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Developing reflective competence in prospective mathematics teachers by analyzing textbooks lessons

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Abstract
Achieving the competence to reflect on the degree of suitability of textbooks is not easy, so it is necessary to train prospective professionals in the knowledge and use of analytical tools to guide them in critically analyzing these curricular materials. In response to this research problem, this paper describes and analyses the implementation of a training intervention that aims to promote reflective competence in prospective Costa Rican teachers through the analysis of textbook lessons. The content of proportionality is chosen because of its central in the curriculum. The training activity was motivating for the participants and helped them to progress in the competence of lesson analysis. However, it is noted that the design needs to be improved in terms of time dedication, as well as attention to the necessary didactical-mathematical background knowledge on proportionality to achieve better results.

Keywords: teacher education, reflective competence, didactic suitability, textbooks, proportionality

INTRODUCTION

An emerging field of research in mathematics education is related to the importance of the role of professional reflection on educational practice and the need to enhance it in teacher education. Through reflective processes, teachers prevent themselves from acting impulsively and enables them to foresee the potential problems arising in the teaching and learning processes, acquiring competence in solving complex situations (Schön, 1987). Reflection is a key element of professionalization, it involves "the reconstruction of experience and knowledge" (Hodgen & Johnson, 2004, p. 223). It should therefore be developed from initial teacher education, which implies the need to stablish a method and conceptual framework for reflection specific to the discipline being taught (Schön, 1987).

Theoretical approaches such as *lesson study* (Fernández & Yoshida, 2004), *professional noticing* (Mason, 2002), *concept study* (Davis, 2008), or *mathematical knowledge for quality mathematics instruction* (Hill et al., 2008), seek to promote teacher reflection on their own or others' teaching practice. However, as Giacomone et al. (2018) highlight, research tends to focus on aspects that allow the teacher, based on certain information, to describe and explain what happens in a given educational context, without developing instruments that guide the assessment and proposal of improvements in study processes, such as those observed or implemented in the classroom, or those described in the curriculum itself or in a textbook.

Breda et al. (2018) argue that reflection on practice should be guided, as "giving teachers the opportunity to reflect on their practice does not suffice". The teachers "need tools to direct their attention to salient aspects of teaching episodes", moreover, "these tools can be taught as part of teacher education" (Breda et al., p. 1914). To address this problem, from the framework of the onto-semiotic approach (OSA) to knowledge and mathematical instruction (Godino et al., 2007), the construct of *didactic suitability* (Godino, 2013) is introduced as a theoretical-methodological tool that guides the macroscopic analysis of the processes of mathematical study, its assessment and progressive improvement (Godino et al., 2017). The use in teacher education processes of didactic suitability criteria, articulated with other didactic-mathematical knowledge

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Contribution to the literature

- Design, implementation, and evaluation of a training action with future mathematics teachers to develop reflective competence through the analysis of textbook lessons.
- Proposal of training strategies that help future professionals to be more critical and precise when using a textbook lesson.
- Extension of specific analysis guides adapted to the content of proportionality, which synthesize didactic-mathematical knowledge on the subject.

and competences, could be a powerful methodological tool to promote and support teachers' reflection on (their own and others') practice (Breda et al., 2018).

From this point of view, this paper shows a methodology to promote the reflective competence of prospective teachers through the analysis of the didactic suitability of the instructional process planned by means of textbook lessons.

The importance of having specific guides for the analysis of textbook lessons and of incorporating into teacher training programs specific actions, aimed at developing the competence to analyze these resources, is essentially justified for two reasons. On the one hand, the role of the textbook as a resource that systematically structures content and guides the planning of study processes is widely recognized by institutions, teachers, students, and families. On the other hand, some teachers encounter difficulties when analyzing textbooks, omit weaknesses in mathematical content or, when they identify them, make inappropriate modifications that alter their meaning (Yang & Liu, 2019). Furthermore, teachers' use of resource materials is often biased by their own beliefs about the mathematics curriculum, teaching and learning (Lloyd, 2002; Yang & Liu, 2019).

As Shawer (2017) suggests, it is important that training programs encourage teachers to question the quality of materials based on evaluation criteria, and even to adapt them to their needs. The teacher who decides to use a textbook must be able to analyze it productively, adopting a critical viewpoint. The didactic analysis of the lesson provides didactic-mathematical knowledge that guides the teacher in making decisions about the management of the text (Godino et al., 2017). Moreover, it is a powerful resource that teacher trainers can use to get prospective teachers in the processes of reflection on the complexity that characterizes the educational reality (Yang & Liu, 2019).

In this study we describe, analyze, and evaluate a training intervention with prospective Costa Rican mathematics teachers, aimed at developing their reflective competence through the analysis of the didactic suitability of a textbook lesson. The main objectives of this experience are that participants reflect on the use of text as a resource in the teaching and learning of mathematics, and that they know and apply in a critical and constructive way a methodology for analyzing a textbook lesson on proportionality.

The selection of this mathematical topic is motivated, firstly, by the fact that it is one of the fundamental learning contents of the Costa Rican curriculum where its applicability is highlighted (MEP, 2012). Secondly, proportionality does not usually receive adequate treatment in mathematics textbooks (Ahl, 2016; Shield & Dole, 2013), which may lead to the fact that the teaching implemented in practice is biased towards rote learning of routines such as the rule of three algorithms (Lamon, 2007). Third, proportionality is a difficult topic for both students and teachers (Fernández & Llinares, 2011, 2012; Van Dooren et al., 2010). We believe that reflecting on the instructional processes provided in texts can be an opportunity to help teachers overcome these difficulties.

THEORETICAL FRAMEWORK

The different theoretical-methodological tools developed in the OSA provide a basis for the knowledge and competences that mathematics teachers should have (Godino et al., 2017).

Pragmatic Meaning

In OSA, the notion of *mathematical practice*, understood as any action that a given subject performs to solve a problem, communicate or generalize its solution, constitutes the starting point for analyzing mathematical activity. Consequently, the *meaning of a mathematical object* refers to the systems of operational and discursive practices carried out by a person (personal meaning) or shared within an institution (institutional meaning) to solve a problem-situation (Godino et al., 2007). In mathematical practices, mathematical objects of different nature and function participate and emerge, which are related to each other forming *onto-semiotic configurations of practices, objects, and processes*. Thus, mathematical objects, problem-situations, languages, concepts, propositions, procedures, and arguments emerge from the systems of practices through the respective mathematical processes of problematization, communication, definition, enunciation, algorithmization and argumentation (Godino et al., 2007).

When the teacher plans an instructional process, he/she must refer to mathematical texts, curricular guidelines and, in general, to what experts consider to be the practices inherent to the object to be taught, which determine the *reference institutional meaning of the object* (relative to the mathematical content). Then, the teacher selects and specifies the system of specific practices that he/she will propose to his/her students to study a particular mathematical content, which defines the *intended institutional meaning* of the mathematical object. At this point, the teacher must consider, among other things, the time available, the students' prior knowledge and the means available for instruction. Learning involves the appropriation of the intended institutional meanings by the students through their participation in the practices generated in the classroom.

In order to explain the errors, difficulties, and obstacles that students may have in learning mathematical content, OSA introduces the notion of *semiotic conflict*. This is understood as any disparity between the meanings attributed to an expression by two subjects (people or institutions) in communicative interaction, distinguishing between *epistemic conflict* (discordance between the reference institutional meaning and the intended or implemented meaning), *cognitive conflict* (disparity that occurs between practices that form the personal meaning of the same subject) and *interactional conflict* (mismatch that arises between the practices of two different subjects in communicative interaction) (Godino et al., 2007).

Didactic Suitability

In OSA, the *didactic suitability* of a teaching-learning process is understood as the degree to which it (or a part of it) meets certain characteristics that allow it to be qualified as optimal or adequate to achieve the adaptation between the personal meanings achieved by students (learning) and the intended or implemented institutional meanings (teaching), taking into account the circumstances and available resources (environment) (Breda et al., 2018; Godino, 2013). Didactic suitability is a gradable characteristic that involves the coherent articulation of the following facets or partial suitability (Godino et al., 2007).

1. *Epistemic*: Degree of representativeness of the implemented institutional meanings with respect to a reference meaning.
2. *Cognitive*: Degree to which implemented meanings are in the learners' zone of potential development; proximity of achieved personal meanings to implemented meanings.
3. *Interactional*: The degree to which configurations (teacher and learner actions and means used to approach the study of a problem-situation) and didactic trajectories (sequence of didactic configurations) allow for the identification and resolution of semiotic conflicts that occur in the instructional process.
4. *Mediational*: Degree of availability and adequacy of material and temporal resources.
5. *Affective*: Degree of involvement (interests, emotions, attitudes, beliefs) of students in the study process.
6. *Ecological*: The degree to which the study process is in line with the educational project of the center, the school and society and the environment in which it takes place.

Didactic suitability criteria should be understood as "standards of correctness that establish how a teaching and learning process should be carried out" (Breda et al., 2018, p. 264). However, in order for these criteria to be operational, it is necessary to define a set of observable indicators that make it possible to assess the degree of achievement of each of these standards (Breda et al., 2018; Godino, 2013). Furthermore, the facets, components and indicators of didactic suitability must be enriched and particularized according to the specific subject to be taught (Breda et al., 2018), as well as to the specificity of the study process.

For this reason, in Castillo et al. (2022a) the system of components and indicators of didactic suitability of Godino (2013) is reviewed and particularized to develop a mathematics textbook lesson analysis guide (TLAG-Mathematics). Subsequently, in Castillo et al. (2022b) the TLAG-Mathematics is adapted to the topic of proportionality yielding the TLAG-Proportionality.

Both guides are composed of a series of indicators for each of the facets and their components, based on an exhaustive theoretical review of research results and on expert judgements made by the educational community. They are a resource to organize trainee and practicing teachers' reflection on the instructional processes planned in textbook lessons.

Mathematics Teacher Knowledge and Competences

In this paper we adopt the mathematics teacher's didactic-mathematical knowledge and competences model (DMKC) (Godino et al., 2017) developed within the OSA framework. This model assumes that the mathematics teacher must be competent to design, implement and assess their own and others' learning sequences, using didactic analysis techniques and quality criteria, in order to make proposals for improvement (Breda et al., 2018; Godino et al., 2017).

To achieve this competence of analysis and didactic intervention, the teacher needs both knowledge that allows him/her to describe and explain what has happened in the teaching and learning process, and knowledge to assess it and make critical management proposals. This global competence is achieved through the articulation of five sub-competences associated with the different levels of analysis of the instructional processes proposed by the OSA (Godino et al., 2017; Pino-Fan et al., 2015):

(a) analysis of global meanings (identification of problem-situations and operational and discursive practices involved in their resolution);

(b) onto-semiotic analysis of practices (identification of the network of objects and processes involved);

(c) management of didactic configurations and trajectories (identification of the sequence of interaction patterns between teacher, student, content and resources);

(d) normative analysis (recognition of the network of norms and meta-norms that condition and support the instructional process); and

(e) didactic adequacy analysis (assessment of the teaching and learning process and identification of potential improvements).

The use of didactic suitability enables teachers to systematically reflect on their own practice or that of others, but also to analyze partial aspects of instructional processes by means of resources, such as educational videos (Beltrán-Pellicer et al., 2018) or textbooks (Castillo et al., 2022a, 2022b; Díaz-Levicoy et al., 2016; Morales-García & Navarro, 2021).

Suitability criteria reflect consensus on what good mathematics teaching should look like, so they function implicitly as regularities in teachers' discourse when they have not yet been trained to use the construct as a guide to their thinking (Breda et al., 2018; Hummes et al., 2019). However, teachers need specific tools and training to direct their attention to the multiple and intertwined factors that affect instructional processes (Seckel & Font, 2020; Sun & van Es, 2015). For this reason, in recent years, several training actions have been developed that use the didactic competence tool to promote the reflective competence of trainee teachers on mathematical study processes (Breda et al., 2018; Burgos et al., 2020; Esqué & Breda, 2021; Giacomone et al., 2018; Hummes et al., 2019; Morales-López & Araya-Román, 2020, among others). The intervention we describe in this paper is part of this line of research in teacher education.

CONTEXT & RESEARCH METHODOLOGY

The training experience was carried out within the framework of the initial training received by students of the mathematics education degree course at the University of Costa Rica (first academic year 2021), within the subject MA-0007 mathematics in the school curriculum. One of its objectives is to "provide specific curricular theoretical elements of mathematics education that allow them to understand and manage their responsibility in the planning, design, implementation and evaluation of mathematics learning processes, from the meso and micro levels" (Escuela de Matemática de la UCR, 2015, p. 1). This subject is taught in the second year of the degree, so the participants, 28 prospective mathematics teachers (PMT), have little or no practical experience.

We adopt a design research approach (Cobb & Gravemeijer, 2008), whose aim is to develop instructional strategies for the improvement of mathematics teaching and learning, implemented in real classroom contexts and based on research. Thus, three phases are considered in conducting a design research experiment (Cobb & Gravemeijer, 2008):

1. *preparation of the experiment* (determining its objectives, in our case the development of prospective teachers' reflective competence; finding out what the participants' prior knowledge is; foreseeing the mediational and temporal constraints, as well as the way of implementing the tasks in the classroom);
2. *experimentation to support learning* (contrasting and testing the learning trajectory formulated in the preparation phase involves deciding on the relevant data types and their collection, as well as the interpretative theoretical framework);
3. *retrospective analysis* of the data generated during the realization of the training experience (documenting participants' learning and reorganizing prior knowledge and competences, producing innovative designs that can be used or generalized to support learning productively in other settings).

What it is designed is a "complete learning environment" with tasks, materials, tools, and other means to guide the learning of groups of students in a particular content domain. The research team is committed to reflect after each implementation in order to readapt the initially proposed design, if necessary. Thus, for the planning and design of the training experience described in this article, we relied on the retrospective analysis of interventions previously implemented with future Spanish teachers of primary education (Castillo & Burgos, 2022) and secondary education (Castillo et al., 2021, 2022c). As we show in the next section, the limitations found in those training actions were considered to improve the design and implementation of the new intervention.

The content analysis methodology (Cohen et al., 2011) is used to examine the participants' response protocols. The PMT delivered the reports in each work session, using the Costa Rica University's official virtual platform. The analysis of the data is oriented towards the identification of significant responses on the initial state of the students' personal meanings, the recognition of difficulties and progress in the development of the intended competence. As part of this analysis, the indicators in the different components of didactic competence allow us to categorize the discourse of the participants, locating fragments of description considered evidence of their implicit use. Moreover, the results of this analysis allow the research team to plan the trainer's interventions to guide or redirect the participants' learning. After each formative session, the participants receive feedback from the trainer-researcher on their individual task results. This enables the prospective teachers to know how far they had progressed in their learning, to raise the necessary questions and to reinforce, or amend, those didactic-mathematical competences that have been found to be insufficient.

DESIGN OF THE TRAINING INTERVENTION

In this section, we describe the design of the training action, specifying the didactic resources, the work methodology followed, and the tasks proposed to the PMT. Initially, the duration of the intervention was planned to be four class sessions of two hours each, in a virtual and synchronous manner. However, the pandemic situation forced a reduction to three effective synchronous work sessions.

In previous experimental cycles, it was observed that participants did not justify the level of compliance with each indicator in the TLAG-Proportionality (Castillo et al., 2021, 2022c; Castillo & Burgos, 2022). For this reason, it was decided to include in the guide given to the PMT, in addition to the column for recording the numerical rating (0, 1, 2) of the degree of compliance with each indicator, another column in which they had to explain the reasons that had led them to assign that rating. Moreover, in the new intervention, PMT were asked to apply the TLAG-Proportionality globally to the whole lesson, whereas in previous experiences the text was broken down by didactic configurations that constituted the units of analysis (Castillo et al., 2021,2022c; Castillo & Burgos, 2022). This change made it possible to alleviate the participants' workload and to see the exercise and evaluation tasks proposed in the lesson (normally separate) together with the conceptualization or exemplification tasks proposed by the author of the textbook.

Session 1. Initial Exploration

In this session the *initial task* (pre-training) was proposed, which the PMT were to start working on individually during the synchronous session and then deliver through the virtual platform, which is commonly used in the MA0007 course.

The initial task instructions included two parts. The first part goal was to determine the participants' beliefs and knowledge in relation to textbook analysis. Therefore, they were asked to answer general questions about how they think a mathematics textbook should be used, what characteristics a good mathematics textbook should have, what are the advantages and disadvantages of using a textbook as a resource in the classroom, and what tools for analyzing a textbook they know. They were also asked about their opinion on the importance and usefulness of knowing book analysis tools for their training as future teachers.

In the second part, the PMT were asked, after carefully reading the whole lesson "proportion" from the mathematics 7 book (Porras et al., 2013, p. 127-136), to indicate whether they considered it suitable or not to address the topic of proportionality at seventh grade level (13-year-old students), pointing out its strengths, weaknesses or errors, and how they would use it in a class to teach the topic. The aim was for participants to make an initial reflection on the suitability of the resource and to recognize implicit suitability criteria in their discourses (Breda et al., 2018; Seckel & Font, 2020).

In addition, as a task to be handed in prior to session 2, they had to answer a questionnaire of didactic-mathematical knowledge in which tasks related to the resolution and creation of problems of direct and inverse proportionality, and the detection of errors in prototypical student responses were posed. The results of this questionnaire allowed us to take into account which didactic suitability indicators needed to be given greater attention in third session, in order to correct or reinforce their knowledge[1] of proportionality.

Session 2. Didactic Suitability as a Resource for Analyzing Textbook Lessons

At the beginning of the session, participants had to access a virtual link to answer a survey via the *Doodle* platform. Based on their previous reflection, they had to rate the appropriateness of the lesson "proportion" on the scale: 5–excellent, 4–very good, 3–good, 2–bad, 1–very bad, according to the positive or negative characteristics identified, the errors encountered and their frequency.

Then, the notion of didactic suitability was introduced, and it was explained how this theoretical-methodological tool can be applied to the analysis and critical appraisal of planned or implemented instructional processes, and in particular, to the analysis of textbook lessons. The teacher included in the presentation the answers that some of the PMT gave to the question "What characteristics should a mathematics textbook have to be a good resource?" (**Figure 1**), inviting participants to reflect on the extent to which they had considered aspects related to one or more facets and components of didactic suitability.

[1] In previous experiences (Castillo et al., 2021; Castillo & Burgos, 2022) prospective teachers found great difficulties in identifying the deficiencies of the lesson in the epistemic facet, which could be explained by limited didactic-mathematical knowledge about proportionality not identified prior to the intervention.

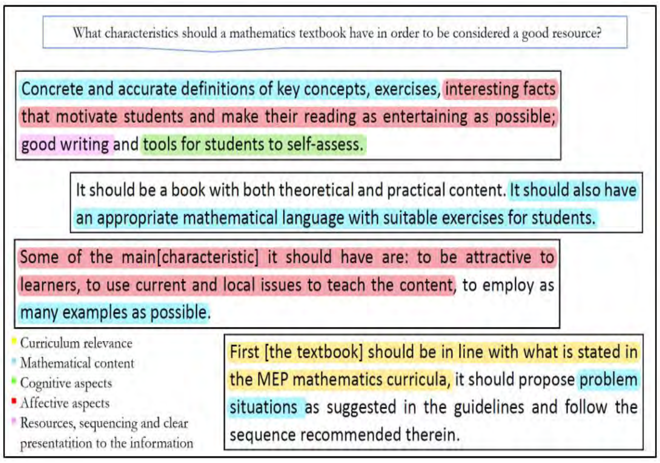

Figure 1. Characteristics of a mathematics book according to the PMT, underlined according to facets

Subsequently, fundamental notions involved in the theory of didactic suitability (Godino, 2013) were explained, such as mathematical practice, (institutional/personal) meaning of a mathematical object and semiotic conflicts (epistemic, cognitive, interactional), showing examples taken from several proportionality lessons in textbooks. The theoretical session concluded with the presentation of the TLAG-Mathematics and reflection on the need for indicators to be adapted to the mathematical content of interest (Breda et al., 2018; Godino, 2013).

At the end of this session, the PMT were provided with a document prepared specifically for the intervention, which included as essential aspects:

(a) research results showing the importance of the mathematics teacher being competent in the analysis of textbooks,

(b) the methodological guideline based on the OSA to carry out such analysis,

(c) the main didactic-mathematical knowledge that must be taken into account to analyze a lesson on proportionality.

Finally, the TLAG-Proportionality adapted to the educational context (secondary education in Costa Rica) is included. Several of the indicators were reformulated in order to avoid the difficulties of interpretation that some prospective teachers had shown in previous interventions, mainly in the cognitive-affective and instructional aspects (Castillo et al., 2022c).

Session 3. TLAG-Proportionality and Didactical-Mathematical Knowledge

The purpose of this session was to familiarize participants with the application of TLAG-Proportionality to analyze a textbook lesson. To exemplify its use, we resorted to the lesson "proportionality" from the textbook by Alvarado (2014), a text selected for its common use in the Costa Rican context, and which had been previously analyzed by the researchers. Thus, in each of the facets and components, specific examples of indicators were presented, which were quantitatively assessed on the scale: 0 if indicator is not met; 1 if it is partially met, 2 if it is fully met.

At the same time, the activity made it possible to reinforce or correct those aspects of didactic-mathematical knowledge that had been detected as insufficient when analyzing the responses to the initial questionnaire (session 1), paying more attention to the suitability indicators involved. We also took into account those indicators that, in the interventions of previous research cycles (Castillo et al., 2021, 2022c; Castillo & Burgos, 2022) had had a lower success rate in their assessment. In the epistemic facet they were related to:

- The different meanings of proportionality: intuitive, geometric, arithmetic, and algebraic-functional (Burgos & Godino, 2020), and how they should be incorporated and articulated.
- The importance of including the fundamental properties of the proportionality relationship: necessary and sufficient conditions that allow determining whether a relationship is of direct proportionality or not; the additive and homogeneous character of the functional relationship (Burgos & Godino, 2020; De Bock et al., 2017).
- The need for proportionality instruction processes to provide opportunities for learners to distinguish multiplicative from additive comparisons (Fernández & Llinares, 2012) and direct from inverse proportionality relationships (Lamon, 2007).
- Expert recommendation to distinguish between *internal ratios* (relating quantities of the same magnitude) and *external ratios* (relating quantities of different magnitudes) and to promote thinking and representation strategies that encourage analysis between and within measurement spaces (Lamon, 2007; Shield & Dole, 2013).

In the remaining facets, theoretical training focused on the following aspects:

- (Mediational) What is considered an appropriate sequencing for the teaching of proportionality according to the existing literature. For example: start the study of proportionality with an informal-qualitative approach, and then move towards formalization and algorithmization, delaying the use of the rule of three and avoiding its indiscriminate use (Burgos & Godino, 2020; Fernández & Llinares, 2011; Lamon, 2007).
- (Ecological) Careful review of the curriculum guidelines to be followed according to the Ministry of Public Education (MEP, 2012).
- (Cognitive) Factors that, from a mathematical and didactic point of view, influence the complexity of

Ana, María, and Luis are planting trees in a "Replanting" camp. Ana and María started at the same time, but María is faster. Luis goes at the same speed as Ana, but he started earlier. When Ana had planted 4 trees, María had planted 12 trees and Luis had planted 8 trees. When finishing, Ana has planted 20 trees. How many trees will Luis have planted? Explain your answer.

Figure 2. Initial mathematical knowledge assessment task

proportionality tasks, such as: the use of integer (numbers are multiples of each other) and non-integer ratios, the way in which the task is formulated, familiarity with the content, and the number of magnitudes involved in the same problem (Fernández & Llinares, 2012; Van Dooren et al., 2010).

Special attention was paid in the discussion to two errors commonly detected in students and teachers:

(a) the *illusion of linearity*, which leads to treating a situation in which quantities do not maintain a direct proportional relationship as if it were one (De Bock et al., 2002) and

(b) the characterization of proportionality between quantities based only on necessary conditions such as the *property* "the more in A, the more in B" (Izsák & Jacobson, 2017).

It was also emphasized that recognizing inverse proportional relationships and solving problems involving this type of relationship are more difficult for students than those involving direct proportionality (Arican, 2019).

To contextualize the discussion and involve the participants in the reflection, the teacher-researcher used the results obtained by the participants in some of the problems posed in the initial questionnaire to assess their mathematical knowledge of proportionality. For example, in the problem shown in **Figure 2**, 13 of the PMT treated the additive relationship between the magnitudes number of trees planted by Ana and Luis as multiplicative, erroneously concluding that Luis managed to plant 40 trees (the correct result being 24).

The solution was discussed, connecting with the illusion of linearity previously explained. In another task (**Figure 3**), the PMT had to formulate a direct and an inverse proportionality problem to be solved by means of a previously established proportional equation.

While 20 PMT answered the first part of the task correctly, no PMT managed to create a problem in which the relationship between the quantities was inverse proportionality and used the given proportion. The aim was to provide space for the participants themselves to understand the errors and difficulties documented in the literature and to clarify their own limitations in solving the problem-situations correctly.

The session ended with a detailed presentation of the final (post-training) task to be delivered virtually. For this task, participants were provided with the TLAG-

Create two problem-situations that can be represented by the proportion,

$$\frac{24}{3} = \frac{x}{6}$$

so that one involves **directly proportional magnitudes** and the other involves **inversely proportional magnitudes**. Justify why the situations-problems you have posed in each case correspond to a proportional or inverse relationship.

Figure 3. Task on creating direct & inverse proportionality problems

Proportionality in tabular format, to which, as we have mentioned, together with the indicators, two columns were added for the quantitative assessment of their degree of compliance and justification of the assigned assessment. Subsequently, they had to make a reasoned judgement on the suitability of the lesson in each facet and overall, taking into account the assessment of the indicators and the conflicts (epistemic, cognitive and interactional) encountered. Finally, they were asked to re-evaluate the lesson by Porras et al. (2013) using the *Doodle* survey and explain whether and why they had changed their opinion regarding the degree of appropriateness assigned in the initial task.

Session 4. Discussion of Results & Sharing of Results

In this session, the synchronous discussion and sharing of the results of the analysis of the textbook lesson was planned. However, given the adjustments made to the MA0007 course schedule due to the health situation, this session could not take place. The actual implementation involved the submission of the individual written report and feedback from the teacher-researcher via the course platform once the reports were submitted. Considering the complexity of assessing teaching and learning processes in their various dimensions and components, we considered it necessary for the PMT to have 10 days to complete their full report.

Assessment of the Training Experience by the PMT

At the end of the design cycle, the PMT were asked to complete a rating survey for each of the following items, using a scale of 1 to 5 (1 minimum value, 5 maximum value):

1. Clarity of tasks and instructions.
2. Adequacy of the methodology followed (way of working, teacher's explanations).
3. Degree of motivation and interest aroused by the activities.
4. Level of learning achieved.
5. Degree of overall relevance of the workshop for their training as mathematics teachers.
6. Degree of relevance of the resources used: TLAG-Proportionality, textbooks analyzed, examples given, readings.
7. Degree of difficulty of the tasks.

Table 1. Pre-training analysis: Characteristics rated positively in each facet by the PMT (n=28)

Facet	Description	Frequency of PMT
Epistemic	Presence of diverse & appropriate examples/exercises Clear presentation of definitions	20
Cognitive	Inclusion of ideas/suggestions for solving the activities Appropriate difficulty of the activities	5
Affective	Use of colors and images that make the presentation of the content more enjoyable Presence of real-life situations	10
Interactional	Clear & organized presentation of the topic	15

Table 2. Pre-training analysis: Negatively rated characteristics in each facet by PMT (n=28)

Facet	Description	Frequency of PMT
Epistemic	Some examples and/or exercises are not clear. Procedures are not presented correctly. Representations are missing or language is inadequate. Some definitions are not clear.	14
Cognitive	No prior study of the concepts involved Some content or exercises may be difficult for 7th graders	10
Interactional	The presentation of topic is not clear because there are disconnected ideas, or the organization is confusing	6
Ecological	Non-mathematical content is included in the lesson	4

Space was also left for them to add any comments about difficulties encountered during the experience, opinions on the usefulness of the TLAG-Proportionality and proposals for improvement.

RESULTS: LEARNING ACHIEVEMENT

In this section we show the results of the evaluation of the learning achieved by the participants. The aim is to confront their results with the a priori analysis of the lesson developed by the researchers and by an external expert collaborator (independently by the three of them and agreed upon afterwards).

First, we describe how the participants assessed the lesson in the initial task. Then, we analyze the PMT reports on didactic suitability in each of the six facets to determine whether there is progress in learning in the following ways:

- *Self-correction:* When PMT that reflected an incorrect assessment of the lesson in some facet in the pre-training analysis, modify them correctly in the post-training task.
- *Increase of referents:* When PMT increase the positive or negative characteristics they correctly take as references to give an assessment of the didactic suitability of the lesson.

Finally, we examine the responses to the *Doodle* surveys to assess the degree of suitability of the lesson, pre and post-training, to determine how it evolves and whether it coincides with score given by research team.

Pre-Training Analysis of the Lesson

In the pre-training analysis of the lesson conducted, PMT implicitly use some components and indicators of different criteria of didactic suitability, in line with research findings such as those of Breda et al. (2018), Giacomone et al. (2018), and Seckel and Font (2020).

When PMT read the lesson for the first time, they focus their attention on concrete elements that are assumed by them as good practices. In this way, there are aspects that are rated positively (**Table 1**) and others that PMT consider as weaknesses of the lesson (**Table 2**), although these are less than the potentialities indicated.

In general, the characteristics that PMT mention positively in the cognitive and affective facets (**Table 1**) are correct. The weaknesses they report in the epistemic, cognitive, and interactional facets (**Table 2**) are also correct. Some of them rightly point out that using the terms "indirect" and "inverse" as synonyms to define inverse proportionality relationships can cause confusion, that some prior knowledge such as "volume, perimeter" is not considered, or that the explanations in relation to inverse proportionality are short and confusing.

However, the contrast of the PMT pre-training analyses with the a priori analysis of the lesson allows us to identify that some of their responses described above are not correct. Specifically, we disagree with the opinions of the PMT in relation to the following facets: epistemic (when valuing the definitions as adequate, 8 PMT), interactional (when indicating that the author makes a clear presentation of the topic, 12 PMT) and ecological (when believing that the inclusion of questions outside the mathematical context is not appropriate, 4 PMT). Indeed, the a priori analysis of the lesson shows that some notions are introduced in a confusing way. For example, as can be seen in **Figure 4**, several terms are used incorrectly as synonyms (ratio-ratio, ratio-proportion, indirect-inverse, ratio-proportionality relationship); a circular definition is

Proporción

En situaciones de la vida cotidiana existen magnitudes medibles que se relacionan entre sí. La razón (o cociente) entre las magnitudes, se llama: proporción.

Estas proporciones se pueden comportar de dos formas: proporción directa y proporción indirecta.

Proporción directa

Cuando existen magnitudes que se relacionan de manera que cuando una aumenta la otra también se incrementa de manera proporcional y viceversa, entonces estamos ante una proporción directa.

Proportion

In everyday situations there are measurable magnitudes that are related to each other. The ratio (or quotient) between the magnitudes is called a proportion.

These proportions can take two different forms: direct proportion and indirect proportion.

Direct proportion

When there are magnitudes that are related in such a way that when one increases, the other also increases proportionally and vice versa, then we have a direct proportion.

Figure 4. Definition of proportion (Porras et al., 2013, p. 127)

given for direct ("increases proportionally") and inverse proportionality. Fundamental concepts such as the constant of proportionality are not defined.

The PMT reviews are based on superficial characteristics, some grounded on their own conceptions and disconnected from each other, reflecting only a small part of the complex network of indicators linked to the different facets of didactic suitability (Godino, 2013). These results, like those of other studies (Seckel & Font, 2020; Sun & van Es, 2015), reflect those participants need specific tools and training to direct their attention to key aspects of instructional processes, a need that participants themselves recognize when responding to the task ("I don't have much knowledge of what I should base my opinions on", PMT28).

Learning Progression

To assess the extent to which the use of TLAG-Proportionality and training allows PMT to progress in reflective competence, we observed to what extent they self-corrected (when incorrect answers were identified in the pre-analysis, i.e., in the epistemic, interactional and ecological facets) and whether, in the post-training analysis, they correctly assessed the lesson by considering a larger number of referents than those considered in the initial task. In the following, the main results of the analysis are described and prototypical examples of correct answers in each of facets are shown.

Epistemic facet

Self-correction: As indicated above, eight of the PMT incorrectly assessed the appropriateness of the concepts in the lesson. After the training, seven of them rightly pointed out that the concepts are not presented in a clear way. For example, PMT16 who was of the opinion in the initial task that "definitions are clearly provided", later states that "...there is also no distinction between ratio, fraction and quotient, on the contrary, in 127, definition of proportion is defined as "the ratio (or quotient) between magnitudes" without making any of this clear".

Increase of referents: After the training, the frequency of participants (12 PMT) who identify that the fundamental concepts of proportionality are not presented clearly and correctly is higher than in the initial task (seven PMT). For example, they suggest:

> ... mention that ratio and quotient are the same. The definition of direct and inverse proportionality is incomplete. It is said that, if we have more in A and more in B then it is direct proportionality, but this is only a necessary and not a sufficient condition (PMT10).

After the training, the PMT have reflected on the lack of introductory situations and the lack of problems that lead to distinguishing multiplicative from additive comparisons (six and seven PMT in each case). In relation to language, in the initial task, only one PMT had recognized the lack of graphical representations in the lesson as a deficiency. After the training, eleven more PMT recognized the limited diversity of representations for modelling direct or inverse proportionality or for distinguishing between such proportionality relationships. For example, PMT26 indicates that "... significant conflicts were found in relation to the [...] poor manifestation of comparative, tabular and graphical representations..."

It is worth mentioning that the participants managed to recognize important deficiencies in the lesson that they had not considered in previous task. For example:

- The solved situations do not involve multiplicative relationships within and between quantities of magnitudes; appropriate representations are not used to distinguish such relationships (eight PMT).
- Fundamental properties of direct and inverse proportionality functions are not presented clearly and correctly (six PMT).
- Situations whose underlying models share certain, but not all, characteristics with the proportionality function are not considered (five PMT).
- The procedures (rule of three and reduction to unity) are not presented in a clear and justified way (five PMT).

Cognitive facet

Increase of referents: After the training, 18 PMT considerably increased their referents when evaluating the lesson from a cognitive point of view, indicating that the lesson does not promote a variety of correct solving strategies (13 PMT), does not anticipate errors or difficulties (11 PMT), or that there are not enough extension and reinforcement activities (five PMT), aspects that were not considered at all prior to the training. For example, PMT9 felt that

There are strengths in relation to the difficulty of the exercises because it is manageable. In addition, it provides extra-class work. As deficiencies we find, for example, the fact of not warning of possible errors or difficulties that students may face when solving them.

Affective facet

Increase of referents: As mentioned above, in the initial activity, ten PMT had identified positive features of the lesson in the motivational aspect, but none pointed out weaknesses in this respect. After the training, 22 PMT pertinently mention weaknesses in the attitude, emotion, or belief components. For example, PMT9 states:

> In the affective facet, important deficiencies were seen in the components of emotions, attitudes, and beliefs. Among the most relevant are the lack of situations that motivate students to solve problems, the absence of a space to express what they feel about the subject, or the interest in generating in students a feeling of love and not hate towards mathematics. This justifies the low level of suitability.

Mediational facet

Increase of references: After the training 22 PMT contemplated relevant weaknesses related to the lack of use of material resources (21 PMT) or references (12 PMT), or that the sequencing of contents is not correct (five PMT). For example, PMT10 indicates that "no use of technology of any kind is requested, or specifically of any manipulative material, and we can see that the sequence of contents is not fully covered". Given that prior to the training no participant assessed the lesson from this point of view, we can consider this as an advance in their learning.

Interactional facet

Self-correction: A total of 12 PMT initially stated incorrectly that the lesson analyzed organizes and presents the topic adequately, based on the structure "first exemplification of the concepts and then exercise", omitting relevant deficiencies regarding the clarity of the examples and the analysis of the relationships in graphical form.

After the training, five of these PMT changed their opinion with respect to their previous assessment. For example, PMT16 who indicated in his initial assessment: "the lesson is suitable for dealing with the topic of proportionality at grade 7 level, because it clearly provides the definitions, as well as providing a large number of examples", later states pertinently "it does not emphasize some key concepts of the topic, such as magnitude, which is mentioned in the presentation, which could detract from the clarity of the presentation of the topic".

Increase of references: After the training, 19 PMT were able to point out deficiencies in relation to the author's poor use of argumentative resources to explain the contents, the fact that flexibility and consensus are not sought based on mathematical arguments, or the lack of tasks that motivate interaction between peers. The PMT suggest similarly to PMT4 that:

It presents quite a few weaknesses when proposing group tasks, the topic is not presented correctly, there are no arguments that capture the student's attention. Most of the indicators have a numerical rating of 0, so I consider that it has a low degree of interactional suitability.

Ecological facet

Self-correction: Initially, only four PMT rated aspects of this facet in the lesson. The descriptions were inadequate; they were based on their own beliefs and disqualified the presence of situations in non-mathematical contexts. In the post-training analysis, two of these PMT reflect on their initial limited view of purely mathematical content, which was later enriched with new aspects of this facet. Thus, PMT21 states that after the training they attach "greater importance to topics that they did not previously consider important in a mathematics textbook, such as democratic values, respect, tolerance, etc."

Increase of referents: In the post-training task, 17 PMT have managed to broaden the features to adequately and objectively specify both weaknesses and potentialities of the lesson. For example, PMT3 states that "...with regard to strengths, the component of socio-professional adaptation can be mentioned, [...] with regard to weaknesses, education in values can be noted". Moreover, on this occasion, seven PMT have considered adaptation to curricular guidelines as a characteristic, an aspect that they did not consider in the previous analysis. For example, PMT10 indicates that, although the curriculum "asks for a varied representation of the contents, the book does not do so, and it does not seek to establish a relationship with other interdisciplinary axes."

Do Prospective Teachers Change Their Opinions When Evaluating the Lesson?

After the initial reading of the lesson, the PMT completed a *Doodle* survey in which they rated the appropriateness of the lesson on a scale of 1 (very poor)-5 (excellent). After applying the guide to analyze the lesson and make a reasoned judgement on its appropriateness, they were asked to respond to the survey again, justifying their change of opinion on the rating if there was any. It is worth mentioning that after carrying out the a priori analysis applying the TLAG-Proportionality, the research team rated the lesson according to this scale as "2 BAD".

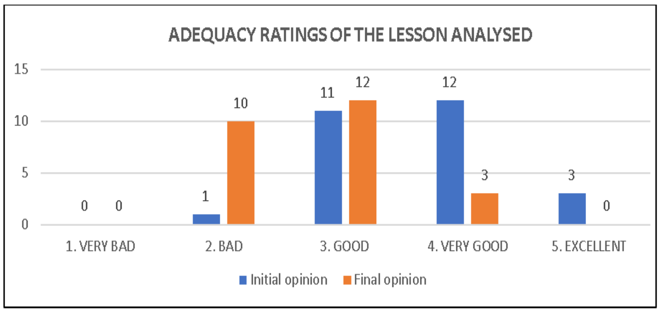

Figure 5. Contrast between initial and final opinions on the adequacy of the lesson

As shown in **Figure 5**, the contrast of the ratings given by the PMT through the survey before and after the training can be seen. The initial survey was answered by 27 PMT and the final survey by 25 PMT. All changes of opinion (16 in total) result in a lower rating compared to the pre-training rating. The highest frequency of changes of opinion occurs in the cases from "good" to "bad" and from "very good" to "good". There are nine PMT who maintain their opinion, of which six do so by rating the lesson as "good".

The majority of PMT (24 in total) claimed to have changed their opinion on the characteristics that a good mathematics textbook should have, mentioning that they had previously had a narrow view on the mathematical aspect. For example, PMT25 stated "I noticed that a textbook should have more than just a definition and a long list of exercises (this was my general idea)". Others attributed their changes of opinion to revising their beliefs that textbooks and teaching resources in general were "well done". For example, PMT8 states that:

> I used to hold the idea that books (in theory) are made by people who are experts in the subject, they are always properly designed for the student to learn. With this analysis it is quite clear to me that [textbooks] are not necessarily going to accomplish everything that is necessary.

The PMT justified their change of opinion on the more comprehensive view on important aspects in the textbooks achieved through the training, which they had not reflected on before. For example, PMT27 stated that

> My opinion did change. Initially I did not consider that one could evaluate the lesson of a book as deeply as one does with the support of the idea of suitability. Now knowing it, I realize that [evaluating the lesson] means going beyond whether what is included is correct or not, but also considering the absence of elements in the lesson.

Synthesis of Results

In general, we can affirm that all the PMT have made progress in the competence of lesson analysis after the training (**Table 3**). The majority (25 PMT) have broadened the criteria they consider important to assess in the different facets, with the affective, mediational, and epistemic facets showing the greatest increase in referents after the training (at least 21 PMT in each case). In addition, for 20 PMT, the extension of criteria is related to changes of opinion, either with respect to how they had rated a partial aspect of the lesson (self-correction) or how they rated the lesson as a whole (*Doodle* survey).

In the category "other achievements" four PMT are considered that give a correct quantitative assessment based on the number of indicators according to their corresponding degree of fulfilment (total: 2, partial: 1, null: 0) or the calculation (sums or averages) of the assessments, in at least two facets that they did not previously consider. For example, PMT16 appropriately stated about the interactional facet that "most of the indicators have a numerical rating of 0, so I consider the degree of appropriateness to be low". Similarly, PMT18 correctly stated that "the cognitive suitability is medium because it scored 7 out of 16 points". As reflected in this section, the results show both the effectiveness of this theoretical-methodological tool and the importance of incorporating reflective learning into teacher education.

RETROSPECTIVE ANALYSIS OF THE DESIGN CYCLE

Following the methodological approach of a design research, it is necessary to be aware of the limitations and challenges to be addressed in future interventions. This implies an assessment of the didactic suitability of the training process implemented with future mathematics teachers, in each of its facets. In the cognitive, affective, interactional, and mediational facets, the information obtained through the experience assessment survey completed by the 28 PMT is important. In general, the median score in all the items was higher than 4 (except for item 7, which obtained a mean of 3.7) and the mode in all of them is 4 or 5.

Table 3. Frequency of participants according to achievement of learning progress (N=28)

Learning achievements	Frequency
They only increase the referents in at least 2 facets	5
They increase referents in at least 2 facets and their overall assessment is more relevant	9
They are self-correcting and increase the characteristics in at least 2 facets	4
They are self-correcting, increase characteristics in at least 2 facets & their overall assessment is more relevant	6
Other achievements	4

Epistemic suitability: With the proposed tasks the participants identified the strengths and weaknesses of the lesson and formulated arguments about their value judgements. In short, the design includes didactic-mathematical knowledge in relation to the mathematical content, in our case proportionality, which the PMT must bring into play when responding to the proposed tasks. Taking into account the a priori analysis and the results of the intervention, we conclude that the overall criterion of epistemic suitability of a teacher training process (Godino et al., 2013) is met, being high.

Ecological suitability: The contents, implementation and assessment correspond to those established in the MA0007 course syllabus; the implementation stages are articulated and coherent with the institutional context given that we are dealing with teachers in initial training. The proposed design shows an openness towards innovation based on research and reflective practice, so we consider the ecological suitability to be high.

Cognitive suitability: The aim of the training experience was for participants to learn a methodology for the analysis of lessons, to apply it critically and constructively, and to reflect on the use of the text as a resource. Learning achievements have been detected in the PMT in three ways: broadening of their analysis criteria, self-correction in the assessment and argumentation of the change of judgement on the overall evaluation of the lesson based on a more critical assessment. However, it was also found that participants had difficulties in carrying out this task. This may be associated with the lack of prior didactic and mathematical knowledge necessary to achieve better results, as shown by the assessment of item 7 "degree of difficulty of the tasks" in the experience assessment survey. The results of the analysis of the complementary reports to the survey show that a total of 19 PMT stated that they had some difficulty related to the proportionality content (eight), the interpretation of the indicators in the guide (seven), or the understanding of the theoretical notions associated with didactic suitability (ten). For this reason, we believe that cognitive *suitability* is medium-low.

Affective suitability: The training experience has considered, at different stages, the PMT beliefs about the teaching and learning of mathematics and, in particular, about the use and management of the textbook. The responses to the evaluation questionnaire reveal good results in terms of the degree of motivation and interest in the activities (item 3, mean of 4.2), the level of learning that participants believe they have achieved (item 4, mean of 4.1), and the relevance of the experience for their training as mathematics teachers (item 5, mean of 4.8). We therefore believe that affective suitability is high.

Interactional suitability: The design of the experience promotes autonomous work by the participants (Godino et al., 2013). In addition, good results are obtained in the assessment of the "clarity of tasks and instructions" (item 1, mean of 4.1). As we have mentioned, the design of the experience described here is a reformulation of previously applied designs (Castillo et al., 2021, 2022c). The results show that reinforcing the training on those indicators that had been conflictive (deficiencies in interpretation and correct assessment by the participants) in previous interventions had positive effects as they were finally assessed in a timely manner by most of the PMT. For example, in the epistemic facet at least 23 PMT correctly assessed two of the indicators on which most attention was paid (presence of problems in distinguishing multiplicative from additive relations, as well as multiplicative relations within and between quantities of magnitudes). Likewise, the indicators emphasizing the cognitive, ecological, and mediational facets were also correctly assessed by at least 21 PMT.

However, the PMT gave the minimum score, 2, to item 2 "adequacy of the methodology followed", so it is necessary to promote greater spaces for discussion and sharing. We believe that it is essential that there be an exchange of final responses, which, although it was foreseen in the design of the experience, could not be carried out in the end. On this basis, we consider the interactional suitability to be medium-high.

Mediational suitability: The evaluation survey responses show satisfactory results regarding the appropriateness of the resources used (item 6; mean 4.3). In addition, 25 PMT rated positively the usefulness of the guide when analyzing textbook lessons. However, as stated by eleven PMT, more time needs to be invested for them to become familiar with the use of resources such as TLAG-Proportionality:

> I loved the workshop, it's a pity that we couldn't spend a little more time on it. [...] I think that with more time we could have worked on more dynamics that reinforce the understanding of didactic suitability which, as such, is an important part of development for the teacher and the student (PMT23).

More time would have been needed to provide opportunities for participants to learn and familiarize themselves with the theoretical framework used in this design. Therefore, the mediational suitability is medium.

CONCLUSIONS

Promoting the competence to analyze instructional processes is not a trivial task for teacher educators. In various contexts, research has investigated ways to encourage prospective teachers to be more analytical when making value judgements about curricular materials, alerting them to the need to help them incorporate theoretical references and use arguments, so that they do not rely solely on their own beliefs or experiences (Lloyd, 2002; Shawer, 2017). In this sense, the interest of our paper has been to describe the design and implementation of a training action with prospective mathematics teachers to develop reflective competence through the analysis of textbook lessons. Considering textbook lessons as instructional processes planned by the author, allows for the application of the didactic suitability construct and its breakdown into facets, components and indicators, as a tool for an overall reflection on the intended didactic practice, assessing it and proposing improvements (Breda et al., 2018).

The training design described here, although not a finished product, is intended to be useful to teacher trainers and in general to those in charge of training programs, as a training strategy to help prospective professionals to be more critical and precise when using a textbook lesson. The results of this experience allow us to affirm that the analysis of textbook lessons, using didactic suitability as a resource, is a motivating training activity for future teachers, which allows them to bring into play a variety of didactic-mathematical knowledge and skills necessary for a reflective prospective education professional. However, the results of the analysis also warn of the importance of carrying out this type of experience in continuous cycles in teacher training. Indeed, future teachers need to overcome difficulties and consolidate their knowledge of mathematical content, in our case, proportionality, while progressively acquiring greater competence in assessing the practice involved.

The specificity of textbook analysis guides for mathematical content can serve as a catalyst for aspects to be considered in teacher education programs with respect to teaching and learning, in our case proportionality. This is an important aspect, given that, as Weiland et al. (2021) point out, there is little research that addresses the knowledge necessary for teachers to be able to teach this subject in a relevant way. Furthermore, it would also be interesting to adapt and apply the instrument presented here to the analysis of other resources, such as educational videos (Beltrán-Pellice et al., 2018; Burgos et al., 2020), which are increasingly in demand.

In this work we have been able to corroborate that prospective teachers are influenced by their beliefs when analyzing texts (Shawer, 2017). Considering that there are few studies that analyze teachers' beliefs about the curriculum, and how these relate to their ideas about mathematics, its teaching and learning, and how they develop in teacher education (Lloyd, 2002), it would be appropriate in future interventions to analyze whether, after training, participants manage to develop rich beliefs about the role of these resources in the teaching of mathematics.

The design we have described here is complemented by action decisions on how the teacher should use the lesson, as well as the proposed resolution of the identified conflicts. However, due to space limitations and given that the interest of the research is to show the progress in reflective competence by confronting the pre and post-training analyses of the lesson, these results will be shown in a future paper.

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