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# How Do We Learn Mathematics? A Framework for a Theoretical and Practical Model

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## Abstract

The purpose of this paper is to propose an effective learning environment for the initial stages of mathematical learning. Basic numerical skills and the objects and actions that trigger those skills are conceptualized as a mathematics-learning environment. We discuss numerical learning mechanism and the basic skills and environments we use to learn numbers briefly within the human cognition system. The three subsystems of number, i.e., exact number system, approximate number system and access to symbol system, are explained with reference to basic number competencies. They are discussed within the framework of "number sense" by drawing evidences from the neuroscience and mathematics education literature. Finally, how to manipulate the components of these subsystems for an effective learning of number is exemplified in a proposed model of mathematical learning environment.

**Keywords:**

Number Sense, Core Systems Of Number, Quantification, Magnitude, Quantity

## Introduction

A In this paper, human cognition system, numerical learning mechanisms and the basic skills and environments we use to learn numbers are discussed within the framework of "number sense". Basic numerical skills and the objects and actions that trigger those skills are conceptualized as the parameters of mathematics-learning environment. First, let us explain some basic concepts used in this paper for a better understanding of the proposed model.

Number Sense: It is the ability to use numbers intuitively, effectively, efficiently and fluently in problem situations.

Human cognition system: The theoretical structure that humans use to acquire any knowledge, skill or habit.

Basic number processing skills: It includes the perception of quantity, magnitude, approximate number estimation, and the ability to establish a symbol quantity-magnitude relationships, which are also known as core skills that enable people to learn mathematics.

Quantity: Quantity is the amount of something, which might be either discrete or continuous. While we call countable quantities as discrete quantities, we call continuous quantities as magnitudes. We use somewhat different actions to quantify a magnitude or a discrete quantity. For example, magnitudes can be measured or estimated, while a discrete quantity can be counted or estimated. Discrete quantities less than 5 are called small, because they can be perceived at a glance through parallel processing, a.k.a., subitizing (Mandler & Shebo, 1982). Discrete quantities greater than 5 are called large since they can be enumerated either by guessing or by counting and/or calculations. As counting methods evolve, arithmetic and other computational methods such as using facts, skip counting, repeated addition, and multiplication are applied on them.

Reaction and response time: The time elapsed between seeing the task and answering it is called reaction or response time. In some studies, reaction time and response time are separated. Special experimental setups are required for this distinction. In this paper, we use the combination and call it reaction time.

Canonic arrangement: The countable quantities are arranged in a way that creates a pattern such as a dice pattern that facilitates perception (Piazza, Mechelli, Butterworth, & Price, 2002).

Random arrangement: The countable quantities are scattered or randomly arranged without any recognizable pattern that facilitates perception.

*What is Number Sense?*

As defined above, number sense is "the ability to intuitively use numbers effectively, efficiently and fluently in problem situations". Although a uniform number sense is mentioned in this definition, we can deduce that it may have different reflections as there will be changes in the concept and types of number at various age and grade levels, and therefore different measurements should also be required. For example, while for a kindergarten or first grader it is meant to use natural numbers fluently from one to ten or one to twenty, it may extend up to 100 or 1000, for a middle school student, on the other hand, this concept naturally includes fractions, decimals and arithmetic facts. In more advanced grades, however, we see that these basic skills are transformed into the use of, for example, algebraic expressions while making transformations, simplifications, and expansions. Therefore, these can also be considered as further extensions of the sense of number.

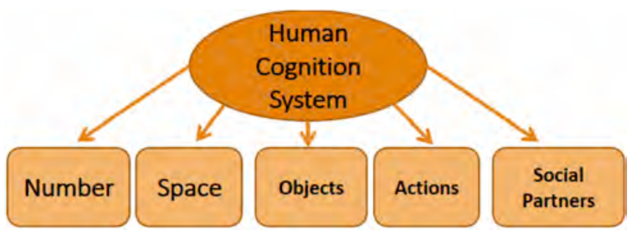
In the following sub-headings, the development of the concept of number in humans from birth will be discussed and the core knowledge, basic number processing systems and skills that enable further numerical learning will be elaborated. Basic numerical skills include the perception of quantity, the relative size and the place of number on a number line, its neighbors, their size relationships with other numbers, and the representation of numbers with symbols. As in all kinds of learning, there is a cognition system for people to learn numerical concepts and relations. This system as a whole mediates the learning, but there are also specialized subsystems for different aspects of numbers. Let us now consider them in detail.

*Human Cognition System*

As shown in Figure 1, the human cognition system consists of a small number of (4 or 5) subsystems (Kinzler & Spelke, 2007). These subsystems are cognitive structures used to represent number, space, objects, actions, and the social environment. It is believed that human beings are born in a way that is programmed to mentally represent the regularities they experience in their environment (Dehaene, 2009), we simply call this learning. The human species achieves this action through these specialized subsystems and networks between them. There are also evidence that some of these subsystems exist in some animal species (Spelke, 2017). However, the cognition system in humans is more comprehensive and complex than the one in other animals. This allows people to learn knowledge that is more abstract as well.

While the human cognition system represents quantity with the number subsystem, it represents shape and space with the space subsystem. It is the task of the objects subsystem to be able to think of one object as separate from another object, and learn the properties of each object building on this core knowledge. The task of imagining and thinking that a moving thing is moving independently from the object falls under the expertise of the actions subsystem. The social environment subsystem, on the other hand, is mostly reserved for the representation of subjects such as language, kinship, and cultural accumulations. Since the focus of this paper is on learning mathematics, the number subsystem and its constituent structures will be emphasized. A more elaborate discussion of the human cognition system can be found in (Kinzler & Spelke, 2007; Spelke & Kinzler, 2007).

**Figure 1**
*Human cognition system*

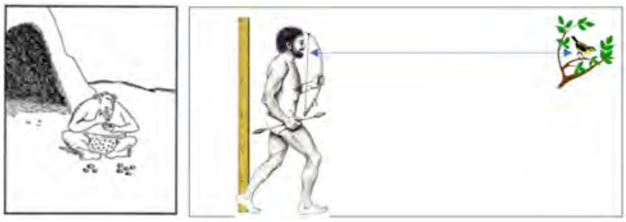



*Source: Spelke and Kinzler (2007) Developmental Science 10:1, ss. 89–96.*

*Number Subsystem*

The number subsystem is built on quantity perception. The amount appears in two different ways. These are "discrete quantity", that is, countable quantity, and "magnitude", that is, continuous or measurable quantity. Humankind has evolved this system, which it shares with some other animal species, and has created more useful forms for its life (Dehaene, Molko, Cohen, & Wilson, 2004). It is claimed that one of the reasons for mathematics learning disability or dyscalculia may be problems in quantity perception (Mazzocco, Feigenson, & Halberda, 2011). Each type of quantity triggers different mathematical actions and processes. Let us now examine these quantity types and their special cases.

**Figure 2**
*Discrete and continuous quantity in primitive times*

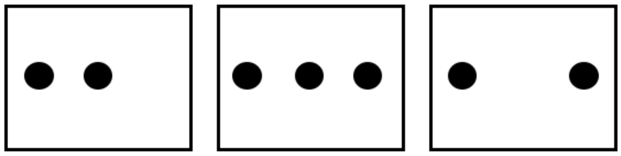


*Discrete Quantity*

Sets of objects that are countable are called discrete quantity. There is a difference in representation and perception depending on whether the quantity is small or large. While we call the quantity whose number we can perceive at a glance as small quantity, we call the quantity that exceeds this limit large quantity. The border between less and more is 4 or 5, also known as the subitizing range (Mandler & Shebo, 1982). When the number of objects is less than five, the human brain can perceive this number at a glance through parallel processing. When the number of objects exceeds five, and if there is no special arrangement, it cannot be detected at a glance, and other actions take place instead. While we use subitizing to enumerate small sets of objects rapidly, counting and/or other calculation operations are needed to enumerate larger sets.

There are evidences that the mechanism of perceiving small quantities is present from birth. In an experiment conducted by Antell and Keating (1983), it was revealed that 7-day-old infants were able to distinguish small quantities from each other, i.e., one from two or two from three. In this experiment, which was carried out using the looking time paradigm, the first group of infants was shown the card A consisting of 2 objects, and after the infants' attention was distracted from the card (practice, habituation), this time the card B consisting of 3 objects was shown (see Figure 3). The infants in the second group were shown the card C after the card A. It was found that the infants in the first group looked longer at the second card. This was shown as evidence that 7-day-old infants noticed the numerical differences in these cards. It was claimed that babies who did not even know number words or even speak yet use a kind of visual perceptual mechanisms to make this distinction.

**Figure 3**
*An experiment with seven days-old infants*

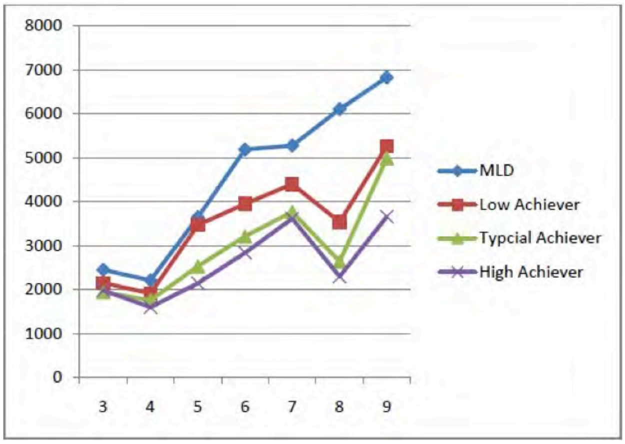



Again, in many experiments with adults, it was found that the responses to small numbers of objects less than five and large numbers more than five were different. Some researchers (Balakrishnan & Ashby, 1992) claimed that they did not find any evidence showing that subitizing is a separate mechanism. On the other hand, many other researchers(Benoit, Lehalle, & Jouen, 2004; Clements, Sarama, & MacDonald, 2019; Desoete, Ceulemans, Roeyers, & Huylebroeck, 2009; Piazza et al., 2002; Schleifer & Landerl, 2011) suggest that there is a different mechanism for perceiving small quantities of less than 5 and that it could act as a stepping stone for learning the cardinal number value and arithmetic facts.

In their study, Olkun, Altun, and Göçer-Şahin (2015) found that primary school children spent almost the same amount of time enumerating three and 4 dots. They were even relatively faster in counting four items. This may be because arrays of four objects are easier to perceive than three objects. After the number of dots exceeds four, not only the response time increases in parallel with the number of objects but also the gap between low-achieving and high-achieving students widens (see Figure 4). Other researchers also found discontinuity between subitizing and counting for dyscalculic children (Schleifer & Landerl, 2011). Another noteworthy detail in Figure 4 is that all groups, except the dyscalculia risk group, were faster in determining the canonically arranged eight dots compared to seven dots. Similar results that spatial arrangements of objects affected enumeration was also reported in the literature (Piazza et al., 2002). This finding also shows that canonically arranged dots facilitate perception and provide the opportunity to use different mental actions such as faster enumeration strategies. In fact, to support this argument, Piazza et al. (2002) claimed that subitizing and counting triggered different neural mechanisms.

**Figure 4**

*The medians of counting the canonically arranged quantities of 3-4-5-6-7-8 and 9 dots according to the achievement groups of the 2nd grade students*




*Source: (Olkun, Altun, & Göçer-Şahin, 2015)*

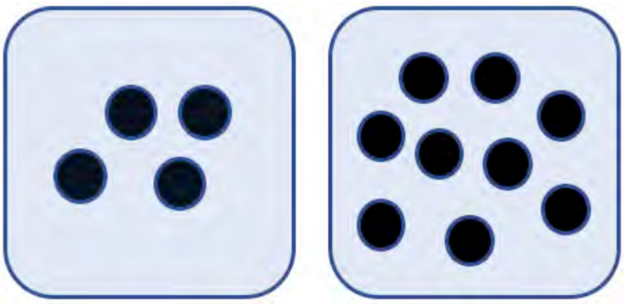
Many studies (Butterworth & Laurillard, 2010; Olkun, Altun, Göçer Şahin, & Akkurt Denizli, 2015) have found strong relationships between quantity perception and mathematics achievement. It has even been claimed that malfunctions in the quantity perception system can be a potential screening tool for mathematics learning disability (Desoete et al., 2009). Now let us do an experiment together to understand the difference in large and small quantity perception. You can repeat the experiment with different number of dots and with different people so that you can experience more reliable information first-hand.

*Experiment 1.*

Cover both of the quantities in Figure 5 with one hand each. Get a friend across you and quickly (in less than a second) open one hand and close it back. Ask how many dots there are. Then quickly open and close your other hand. Ask how many dots there are. Evaluate your friend's answers. Which one did s/he answer more correctly? In which one did s/he say a close number? Ask what actions s/he used to enumerate each "quantity".

**Figure 5**

*Small number and large number*



Another study examined whether 6-month-old infants (Wynn, 1992) understood the consequences of simple arithmetic actions. It was found that infants noticed when a new object was secretly added or removed from a small set of (>4) objects and showed a longer reaction time that could be regarded as astonishment to the incorrectly displayed result. For example, they responded with surprise that when two objects were shown and one object was added to it behind the scenes, the result was shown as two. However, the same infants remained indifferent when the number of objects treated was four or more. These experiments show that the mechanism of dealing with small numbers is present at very young ages, perhaps with birth, and the same system continues to be used in some form in adulthood. Some researchers tried to replicate the Wynn's study but found little or no evidence that infants can do addition or subtraction (Wakeley, Rivera, & Langer, 2000). It was concluded that simple adding and subtracting develops gradually throughout infancy and early childhood.

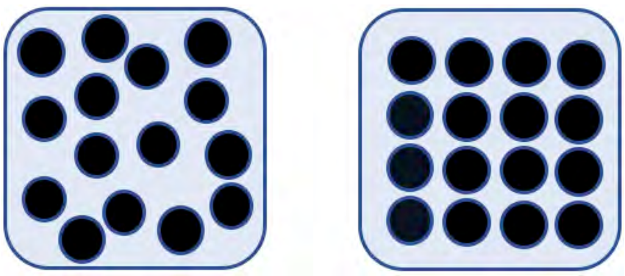
While trying to determine the numerosity of a "quantity", one of the factors affecting this is the arrangement of the objects that make up the quantity (Benoit et al., 2004). Differently arrayed objects trigger different actions, and different actions can reveal different mathematical processes (Olkun, Karslı-Çalamak, Sözen-Özdoğan, Solmaz, & Haşlaman, 2018). To examine this situation, repeat the following experiment with a friend.

*Experiment 2.*

Cover both of the sets in Figure 6 with one hand each. Take a friend in front of you and quickly (about 1-1.5 seconds) uncover one hand and cover it back while you ask how many dots there are. Then quickly (approximately the same passage of time) open and close your other hand. Ask how many dots there are.

**Figure 6**

*Random and canonically arranged discrete quantities*



Evaluate your friend's answers. Which one did s/he answer more correctly? In which one did s/he say a close number? In both experiments, ask your friend what actions s/he used to determine the number of sets. If s/he finds it difficult to answer, you can show her/him to choose the action listed below.

List of actions in quantifying a quantity:

Counting: Counting objects one by one

Subitizing: Perception of the numbers of groups of less than five at a glance

Grouping: Seeing objects in perceptible small quantities at a glance

Calculation: Finding the total number of objects in groups by using number facts

Estimating: Approximating the quantity or magnitude

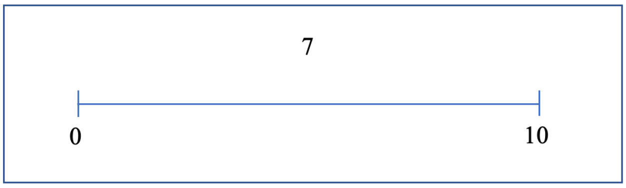
Measuring: Finding the size of a continuous quantity using a natural or a standard unit and unit iteration

These are the basic actions for quantifying a quantity; however, some combinations of the actions above might be used for enumerating large size quantities.

*Magnitude*

Another type of quantity that mathematics tries to quantify is magnitudes. Magnitude is also known as "continuous quantity". Concepts such as length, area, volume, and time are considered as continuous quantities. Continuous quantities, that is, magnitudes, trigger different actions and processes than that of discrete quantities. For example, while determining the number of a countable "quantity", it is necessary to use actions such as counting, grouping, calculating, estimating, however we use estimating or measuring for quantifying a continuous quantity. If we want to count or calculate continuous quantities, we must first make them countable by using a unit (hand span, meter, square unit, minute, hour, etc.).

**Figure 7**
*A typical number line estimation task*



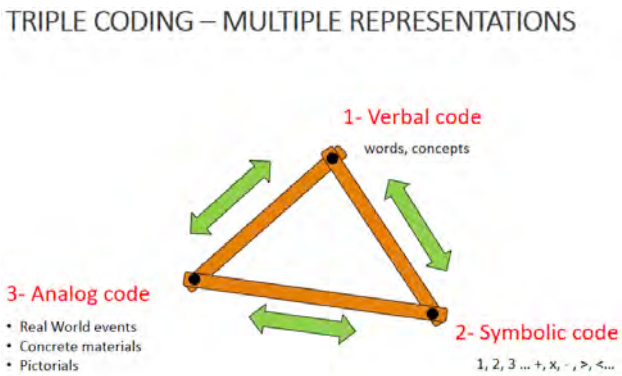

The most commonly used analog quantity in research and educational settings is the number line (Booth & Siegler, 2006). For example, a number line used for preschool and primary school first grade students is shown in Figure 7. By showing a number line, a child is asked "This number line has zero at the beginning and ten at the end. Where do you think seven is on this number line? Do you make a hash mark?" Thus, it is tested whether the child knows numerical concepts such as the reading, location, symbol, relative size, and positioning of numbers in the range of 0-10. Here, the child is expected to find the approximate location of the number rather than providing an exact hit. By finding the amount of error in the predictions made by the children, the estimation skills on the number line, in other words, the number sense skills are evaluated.

*Access to Symbol System*

Another task of the number subsystem is to establish a connection between Arabic number symbols and quantity. In other words, it is to be able to think of the symbol equivalent of the quantity shown, or vice versa. According to the triple coding theory (Dehaene, 1992) any mathematical knowledge is coded (represented) in three different codes or modalities. These are symbolic code, analog code and verbal (see Figure 8). This subsystem is also used to make transitions between the codes we use for a quantity fluently.

All kinds of concrete tools, drawings, graphics, or real-life situations are called analog representations. The word analog comes from the word analogy, which has been used to mean similarity. Its usage here means similar to the original event. In other words, quantity comes first either visible or hearable as in dram beats. That is, the amount is perceived first as analog quantity, and then this perception is converted into symbol(s) and word(s). In the future, it is constantly transcoded from one to the other.

**Figure 8**
*Triple coding or multiple representations of mathematical knowledge*




It is claimed that the triple coding theory is also a suitable framework for examining performance in complex mathematical problem solving from neuro-psychological perspectives (Schmithorst & Brown, 2004). If we show the issues discussed so far regarding the number system on a diagram, we can say that the number system in human cognition consists of 3 sub-systems, and these are Approximate (or large) Number System (ANS), Exact (or small) Number System (ENS), and Access to Symbol System (ATS), (see Figure 9).

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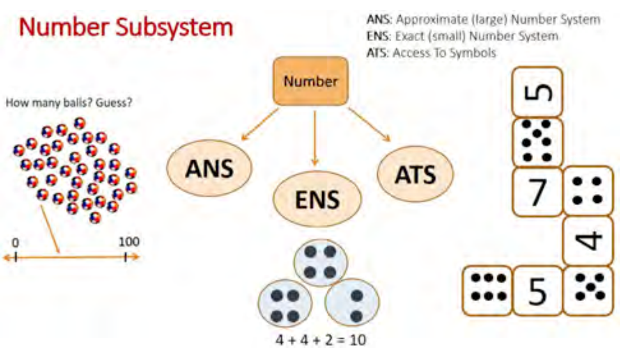
**Figure 9**

*Subsystems of number in human cognition and sample tasks for measuring each subsystem*




The time given for enumerating a "quantity" or "magnitude", whether presented as an analog quantity, as a symbol or verbally, is also affective on the action to be used for the quantification. For example, if the quantity is large and the given time is very short, a wild guess is used, in case the time increases a little, educated guessing based on the use of some strategies (i.e. estimation) can be used.

### Summary and Conclusions

In the simplest terms, we can define learning as "recognizing or discovering the regularities in our experiences". We can define learning mathematics as noticing the patterns between numbers and shapes and expressing them with the language of mathematics. Expressing in the language of mathematics, or in short, mathematization, is the process of representing recognized patterns using numbers and other symbols, or transcoding between representations. We can think of a large part of the mathematization process as a quantification process.

As summarized in Figure 10, it is seen that the first external factor triggering mathematical perception and thinking is the quantity in the first column. As seen in the second column, the amount can take two different forms. Countable quantities are called discrete quantities, while continuous quantities are called magnitudes. The small (<5) or large (>5) discrete quantities also affect the action to be used for the enumeration. While small amounts can be detected at a glance without counting or estimation by means of parallel processing, estimation and calculation can be activated for large quantities. Perception of small quantities are exact and present in infants possibly at birth (Antell & Keating, 1983). Large quantities on the other hand is not exact and perceived approximately (Lipton & Spelke, 2003). There is an interaction between exact and approximate number system and practicing non symbolic approximate number leads to an improvement in exact arithmetic in school (Hyde, Khanum, & Spelke, 2014)

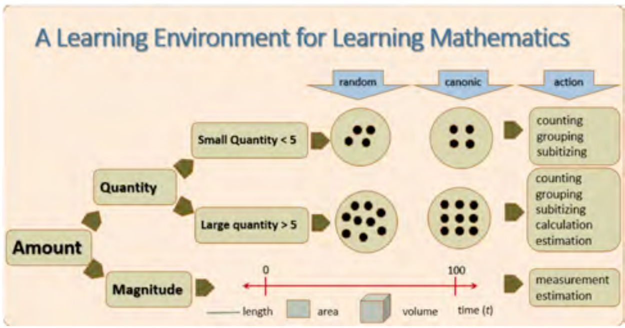
Canonically or randomly arranged discrete quantities also affects the action to be used for enumerating (Krajcsi, Szabo, & Morocz, 2013). Randomly arranged quantities, which make grouping and calculation relatively difficult, encourage the individual to guess if the given time is short, count if the time is sufficient. However, noticing the regularities in the canonically ordered quantities can create the opportunity to use the groupings and different calculation actions. We use the actions of measuring and estimating to determine the amount of things called magnitudes, such as length, area, volume, time etc. We can summarize the quantification process of mathematics as in Figure 10.

If we think of mathematization as a quantification process; we can say that the main triggering thing used in this process is the amount. The main action that governs this process is the determination of this amount. There are various sub-actions used to perform this main action. These actions may differ according to the type, shape and time of appearance of the quantity or magnitude. For example, estimation, measurement and calculation actions can be performed when determining a continuous quantity, while counting, grouping and calculation actions can be used to determine a discrete quantity. If the number of a canonically arranged and large "quantity" needs to be found in a very short time, the estimation action is triggered, while grouping and calculation can also come into play as the given time increases.

As a result, as can be seen, this context provides the framework for an important part of basic mathematics. It can be said that it will be possible to conduct mathematics education more effectively and efficiently in learning environments where the variables mentioned in this section can be controlled and manipulated. It is hoped that this framework, which is theoretically at the beginning and quite crude, will mature with additional research and theoretical studies.

**Figure 10**

*A learning environment that triggers mathematical actions and thinking*



When we consider the issue from the perspective of mathematics learning disability or dyscalculia, we see that individuals can perceive mathematical concepts or relationships at different levels and forms. As different representations activate different parts of the brain (Dehaene, Piazza, Pinel, & Cohen, 2003; Vogel, Goffin, & Ansari, 2015), the probability of learning the concept increases. For this reason, we can say that each mathematical concept to be taught will be more effective and productive with a teaching environment prepared in accordance with the triple coding theory or multiple representations of content (Sankey, Birch, & Gardiner, 2011). In fact, there are studies in this direction (Cohen Kadosh, Dowker, Heine, Kaufmann, & Kucian, 2013; Kucian et al., 2011; Ozdem & Olkun, 2019) in the current literature that show the efficacy of the basic mathematical skills training, such as subitizing and conceptual subitizing (Clements et al., 2019). It is seen that such intervention studies (Groffman, 2009; Olkun & Özdem, 2015; Ozdem & Olkun, 2019), which aim to develop different aspects of the basic number processing system as a whole, are more effective than traditional methods.

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