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The Heterogeneity of Mathematical Learning Disabilities: Consequences for Research and Practice

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Abstract

This paper argues why children with Mathematical Learning Disabilities (MLD) do not form a unitary group. Instead, they should be regarded as individuals with unique profiles of strengths and weaknesses that explain their mathematical difficulties. To build this argument, we shortly recapitulate the research on MLD, which has mainly been focused on characterizing the group of children with MLD– as compared to control groups. However, these general characteristics are not applicable to all children with MLD. Furthermore, attempts to define separate, relevant subgroups merely failed. Based on some recent studies, we show how individual profiles of strengths and weaknesses might help in understanding the specific mathematical difficulties of a child. We propose a new multidimensional framework of MLD, in which both strengths and weaknesses are recognized. We argue that both research and practice are in need of further research that takes individual differences into account.

Keywords:

Mathematical Learning Disabilities, Dyscalculia, Strengths-and-weaknesses, Multidimensional Approach

Introduction

Mathematical Learning Disabilities (MLD) refer to specific, severe and persistent difficulties that children can encounter in learning mathematics. In general, children with MLD have both difficulties with learning and remembering arithmetic facts and difficulties in executing calculation procedures (Landerl et al., 2004). Whereas MLD is a common term, different terminologies have also been used for similar concepts, such as mathematical learning difficulties (e.g., De Smedt, & Gilmore, 2011; Mazzocco, 2007), mathematical learning disabilities (e.g., De Smedt et al., 2012; Desoete, 2007; Geary, 2011; Mazzocco, 2007; Szűcs, 2016), mathematical learning disorders (e.g., Desoete & De Weerdt, 2013) and dyscalculia (e.g., Butterworth et al., 2011; Van Luit, 2019) or developmental dyscalculia (e.g., Butterworth, 2008; Dehaene et al., 1993; Shalev, 2004; Van Luit, & Toll, 2018). It should be noted that these terminologies might seem interchangeable, and are sometimes used as such. However, in the literature, some (variable) distinctions

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are often made to differentiate between the various terms. For example, different nuances to the term MLD usually indicate gradations in the severity of the math learning problems (e.g., a difficulty is less severe than a disability), whereas developmental dyscalculia might indicate more specific impairments such as a core deficit in number sense (Butterworth et al., 2011; Dehaene et al., 1993; Shalev, 2004).

Next to differences in terminology, there are different definitions and operationalisations of MLD in the literature. A closer look on empirical studies shows that most often MLD samples are only based on the seriousness of math difficulties (e.g., performance below a certain cut-off score), although sometimes accompanied by the criterion of persistence (e.g., the problems exist at least for a certain period; Kroesbergen et al., 2021). Comorbid learning, behavioral or developmental problems are excluded in some studies (i.e., specificity criterion), but are the subject of study in others. The fact that different cut-off criteria are used (ranging from percentile 2 to 40; Kroesbergen et al., 2021), makes the comparison of different studies even more difficult, and the implications for practice at least vague. Nevertheless, despite these limitations, some conclusions can be drawn from former research about MLD.

A growing body of research has tried to find explanations for the phenomenon of MLD, based on the assumption that underlying cognitive deficits cause the specific mathematical problems. In the next section, we will give an overview of this line of research, mainly based on review studies and meta-analyses. The existing literature on underlying deficits has mainly focused on characterizing children (or adults) with MLD, by comparing groups of children with and without MLD. Other studies have used more descriptive methods. An interesting idea in this line of research is that there is not one type of MLD, but that several subtypes exist (e.g., Geary, 2004; Moeller et al., 2012), which might have different origins or manifestations. Furthermore, different lines of MLD-related research have turned their focus towards the brain, in order to investigate the neural underpinnings of MLD, using neuroscientific methods. Despite the promising advances in this field, no clear brain structures or networks have yet been identified in children with MLD. For this reason, we will not include the neural models in our current paper.

After reviewing these different lines of research, we propose an alternative view on MLD, which takes not only into account the possible deficits, but also the (compensating) strengths that could potentially be related to children's math performance. Importantly, this approach assumes that there is no inherent distinction between children with and without MLD. Rather, we propose that children's mathematical performance should be regarded as a continuous scale ranging from very poor to very strong math performance. Hence, a clear distinction between children with and without MLD is at least difficult if not impossible to make. Following this alternative model, the present paper concludes with implications for both research and practice.

Characterizing MLD: Results from Group Comparison Studies

A large body of research has shown that mathematical performance is related to a number of domain-general and domain-specific cognitive skills. The most salient cognitive skills involved in math learning are number sense, working memory, attention, processing speed, and phonological processing (e.g., Geary, 2004; Mammarella et al., 2021; Peng et al., 2018). Not surprisingly, these are also the skills that are suggested to play a role in MLD. Many studies on the characteristics of MLD have compared MLD groups with control groups, to find in what way children with MLD differ from children without MLD. Based on these studies, some conclusions can be drawn about the cognitive characteristics of MLD in general, in which a distinction can be made between domain-specific and domain-general cognitive skills.

Number sense - the only domain-specific skill related to mathematics and MLD - can be defined as the ability to recognize and understand non-symbolic numerosity (quantities) and symbolic numbers (number words and Arabic digits), and mapping between these numerical representations (Dehaene et al., 2003; Geary, 2011). More comprehensive definitions of number sense also include skills like counting, nonverbal calculations and number patterns (e.g., Jordan et al., 2007). Berch (2005) proposed to make a distinction between lower order and higher order number sense. Lower order number sense refers to the intuitive perceptions of quantity, as described above. Berch considers higher order number sense much more complex and multifaceted, comprising a deep understanding of mathematical principles and relationships and a high degree of fluency and flexibility with operations and procedures. Number sense is thought to play a crucial role in MLD (e.g., Jordan et al., 2007). More specifically, a core deficit in numerosity processing has been proposed to underlie MLD, or at least a specific form of MLD, namely pure or developmental dyscalculia (Butterworth, 1999; Mazzocco, et al., 2011). It should be noted however, that the core-deficit view has been criticized by scholars, because it might not be a deficit in processing numerosity itself, but in accessing numerical meaning from symbolic digits (De Smedt & Gilmore, 2011; Rousselle & Noel, 2007). In a meta-analysis on the differences between MLD and typically developing (TD) children in number sense, effect sizes for symbolic skills were indeed significantly larger than for non-symbolic skills (Schwenk et al., 2017). These results were replicated in a recent meta-analysis (Kroesbergen et al., 2021). The latter study, however, also found that effect sizes for higher-order number sense skills were even higher. It is interesting to note, that according to the meta-analyses by Kroesbergen et al. (2021) and Schneider et al. (2017), the differences in number sense between people with and without MLD, seem to decline with age.

Studies that compared children with MLD to typically developing control groups, have pointed to several domain-general cognitive skills that are weaker in children with MLD. An extensive meta-analysis on 75 cognitive profiling studies on MLD showed that these differences are especially apparent in working memory and processing speed, but also in phonological skills and attention (Peng et al., 2018). Other domain general skills that have been found to be weaker in groups of children with MLD are spatial skills (e.g. Peters et al., 2020; Träff et al., 2020), visual perception (e.g. Cheng et al., 2018), ordering/ order processing (e.g. Morsanyi et al., 2018; Sasanguie et al., 2017), inhibition (e.g. Szűcs et al., 2013), number-specific executive function (Wilkey et al., 2020), and logical/ non-verbal reasoning (Huijsmans et al., 2022; Träff et al., 2020). The main weaknesses found are shortly elaborated below.

Working memory involves the temporal storage, processing, and recollection (i.e., the executive function of updating) of verbal and visuospatial information (Alloway et al., 2009; Passolunghi, & Siegel, 2004). A vast amount of research has identified working memory as a domain general cognitive factor in learning mathematics. Strong working memory skills facilitate stepwise solving multiple-component math problems. Poor working memory skills on the other hand, have been associated with MLD (e.g. David, 2012; Klesczeweski et al., 2018). An interesting finding is that especially the processing of numerical information in working memory (as compared to non-numerical verbal information) is often impaired in children with MLD (e.g. David, 2012; Peng & Fuchs, 2016; Peng et al., 2012; Raghubar et al. 2010; Wilkey et al., 2020), which also points to a domain-specific deficit. However, visual-spatial working memory seems to be more affected than verbal working memory in children with MLD (David, 2012), and especially spatial working memory (e.g., Mammarella et al., 2018, Szűcs et al., 2013).

Attention refers to the allocation of cognitive resources to relevant stimuli (Posner & Petersen, 1990) and has often been found to be impaired in children with MLD (e.g. Peng et al., 2018). Attention might especially be necessary to support the executive process when doing (complex) calculations, especially when arithmetic facts are not (yet) automatized (Peng et al., 2018). However, attention is also required in learning and automatizing arithmetic facts, which requires an active engagement.

Processing speed can be defined as the speed at which a person is able to encode, transform, and retrieve information (Conway et al., 2002). Processing speed is found to play a role in the development of mathematics as it facilitates the temporary storage on answers of simple sums and counting words in working memory (Geary, 1993). In their meta-analysis, Peng et al. (2018) showed that shortcomings in processing speed and short-term memory were related to problems in higher-level cognitive skills such as working memory and attention, supporting the idea that processing speed plays a central role in explaining mathematical deficits in all children with MLD.

Phonological processing has emerged as a domain general factor in mathematics as well. Phonological awareness (Vellutino et al., 2004) and rapid naming (Donker et al., 2016; Willburger et al., 2008) have been identified as relevant components of phonological processing. Quick access to verbal codes stored in long-term memory (i.e., rapid automatized naming) that correspond to number facts, and effective recognition and manipulation of those verbal codes (i.e., phonological awareness) are required for arithmetic fact retrieval (Simmons, & Singleton, 2008). Although weaker phonological skills have been found in children with MLD (e.g., Peng et al., 2018), deficits in phonological skills are most often found in children with both mathematical and reading difficulties and could thus possibly explain the frequent comorbidity between mathematics and reading problems (Peng et al., 2018; Slot et al., 2016). It has been suggested that affected children may have difficulties with fact retrieval, which interferes with their mathematical abilities (Landerl et al., 2009).

Although in general some characteristics of MLD can be determined, the heterogeneity within the group of MLD is large. To demonstrate this heterogeneity, we will describe some recent case studies, which have demonstrated that very different (specific) factors may play a role in different people with mathematical learning problems. This supports the idea that not all people with MLD experience the problems described above.

Characterizing MLD: Results from Case Studies

Several case studies on MLD have been described in the literature. They all have a specific focus, and inherently different descriptions of MLD. All of them measured at least some form of number sense as potential underlying deficit. In addition, domain general skills are described as possible explaining variables. First, we will shortly summarize these case studies.

De Visscher and Noël (2013) describe a case study of an adult woman with a specific form of dyscalculia, in combination with high general cognitive capacities. She has specific arithmetic fact retrieval deficit, as shown in very long reaction times (but accurate performance), most visible in multiplication facts. The authors used this case to investigate the specific hypothesis of hypersensitivity-to-interference in memory. Being highly sensitive to interference means experiencing difficulties in retrieving the exact context of similar items which have been processed recently. In the case of arithmetic facts, the context is the problem, which has to be associated with the answer, all consisting of numbers. The results of this woman were compared to a reference group of 11 women matched on educational level and age. Remarkably, this woman performed above average on other cognitive skills related to MLD: attention, executive functions, phonological processing, and verbal and visual working memory. Number sense, as measured with a dot estimation task and a comparison task, was also not impaired. However, she did show a high sensitivity to interference, as measured with learning-associations tasks. Thus, in this specific case, high sensitivity to interference caused the specific mathematical difficulty.

Two other case studies focused on specific forms of number sense, namely subitizing and number lines (Moeller et al., 2009; Van Viersen et al., 2013). Both studies investigated in depth the performance of children with MLD on the respective tasks, applying eye-tracking. Moeller and colleagues (2009) report on two 10-year-old boys with dyscalculia, without problems in reading, general cognitive abilities, or attention. They investigated the subitizing skills (range 1-8) of these boys, and compared these to a reference group of 8 age-matched typically developing children. The boys were impaired in subitizing (range 1-3) as well as enumeration in the counting range (4-8). By applying eye-tracking, the researchers were able to show that even with the smallest numbers, the boys often used counting strategies instead of subitizing. They conclude that the problem lays in quick automatic and parallel encoding of non-symbolic quantities.

Van Viersen and colleagues (2013) applied eyetracking to investigate the strategies of a 9-year-old girl with MLD on a symbolic and a nonsymbolic numberline task (compared to a reference group of 10 typically developing children). In addition to poorer performance on the numberline tasks (0-100 and 0-1000), the child also performed lower on visual-spatial working memory, but showed average performance on verbal working memory. The analyses of eyetracking showed that she used less clear strategies and that her strategies were often less efficient and atypical as compared to those of the reference group.

Other case studies have focused on both domain-specific and domain-general cognitive factors related to MLD (Davidse et al., 2014; Träff et al., 2017). Davidse et al. (2014) report on two 9-year-old monozygotic twin girls, who had severe mathematical learning disabilities, but scored above average on word reading tests. They were not able to learn even basic mathematical skills. Their performance on a series of number sense skills was investigated and compared to a reference group of 8 age-matched girls. These girls scored significantly lower on all number sense tasks (numberline 0-10, magnitude comparison 1-16, and subitizing 1-4). They even scored at chance level on the comparison and subitizing tasks. In addition to their number sense deficits, these girls also showed poor working memory performance and poor visual-spatial skills, as well as poor spelling performance.

Träff and colleagues (2017) administered a comprehensive cognitive test battery to four children (two boys, two girls) with MLD aged 8-9 years old. Perhaps the most interesting finding was the heterogeneity in the profiles of the four children. Two of them showed number sense deficits and domain-general deficits (especially visual-spatial working memory). One only showed problems with the symbolic number sense tasks, but not with the non-symbolic, in combination with a general cognitive deficit (visuospatial working memory and executive functions). And one of them had only general cognitive deficits (verbal working memory, executive functions). The authors concluded that MLD cannot be attributed to a single explanatory factor, but that a multiple deficit account should be applied.

Although it is difficult to compare these case studies, due to the differences between participants and methodologies (measures, constructs), the conclusion seems justified that these studies do not converge to one conclusion. Although in most of the described cases number sense was impaired, this was not found in all cases (De Visscher & Noël, 2013; one out of four cases in Träff et al., 2017). Furthermore, in most cases some domain general deficits were found, but again not in all. According to Träff et al. (2017), this can be explained by recognizing different subtypes within MLD. When considering these different case studies, it indeed seems obvious that large differences exist between different people affected by MLD, making one general description almost impossible. Distinguishing between subtypes could be an interesting alternative to describe the characteristics of MLD. The next section will discuss research that has focused on these possible subtypes of MLD.

The Search for MLD Subtypes

To explain the large heterogeneity within the group of children with MLD, some have argued that several subtypes of MLD exist. Probably the most common distinction is the tripartite that Geary (2004) conceptualized: 1) a procedural subtype: Difficulties with strategies and concepts involved in advanced mathematics, 2) a semantic subtype: Reduced accuracy and speed for arithmetic fact retrieval, and 3) a visuospatial subtype: Difficulties in visuospatial skills. These three subtypes are assumed to be different not only in their mathematical problems, but also in their underlying cognitive characteristics and developmental patterns. Desoete (2007) proposed a fourth subtype, in which children's numerical cognition is impaired. Karagiannakis et al. (2014) continued on these profiles and distinguish between deficits in (1) core number, (2) memory, (3) reasoning, and (4) visual-spatial. They also link these deficits to specific cognitive characteristics and mathematical outcomes. Other profiles have been suggested as well, for example by differentiating between various numerical representations (Moeller et al., 2012). Others have focused on the comorbidity with for example reading or motor disabilities to distinguish between profiles (e.g., Pieters et al., 2015; Szűcs, 2016).

However, the various subtypes described above are theoretical in nature, and only limited empirical evidence exists. Pieters et al. (2015) have identified two subgroups based on data-driven model-based clustering: They found evidence for the procedural and for the semantic subtype. Bartelet et al. (2014) distinguished six profiles based on numerical abilities, although it is remarkable that the mathematical performance of these profiles barely differed. Salvador et al. (2019) used cluster analysis on a mixed group of MLD and typically developing students and found two profiles with weak arithmetic skills: one with number sense problems and one with visual-spatial problems, but again the two subtypes performed similarly on arithmetic achievement, although the within-group variance was large. Szűcs (2016) focused on working memory. Based on a meta-analysis of 36 studies, he found one subtype with weak verbal working memory, this subtype is also related to reading problems, and another subtype with weak visuospatial working memory (without reading problems).

To conclude, some subgroups might indeed exist within the group of MLD, although the results vary over studies and further research is needed to find more converging evidence. However, the results also show that it is difficult to find distinct cognitive profiles that are related to specific mathematical abilities, and that even within subgroups there is still much variability.

A Critical Reflection on former MLD Research

Sofar, we reviewed the evidence from different types of research on MLD. Below we will elaborate on the conclusions and provide our explanations for the main findings. In addition, we will critically reflect on the methods used in former research and how this might have affected the results of these studies.

First of all, MLD is a heterogeneous concept. Terms such as mathematical learning disabilities, mathematical learning difficulties and dyscalculia are used, sometimes with distinguished meanings, but without consensus about the specific definitions. In general, all of these terms point to serious problems in mathematical abilities, mostly to specific math problems and often to intervention-resistant problems. However, these three criteria (seriousness, specificity and resistance) are not always applied in the same way. Different cut-off criteria are used, both in practice and in research. The specificity criterion (i.e. children should not have additional disabilities and at least average performance on other academic skills) is often used in research. However, because of the high comorbidity rates (e.g., around 20-25% for dyslexia, spelling problems and attention disorders; Capano et al., 2008; Moll et al., 2014), in practice this criterion is less usable. Applying such criteria in research selects relative homogeneous samples that are not representative of the population, with - consequently - possibly faulty conclusions about characteristics of MLD. In contrast, the resistance criterion is very important in educational and clinical practices, but often not applied in empirical research. However, it should also be noted that in a recent meta-analysis it was shown that these criteria do not seem to make a major difference for the conclusions about MLD characteristics (Kroesbergen et al., 2021). Nevertheless, the heterogeneity in definitions and selection criteria makes drawing conclusions based on empirical studies quite difficult.

Secondly, conclusions about cognitive characteristics of MLD are mostly based on group comparisons. It has indeed been found that groups of children with MLD score on average lower on a number of skills, compared to groups of typically developing peers. These skills include - but are not limited to - number sense, working memory, processing speed, phonological skills, attention, spatial skills, ordering/ order processing, inhibition, number-specific executive function, and logical/non-verbal reasoning. It should be noted that most studies have compared groups of children with MLD, selected on strict criteria, with control groups (Astle & Fletcher-Watson, 2020). Differences between groups are then taken as evidence for a specific cognitive profile in the MLD group. These groups are often based on an (arbitrary) cutoff point along the normal distribution,

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while the children performing below this cut-off point are not necessarily qualitatively different from those who scored above that criterion (Peters & Ansari, 2019). Mammarella and colleagues (2021) tested the hypothesis that children with MLD are at the end of a developmental continuum, visible in impairments in many cognitive skills rather than having a core deficit in basic number processing skills. Data from a large sample were compared to simulated data to investigate the diagnostic power of possible underlying factors. They indeed found that none of the measured factors exceeded the diagnostic power that could be derived via simulation from the dimensional characteristics of a population. Applying a dimensional approach to learning disabilities might therefore be more valid (Peters & Ansari, 2019; Szűcs, 2016; Zhao & Castellanos, 2016). The assumption in this approach is that there is no qualitative discontinuity in the distribution from low to high performers.

Another problem with the method of group comparisons is that only means between groups are compared, and it can be questioned whether all children within these groups can be characterized by such a cognitive profile. As far as reported, this often seems not to be the case. For example, Kroesbergen and Van Dijk (2015) showed that a group of children with MLD significantly differed from their same-age peers in terms of working memory as well as number sense. However, when considering the specific individuals within the MLD group, only 38% of these MLD children indeed scored low on both working memory and number sense (and 23% showed neither working memory nor number sense problems). So, in this case, the group description could only be used to correctly describe about one-third of the individuals within that group. It might therefore be hazardous to use group comparisons to draw conclusions about characteristics of MLD problems in individuals. Qualitative analyses of individuals with MLD might provide a more nuanced understanding of their disability (e.g., Lewis et al., 2020), although the generalizability of case studies is small, and the review of case studies described here only stresses the variability between individuals with MLD.

This relates to a third conclusion: The heterogeneity within the group of individuals with MLD is enormous. As described above, group means are not applicable to all individuals, and conclusions drawn from group comparisons cannot always explain individual characteristics. The large variability between individuals within groups is often not studied, in contrary, heterogeneity is more often approached as 'noise' that should be controlled for. Even when the within-group differences are studied, the same approach usually leads to grouping individuals, i.e. research into MLD subtypes. However, for these subgroups, the same criticisms hold as for more inclusive groups. Even when a data-driven approach was used to distinguish between subgroups, the variability within subgroups is large (e.g. Huijsmans et al., 2020; Szűcs, 2016). It would do more justice to reality to take this variability into account and to use heterogeneity as evidence that a simple, uniform explanation of MLD is not possible and should be replaced by other theoretical models that take the variability in both cognitive and mathematical skills into account. According to Pennington (2006), development occurs through an interconnected network of (cognitive) skills. The development of mathematical learning difficulties could therefore depend on a different profile of cognitive deficits for each child. In addition, next to cognitive deficits, cognitive strengths might function as compensatory mechanisms and should be considered as well. Studying unique profiles of cognitive weaknesses and relative strengths might enlarge our understanding of MLD (cf. Huijsmans et al., 2021; Koriakin et al., 2016; Lewis & Lynn, 2018a).

The fourth, and probably most important, conclusion is that the causes of MLD are still not (fully) understood. The empirical evidence does not point to a single or fixed combination of factors that are apparent in all children with MLD. Although some cognitive characteristics have been described that might play a role, such deficits do not always lead to lower math performance. For example, not even half of the children with specific cognitive deficits (e.g., deficits in number sense, working memory, or rapid naming) have mathematical learning difficulties (Huijsmans et al., 2021; Kroesbergen & Van Dijk, 2015). These findings seriously challenge the assumed causal relations between the cognitive deficits and MLD. Furthermore, although group comparison studies generally assume such causal relations, the direction of these relations if often not examined. Peng and Kievit (2020) show, based on a review of both longitudinal and intervention studies, that the relation between cognitive abilities and academic achievement could best be described by bidirectional relations. This has indeed also been found for the relations between number sense and mathematics (e.g., Elliot et al., 2019; Friso-van den Bos et al., 2015). The assumption of a core deficit thus seems outdated, although the simplicity of this model might have had a strong appeal to both researchers and practitioners (see also Astle & Fletcher-Watson, 2020).

In our opinion, the heterogeneity in definitions and selection criteria as described above, as well as the variability in individuals with MLD, clearly point to the underlying problem that no evidence exists that MLD is a disability that is qualitatively different from (extremely) low performance, because no specific causes leading to specific symptoms in MLD have been found. Consequently, it depends on the used definition which children are labeled with MLD or not, with arbitrary and undesirable dichotomization of children with less or more severe mathematical learning difficulties as a result. This way of categorizing children is undesirable, because such labels could have important consequences for the education and interventions children receive.

Variation in Mathematics Revisited: A Multidimensional Model of MLD

Following our conclusions, dominant frameworks that help us to understand individual differences in mathematics learning may be in need of revision. According to the Multiple Deficit Model (MDM; Pennington, 2006; see also left panel of Figure 1) neurodevelopmental disorders, such as learning disorders, can be better understood, by studying their underlying etiology (genes, environment, and gene x environment interactions), brain mechanisms, neuropsychology, and behavioral symptoms. Cognitive multiple deficit models have been most successful for reading and mathematical disorders (McGrath et al., 2019), but have been criticized as well because they do not account sufficiently for heterogeneity in learning disabilities. Indeed, research from Huijsmans and colleagues (2020) suggested that children with MLD should be regarded as individuals with unique profiles of strengths and weaknesses that explain the way they learn mathematics in a similar fashion as their typical developing peers. In a cross-sectional study, they investigated to what extent specific profile(s) of mathematics difficulties and associated cognitive skills could be identified in a sample of 281 fourth grade children, using latent profile analysis (LPA). The results showed that children with MLD could not be separated from (low-achieving) typically developing children based on their profile of mathematics performance alone: 34% of the whole sample was grouped together into one profile consisting of weak performance on arithmetic and mathematics. Additionally, contrasting the cognitive skills of children with MLD to those of typically developing children did not result in separate profiles either. They stress that although their data-driven approach yielded different subgroups, the heterogeneity within the identified subgroups was still large. We propose that various sets of cognitive strengths and weaknesses are related to a wide variety in mathematical profiles, as visualized in the right panel of Figure 1.

Another point of criticism regarding the use of the MDM-framework in its current form, is that it does not fully recognize specific cognitive strengths that may compensate for cognitive weaknesses (McGrath et al., 2019). As a result, such cognitive strengths are often overlooked in research nowadays. To illustrate, children with MLD but without reading problems (but with deficits in number sense skills) were able to (partly) compensate their lower scores on fact retrieval and mathematics when they had high rapid naming skills (Huijsmans et al., 2021). Although it should be acknowledged that groups were relatively small, these results point in the direction that the consequences of cognitive risk factors for mathematical difficulties might be reduced through compensatory protective factors. It thus seems unfeasible to think about mathematics performance as a singular cause-effect relation wherein one (or few) core deficit causes math difficulties. In addition, the unidirectional relations from cognitive to behavioral characteristics also is a simplistic representation of the complex interaction of factors involved (Peng & Kievit, 2020).

We therefore propose a new multidimensional approach to MLD, in which both strengths and weaknesses on an individual level are recognized (see Figure 1). The rationale for this model is that children with MLD do not show different patterns of (cognitive predictors of) math development compared with typically developing children (Peters & Ansari, 2019; Szűcs, 2016; Zhao & Castellanos, 2016). Therefore, neither mathematical profiles nor cognitive profiles appear to be suitable to divide children with weaker math performance into separate groups, but mathematical abilities should be regarded as a dimensional construct. Furthermore, not all children within a group show the same behavioral or cognitive difficulties (Huijsmans et al., 2020). Such a complex interaction between multiple cognitive and behavioral factors requires a multidimensional approach. Additionally, learning difficulties do not result from cognitive weaknesses alone, but co-exist with strengths in other cognitive skills. That is why a multidimensional model of MLD might be a better representation of reality than a multiple deficit model. This model recognizes both cognitive weaknesses and strengths as relevant cognitive processes in the interaction with neural and behavioral characteristics. In such a model, certain combinations of cognitive weaknesses can result in the development of a specific learning difficulty, but on the other hand, specific strengths may partly compensate for such difficulties as well. As a result, the child's mathematical profile (i.e., behavioral phenotype) may also have specific strengths and difficulties. As a case in point, Huijsmans and colleagues (2020) showed that some children with mathematical learning difficulties had problems in fact retrieval, but not in advanced math (i.e., geometry, fractions), whereas for other children it was the other way around. Furthermore, a child that has both reading and mathematical difficulties, may have less cognitive strengths to compensate their mathematical abilities because of overlapping cognitive factors that account for the development in both reading and mathematics (i.e., phonological processing skills). Our model allows for such unique combinations of strengths and/or weaknesses on a cognitive level that relate to unique strengths and/or weaknesses of the child's mathematical profile.

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Of course, combinations of other cognitive strengths or weaknesses can be considered as well. For example, as both logical reasoning and working memory skills are needed to solve multi-component math problems (e.g., Kleemans & Segers, 2020) a child with relatively high logical reasoning skills may partly compensate for their relatively weak visual spatial working memory skills and, as a consequence, partly overcome their mathematical learning difficulties. Furthermore, as it turns out that affective processes such as a high self-efficacy can increase mathematical problem solving, despite having relatively weak working memory capacity (Hoffman & Schraw, 2009), factors including motivation and personality of the child, combined with domain-general and domain-specific cognitive factors should be considered as potential candidates for cognitive strengths/weaknesses as well.

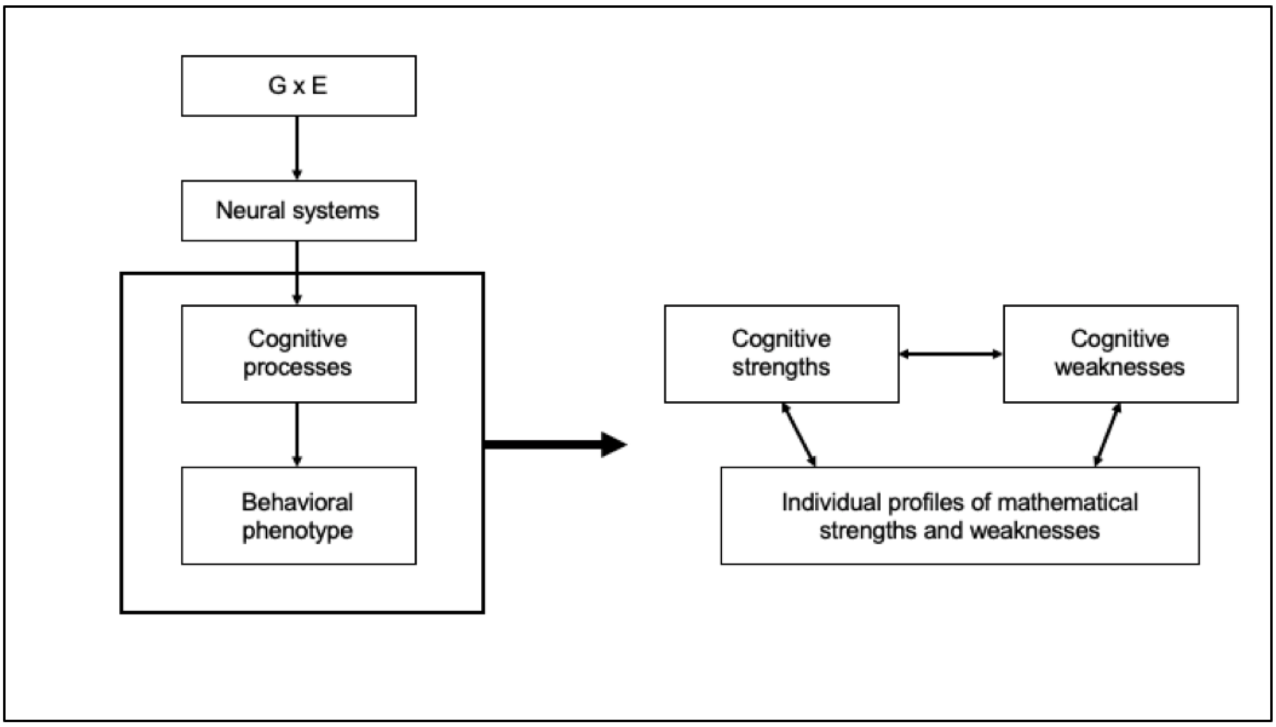
To summarize, we view the development of disabilities such as MLD as a result of a unique combination of factors, fully recognizing both strengths and weaknesses, that impact on and work together in the process of learning a complex skill such as mathematics. As a consequence, mathematical learning difficulties should be seen as a system of causally connected symptoms rather than as effects of a fixed set of causal cognitive mechanisms. One of the challenges that needs to be addressed in future research is which specific combinations of strengths and weaknesses can account for individual differences in mathematical learning disabilities. Below we further elaborate on the implications this view has for both research and practice.

Implications for Research

A number of general implications for future research can be derived from our multidimensional model on individual differences in mathematical skills. First, the dichotomous definition of MLD (as opposed to non-MLD) should be reconsidered in scientific research. Commonly used methodological and statistical techniques aid differentiation between MLD and typical development as if they are two separate categories, but no evidence exists for such a qualitative difference. In addition, choices regarding sample selection are often ambiguous. This could have resulted in inconsistent conclusions across studies regarding the academic and cognitive profiles of MLD, and may have impeded the generalizability across empirical studies (Murphy et al., 2007). A more elegant perspective on developmental disabilities is the dimensional approach that views mathematics performance on a continuous scale (Hudziak et al., 2007; Moll et al., 2014), wherein some people perform somewhat better on this scale than others. The lowest range of weak performance on such a continuous scale would then be defined as MLD, preferably in interaction with one's profile of (cognitive) skills associated with the math difficulties. This dimensional approach to learning difficulties is different from a binary approach, because it is not based on one (set of) skill(s) or characteristic(s) that defines whether one has MLD or not. Instead, it does justice to the complexity of a skill such as mathematics by taking into account the large amount of individual variability within people.

Figure 1

Overview of the original Multiple Deficit Model by Pennington (2006) (left panel), and the elaborated multidimensional model (right panel). G x E = gene-environment interactions



When conducting research on learning difficulties, one should therefore be aware that although a definition like MLD can be used to describe the lowest-achieving group, not every individual within that group will have the same characteristics, because of the considerable amount of individual variation within the mechanisms associated with each child's math performance. In light of these reflections, future research on MLD ideally focuses on a variety of weaknesses but also strengths related to mathematics in line with our proposed multidimensional model, as well as environmental risk or protective factors. This approach will allow researchers to gain a comprehensive understanding of the development of MLD both at the group and the individual level (cf. Lewis & Lynn, 2018a; Mammarella et al., 2021)

As a consequence, alternative statistical methods, such as network analysis (Astle et al., 2019; Borsboom & Kramer, 2013; Fonseca-Pedrero, 2017; Zhao & Castellanos, 2016) may be a better way to fully account for intra- and interindividual differences in mathematics learning, next to the use of qualitative in-depth case studies (see e.g., Lewis & Lynn, 2018b) to better understand the cognitive strengths and weaknesses in mathematical learning on an individual level. Another option is to include larger variability in the samples: Not comparing selective groups (such as specific learning disabilities) but including participants with a range of mathematical - and other comorbid - problems (see also Astle & Fletcher-Watson, 2020). Only large samples enable research to find data-driven neurocognitive dimensions that might underlie learning problems (Astle et al., 2019) and improve statistical power and lower the risk of overestimating effect sizes (Mammarella et al., 2021).

Secondly, instead of investigating cognitive differences and similarities between MLD and typically developing samples, future research should elucidate how individual profiles are related to the differences and—more likely—the similarities in the educational needs of individual children. Given the fact that each child with or without MLD needs the same set of skills (with different degrees of reliance on each of the skills), it can be questioned how children can best be taught to become proficient in mathematics. Mapping a profile of an individual child's strengths and weaknesses in mathematics and related skills such as reading and cognition may seem promising in that respect, but is not easily integrated into treatment programs for MLD. More research is needed to find out how diagnostic criteria should be applied and when it is necessary to further investigate cognitive profiles. In addition, environmental factors such as education probably offer good potential to decrease differences between children's math skills. The emphasis of research on MLD could therefore not only focus on the identification of cognitive factors related to the differences between children, but on the tools that help a diversity of students to learn mathematics well. Potential questions in this regard could for instance be what type of instruction works best for those children; which (digital) methods aid the development of math skills; and which degree of differentiation is desirable. Research outside the classroom could further identify the elements that improve implicit learning at home; and how the school board and the nation's government can facilitate learning mathematics within schools.

Finally, the tension between desirability and feasibility of an individual variation perspective within primary schools should be considered in future research as well. Although such an approach would be desirable for all children, the question arises whether teachers and other educational professionals have sufficient resources (e.g., knowledge, time, and money) to implement an individual variation approach in the near future. Moreover, the question rises how desirable it is to regard every child within the classroom as an individual. Each child has to achieve the same curriculum-based goals at the end of primary education, so they must participate in instruction together as much as possible. Differences in the educational needs of children with weaker math performance as opposed to children with (above) average math performance probably are more quantitative in nature than they are qualitative. To elaborate, these children may need more instruction time, but what they are being taught should be unified. Future research should investigate how teachers can best be supported in employing differentiation in instruction to give each child the challenge and support they need. Other approaches, such as peer-assisted learning wherein stronger learners collaborate with weaker learners, have also shown promising results (Fuchs et al., 2019). In this way, policy makers and school principals can be assisted to make informed decisions about best practices on the implementation of an individual differences perspective.

Implications for Practice

Next to implications for research, some implications for clinical practice can be mentioned as well. To begin with, educational professionals in the field of primary school mathematics are recommended to move away from their existing frameworks of learning that views the worst performance on a continuous scale as a learning disability (e.g., a discrete group that is intrinsically different from children that belong to another group). Once a child has been identified as learning disabled, teacher expectations and learning goals are generally adjusted downwards for those children (Szumski & Karwowski, 2019). However, as it appears that the mathematical and cognitive profiles of children with weak math performance are quite similar to those of average to high achieving children, all children may most probably benefit from the same education within the classroom. The children at the lowest end of the continuum might need additional guidance and time in small, heterogeneous groups regarding topics they do not master yet. This could entail increased practicing with automatizing arithmetic operations, or systematically writing down intermediate steps when solving complex math problems (Gelderblom, 2010; Ruijssenaars et al., 2021).

Next to the fact that there are similarities in the educational needs of children with MLD, it should be noted, however, that evidence from various case studies suggest that large differences do exist between children, making one general description for what is needed to remediate their difficulties almost impossible. As a consequence, when mathematical learning difficulties are more severe and persistent, educational professionals are recommended to first carefully map the cognitive strengths and weaknesses of the child and then make adaptations to the educational context to match their educational needs. By observing children during math instruction, by examining patterns of errors within children's math work, and by discussing the strategies children use to solve math problems, teachers or other professionals could identify the cognitive strengths and weaknesses of the child. For instance, a child that cannot seem to remember intermediate steps or intermediate answers might have difficulties with his working memory, and a child that does not seem to grasp how to work with a number line might have problems with his number sense, and vice versa. Only when their unique profiles of weaknesses and strengths are being fully acknowledged, all children (including those at the higher end of the mathematics continuum) are able to receive high-quality education and will ultimately have the potential to meet their countries' national requirements for mathematics (Vaughn, & Fuchs, 2003).

Furthermore, the way MLD is currently being diagnosed in clinical practice appears to be somewhat ambiguous. Diagnostic criteria have been described in widespread manuals (DSM-5, APA, 2013; ICD-11, WHO, 2018), but these are based on a descriptive behavioral pattern only: Severe, persistent, and specific difficulties with learning mathematics. This descriptive diagnosis does not indicate possible causes that may have induced the learning problem for individuals with MLD, and discrepancies in the definition of MLD between research and clinical practice exist. The scientific basis for the way MLD is currently being diagnosed is quite weak (Peters & Ansari, 2019), and does not sufficiently differentiate between children with and without mathematical learning disabilities. As a result, this may have hindered the development of successful prevention and remediation programs for clinical practice. Abilities related to mathematics such as reading and cognition should not be overlooked in clinical practice either, and it is therefore advised that interventions for math difficulties become available for all children with mathematical difficulties, ranging from mild to serious. Furthermore, the interventions should emphasize a broad spectrum of strengths and weaknesses related to mathematics, again in line with our proposed multidimensional model.

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