowed the researchers to explore the changes in individuals' perceptions before and after the teaching module (Allen, 2017). This study was reviewed and approved by the Institutional Review Board for the protection of human subjects in research. All the participants' names have been replaced with pseudonyms.

Setting and Background

The teaching module was first piloted in the *Probability and Linear Models* course. This is a three-credit hour course (i.e., meets three hours a week for 14 weeks) at a northwestern institution in the United States. The course is for undergraduate students in their first, second, third, or fourth years, generally from health, business and social sciences majors, to help them effectively use mathematical and statistical reasoning in problem solving. The teaching module was initially used in one section of the course with 15 out of a class of 25 students volunteering to participate. During the implementation, the researchers observed that (1) students were justifying that it was possible that both strategies for winning were equiprobable regardless of the empirical probability computed while playing the game and (2) the presurvey question needed clarification. After this pilot implementation, the researchers made modifications to the teaching module to include an introduction of the empirical LLN and to the presurvey to have the same context but different question than the postsurvey.

After the changes mentioned above, the revised teaching module and assessments were implemented in the *Introduction to Statistics* course, which is a three-credit course that has *Probability and Linear Models* as a prerequisite. Students in this introductory statistics course usually major in health, business, social sciences, communication, and biology. This statistics course is one of their degree requirements. The teaching module was implemented in three sections, with a total of 55 students. Students were recruited by their instructors who offered a minor homework score grade adjustment for participation in this study. None of the researchers were course instructors during this study.

Conceptual Framework for the Development of the Teaching Module

The conceptual framework that guided the development of the teaching module was an integration of two constructs: the guess – experiment – discussion approach and game-based instruction. The guess – experiment – discussion approach was adapted from Sáenz Castro (1998) as a structure for teaching probability. This framework utilizes a focus of attention on (a) clarifying students' ideas, (b) carrying out random experiments to encourage cognitive conflict, (c) applying the new ideas to new contexts, and (d) revising the previous ideas to obtain new knowledge. Researchers of this study modified this framework by integrating game-based instruction (Offenholley, 2012; Wu et al., 2012) during the experiment stage. Details on the modification are:

(1) *Guess* an answer - During the guess phase, a facilitator presents a question to the students. Students make a prediction based on their existing knowledge and ways of thinking. This allows researchers to understand students' previous understanding or primary perceptions of conditional probability.

(2) *Experiment* through game play - During this stage, students conduct experiments to verify their hypothesis. Often they encounter cognitive conflict if the results of the experiment did not support their hypothesis (Sáenz Castro, 1998; Shaughnessy, 1977). Our study used a game-based learning model (the Monty Hall Problem game) in the experiment stage. Learning through play is fundamental to humans (Offenholley, 2012). A game learning model guided by constructivism stresses interaction among players and games and views learning as a social process (Wu et al., 2012).

(3) *Discussion* - The discussion phase gave learners opportunities to verbalize their findings and listen to their peers' arguments, including those who may disagree. This study used both small and whole group discussions. During these conversations, learners had opportunities to construct arguments explaining their findings during the first two phases and modifying their primary perceptions.

Implementation of the Teaching Module

Based on the above conceptual framework, the researchers developed the teaching module found in the Appendix. This module was implemented by two of the three authors. It started with an introductory explanation of the LLN, which is an important element for promoting connections between the empirical and theoretical probability (Ireland & Watson, 2009). To explain the LLN, researchers used a graph to illustrate the probability of obtaining heads when tossing a coin. After explaining the LLN, the researchers introduced the main activity, which consisted of playing cards simulating the Monty Hall problem and discussing a strategy for winning the game in small groups: *Is it better to switch or stay?* First, researchers presented the problem in front of the class with a set of large cards and students discussed as a whole group if there was a best strategy to win. After this discussion, students were divided into small groups of three or four students. Each group received a set of three cards and was instructed to play the game at least fifteen times. There were three roles a student could play: host, player, and recorder. Students were encouraged to rotate the roles every few trials so every student had the experience of having each role. Students discussed their recommendations for a winning strategy. Then the groups were brought together. During discussion, each group reported their game results, and the results were combined for a whole class result.

To help students envision the trend in the game results, the researchers then introduced a simulation of the Monty Hall game (Jones, 2011). Students played the online game as a class, viewed the results of the game played more than 200,000 times by other players, and discussed the general results and the strategies of stay or switch. The use of technology helped facilitate generation of large samples to promote discussion to examine the results (Stohl & Tarr, 2002). To wrap up this activity, the investigators and students engaged in a whole group discussion about the reasoning behind the results to introduce the concept of conditional probability.

Data Collection

To answer the research question, the researchers collected demographic information and pre-and-post surveys. The demographic information (gender, age, year of study, major, and highest prior math course with grade) helped the researchers understand students' background and the pre-and-post surveys provided evidence of students' perceptions of conditional probability and problem solving strategies. The presurvey was given one week before the teaching intervention and the postsurvey was given immediately following the teaching intervention was complete. The pre-and-post surveys each consisted of one card game problem. The problem is:

> *Three cards are in a hat. One is blue on both sides, one is green on both sides, and one is blue on one side and green on the other. We draw one card blindly and put it on the table as it comes out. It shows a blue face up. What is the probability that the hidden side is also blue?* (Falk, 1986, p.293).

Students were encouraged to explain their reasoning in their response. This exact question was used in the postsurvey. For the presurvey, the question was modified slightly to read: *It shows a green face up. What is the probability that the hidden side is blue?* The colored card question in the pre-and-post surveys was chosen because it can be solved with a similar method as the Monty Hall question while appearing as a new scenario to students. In addition, both questions are based on Bertrand's Paradox (Batanero et al., 2004).

Analysis of Data

To analyze the pre-and-post surveys, the researchers used the SOLO taxonomy (Biggs & Collis, 1982). This taxonomy describes the performance of a student at a particular time, shedding light on the sophistication of student reasoning even if their final reasoning is not yet a correct response. The taxonomy was also chosen because it had been implemented in previous studies examining student reasoning about conditional probability (e.g. Tarr & Jones, 1997; Watson & Kelly, 2007).

This taxonomy has five levels: prestructural, unistructural, multistructural, relational, and extended abstract. A detailed description of these levels is in Table 1. In general, prestructural responses are represented by repetition of the question or a non-response. This could also be represented by a response without justification in this study. Unistructural and multistructural responses are represented by one aspect or multiple disconnected aspects, respectively. In this study, a unistructural response could have been a student who computed a simple probability instead of a compound probability. A multistructural response may have been students attempting a tree diagram or using the multiplication rule but making errors when carrying this out. Generally, when a response includes an interrelation of multiple ideas, the response is considered relational. In this situation, many students who showed appropriate work such as a tree diagram or using the multiplication rule and then also arrived at the correct answer, making the connection between the work and the answer, fell into this relational response. The extended abstract response entails an application to a different situation.

Researchers of this study first met to look at responses and the alignment of the SOLO taxonomy to this scenario. Then the researchers categorized the characteristics of the students' responses into the levels of the taxonomy shown in column 3 in Table 1. Using these descriptions, two of the three researchers coded all the pre-and-post surveys independently and then met to discuss their coding results. All items with code disagreements were discussed until a consensus was reached.

Table 1.

Levels in the SOLO taxonomy (Adapted from Biggs & Collis, 1982)

Response level	Description on Characteristics of Student Response	Illustrative Example for Conditional Probability
Prestructural	The participant's response focused on irrelevant features of the problem or was not justified.	The student provided the result without showing reasoning or gave a result that was not a probability.
Unistructural	The participant's response focused on one relevant aspect of the situation without taking other aspects into account.	The student computed a simple probability when a compound probability was required.
Multistructural	The participant's response gave multiple relevant aspects but did not link these together.	The student showed more than one approach to compute probability such as a tree diagram and multiplication rule but did not make a correct connection between the approaches.
Relational	The participant's response linked multiple relevant aspects to correctly support the argument.	The student used several approaches to compute probability such as a tree diagram and the multiplication rule and showed the connection between these tools.
Extended Abstract	The participant's response abstracted or generalized the structure created within a relational response.	The student generalized a relational level response to other probability scenarios.

Results

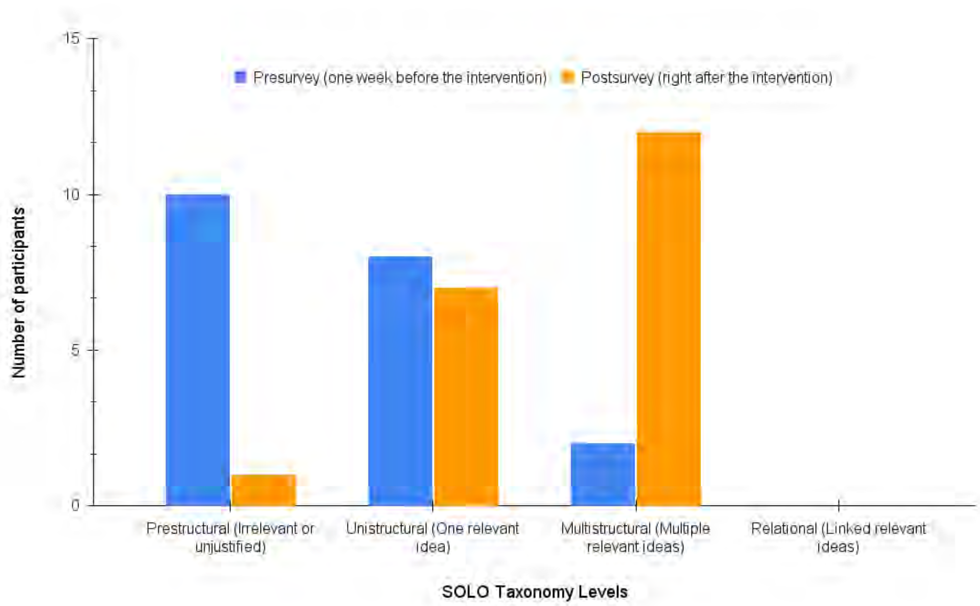
Thirty-seven students agreed to participate in the study, all of whom took the presurvey, and 20 students went through the teaching module and took the postsurvey. Participants were majoring in health fields, natural sciences, business, or social sciences. Researchers analyzed the data of the 20 participants, of whom 45% were females and 55% males, and 35% were 24 years old or older.

Researchers observed changes in the presurvey and postsurvey outcomes, which reflected changes in students' perspectives about conditional probability. Researchers used the SOLO Taxonomy to compare the results of the pre-and-post surveys. The initial coding had an interrater reliability of 82.5%. Researchers then met to discuss differences and were able to reach consensus on the codes.

On the presurvey, half of the students' answers were at the prestructural level. Figure 2 shows that 12, or 60%, of the students' answers on the postsurvey were at the multistructural level. On the other hand, only one student's answers were at the prestructural level during the postsurvey. These results showed that after participation in the teaching module students' perceptions of conditional probability measured by the SOLO taxonomy improved. More specifically, five students increased two levels, ten students increased one level, four students stayed at the same level and one student decreased one level.

Figure 2:

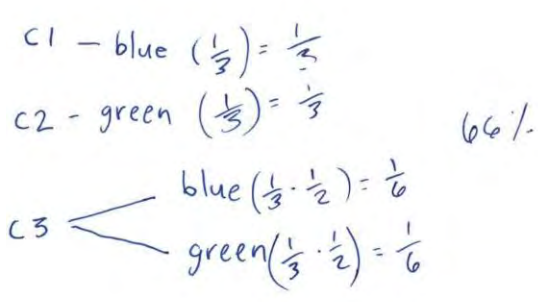
Results on frequency of SOLO levels on the pre-and-post survey (n=20)



The presurvey results in Figure 2 showed that 10 of the responses were in the prestructural level before the teaching module. These were non-responses or answers without explanations. All students who performed at a prestructural level in the presurvey were able to increase their level of perceiving conditional probability. Specifically, on the postsurvey, half of these students performed at the unistructural level and half at the multistructural levels. Alex's work, in Figure 3, exemplifies a student's postsurvey multistructural level response, which was two levels above Alex's presurvey result, as he did not respond to the question. Alex's diagram for the postsurvey shows that he had several ideas such as constructing a tree diagram and considering if the events were independent, but he was unable to connect the 66% response to his work.

Figure 3:

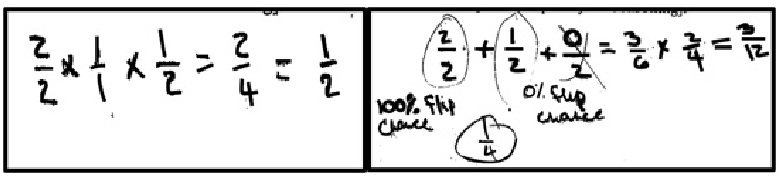
Alex's postsurvey work at the multistructural level



There were eight responses at the unistructural level when working with the presurvey. Students who provided responses at the unistructural level for the presurvey mostly increased one level during the postsurvey, 62.5%. Only two students remained at the same level and one decreased one level. Figure 4 shows an example from Phoebe, a student who progressed to a multistructural level in the postsurvey. While her work is similar, there is a key difference between the responses. For both responses, this student is considering a simple probability of the success of the event for each card. However, in the postsurvey, the student is adding a new structure, specifying the 100% chance of getting the desired color when the card is flipped, while there is a 0% chance in the other two cards. Phoebe also discarded the card with a car. This student was able to see a new idea in the postsurvey but was not able to make connections between the ideas, as she did not use the information.

Figure 4:

Phoebe's work for the presurvey at the unistructural level and the postsurvey at the multistructural level



On the presurvey only two students were at the multistructural level, and both remained at the same level on the postsurvey.

Conclusions and Discussions

In this study, the researchers developed and implemented a teaching module based on a guess – experiment – discussion framework integrated with game-based instruction. The teaching module was designed as an *introductory activity* for exploring the concept of conditional probability.

The presurvey results in Figure 2 showed that most of the responses were in the lowest level of the SOLO taxonomy, although all students in this course had taken the prerequisite mathematics course that includes the concept of conditional probability. These results relate to the challenges in reasoning and understanding conditional probability (e.g., Díaz & Batanero, 2009; Falk, 1986). The teaching module in this study intended to counter these challenges as an initial module when introducing conditional probability.

Seventy-five percent of the students demonstrated some positive changes in perceiving conditional probability after the teaching module. The researchers found evidence of students' perceptions at the prestructural, unistructural, and multistructural levels, but not at the relational level, after the teaching module. As this is an introductory activity, researchers find that follow-up activities may help students' comprehension of the concept move to a higher level.

In addition, the results of this study provide evidence for other instructors or teachers to adapt the module. It is important to be aware that such a one-time introductory teaching module is just the beginning. Continuous effort to implement effective teaching interventions is needed — for example, using PLINKO and the Money Game (Butterworth & Coe, 2012) and/or using "with" and "without" replacement situations (Tarr & Jones, 1997).

This study has limitations. Students' participation was voluntary, although an incentive of replacing the lowest homework score was offered. The small sample size did not allow comparison between different groups of participants such as sex or age. The design of the study only allows for suggestive results and these are not implications. In addition, the researchers did not measure students' motivation since the research question focuses on students' conceptual perceptions of conditional probability before and after the teaching module. For researchers who study the teaching and learning of conditional probability, the researchers recommend (1) studying the impact of a Monty Hall intervention with a larger population; (2) studying the impact on students' motivation, confidence, and attitudes associated with game-based instruction; and (3) developing sequential teaching modules and investigating their impact on students' learning and understanding of conditional probability.

While this study did not measure students' motivation, the researchers observed that students were motivated and engaged while discussing its counterintuitive results of the game. Students were talking in small groups, agreeing and disagreeing with their peers, and constructing arguments to support their responses. The element of having a game was motivational for them (Butterworth & Coe 2004) because through their exploration, they were strategizing ways to win the game through the stay or switch options. The researchers perceived that students' motivation and engagement in solving the task in small groups were key for the effectiveness of the whole group discussion and the success of the activity to introduce the concept of conditional probability.

In summary, this study developed, implemented, and studied a teaching module using the Monty Hall game through a guess – experiment – discussion approach. Results from the pre-and-post surveys indicated improvement of students' perceptions of conditional probability. Adaptation of the teaching module and advice for further research activities are recommended to the larger teaching and research communities.

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Appendix

Teaching module

A. Materials:

   a. A big set of cards of two goats and one car.

   b. Sets of cards of two goats and one car. One set per group.

   c. Worksheet with information about the game, small group roles, and recording table.

B. Step by step - 50 minute intervention

   a. Explanation of the LLN using a graph to illustrate the probability of obtaining heads when tossing a coin.

   b. Introduction of the Monty Hall problem, play the game with a set of large big cards and ask the whole group *Is it better to switch or stay?* Is there a best strategy to win?

   c. Create small groups of three or four students. Provide a set of three cards and a worksheet for each group

   d. Clearly share the instructions of the small group task:

      i. Play the game at least fifteen times.

      ii. There are three roles a student could play: host, player and recorder.

      iii. The recorder collects the data of each game specifying the strategy used and the result of the game.

      iv. Rotate the roles every few trials so every person has the experience of having each of the roles.

      v. Discuss a group recommendation for a winning strategy.

      vi. After discussion, each group reports their game results on the whiteboard.

   e. Students will share the group results on the white board. Calculate the whole group results and discuss strategies.

   f. As a support, you can introduce and play a simulation of the Monty Hall game on a web page designed by Jones (2011) to compare the whole group results to the ones in the simulation.

   g. Discuss the reasoning behind the results to introduce the concept of conditional probability.

C. Teaching module considerations

   a. During the whole group discussion students' arguments were focused on understanding that the best strategy was switching. However, they did rely on the LLN, but not in conditional probability. As the researcher guided the whole group conversation, students were encouraged to explore ways to explain why the theoretical probability to win while switching was $\frac{2}{3}$. During the discussion, one of the researchers highlighted that there was information provided by the host by showing a goat. The introduction to the conditional probability included analyzing the problem step by step: “I give you information, it is a condition now, what is the probability that you win the car?” After this brief introduction, the researcher asked students to represent the situation with a tree diagram.

b. A student, Charles, recognized that the tree diagram will start with three branches, “there are three of them,” referring to the first three options car, goat one, and goat two each with a probability of “one out of three.” Another researcher labeled each branch with ⅓, and there was agreement that this was the first step or the “original choice.” Students, then, discussed in small groups what the next step in the tree diagram would be. The researcher guided the conversation to construct the tree diagram together, for example, “[if you select the car on your first choice] what would be the switch choice? [...] what is the probability for each one?” Students said that there were two choices: goat one or goat two with ½ probability for each of them. For each of the goat branches, the switch will just include a car with a probability of one. The conversation continued until the tree diagram was completed. By construction, the second set of branches represented the option of staying.

Presurvey

Thank you for your participation!

1. ID: ____________ First letter of your last name and last 3 digits of your cell phone or home phone number: (For example, John Smith’s cell phone number is (406)555-1234. His ID would be “S234”)
2. Gender: _______F ________M
3. Major: ______________________________
4. Year of study: _____________________
5. Age: ________
6. Highest Math Class Taken before this Class: _________________ Grade earned: _____ Where did you take the class? ____
7. Three cards are in a hat. One is blue on both sides, one is green on both sides, and one is blue on one side and green on the other. We draw one card blindly and put it on the table as it comes out. It shows a green face up. What is the probability that the hidden side is blue? [Please explain your reasoning].

Postsurvey

Thank you for your participation!

ID: ____________ First letter of your last name and last 3 digits of your cell phone or home phone number: (For example, John Smith’s cell phone number is (406)555-1234. His ID would be “S234”)

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1. Three cards are in a hat. One is blue on both sides, one is green on both sides, and one is blue on one side and green on the other. We draw one card blindly and put it on the table as it comes out. It shows a blue face up. What is the probability that the hidden side is also blue? [Please explain your reasoning].

Worksheet for Students

The game

We have three cards (one car, two goats). The contestant wins the game when picks a car. The host will reveal one of the goats to the contestant, who then will have the option to switch to the other card or stay with the original selection.

Description of roles

You can rotate the roles, so everyone can have the chance to play!

1. Host: The host always knows where the car is. After the first selection of the contestant, the host shows one goat to the contestant (this goat cannot be the selected card). After showing the goat, the host will ask: Do you want to switch or stay with your card?
2. Contestant: The contestant doesn't know where the car is. The objective is to win the car at the end of the game. The contestant will pick a first choice card, wait for the host to reveal a goat, and then decide to switch or stay with the first choice card.
3. Secretary: The secretary observes the role of the contestant and records the results in the recording table.

Table A1.

Recording each game

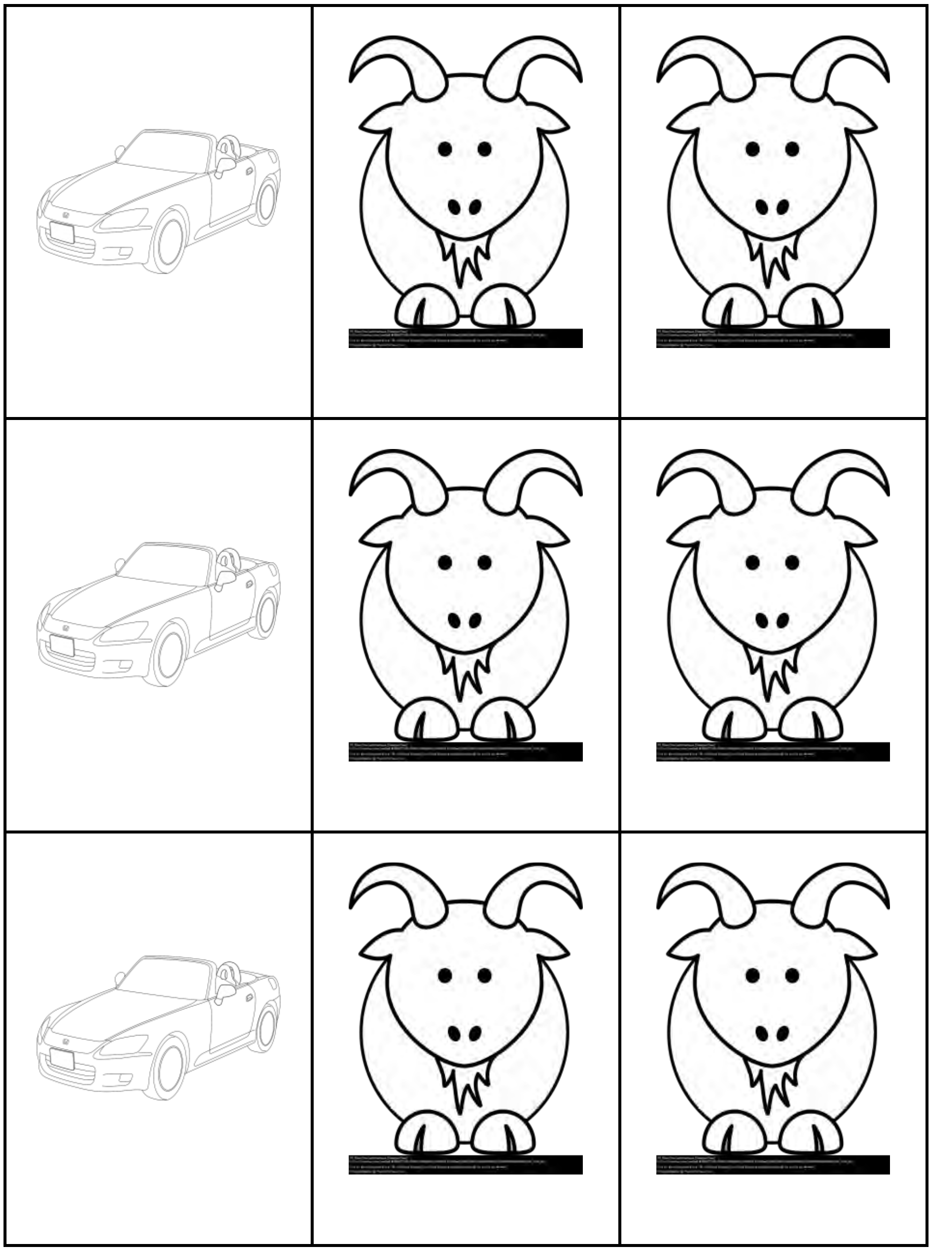
	Stay or Switch?	Win or Lose?
Game #1		
Game #2		
Game #3		
Game #4		
Game #5		
Game #6		
Game #7		
Game #8		
Game #9		
Game #10		
Game #11		
Game #12		
Game #13		
Game #14		
Game #15		

Table A2.

Summary of the results

	Stay/Win	Stay/Lose	Switch/Win	Switch/Lose
Group #____				

Game Cards



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