AN EXAMPLE TEACHING ACTIVITY BASED ON THE COMPARISON MEANING OF THE SUBTRACTION OF INTEGERS[1]

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ABSTRACT

This paper reports on the design, implementation, and evaluation processes of an activity that focused on the comparison meaning of the subtraction of integers and included the number line model as the primary representation. Finding the answer and determining the sign of the answer in the subtraction questions were targeted as the learning objectives. Four different situations that can be encountered in the subtraction questions were determined considering the magnitude and sign of minuend and subtrahend. Through the designed activity, it is expected that the students work on example questions first and then make generalizations by reflecting on the questions they solve. The activity was implemented in an elementary school with the participation of 27 seventh grade students. The analysis of the implementation process revealed that the activity was effective in supporting the students' performance of finding the difference and the sign of the difference when subtracting integers.
Keywords: integers, subtraction, the number line model.

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ÖZ

Bu çalışmada, tam sayılarla çıkarma işleminin fark anlamına uygun olarak hazırlanmış bir etkinliğin tasarım, uygulama ve değerlendirme süreçleri sunulmuştur. Bu kapsamda tam sayılarla çıkarma işleminde sonuç ve sonucun işaretinin bulunması öğrenme nesnesi olarak belirlenmiştir. Etkinlik sayı doğrusu modeli kullanılarak tasarlanmıştır. Tam sayılarla çıkarma işleminde eksilen ve çıkanın, işaret ve büyüklüklerine göre karşılaşılabilecek dört farklı durum belirlenmiştir. Tasarlanan etkinlikte öğrencilerin her bir durum için öncelikle uygulama yapmaları ve daha sonra ulaştıkları sonuçları genellemeleri beklenmektedir. Etkinlik uygulamasına 27 ortaokul 7. sınıf öğrencisi katılmıştır. Uygulamadan toplanan verilerin analizi sonucunda, çıkarma işleminin fark anlamına uygun olarak hazırlanan etkinliğin, tam sayılarla çıkarma işleminde doğru sonucu bulmada etkili olduğu tespit edilmiştir.
Anahtar kelimeler: tam sayılar, çıkarma işlemi, sayı doğrusu modeli.

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INTRODUCTION

Integers include a variety of conceptual meanings such as directional numbers, negative numbers, absolute value, and relative zero. These concepts are abstract and, therefore, integer numbers and operations are one of the subjects that students have difficulty in learning (Van De Walle et al., 2013). For example, it is claimed that it is more difficult to make sense of negative numbers than to make sense of positive numbers (Fischer, 2003; Kilhamn, 2011). According to Kilhamn (2011), the reason for this is that students have difficulty in transferring the natural number perception they learned intuitively to negative numbers. Therefore, it becomes a difficult task for students to understand integers and to operate with integers.

Makonye and Fakude (2016) state that students encounter various difficulties in subtracting integers, and these difficulties mostly arise from the misconception that subtracting greater numbers from smaller numbers is not possible. Subtraction has two meanings that are commonly used: separate and compare. The first meaning focuses on separating a group of objects. The second meaning involves comparison and is the difference or distance between two numbers (Murdiyani, et al. 2013). In Turkey, subtraction operations are limited to natural numbers during the first 6 years of the formal education process as required by the mathematics curriculum (Ministry of National Education, 2018). During this time, the "difference" meaning of subtraction may remain in the background, since only the cases where the difference is positive are examined (as it should be) in subtraction with natural numbers (Tezcan et al., 2018). In the subtraction with integers, however, the cases where both the minuend greater than subtrahend and the subtrahend greater than minuend are examined. Especially when examining the cases where the minuend is less than the subtrahend, the difference meaning of subtraction is needed.

In most of the studies on subtraction with integers, it is stated that using the contexts that students are familiar with and making associations with real life will help them to make sense of negative numbers and subtraction (e.g, Stephan et al., 2003). During the instruction of integers, the *number line* model is frequently used (Stephan & Akyüz, 2012). The usage of number line model is common because it contains all the meanings of the negative symbol. This model is crucial because its usage starts with natural numbers and proceeds with whole numbers, fractions, integers, and rational numbers. In addition to the number line model in current textbooks, the analogies such as "*paying debt, the enemy of the enemy is my friend, the reverse of the reverse is the straight*" is also used to determine the sign of the difference (Boz Yaman, 2019).

Another tool that is used in the textbooks during the instruction of operations on integers is the counting chips (Bozkurt & Polat, 2011). In Figure 1, a subtraction performed with the counting chips is given.

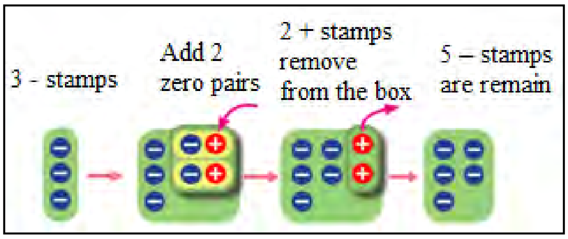

Figure 1. Subtracting Integers: The Example of (-3) – (+2) (Boz Yaman, 2019, p.30)

The operation given in Figure 1 demands removing two + chips (representing +2) from three – chips (representing -3). For this purpose, 2 zero pairs, containing 2 + chips need to be added to the box. Here, one + chip and one – chip create zero pair together. There would be 5 – chips and 2 + chips in the box in the last case. When 2 + chips are removed from the box, 5 – chips will remain. Notice that 2 – chips were added to the box to remove 2 + chips. So, the operation is: (-3)-(+2)=(-3)+(-2)=-5

While the operation modelled with counting chips in Figure 1 is sufficient to explain the proposition that "Subtraction of integers is to add the minuend with the additive inverse of the subtrahend", the explanation made by modelling the subtraction on the number line can become confusing. To fix this problem, the "reduce" meaning of the subtraction of natural numbers has been transferred to the subtraction of integers. However, it is seen that the number line model of the subtraction used in some textbooks are based on the proposition that "add the minuend number with the additive inverse of the subtrahend". Figure 2 shows the number line modeling for subtraction in Boz Yaman (2019, p.32):

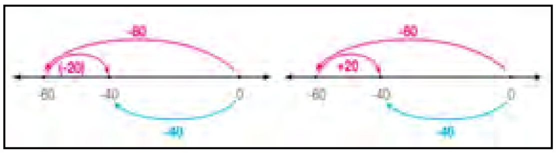

Figure 2. Modeling Subtraction on Number Line

The model, given in Figure 2, is based on the assumption that "subtracting -20 is adding +20". Let us think about the meaning of subtraction here. The operation of $a-b=c$ is equivalent to the operation of $b+c=a$. In other words, subtracting a from b is to answer the question "What should I add to 'b' to reach 'a' on the number line?" This idea is different from the modeling done by the counting chips that rely on zero pairs. The activity that was designed within the context of this study contains the number line model based on this idea.

DESIGNING THE ACTIVITY

In the activity designed within the scope of this study, the subtraction of integers is demonstrated with a new model, adhering to the concept of difference. Yakes (2017) argues that the number line model based on the meaning of difference is consistent with students' previous learning. The models here are consistent with the way students model operations on natural numbers that they have learned before. For this purpose, subtraction operations with first two positive numbers, then two negative numbers, and finally one positive and one negative number were used in the activity. Thus, modeling was done on all possible cases of the subtrahend and the minuend integers. In Figure 3, the subtraction of two positive numbers that subtrahend is smaller than the minuend is examined.

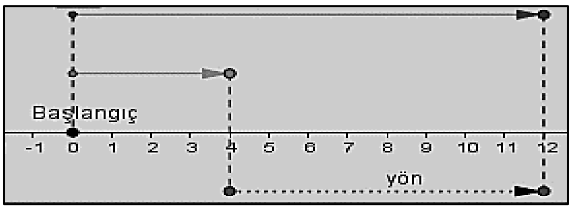

Figure 3. The Subtraction of Two Positive Numbers That the Subtrahend is Smaller Than the Minuend($(+12)-(+4)=?$)

In Figure 3, the minuend (+12) and the subtrahend (+4) are located on the number line. After determining the difference (the difference is 8 units), the direction of the difference is accepted as the direction from the subtrahend to the minuend. Therefore, the answer is +8. In Figure 4, the subtraction of two positive numbers in which the subtrahend is greater than the minuend is examined.

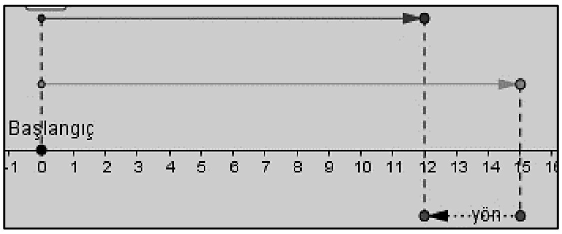

Figure 4. The Subtraction of Two Positive Numbers in Which the Subtrahend is Greater than the Minuend ($(+12)-(+15)=?$)

In Figure 4, the difference between the minuend $(+12)$ and the subtrahend $(+15)$ is 3 units. While determining the direction, as in Figure 3, the answer is found as -3, since we have to move to the negative direction when we head towards the minuend from the subtrahend. The number line model of subtraction of two negative numbers is shown in Figure 5.

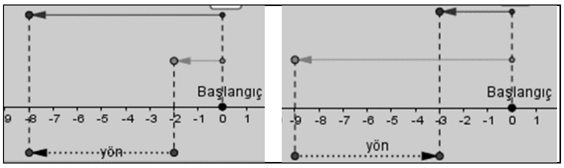

Figure 5. Subtracting Two Negative Numbers ($(-8)-(-2)=?$; $(-3)-(-9)=?$)

In Figure 5, first, the places of the integers in the question are shown on the number line. In order to calculate (-8) – (-2), the question on the left, it is seen that the distance between the two numbers is 6 units. To determine the direction of the answer, we move from the subtrahend to the minuend by drawing an arrow and notice that the direction is negative, hence the answer is -6. Although proceeded in the same way for finding the answer to the question (-3) – (-9) on the right, it is clearly seen that the answer is 6 positive units. In Figure 6, the modelling of subtraction with integers with opposite signs is given:

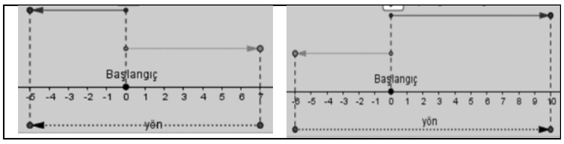

Figure 6. Subtraction of Two Integers with Opposite Signs

During the modeling of opposite signed integers' subtraction adhering to the difference meaning, firstly the integers should be placed on the number line and the distance between them should be calculated. Then the direction from the subtrahend to the minuend should be determined to designate the sign of the difference. Thus, in Figure 6, the result of the operation $(-5) - (+7)$ on the left hand side can be determined as negative 12 units. Similarly, the answer to the question $(+10) - (-6)$ on the right hand side can be determined as positive 16 units.

In this study, the idea of using a number line model in which the students can perform subtraction with integers based on their prior knowledge of operations with natural numbers was featured. The model was created to focus on reckoning the difference between the subtrahend and the minuend on the number line. In this context, the first draft was created by the authors while considering the conceptual richness of the subtraction operation aforementioned. Then the activity plan was given to 26 elementary mathematics teachers and they were asked to write their interpretations and criticisms of the script. The new model and the existing model in the textbook were presented to the teachers and their opinions were taken in writing. It was seen that the teachers generally gave positive feedback. The aspects that the teachers thought were missing were evaluated by the researchers, and the activity was revised accordingly. The teachers stated that the activity was sufficient to teach the logic of subtraction of integers. One of the teachers expressed his opinion as follows: “I think it's quite enough. Students will understand better because it is more embodied. Also, because it is grounded on the knowledge from elementary school, that will be better for students.” Another teacher wrote, “If a student knows the place of the numbers on the number line and their directions (+) or (-) and is able to compare the numbers, then this model will be very appropriate and adequate.”

The revised activity based on the teachers’ feedback was piloted with a group of students. And the students’ achievement in reaching a generalization after following the instructions given in the activity was observed. It was seen that even though the students could reach a generalization, they struggled to follow the instructions. To overcome this problem, a letter was given to every instruction in alphabetic order. In the pilot study, it was observed that this activity could be completed in a period of 28-33 minutes, and it was planned to allocate one lesson hour (40 minutes) for the actual implementation.

ACTIVITY IMPLEMENTATION

The activity was implemented in one of the researchers’ classrooms in the fall semester of the 2021-2022 academic year. The ethics committee permission required for the study was given by the "Gaziantep University Ethics Committee" (Date: 29.12.2021, document no: 131285). Parent permission documents were requested from the students before the activity started. After these documents were collected, the students were reminded that if they did not want to participate in the activity, they could leave. In total, 27 secondary school 7th-grade students participated in the activity. In the class where the activity was applied, the students had not learned how to do subtraction with integers before at school. Before starting the activity, the teacher reminded the concepts of the minuend, the subtrahend, and the difference in the context of natural numbers, and consolidated them with an example.

In the introductory part of the activity, the teacher explained that they would try to reach a generalization that they could use when subtracting integers by engaging in this activity. The worksheet (Appendix 1) containing the activity script was then distributed to each student, one at a time. The teacher read and explained the instructions in the activity script aloud. In line with the guidelines given in the activity, a sample application was made on the board to ensure that the instructions were understood. However, this sample application was made to understand the instructions, and the mathematical output targeted by the activity was not disclosed. The stages of checking prior knowledge, informing the students about the context and expected outcome, and distributing the activity script took a total of 5 minutes.

After the introduction phase, the teacher asked students to answer the question in the sections “a” and “b” and perform the proper operations in line with the instructions. The teacher asked if there was anything not understood at this point and clarified any questions from the students about the instructions. During this stage, the teacher answered the questions individually as some students did not understand some instructions by walking around. In the first part of the activity, the students worked individually and completed the instructions in the sections "a" and "b". This part lasted about 8 minutes.

The first instruction of the activity was (a) “*Pick two integers with the same sign so that the greater one is minuend, and the smaller one is the subtrahend.”* The students were asked to place them on the number line, measure the difference between the points that represent these numbers, and express the direction from the point that represents the subtrahend to the point that represents the minuend, as explained in the introduction section of the activity.

The second instruction of the activity was (b) *"Pick two integers with different signs, so that the greater one is minuend, and the smaller one is the subtrahend."* The students were asked to place these numbers on the number line, measure the difference between the points that represent these numbers, and express the direction from the point that represents the subtrahend to the point that represents the minuend, as explained in the introduction section of the activity.

The teacher started the whole class discussion by telling students, not to change their first answer on the worksheet. The teacher continued the whole class discussion by asking students to share the numbers they had picked, and the results they had found. Meanwhile, he drew a number line model on the board and asked students to say the numbers they had selected. The students told the numbers and the teacher plotted those numbers on the number line model. Then he asked the difference, the direction from the subtrahend to the minuend, and received the answers from the students. He repeated the same process for instruction “b” after having received answers from two students. Then the teacher introduced the instruction G1 “*Compare the sign of the difference that you find to your classmates' answers, what do you notice?*” by saying, “Let's read the next one (reading the G1 instruction).What do you notice? Let's write.” After a while, the following dialogue took place:

> Teacher (T): What did you find?
> Students (S) (en masse): Positive (+)
> T: Did everybody find positive?
> S: Yes!
> T: Hum.

In the meanwhile, a student (Students are coded as S1, S2 and so on. All the names used in the dialogues are pseudonyms.) argued that she the answer should be negative, and the following dialogue took place:

> S1: Sir, I find negative at “a”.
> T: Tell the subtrahend and the minuend.
> S1: Sir, the minuend is +7, and the subtrahend is +3.
> T: (indicating numbers on the number line) the minuend is +7, and the subtrahend is +3
> S1: There are 4 units.
> T: Now, go from the subtrahend to the minuend.
> S2: You are going towards positive (direction).
> S1: Aah!, I thought the thing,... from the minuend to the subtrahend.

In the meanwhile, another student claimed that he found a negative value, and the teacher helped him notice his mistake by writing on the board like in the previous case.

Subsequently, the teacher ended the whole-class discussion and started individual work by saying “Ok, let’s move on to the second phase.” He made explanations while walking among the students, and the students were working on the worksheet individually. The students wanted to share their answers from the first minute, but the teacher asked them to wait and suggested checking their answers. The teacher finished this phase by saying “Let’s share the answers.” after he observed the students had finished the work.

The fourth instruction of the activity was (c): *“Pick two integers with the same sign so that the greater one is subtrahend, and the smaller one is the minuend.”* The students were asked to show these numbers on the number line, measure the difference between the points that represent these numbers, and express the direction from the point that represents the subtrahend to the point that represents the minuend, as explained in the introduction phase of the activity.

The fifth instruction of the activity was (d): *"Pick two integers with different signs so that the greater one is subtrahend, and the smaller one is the minuend."* The students were asked to show these numbers on the number line, measure the difference between the points that represent these numbers, and express the direction from the point that represents the subtrahend to the point that represents the minuend, as explained in the introduction section of the activity.

After the students completed the sections "c" and the "d", a process that was similar to the process held during the transition from "a" and "b" to the section G1 was done. The students' responses were shared and discussed.

The teacher read the instruction after the students answered the questions individually:

T: What did you notice, Sibel?

S3: All are negative, Sir.

T: Okey, let's listen to Derya.

S4: It says larger is the minuend and smaller is the subtrahend in "a" and "b"; larger is the subtrahend and smaller is the minuend in "c" and "d". That means if we want the difference to be positive, then we should pick the big one as minuend; if we want it to be negative, then we should pick the big one as the subtrahend.

S5: I would say the same.

T: Zilan has thought the same, did you understand what Derya has told? Do you agree?

S: Yes.

The teacher moved to the last instruction after asking students if anyone found a counterexample and seeing there was no one.

The teacher concluded the activity with the whole class discussion after he obtained the answers from the students and asked them to criticize each other's claims. The dialogues during this phase were as follows:

S7: Sir, if the minuend is greater and the subtrahend is smaller, then they all go to the positive direction but if the minuend is smaller and the subtrahend is larger, then all go to the negative direction.

T: This is the direction of the difference, right?

S7: Yes.

T: Alright, it also asks about the difference, what can we tell about the difference?

S4: To me, it depends on the direction. I mean, if we go to the positive or negative, it depends on this.

T: You have already decided on the direction of the difference, but it says the "difference" in the instruction, what about the difference itself?

S4: The numbers of the units between, sir.

S8: It's as much as the number of units.

The teacher concluded the activity after asking students "Is there something that you want to ask? And is there anything you are not convinced of?" The students replied that they did not have any questions.

EVALUATION OF THE ACTIVITY

During the implementation of the activity, which was designed based on the "difference/comparison" meaning of subtraction of integers, it was observed that the students engaged in the activity in a highly motivated way and they worked on the activity without needing any intervention. They seemed to be eager in terms of sharing ideas and criticizing their classmates' ideas during the whole class discussion. In addition, considering both the time allocated and the number of correct answers, it can be said that the students had difficulty following the instructions in the first part of the activity, but they progressed more fluently in the second part. The findings that emerged as a result of the evaluation of student answers for each of the activity instructions are detailed below.

The students' responses to the questions on the activity worksheet were collected and analyzed by the authors in terms of the correctness of the answers after the implementation. It was found that 18 students (68%) reached the correct answer for the first instruction (instruction a). The following conditions were required in order for an answer to be correct: 1) The larger of the selected numbers must be the minuend. 2) The numbers must have the same sign. 3) The representation of the number line model, the difference, and the direction of the difference must be correct. For instance, S12's answer (Figure 7) is an example of a correct answer.

Figure 7. S12's Answer to the Instruction A

S20 didn't write the minuend and the subtrahend clearly and made operations on the number line only. Because of these reasons, his answer was evaluated as partly correct (Figure 8). Picking up numbers not aligned with the instructions and operational errors were evaluated as wrong answers (Figure 9).

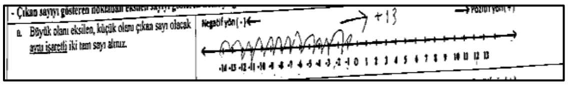

Figure 8. S20's Answer to the Instruction A

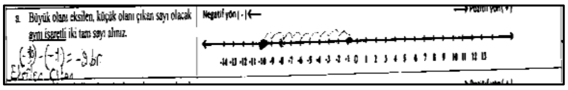

Figure 9. S8's Answer to the Instruction A

Fifteen students' (65%) responses to the second instruction (instruction b) were evaluated as correct. The following conditions were required in order for an answer to be correct: 1) The larger of the selected numbers must be the minuend. 2) The numbers must have opposite signs. 3) The representation of the number line model, the difference, and the direction of the difference must be correct. For instance, S22's answer (Figure 10) can be an example of a correct answer.

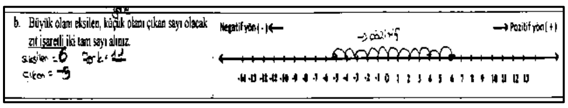

Figure 10. S22's Answer to the Instruction B

In the scope of this instruction, two students did the operation on the number line correctly but wrote the result wrong (Figure 11). These responses were evaluated as partially correct. S1, on the other hand, picked +10 and -5, modeled the operation on the number line in a correct way but wrote +13 as the result. Since the number line model, the direction of picked numbers, and the direction of the result were correct, that response was evaluated as partly correct because of the aim of the study even if the result was not correct. Picking up numbers not aligned with the instructions and errors in the operations were evaluated as wrong answers.

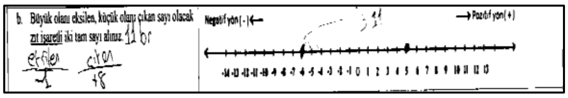

Figure 11. S21's Answer to the Instruction B

For the G1 instruction, 19 students (70%) gave the correct answer. An emphasis on the positive direction was sought in the students' answers in order for the answer to be correct. For instance, S21's response (Figure 12) is preferable.

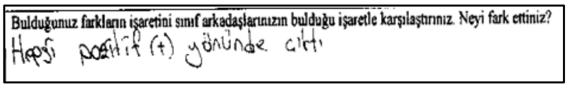

Figure 12. S21's Answer to the Instruction G1

Two responses to this instruction (7%) were evaluated as partially correct. These students wrote only "positive" as a one-word answer. Four responses (15%) were evaluated as wrong answers. One of them (S9) was "There was a difference." another one (S10) was "There was no subtrahend." Four students (15%) didn't write any answer.

Twenty-two students (82%) provided correct answers in their response to the first instruction (instruction c) of the second phase of the activity. It can be thought that the reason for the increase in the number of correct answers at this stage is that the students probably better understood what was expected of them in the class discussion that took place after the instructions "a" and "b". For an answer to be evaluated as "correct", the following conditions were required: 1) The smaller of the selected numbers must be the minuend. 2) The numbers must have the same sign. 3) The representation of the number line model, the difference, and the direction of the difference must be correct. For instance, S18's answer (Figure 13) can be an example of a correct answer.

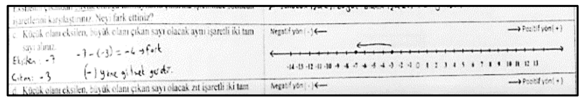

Figure 13. S18's Answer to the Instruction C

Five students (18%) gave a wrong answer to this question. One of them chose proper numbers but wrote the incorrect sign; the other four students, on the other hand, chose incorrect numbers (the numbers with different signs or picked the numbers in an improper order).

There were 20 students (74%) who gave correct answers to the second instruction (instruction d) of the second section of the activity. For an answer to be evaluated as "correct", the following conditions were required: 1) The smaller of the selected numbers must be the minuend. 2) The numbers must have opposite signs. 3) The representation of the number line model, the difference, and the direction of the difference must be correct. For instance, S22's answer (Figure 14) is an example of a correct answer.

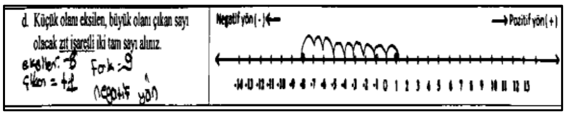

Figure 14. S22's Answer to the Instruction D

There were one (4%) partially correct and five (19%) incorrect answers for this question and one student didn't write any answers. The partially correct answer included correctly selected numbers and the correct result, however, there was no number line representation (Figure 15). In three of the wrong answers, the numbers chosen were not suitable for the given criteria, and although the numbers selected in two of them were appropriate, the sign of the result was written incorrectly.

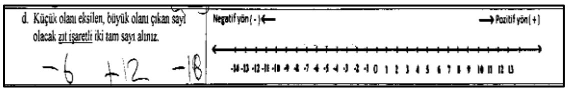

Figure 15. S19's Answer to the Instruction D

There were 22 students (81%) who reached a correct generalization in their response to the instruction G2. An emphasizes on the negative direction was sought as the condition of correctness. For instance, S21's response (Figure 16) was accepted as correct.

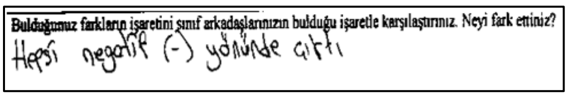

Figure 16. S21's Answer to the Instruction G2

Related to the instruction G2, the following answers were evaluated as incorrect: "All the same." (S9 and S8), "Generally positive." (S1). After a closer examination, it was seen that S1 did not choose proper numbers within "c" and "d", so she found a positive value as the result. S8 and S9, on the other hand, reached the correct answer for "c" but wrote incorrect answers for "d". Two students did not write any answers to this question.

For the instruction G3, 14 students (52%) reached a correct answer. For instance, S21's response in Figure 17 was one of them.

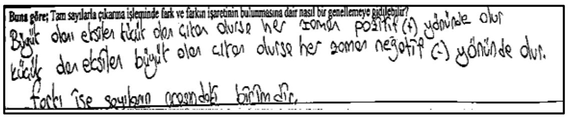

Figure 17.S21's Answer to the Instruction G3

Three answers were evaluated as partially correct in the scope of G3. Responses that only mentioned the difference or the direction were evaluated as partially correct. For instance, S27's answer in Figure 18 is one of them.

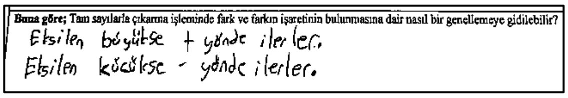

Figure 18. S27's Answer to the Instruction G3

Four of the seven incorrect responses included incomprehensible sentences (e.g., The difference is the generalization there [S1]). The other three responses included false generalizations.

CONCLUSIONS and SUGGESTIONS

The design, implementation, and evaluation processes of an activity that was designed based on the "difference/comparison" meaning of subtraction were presented in this study. In the first section of the implementation, when considering both time and the rate of correct answers, it is seen that students experienced an accommodation process. Hence, an increase in the correct answers and a decrease in time spent in responding the questions have been observed in the second section.

The majority of the students could reach the expected generalizations. In this regard, we can claim that the activity is effective to find both "the difference" and "the direction of the difference" in subtraction of integers.

The rate of reaching a correct generalization about the difference and the sign of the difference in the last instruction of the activity

(G3) was lower than it was in the G1 and G2. Considering the situation that the students learned the subtraction of integers for the very first time and the epistemological difficulties of the content (Altiparmak & Özdoğan, 2009; Fisher, 2003; Hefendehl-Hebeker, 1991; Kilhamn, 2011), this low rate is expectable.

We recommend to add the justifications alongside to the generalizations ((e.g., Why do you think that way? How can you convince us? How have you been convinced?) so that justifications may guide the generalizations.

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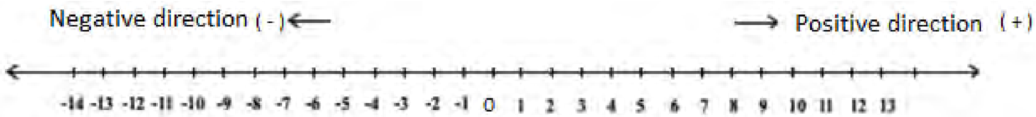
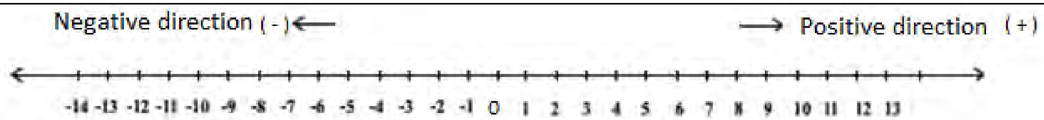
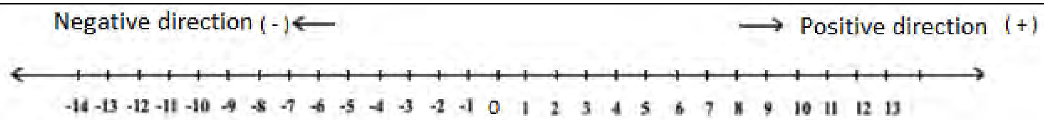
Appendix 1

Activity Worksheet

<table>
<tr><td colspan="2">In the a, b, c, and d sections, make the following operations after picking up numbers.
- Show those numbers on the number line.
- Measure the difference in units between the points that represent those numbers (difference).
- In which direction you should move to go from the subtrahend to the minuend? (+ or -)</td></tr>
<tr><td>a. Pick two integers with the <u>same sign</u> so that the greater one is minuend, and the smaller one is the subtrahend.</td><td>
</td></tr>
<tr><td>b. Pick two integers with <u>different signs</u> so that the greater one is minuend, and the smaller one is the subtrahend.</td><td>
</td></tr>
<tr><td colspan="2">(G1) Compare the sign of the difference that you find to your classmates' answers, what do you notice?</td></tr>
<tr><td>c. Pick two integers with the <u>same sign</u> so that the greater one is subtrahend, and the smaller one is the minuend.</td><td>
</td></tr>
<tr><td>d. Pick two integers with <u>different signs</u> so that the greater one is subtrahend, and the smaller one is the minuend.</td><td>
</td></tr>
<tr><td colspan="2">(G2) Compare the sign of the difference that you find to your classmates' answers, what do you notice?</td></tr>
<tr><td colspan="2">(G3) Accordingly; How can one generalize about finding the difference and the sign of the difference when subtracting integers?</td></tr>
</table>