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Mathematical connections made during investigative tasks in statistics and probability

Liezl Joy Lazaro Quilang[1], Lyndon Laborte Lazaro[2]
[1]College of Education, Bachelor of Secondary Education Program, Isabela State University, Cauayan City, Philippines
[2]Mathematics Unit, Curriculum Instruction Division, Philippine Science High School, Bayombong, Philippines

Article Info	ABSTRACT
	The Philippines has undergone a fundamental overhaul of its educational system to highlight basic education and overcome its deficiencies. The educational reform prompts prospective teachers to connect the concepts taught to instill a more profound understanding. As such, the researchers explored the mathematical connections made by prospective secondary mathematics teachers while completing investigative tasks. The study employed a concurrent triangulation mixed-methods design. Data were gathered from 39 prospective secondary mathematics teachers enrolled in the state universities' flagship campuses in Northeastern Philippines. Data were gathered using mathematical connections evaluation, think-aloud strategy, and interviews. Frequency counts, percentages, mean percent scores, Pearson product-moment correlation coefficient, and analysis of the interviews' transcriptions were employed in the study. Results showed that the prospective teachers performed best in making instruction-oriented connections but have difficulty in making implications connection. Also, the prospective teachers' ability to successfully make the mathematical connection is directly linked to their knowledge of the concept, the algorithm, and the part-whole relationship. As such, reform efforts should be made to enhance mathematical connections, emphasizing statistical thinking, and reasoning. Moreover, higher education institutions should incorporate connecting as one of the intended learning outcomes for prospective mathematics teachers. *This is an open access article under the CC BY-SA license.*

Corresponding Author:

Liezl Joy Lazaro Quilang
College of Education, Bachelor of Secondary Education Program, Isabela State University
San Fermin, Cauayan City, Isabela, 3005, Philippines
Email: liezel.l.quilang@isu.edu.ph

1. INTRODUCTION

Emerging trends and developments in the society require educational systems worldwide to equip new-age learners with skills and competencies to make substantial contributions to the economic development of the society where they belong. The initiatives on the globalizing 21st century world originate in the belief that this century will demand a very different set of competencies from people to function well at work [1]. Among these competencies are critical thinking, connection making, problem-solving, and innovation [2]. The Philippines launched the K to 12 program by the Enhanced Education Act of 2013 to catch up with international standards in basic education. Through this reform, statistics and probability has become a key component as early as the elementary grades [3]. As a result, fostering connections among prospective mathematics teachers could help improve the abilities and capacities they need and which are required by the educative reform.

The K to 12 curriculum's mathematics framework illustrates that problem-solving is at the core of the curriculum in which connecting is one of the skills and processes involved [4]. Connecting is one of the essential skills to acquire because when learners connect mathematical concepts, their understanding becomes more profound and more lasting, and they come to see mathematics as a coherent whole [5], [6]. The mathematical connection (MC) is a tool to help the learners "recognize and use connections among mathematical ideas" in solving mathematical problems, rather than approaching mathematics as a disconnected collection and separate concepts and skills [6]. MC is "a cognitive process through which a person relates two or more ideas, concepts, definitions, theorems, procedures, representations, and meanings among themselves" [7]. The reform has advocated that students should make connections between various mathematical concepts and procedures and various kinds of mathematical representations [8].

On an international scale, MC were proven to have a positive direct influence on problem-solving abilities [9]. Students who have good mathematical connection skills succeed in solving mathematical problems well, while poor mathematical connection skills cause students to fail [10]. However, the teachers' representations of MC were either faulty or superficial, which compromised the learners' making meaningful connections [11]. Also, the teachers tend to present lessons in isolated pockets that are not sufficient in building a broader, connected understanding of mathematics [12]. The prospective secondary mathematics teachers must make MC between mathematical concepts and between concepts and representation for the concepts.

Mathematics teachers must be capable of doing the mathematics they will teach and possess a profound conceptual understanding of mathematics. Thorough knowledge of mathematics can be shown through connections between different mathematical ideas, various representations, reasoning [13], linking algorithms, and determining the hierarchical relationship between concepts [14]. However, a lack of understanding of the connections among the relevant functional concepts in mathematics may lead them to ineffectively engage high school students in mathematical connection making, reasoning, and problem-solving [15]. Given that examining someone's understanding of a mathematical concept entails examining the connections that the person makes to that idea [13], it can then be claimed that effective teaching demands the ability to make connections among topics [16]. As such, learners' holistic development requires future mathematics teachers to be knowledgeable in making connections across content areas.

The theoretical framework of the study is based on the structure for thinking about MC [14]. There are five types of connections between mathematical concepts/ideas, namely: Different representations (DR), Implication connection (IC), Part-whole relationship (PWR), Procedural connection (PC), and Instruction-oriented connection (IOC) in his framework. In particular, the DR mean the same concept are represented in two or more ways. It could be alternate representations for different modes of representation or equivalent representations for the same mode. IC inject that one concept leads to another in logical form, while PWR means one concept is linked to another in some sense of part and whole. Moreover, PC pertain to the algorithm associated with a particular concept, and IOC denote those mathematical objects are linked not because of any mathematical association but because they share some pedagogical purpose.

Although some explored the area of MC or interconnectedness within mathematics [14], geometry [17], [18], abstract algebra [19], [20], pre-calculus [21], calculus [22], trigonometry [23], [24], and statistics [25], there are insufficient explorations or studies conducted on the ability of the Filipino learners to understand the connection between and among the concepts learned in statistics and probability and the link between the different types of MC [26]. Given this scenario, the need to explore the prospective secondary Filipino mathematics teachers' ability to construct connections between and among concepts is warranted. This study investigated the MC made by future mathematics teachers while engaged in investigative tasks in statistics and probability and determine the relationship between and among the MC.

2.   RESEARCH METHOD

The concurrent triangulation mixed-methods design was applied in the study. It examined the MC made by prospective teachers while completing investigative tasks in statistics and probability. Purposive sampling was employed to select the participants. They were prospective secondary mathematics teachers enrolled in the state universities' teacher education programs in Northeastern Philippines.

2.1. Data collection

Data were gathered using evidentiary sources such as mathematical connections evaluation, think-aloud strategy, and interviews. Mathematical connections evaluation was designed to measure the participants' mathematical connection making while engaged in a problem-solving activity. Furthermore, it was scored using rubrics in which a maximum score of 2 points for correctly made connection; 1 point for partially made connection; and 0 points otherwise. The instrument was tried out to fourth-year Bachelor of

Secondary Education major in mathematics students enrolled in the five campuses of Isabela State University. It was validated by mathematics specialists from various higher education institutions and secondary schools. For the test of reliability, 60% of the types of mathematical connections, implications, procedural, and instruction-oriented connections, had intraclass correlation values greater than 0.9 which means that these components had excellent reliability while the remaining 40% had good reliability [27].

The participants were oriented on the nature of the study and were given the option to stop their participation anytime without being required to explain in compliance with the research ethics protocol. They were assured that their identities would be kept confidential. Initially, every participant was asked to answer problems one and two independently while the think-aloud strategy for questions three and four were employed. Then, the researchers engaged each participant in a 20 minutes interview after the problem was done. After gathering the data from the participants, the interviews were transcribed and validated by the researchers.

2.2. Data analysis

Frequency counts, percentages, mean percent scores, standard deviations, and Pearson product-moment correlation coefficient were employed in the study. Furthermore, a qualitative analysis was done through the within-case and cross-case analyses in which the MC were described as successfully made, partially made, did not make, and cannot tell or no answer.

3. RESULTS AND DISCUSSION

Connections are the result of understanding and understanding can be manifested through making connections [28]. As such, connections are vital for future teachers and all students in general because they are the foundation of mathematical understanding from the constructivist perspective. Hence, this section presents the connections made by prospective secondary mathematics teachers during investigative tasks.

Table 1 presents the distribution of the DR made by the prospective secondary mathematics teachers. As shown in the table, almost one-fourth of them successfully identified that scatter plot should be utilized in presenting the relationship between two variables which is apparent in the response of Alice (Figure 1). Meanwhile, the rest of them had difficulty in making DR. Erwin mistakenly employed a line graph (Figure 2), while Aaron utilized a bar graph (Figure 3). This implies that they had difficulty in linking mathematical concepts or ideas to their graphical form. The inability to link concepts to its graphical form may pose a great challenge in the learning of students because representations connection may provide a coherent solution to some of the most difficult problems [29]. The fact that most of the DR made were faulty [30] means that there is a need to strengthen their ability in making DR connection.

Table 2 shows the IC made by the prospective secondary mathematics teachers. As presented in the table, the prospective teachers performed best in making IC in data presentation. In particular, one-fifth of them were able to give the correct interpretation of the graphical display. This could be seen in Abraham's case, wherein he inferred no relationship between the two variables based on the graph (Figure 4). Meanwhile, almost 80% of them failed to make an IC. Aida inferred that the graph represented a direct relationship (Figure 5) and Accacia described the graph relative to the highest values (Figure 6). Their difficulty in interpreting the graphical display may be because of their typical error in which they interpreted graphs as a literal picture without understanding the mathematical relationship represented [31].

Table 1. Distribution of the different representations connection made by the prospective secondary mathematics teachers

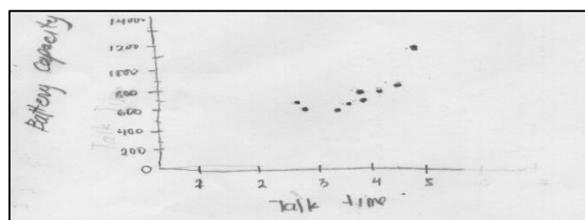| Level	f	%
Successfully made different representations connection	9	23.08
Did not make different representations connection	30	76.92
Cannot tell/no answer	0	0

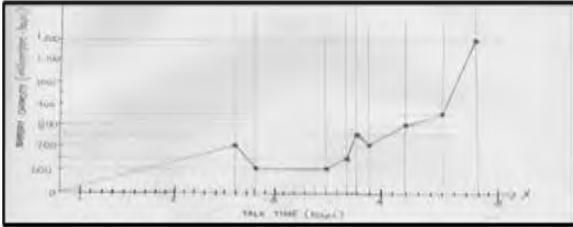Figure 1. Alice's response to data presentation
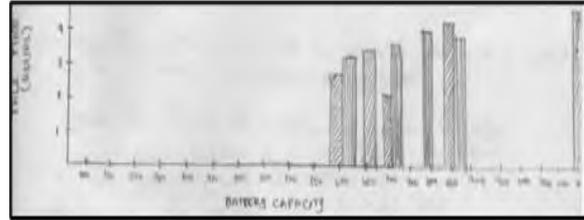
Figure 2. Erwin's response to data presentation

Figure 3. Aaron's response to data presentation

Table 2. Distribution of the implications connection made by the prospective secondary mathematics teachers

Level	Data presentation		Measures of central tendency		Measures of variability	
	f	%	f	%	f	%
Successfully made implications connection	8	20.51	2	5.13	1	2.56
Did not make implications connection	29	74.36	37	94.87	25	64.10
Cannot tell/no answer	2	5.13	0	0	13	33.33

Figure 4. Abraham's response to data presentation

Figure 5. Aida's response to data presentation

Figure 6. Accacia's response to data presentation

Meanwhile, only 3% of them were able to come up with a justifiable reason in selecting the most appropriate measure of variability. Ajay justified that there were no extreme values which might mean that he knew that one of the considerations in choosing the most appropriate measure of variability was the data set's values. However, some participants failed to give a justifiable reason for selecting standard deviation. Grace answered that she used standard deviation to determine the relationship between the two variables.

"No extreme values." (Ajay)
"Para Makita ko pa rin still ma'am kung may relationship ba. Halimbawa 'pag napalitan ba 'yung battery capacity magkakaroon ba siya ng effect dun sa talk time. (To still determine, ma'am, if there is a relationship. For example, if the battery capacity would be changed, would there be an effect on the talk time)" (Grace)

In general, the prospective secondary mathematics teachers had difficulty in making IC. The fact that they could not provide correct inference [32] could be because they could not develop statistical reasoning to justify their answers and make inferences about the data [33], [34]. Hence, IC should be strengthened for more chances to succeed in a problem-solving activity.

Table 3 presents the distribution of IOC made by the prospective secondary mathematics teachers. As shown in the table, most of them were able to make IOC especially in probability. Almost 90% of the participants successfully identified the principle that should be applied in determining the probability of selecting a model with a battery capacity of 600 mAh. This could be observed in the case of Grace who successfully determined that simple probability should be applied (Figure 7). In general, it could be claimed that they manifested strong knowledge in making IOC in probability. This implies that they were well-versed in identifying the concept to be applied to the problem.

Table 3. Distribution of the instruction-oriented connection

Level	Measures of central tendency		Measures of variability		Probability		Fundamental counting techniques and combinatorics		Random variables and probability distributions	
	f	%	f	%	f	%	f	%	f	%
Successfully made instruction-oriented connection	27	69.23	20	51.28	35	89.74	21	53.85	0	0
Did not make instruction-oriented connection	12	30.77	13	33.33	4	10.26	18	46.15	39	100
Cannot tell/no answer	0	0	6	15.39	0	0	0	0	0	0

Figure 7. Grace's response to probability

However, none of the participants made an IOC in random variables and probability distributions. They were asked to compute the expected value if the cellular phone is selected at random based on its battery capacity. In detail, Karlo associated that he encountered "expected value" in the subject probability but was not able to recall what it meant. Mary optimized permutation in computing for the expected value (Figure 8), but during the interview, she associated random variables with combination because of the term "selected at random" and Ian even utilized the mean in computing for the expected value. They could not determine the expected value because they may not be familiar with the concept. As such, it could be claimed that the prospective secondary mathematics teachers manifested instruction-oriented disconnection in random variables and probability distributions.

"I think 'yung expected value nila is sum of the probability tapos iyung total probability? (I think their expected value is sum of the probability and the total probability)" (Karlo)
"Formula lang siya ng combination in simple events. Random kasi siya selected. (It is only the formula of combination in simple events. Because it is selected at random)" (Mary)
"Kukunin ko yung mean ng battery capacity. (I will get the mean battery capacity)" (Ian)

Figure 8. Mary's response to random variables and probability distributions

As a whole, the inability of the prospective secondary teachers to link the problem to the concept may be due to their unfamiliarity with the problem's words. Poor word recognition was one of the challenges in solving worded problems [35]. Given that vocabulary knowledge is a predictor of word-problem solving skills [36], this means that there may be a need to improve the vocabulary knowledge of the prospective teachers for them to be successful in making the instruction-oriented connection.

Table 4 presents the distribution of the PWR made by the prospective secondary mathematics teachers. The table shows that almost 80% of the prospective teachers were able to determine at least one of the tests of relationship but none of them were able to determine more than two tests of relationship. This was apparent in the case of Ulysses in which he only identified Spearman's rho and Pearson r as the statistical tools in determining the relationship between two variables. However, more than 20% of the prospective teachers failed to name the different tests of correlation. This could be seen in the response of Stephanie in which she answered variance and justified that she is not familiar with the different tests of relationship.

Table 4. Distribution of the part-whole relationship connection

Level	f	%
Successfully made part-whole relationship connection	0	0
Partially made part-whole relationship connection	31	79.49
Did not make part-whole relationship connection	8	20.51

"Spearman rho ma'am. Pearson r." (Ulysses)
"Variance. 'Di ako familiar ma'am. (I am not familiar with it, ma'am)" (Stephanie)

As a whole, it could be said that the prospective teachers had slight difficulty in making PWR. Their faulty responses could be because they are not familiar with the different tests of relationship. As such, there is a need to strengthen the ability of the prospective teachers in making PWR for failure in doing so means they could not link one concept to another in some sense of part and whole.

Table 5 shows the distribution of the PC made by the prospective secondary mathematics teachers. As presented in table, almost 90% of the prospective teachers were able to compute correctly the probability of selecting a model with a battery capacity of 600 mAh as could be observed in the response of Grace (Figure 9). This is a manifestation that the prospective teachers were capable of performing the algorithm involved. In general, it could be claimed that the prospective teachers manifested strong knowledge in making PC along probability. This implies that the prospective teachers were well-versed in applying the algorithm to the problem. Their knowledge in making a PC could be because the teachers tend to focus on teaching algorithms even though they value the conceptual approach [14].

However, none of them were able to make PC in random variables and probability distributions. Ian even utilized the mean in computing for the expected value if the cellular phone is selected at random based on its battery capacity (Figure 10) because he did not know the formula to be applied. Therefore, they were unable to determine the expected value because they were not familiar with the concept and they did not know the formula to be applied in the problem as reflected in the interview with Ian. As a whole, it could be claimed that the prospective teachers had poor PC in random variables and probability distributions.

Table 5. Distribution of the procedural connection made by the prospective secondary mathematics teachers

| Level	Data presentations		Measures of central tendency		Measures of variability		Probability		Fundamental counting techniques and combinatorics		Random variables and probability distributions	
	f	%	f	%	f	%	f	%	f	%	f	%
Successfully made procedural connection	4	10.26	13	33.33	6	15.39	35	89.74	3	7.69	0	0
Partially made procedural connection	0	0	8	20.51	4	10.26	0	0	0	0	0	0
Did not make procedural connection	35	89.74	10	25.64	5	12.82	4	10.26	36	92.31	39	100
Cannot tell/no answer	0	0	8	20.51	24	61.54	0	0	0	0	0	0
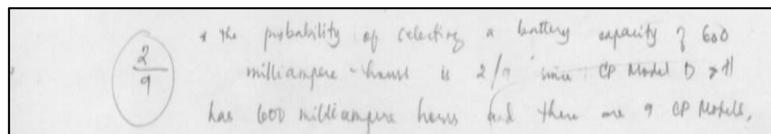
Figure 9. Grace's response to probability

Figure 10. Ian's response to random variables and probability distributions

Okay, could you have chosen another method to answer that problem? (Interviewer)
Yes, ma'am kung alam ko 'yung formula. (Yes, ma'am if I know the formula) (Ian)

Generally, their difficulty in making a PC may be due to their unfamiliarity with words used in the problem or because of their strong pull toward formulas. Unfamiliarity with words used in the problem was one of the challenges encountered by students in solving worded problems [35]. Also, the use of formulas as the basis of their work could make the progress very slow, especially if the formulas were not remembered [37]. Hence, the PC should be strengthened for more chance to succeed in a problem-solving activity.

Table 6 presents the mean percent scores and standard deviations of prospective secondary mathematics teachers' knowledge of MC. As shown in the table, the participants could successfully make connections in less than 50% of the cases (overall MPS=34.42, overall SD=12.09). Their below acceptable level of cognition in MC [38] might be because those with less teaching experience view mathematics as fragmented and purely linear [12].

Table 6. Mean percent scores and standard deviations of prospective secondary mathematics teachers' knowledge on mathematical connections

Mathematical connection	Mean percent scores (MPS)	Standard deviation (SD)	Qualitative description
Different representations (DR)	23.08	42.68	Weak
Implication connection (IC)	9.19	13.76	Weak
Instruction-oriented connection (IOC)	56.15	24.24	Strong
Part-whole relationship (PWR)	38.46	20.56	Good
Procedural connection (PC)	30.13	16.01	Good
Overall	34.42	12.09	Good

Legend: 75.1–100.0%=very strong (VS); 50.1-75.0%=strong (S); 25.1-50.0%=good (G); 0.0-25.0%=weak (W)

In detail, the participants were strong in making IOC (MPS=56.15, SD=24.24). Their high cognition in making IOC implies that they were knowledgeable in identifying the concept to be applied to a given problem. This could be because students could associate situations with particular concepts learned in school [39]. The participant's ability to connect problem information with appropriate concepts is integral in developing their mathematical understanding [25] because the correct association of the topics to be learned increases their ability to identify correctly and carry out the appropriate analysis for a given problem [40].

On the other hand, they had the weakest understanding of the IC because they found it challenging to justify. Their difficulty in justifying their answers may be because teachers were least likely to provide explanations based on theories, structures, or mechanisms [41]. Difficulty in providing a clear explanation implies the need to strengthen their reasoning ability because understanding the insides of ideas and their reasons are among the expected competencies of 21st century teachers [16].

The prospective secondary mathematics teachers' strong IOC but weak IC implies that their ability to identify the most appropriate statistical tool is not enough for them to succeed in a problem-solving activity, for they should provide rationalization of their solution for deeper understanding. This could be a sign of rote learning based education, which failed to see that conceptual learning is more important in mathematics education [42]. As such, mathematics education should not focus on computational understanding alone but on the development of learners' conceptual knowledge.

As a whole, they could make different types of connections [2]. However, their ability to make MC should be strengthened, especially because they manifested some misconceptions in connecting the concepts, which may lead to the inability to solve problems [43]. The faulty connections might explain why most learners probably lost opportunities to develop a deep understanding of MC [11].

Table 7 shows the Pearson product-moment correlation coefficient on significant relationship between and among the MC made by the prospective teachers. As shown in the table, their ability to make MC is associated to their knowledge in PC (r-value=0.882, p-value<0.0005), IOC (r-value=0.838, p-value<0.0005), and PWR (r-value=0.472, p-value=0.002). The positive r-value indicate that the prospective teachers' ability to make connections is directly associated with their knowledge of the algorithm involved, mathematical principles, and hierarchical relationship.

The direct relationship between MC and PC means that those with deeper mathematical understanding were more likely those who apply the formulas correctly. However, this may pose a challenge because of the tendency to make progress very slow, especially when the formulas were not remembered [37], and to revert to memorized algorithms without correctly identifying the data [44]. Also, improving MC ability entails strong contextual understanding [45]. Henceforth, the ability to relate the new topic to prior experience and linking groups of mathematics concepts and procedures as prerequisites [14] may be useful in developing a more holistic understanding of mathematics. Lastly, MC could be improved by fostering the ability to link concepts to one another through a hierarchical relationship like examples, inclusions, and generalizations [14].

Table 7. Pearson product-moment correlation coefficient on significant relationship

Mathematical collections		Implication connection	Instruction-oriented connection	Part-whole relationship	Procedural connection	Mathematical connections
Different representations (DR)	r-value	-0.137	-0.167	0.051	-0.170	-0.038
	p-value	0.405	0.311	0.759	0.300	0.818
Implication connection (IC)	r-value		-0.141	0.156	0.134	0.276
	p-value		0.391	0.343	0.416	0.089
Instruction-oriented connection (IOC)	r-value			0.314	0.681**	0.838**
	p-value			0.051	0.000	0.000
Part-whole relationship (PWR)	r-value				0.309	0.472**
	p-value				0.055	0.002
Procedural connection (PC)	r-value					0.882**
	p-value					0.000

**significant at 0.01 level

In a nutshell, it was not enough that prospective secondary mathematics teachers could make the different types of connections because what is essential is their accuracy in connecting formulae, concepts, and hierarchical relationships in statistics and probability that would facilitate better ability in making MC for them to be fully prepared in joining the teaching profession. This is because 21st century teachers are expected to be knowledgeable of unifying principles in mathematics and its connectedness [46].

Furthermore, there is also a positively high correlation between IOC and PC (r-value=0.681, p-value<0.005). The direct link means that those who successfully identified the concept to be applied in the problem were also those who applied the correct algorithm in solving the problems. Alvin identified that linear permutation should be used to determine the number of ways cellular phones could be displayed on the shelf if the phones with the same battery capacity were adjacent (Figure 11). Also, he successfully answered that there were 5,760 ways to display cellular phones on the shelf, given that the phones with the same battery capacity were adjacent. He even expounded during the think-aloud that he arrived with the final answer by initially counting the number of models with the same battery capacity.

Figure 11. Alvin's response to mathematical connections evaluation

"So, first I'll see first the CP models with or I will count the CP models with unique battery capacity. So, there are three, then I'll count the CP models with the same battery capacity. So, if there are two CP models with the same battery capacity serves as one, they'll be counted as one." (Alvin)

The strong relationship between IOC and PC [22] plays a valuable role in MC because improving conceptual knowledge may lead to better procedural connection and vice versa [21]. Henceforth, higher education institutions should focus on building the learners' procedural and conceptual understanding because procedural knowledge coupled with conceptual understanding promotes rich discussion that leads instrumental understanding to the big idea of the lesson and makes the learning goal visible [47].

4. CONCLUSION

The ability to make a mathematical connection is a powerful tool in the improvement of mathematical comprehension among prospective secondary mathematics teachers. They successfully made connections to less than 50% of the cases. This means that tertiary level institutions should consider the ability to make MC as one of the essential skills for future teachers' holistic development. Particularly, they have the highest cognition in making an instruction-oriented connection. Meanwhile, they had the weakest understanding of the implications connection because they found it challenging to give justifications and interpretations of their results. These findings imply that while they are skilled in accurately identifying the

concept to be applied to a given problem and they have the potential to inculcate a richer and deeper understanding of the problems among their learners, reform efforts should be made to enhance MC, emphasizing statistical thinking, and reasoning.

In a broader perspective, the prospective secondary mathematics teachers are capable of making different types of connections – a strong tool that they could use in the field because as the learning progresses in the Philippine K to 12 Enhanced Basic Education Curriculum, details are gradually increased, and its connections with concepts in other learning areas are established, and the basic and preceding concepts are emphasized many times. This ability is directly linked to their knowledge of the concept, the algorithm, and the part-whole relationship. As such, their ability to make MC should be enhanced by providing them activities that would strengthen their capability to link mathematical procedure, method, or algorithm, connect the new topic to students' prior knowledge, groups of concepts, procedures as prerequisite concepts, skills, or vocabulary, and particular problem to the concept, and relate concepts in some sense of part and whole. Also, mathematics educators should incorporate connecting as one of the intended learning outcomes for prospective mathematics teachers.

Continued research on the MC should be done to produce more sound results for the development of secondary teachers' ability to connect mathematical concepts, procedures, representations, and meanings. In particular, designing a learning model to improve the MC of the prospective teachers may be done. Also, follow-up studies may be done with the prospective teachers who manifested strong MC on the relationship between their knowledge in mathematical connection and mathematics knowledge for teaching.

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BIOGRAPHIES OF AUTHORS

Liezl Joy Lazaro Quilang ⓘ 🅖 ⓢⓒ Ⓟ is an Associate Professor III at Isabela State University Cauayan Campus, Cauayan City, Isabela. She completed her Doctor of Philosophy in Science Education, Major in Mathematics at Saint Mary's University, Nueva Vizcaya, Philippines and earned both her bachelor's and master's degrees in Mathematics at Philippine Normal University North Luzon, Isabela, Philippines. She can be contacted at email: liezel.l.quilang@isu.edu.ph.

Lyndon Laborte Lazaro ⓘ 🅖 ⓢⓒ Ⓟ is a Special Science Teacher IV at Philippine Science High School Cagayan Valley Campus, Nueva Vizcaya, Philippines. He finished Master of Science in Teaching Mathematics at Saint Mary's University, Nueva Vizcaya, Philippines and Master of Arts in Curriculum Design, Development and Supervision, Major in Mathematics at Saint Paul University Manila, Philippines. He can be contacted at email: lllazaro@cvc.pshs.edu.ph.