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Pedagogical Content Knowledge of Teachers in Teaching Decimals through Realistic Mathematics Education

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Abstract: Pedagogical Content Knowledge (PCK) as combines a teacher's knowledge of teaching and content so that the specific content is easy for students to understand. This study aimed to investigate the teacher's PCK in teaching decimals after mentoring by the research center of the Indonesian Realistic Mathematics Education (PRP-PMRI) team. This qualitative study involved two teachers teaching fourth grade at the school partner of PRP-PMRI, Aceh Province, Indonesia. The participation was voluntary, and the teachers had attended training related to the implementation of RME. Data were collected by observing during learning, interviews, reflection, tests, and interviews. This study showed that teachers' PCK in teaching focused more on the reality principle, activity principle, interactivity principle, and guidance principle of RME. They paid less attention to the other RME principles: level principle and intertwinement principle. The teacher also lacked experience making students' conjectures and did not prepare strategies to anticipate them. The teacher also paid less attention to identifying students' ways of thinking. In addition, the teacher's knowledge about the content was also unsatisfactory. This study also revealed that teachers' content knowledge also influenced teaching strategies, leading to students' misconceptions. This research implies that mentoring to improve teachers' PCK needs to be carried out continuously.

INTRODUCTION

Professional teachers carry out their teaching activities through special skills to create active, innovative, creative, effective, and fun learning (Iru & Ode, 2012). Teachers' work is not just to convey material but must carry out educational activities and help students with learning difficulties (Shulman, 1987). Shulman (1986) defined pedagogical content knowledge (PCK) as ways to represent and formulate specific content so that it is easy for students to understand. Good quality teachers are teachers who know the content and know how the specific content is taught (Kennedy 1998, Pavinee, Jari, & Kalle, 2013; Shulman, 1987).

PCK is a special fusion of content and pedagogical knowledge built from time to time through various experiences to produce professional teachers (Pavinee, Jari, & Kalle, 2013). Research by Loughran, Berry, and Mulhall (2012) showed that one of the possible factors increasing the effectiveness of teachers' work is to enrich their PCK. Observing and analyzing the PCK of a

teacher, either during the learning process or when planning the lesson, can provide an overview to examine and understand the teacher's competence (Depaepe, Verschaffel, & Kelchtermans, 2013). Therefore, there is a need for a more in-depth investigation of the teacher's PCK in preparing learning tools and their implementation in the classroom.

Teachers having problems with PCK can cause students' misconceptions or the emergence of student reluctance to learn mathematics (Fianga, Khabibah, Amin, & Ekawati, 2020). One of the topics that students often hold misconceptions about is decimals. Steinle and Stacey (2004) found misconceptions of Grade 4 to 10 students who consider 0.73 less than 0.6 by focusing on the size of the denominator. Their knowledge that one hundredth is less than one-tenth is incorrectly generalized to any number of hundredths less than any number of one-tenth. Desmet, Grégoire, and Mussolin (2010) and Sackur-Grisvard, and Léonard (1985) presented that many students of Grade 4 to 7 decided that 4.25 > 4.3 because 25 >3. Ubuz and Yayan (2010) reported that the most common mistakes in decimal addition problems were reading scales, ordering numbers, and operating decimals. When adding the last digit after the point, for example, adding 0.1 to 4.256 and 6.98, the student then gave an incorrect answer of 4.257 and 6.99, instead of 4.356 and 7.08.

One of the efforts to overcome students' misconceptions about decimals is to implement Realistic Mathematics Education (RME). RME is an approach that starts learning with challenging realistic problems. Students solve these problems informally, and then gradually, the teacher scaffolds them to reach the formal thinking stage through horizontal mathematization and vertical mathematization processes (Gravemeijer, 1994; van den Heuvel-Panhuizen and Drijvers, 2014). Freudenthal (1991), the founder of RME, explained that, in RME, students should reinvent mathematical ideas, such as concepts, strategies, procedures, formulas, definitions, etc., with teacher guidance because mathematics is a human activity, not a ready-made.

Treffers (1987) and van den Heuvel-Panhuizen and Drijvers (2014) stated six principles of RME, namely activity principle, emphasizing that students are treated as active participants in the learning process; reality principle, emphasizing the importance of learning mathematics that begins with problems in everyday life or real problems, students carry out the mathematization process to solve them; level principle, emphasizing that students learn mathematics through various levels of understanding, from informal understanding related to the situation gradually through model of, model for, and formal knowledge; intertwinement principle; emphasizing the interrelationships between topics in solving rich problems such as numbers, geometry, measurements, and data; interactivity principle, views mathematics not only as an individual activity but also as a social activity; guidance principle, emphasizing the active role of teachers in developing scenarios of learning activities that can facilitate or guide students to achieve the expected level of understanding.

Several studies have been conducted on increasing the PCK of prospective teachers or teachers in overcoming students' misconceptions about decimals through RME. Widjaja (2008) designed learning instruction theory (LIT) assisted linear arithmetic block (LAB) for teaching decimals for preservice teachers. LIT made substantial improvements in both content and PCK of preservice teachers. Pramudiani, Zulkardi, Hartono, and van Amerom (2011) compiled a learning trajectory along with weight and volume measurement activities, in which a number line is used as a model. The students could discover decimals by themselves and develop ideas to come to the number line

as a model for placing the value. Furthermore, Wirda, Johar, and Ikhsan (2015) adapted these activities to suit the 2013 Indonesian curriculum using thematic learning and made video lessons involving expert teachers. Yet, research examining teachers' PCK in teaching decimals by video lessons and implementing RME principles is limited. The research question of this study is how is the primary school teachers' PCK in teaching decimals related to RME principles?

RESEARCH METHODS

This qualitative study involved two of the teachers participating in the PMRI teacher workshop-assisted video lessons. Both teachers teach at partner schools of the research center of Indonesian Realistic Mathematics Education (*Pusat Riset dan Pengembangan Pendidikan Matematika Realistik Indonesia*/PRP-PMRI). The teacher was involved voluntarily and had attended training related to the implementation of RME. They hold an undergraduate education; teacher 1 (T1) is an alumnus of the mathematics education department, while Teacher 2 (T2) graduated from the Islamic Education Department. T1 has been teaching for ten years while T2 for 21 years. The two teachers attended training on decimals learning through RME.

The PMRI workshop assisted video lessons were carried out by researchers as the PRP-PMRI team in Aceh Province, Indonesia. The video lesson was obtained from the recording of decimals learning by the expert teacher. Also, researchers adapted the lesson plan and student worksheet designed by Wirda, Johar, and Ikhsan (2015). The workshop was held for two days (12 hours) with the following activities: a) Pre-test on ordering decimal numbers, b) Discussion of teachers' experience in teaching decimals, c) Discussion on the principles and characteristics of RME, d) Discussion on the scope of decimal topic for Grade 4, e) Watching a video lesson of decimals learning through RME, f) Discussion on critical moments in the video associated with RME principles, and g) Discussion on the improvement of lesson plans for implementation. Furthermore, the teachers implement decimals learning with the RME approach as in the video lesson in their classes.

Teachers copied the files of two video lessons to their laptops so they could watch them many times. Video lesson 1 was about exploring the meaning of decimals, and video lesson 2 was about ordering decimals. The activities on video lessons are presented in Table 1.

Day	Content	Goals and Activities
1	Exploring the meaning of decimals	(i) Understanding text containing decimals The text of the Muara Takus Temple is found in the student textbook with the theme My Hero. In the text, it is written that the length of the temple fence is 1.5 km. This activity allows students to predict the meaning of 1.5 km and estimate how long the 1.5 km is. (ii) Measuring the length of objects in the classroom This activity gives students the experience that the length of an object is not always a natural number. If the object's length is not in integer,

		the students write in their language, for example, 21 cm less or 21 cm more than two small lines. (iii) Measuring the weight of items A total of five nutmegs were weighed using two types of scales. On the first scale, the weight of the nutmeg is 500 grams, while on the second scale, it is 0.5 kg. This activity stimulates students to find the relationship between 0.5 kg and 500 grams. Next, in groups, students weigh an imitation of nutmeg, small stones, each weighing 100 grams or 0.1 kilograms. Students are free to determine how many stones will be weighed. (iv) Rewriting the length of the object in an activity (ii) using the decimals symbol This activity allows students to use an analogy between the length of the object '21 cm and two small lines' and the number symbol on the scale so that with the teacher's guidance, students are expected to write down the length of the object to be 21.2 cm.
2	Ordering decimal number	(i) Sorting the distance of a location written on the signpost This activity allows students to predict which locations are far or near the sign (ii) Weighing and sorting the weight of students using a digital scale This activity leads students to order their friend weigh in decimals (iii) Hanging the decimal number card and doing the exercise problem This activity leads students to order decimals using cards and write them on the number line
	Convert fraction to one-digit decimal and vice versa	(i) Shading part of a whole These activities lead students to remember a fraction of the tenth, such as 1/10, 3/10, etc. Students shade some parts then write them in the fraction with denominator 10 (ii) Weighing several pieces of green bean packets This activity guide students to find the relationship between fraction and one-digit decimals so that they convert fraction to one-digit decimals and vice versa

Table 1: Decimal number activities

There were two additional activities other than the video lesson such as using imitation of scale face and pouring seeds into a block glass.

The implementation was carried out by the teachers and mentored by the researchers. Mentoring was done when the teachers made the preparations and after implementation in the classroom through reflection. Assistance during preparation aimed to provide opportunities for teachers to adapt learning activities to the conditions of their students. Mentoring after implementation was intended to help teachers reflect on the learning done related to the principles of RME and aspects that need to be considered for further learning. Reflection was carried out through interactive discussions between teachers and researchers.

Teachers' PCK data were obtained through observations, tests, and interviews based on the PCK framework proposed by Chick, Baker, and Cheng (2006). The PCK indicators are integrated with the RME principle, as shown in Table 2.

PCK Category	Evidence when teachers...
1. Learning strategies	Discussing or using strategies or approaches for learning certain mathematical concepts or skills based on RME principles.
2. Student's Way of Thinking	Identifying a certain level of understanding of students or students' ways of thinking about concepts.
3. Demonstrating a deep understanding of the foundations of mathematics (content knowledge)	Demonstrating understanding of concepts while identifying mathematical aspects in depth and detail.

Table 2: A Framework for Analyzing Teachers' PCK

The 1 and 2 categories data (learning strategies and identifying students' ways of thinking) was collected through observations when the teacher taught decimals in their classes based on the video lesson. Furthermore, the data was confirmed to the teacher's reflection after the learning process. Both data were also reconfirmed through semi-structured interviews conducted after the learning. Data concerning category 3 of teacher content knowledge was obtained through tests conducted before the discussion about the scope of the decimals and during the teaching of decimals.

Data analysis was then undertaken through a data simplification process for more comprehensive research (Fraenkel & Wallen 2006). Also, the data of this study were analyzed through the stages of data reduction, data presentation, and conclusions (Milles & Huberman, 1994).

RESULTS

The following explanation describes the PCK of the two teachers (T1 and T2) for each PCK category, learning strategies, students' way of thinking, and content knowledge. The two teachers taught 70 minutes for the first meeting and 90 minutes for the second meeting.

Learning strategies

The observed aspect is how the teacher carries out learning in line with other RME principles: the activity principle, level principle, intertwinement principle, interaction principle, and guidance principle. Before teaching, the researchers conducted interviews with teachers regarding learning strategies.

The first meeting

The question asked during the interview was, "what is the planned teaching strategy for the class tomorrow?". The answers of the two teachers are as follows.

The first meeting aims to find the meaning of decimals. My strategy is to give students activities to weigh spices, such as garlic, ginger, and cinnamon. Students bring the spices per group before the learning. I have put it in a plastic bag that weighs 0.5 kg; then, it will be weighed using two scales that say 500 grams and 0.5 kg. Then, students weigh the objects around them, such as books, drink bottles, etc. From this activity, students see themselves where the decimals came from. Before weighing, I will also ask students to measure a string whose length is not a natural number, for example, 2 cm over or 2.4. .... I plan students to work in groups so that they interact with each other... I will guide them (T1)

Since this lesson has the theme of My Hero and is related to the ancient kingdom, I try to relate it to decimals. For example, my students and I will weigh rice to get decimals. I will tell the students to bring the rice later, or I will just bring it myself. I happen to have rice at home. Then, students will be asked to measure the sticks that I have prepared with a ruler that closes the original number then we write down the others. The original one is 1 to 100; later we will just make scale 1 to 10. The goal is that when we ask students to measure the sticks that I have prepared later, the size is not exactly integer.... . I plan students to work in groups so that they interact with each other... I will guide them (T2)

Next, the researcher asked the question, "why did you choose this strategy?" and the teacher's answer is as follows.

T1: So that students learn through experiences directly and real, in accordance with RME.

T2: So that students find the meaning of decimals according to the RME, students understand better if they see directly the number 0.5 on the scales; it is more real than just reading the writings in textbooks.

From the answers of the two teachers above, it is known that teachers T1 and T2 applied a strategy based on the video lesson and related it to the teaching principles according to RME. We conclude that the teacher only knew four of the RME principles: reality principle, activity principle, and interactivity principle, and guidance principle. They did not link their strategy to other RME principles: level principle and intertwinement principle.

Based on observations in class, it is known that T1 began the lesson by showing the story of the Muara Takus Temple taken from a textbook. In the story, there is an article about the size of the temple wall fence, which is 1.5 km. So the two teachers challenged the students to express their opinions regarding the number of 1.5. The following is the dialogue between T1 and the students.

T1: Who knows what 1.5 means?

S1: Half

S2: One plus half

S3: One fifth

T1: Thank you, many of you have expressed your opinion. How it looks if it is on a number line? Who can come to the front of the class?

S4: (Drawing the number line below)

T1: There are one, one point five, and two. Aren't there?

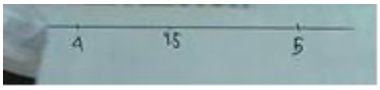
S: (chorus) Yes

T1: (The teacher draws a number line containing 4 and 5). Does anyone know if there is a number in the middle of this?

S: (chorus) Yes.

T1: Come on, who wants to come and write the number in the middle?

S5: (Drawing)



From the script above, it is known that students can write numbers between two consecutive integers.

Next, T1 asked the students to measure the length of a rope, sketch the length in a number line, and write the length in their language. At that time, the teacher asked students directly to write in the symbol of decimals. Some students had difficulties, T1 guided them, as shown in the following dialogue.

T1: What is the length of your rope?

S: (Put her rope on the ruler)

T1: What is the length?

S: 8

T1: 8 point… ?

S: (Silent)

T1: 8 point 9, isn't it?

Based on the dialogue above, the teacher directly informed the length of the rope in decimals. It should be the student using her expressions, such as 8 cm over 9 small lines or almost 9 cm. Next, the teacher asked students to do the activity (iv): rewriting the object's length in the activity (ii) using the decimals symbol. Students' answer is seen in Figure 1.

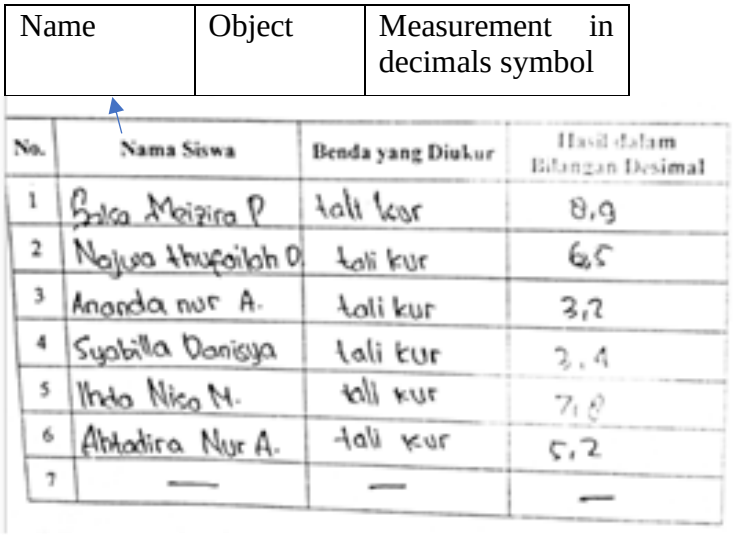


Figure 1. The results of measurement of length by students

After activity (iv), T1 was back to activity (iii): measuring the weight of items. Students weighed the items and saw the number of 500 grams on the first scale and 0.5 kg on the second scale. Then, the teacher instructed students to write the display of the scales on the whiteboard, as shown in Figure 2.

Figure 2: Weighing Results (i) 500 gram, (ii) 0.5 kilograms, and (iii) representation

After the teacher introduced decimals to students using scales, T1 asked students to work on the activities on the worksheet in groups. One of the results of students' work is displayed in Figure 3.

Object	Representation of weight	Measurement in decimal symbol

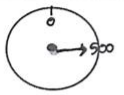
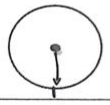
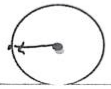
NO	Jenis rempah	GAMBAR SKALA TIMBANGAN	HASIL (dalam desimal)
1	Rempah-rempah		500 gram = 0,5kg
2	jahe/b. putih		1 kg = 1 kg
3	Gabungan rempah dan jahe/b.putih		1,5 kg = 1,5 kg

Figure 3: Answers in Student Worksheet

Before ending the lesson, the teacher asked students, “what does 0.5 mean?”. Most students said ‘a half’, “one-second”. ‘What is decimals?’, one of the students answered that decimals are numbers with points. The dialogue of T2 with the students during the lesson is as follows.

T2: From the story, we know that the temple is surrounded by a wall of 1.5 x 1.5 m. Does anyone know the meaning of 1.5?

S1: One and a half

T2: Yes, one and a half kilometers. Does anyone else know?

S: (Silence)

T2: If you don't have one, I'll bring a ruler, we will measure the sticks. We will it measure from zero. Please, one student comes forward.

S2: (Holding the stick)

T2: How long is it?

S2: Six and a half

T2: Try writing it, how should it be written?

S2: (Writing 6.5)

Then the teacher asked other students to measure the stick, and the length was 3.5. The teacher drew a number line containing 7 and 8.

T2: What is the value between 7 and 8?

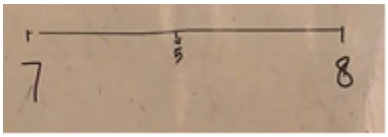
S: (Silence)

T2: Come on, go ahead, don't be shy.

S3: Between 7 and 8?

T2: Yes, let's write on the whiteboard, not just sitting down.

S3: (Writing)



T2: Between 7 and 8. Between 3 and 4, the middle is 3.5. What is this between 7 and 8?

S3: (Silence)

T2: Between 7 and 8!

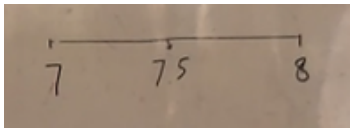
S3: (Silence)

T2: Can S3 answer?

S3: (Shaking his head)

T2: If you can't, you can sit down. Come on, who have the answer?

S4: (Writing)



T2: Is the answer correct?

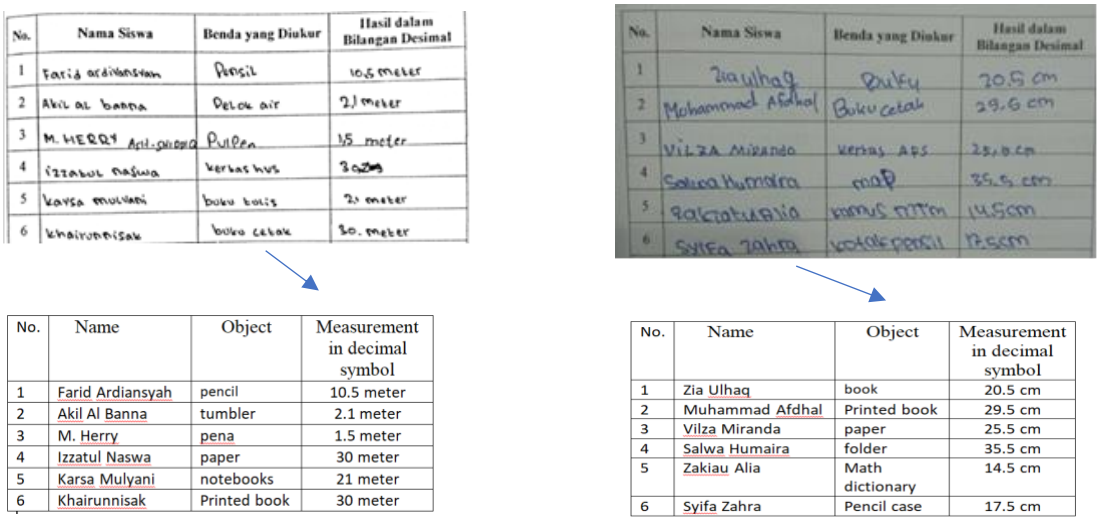
S: (Chorus) Yes...

The dialogue above indicates that the teacher only repeated the number in the middle but did not associate it with fractions so that some students had difficulty in making sense of it. The teacher also did not ask why S4 answered 7.5.

Next, the teacher asked students to measure the objects around them using a ruler, as shown in Figure 4.

Figure 4: Wooden Ruler for Measuring the Length

The numbers written on the rulers are multiples of five, and the lines provided are only for integers and their halves. So, students' answers in the group worksheets were only two kinds: half fractions and whole numbers. There was even a group writing the length of all items in the symbol of decimals when it should be in integers. For instance, the length of "*pelok air*" (tumbler) was 2.1 m but it should be 21 cm (see Figure 5). However, the teacher did not have time to respond and the students did not present their answers.



No.	Nama Siswa	Benda yang Diukur	Hasil dalam Bilangan Desimal
1	Farid ardiansyah	Pensil	10,5 meter
2	Akil al banna	Pelok air	2,1 meter
3	M. HERRY Akil.albanna	Pulpen	1,5 meter
4	izzatul naswa	kertas hvs	30
5	karsa mulyani	buku tulis	21 meter
6	khairunnisak	buku cetak	30. meter

No.	Name	Object	Measurement in decimal symbol
1	Farid Ardiansyah	pencil	10.5 meter
2	Akil Al Banna	tumbler	2.1 meter
3	M. Herry	pena	1.5 meter
4	Izzatul Naswa	paper	30 meter
5	Karsa Mulyani	notebooks	21 meter
6	Khairunnisak	Printed book	30 meter

No.	Nama Siswa	Benda yang Diukur	Hasil dalam Bilangan Desimal
1	Ziaulhaq	Buku	20,5 cm
2	Muhammad Afdhal	Buku cetak	29,6 cm
3	VILZA MIRANDO	kertas A4	25,0 cm
4	Salwa Humaira	map	35,5 cm
5	Zakiau Alia	kamus matematika	14,5cm
6	Syifa Zahra	kotak pensil	17,5cm

No.	Name	Object	Measurement in decimal symbol
1	Zia Ulhaq	book	20.5 cm
2	Muhammad Afdhal	Printed book	29.5 cm
3	Vilza Miranda	paper	25.5 cm
4	Salwa Humaira	folder	35.5 cm
5	Zakiau Alia	Math dictionary	14.5 cm
6	Syifa Zahra	Pencil case	17.5 cm

Figure 5: The Length Measurement Results

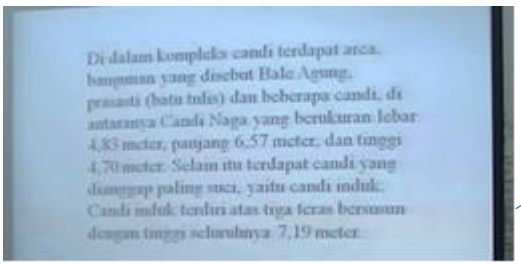
Another activity carried out by T2 to introduce decimal numbers is to carry out weighing activities. T2 used rice to weigh and compare its size as T1 in Figure 2.

The second meeting

T1 and T2 displayed a picture of the location pointer to start the lesson, as shown in Figure 6.

Figure 6: Location Pointer

Teachers (T1 and T2) asked which one was the furthest and the closest, "who can sort from closest to furthest?". Next, the teacher shows the continuation of the temple story in the textbook, as shown in Figure 7. Then, the teacher asked one of the students to write down the decimals in the story on the board "How can we sort from the smallest to the largest? We will discuss it today".



Inside the temple complex, there are statues, a building called Bale Agung, inscriptions (slate) and several temples, including the Naga Temple, which is 4.83 meters wide, 6.57 meters long, and 4.70 meters high. In addition, there is a temple that is considered the most sacred, namely the main temple. It consists of three terraces with a total height of 7.19 meters.

Figure 7: Text about Temples which Containing Decimal Numbers

T1 asks two students poor seed to two block glasses. The first student filled the seeds in the first block glass of 3.2; then the second student filled the seeds in the second block glass block of 3.7. After the two-block glass are filled, the teacher shows the difference between the two-block glasses. This activity is carried out several times with other numbers so that students can understand the location or sequence of decimal numbers. This activity can be seen in Figure 8 below.

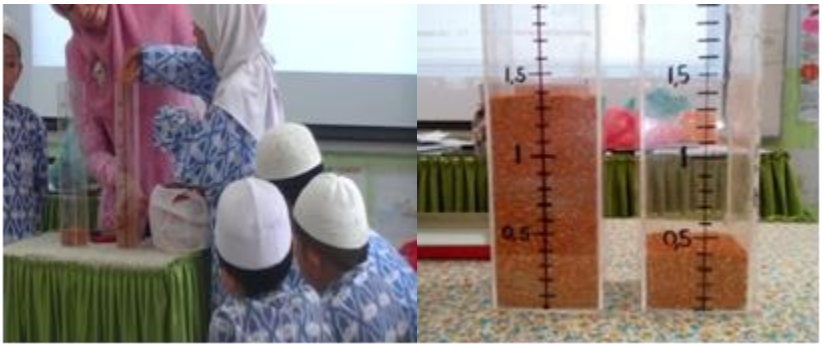


Figure 8: Teacher T1 Demonstrating the Sequence of Decimal Numbers

After the demonstration, students were asked to order the decimal numbers on the worksheet. Student worksheet answers of T1s' class and T2s'class can be seen in Figure 9(i) and (ii).

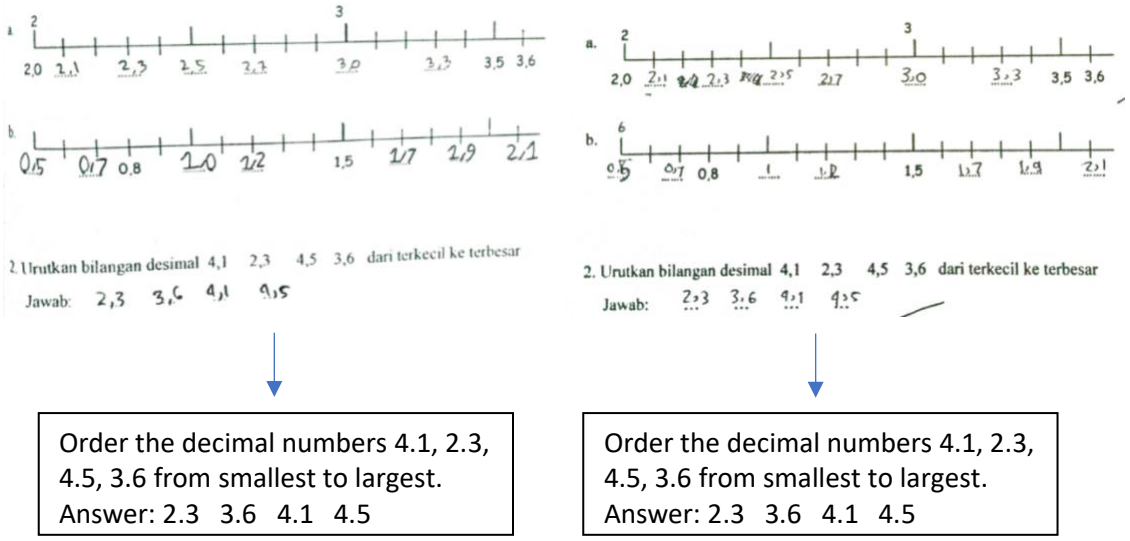
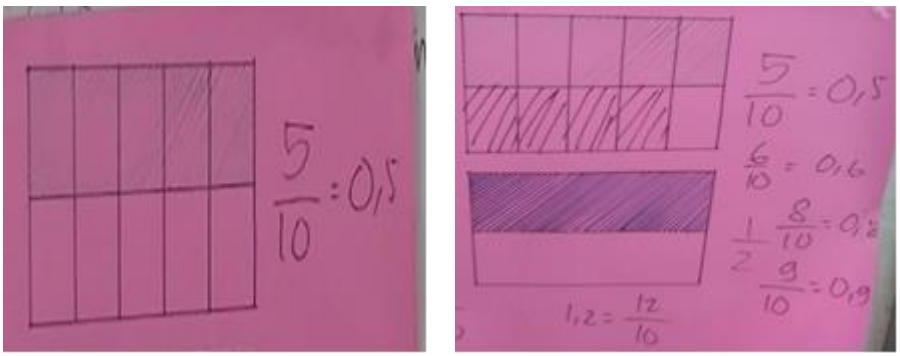


Figure 9: The Answers of Student Worksheet in T1 and T2 Classes about Ordering

Next, the teacher introduced the relationship between decimals and fractions. The teacher used cardboard media containing shaded boxes. The teacher explained about $\frac{5}{10}$ by showing the students ten squares with five squares shaded as 0.5. Next, for $\frac{6}{10}$, the teacher showed ten squares with six boxes shaded as 0.6 (Figure 10).



(i) (ii)

Figure 10: Relationship between Fraction and one-digit Decimals

The answers of the worksheet about converting fractions into decimals and vice versa for the students of T1 and T2 are presented in Figures 11 (i) and (ii).

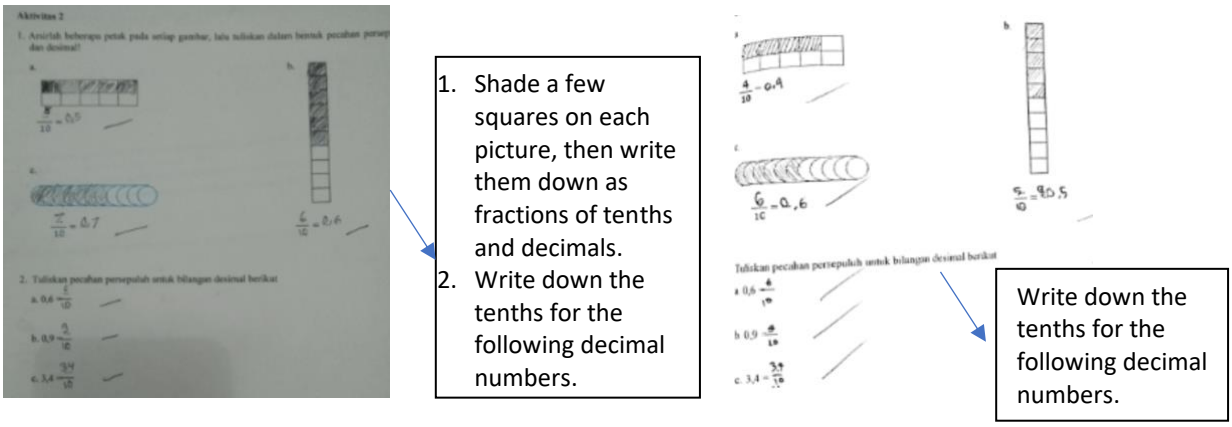


Figure 11: The Answers of Student Worksheet in T1 and T2 classes about Fraction and Decimals

Students' way of thinking

Data about student thinking obtained through observations and quizzes given to students at the end of each lesson. The quiz results showed that four out of 30 students in the T1 class had misconceptions about the location of decimals. Figure 12 describes the example of students' misconceptions.

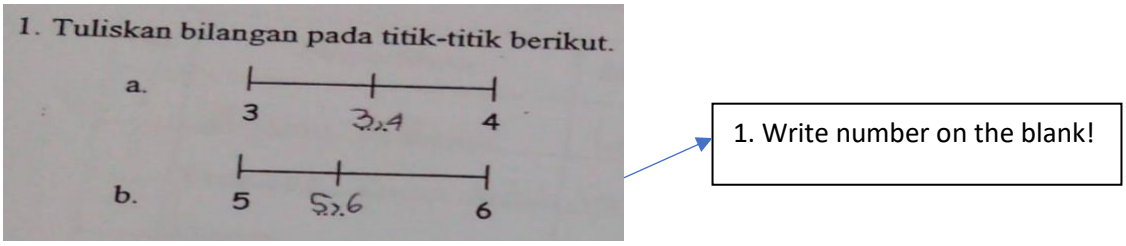


Figure 12: The Example of Students' Misconceptions

Figure 12 shows students' understanding of decimals between 3 and 4, namely 3.4, even though the specified line mark is located right in the middle between the numbers 3 and 4. Furthermore, in the second question, students wrote that the decimals between 5 and 6 was 5.6, even though the specified dash did not reach the middle between 5 and 6.

Based on the reflections made by researchers and teachers at the end of the meeting, T1 and T2 planned to overcome students' misconceptions using an image of a scale face. Here's a dialogue between T1 and her students.

T1: I will display an image of the scale face (using PowerPoint) so that all of you can see it. What is the maximum weight on the scale?

S: (Answering in unison) 2 kilograms.

T1: Let's read it together.

S: (reading in unison) 100 grams, 200 grams, …. 1 kg, 1.8 kg, 1.9 kg, 2 kg.

Then the teacher showed an imitation of the scale face and asked several students in turn. The teacher asked the students, "What is the number after 1.5?. The student answered 1.6 by pointing the arrow at the image of the scale face. Then the teacher asked the other students one by one, “What are other numbers between 2 and 3?. The students answered 2.1; 2.2; 2.3; 2.4; 2.5; 2.6; 2.7; 2.8 and 2.9. The teacher asked one of the students to write down the decimals on the scale face, as shown in Figure 13.

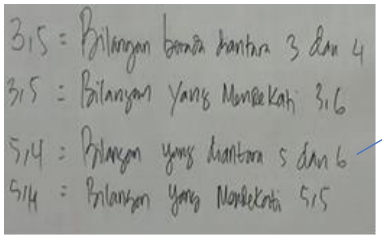
(i)

(ii)

Figure 13. Teacher Using Scales Face as a Model

The decimal numbers that students have written on the scale face were then released from the scale to form a number line (see Figure 14(i)). This activity was done by T1 so that students understood that the numbers on the scales could also be put into the number line. Then the teacher and students write the meaning of decimals using various ways, as in Figure 14(ii).

(i)



(ii)

3.5=number between 3 and 4
3.5=number close to 3.6
5.4= number between 5 and 6
5.4= number close to 5.5

Figure 14: (i) Students Writing Down Decimals (ii) The Meaning of Decimals

Content knowledge

Regarding the teacher's answers of the test about ordering decimals, the researchers interviewed T1 and T2.

The excerpt of the interview with T1 is as follows.

R: Which is the greater decimals, 4.7 or 4.18 and 2.007 or 2.0045. What is your answer and why?

T1: 4.7 is greater than 4.18 and 2.007 is greater than 2.0045. The reason is that we look at the first number after the point. If it has same main number and so on, let's look at the numbers behind it. Like 2.007 and 2.0045, first, let's look at the number behind the point, which is 0, look again behind it, which is also 0. Then, look at the next numbers. For 2.007, it shows 7, and for 2.0045, it shows 4. Because 7 is greater than 4, then 2.007 > 2.0045

The excerpt of the interview with T2 is as follows.

R: Which number is greater, 0.75 or 0.8?

T2: 0.75 is less than 0.8. If it is sorted 0.75; 0.76; 0.77; 0.78 and so on, 0.75 does not reach 0.8.

R: Which number is greater, 3.92 or 3.480?

T2: 3.92 is greater than 3.480 because of the second number after the point. For example, for 3.92, the second number is 9 and for the number 3.480, the second number is 4. Because 9 is greater than 4. So, 3.92 is greater than 3.480.

Both teachers misunderstood decimals as point numbers. Even at the end of the first meeting, T1 concluded that decimals are numbers based on ten, namely 0,1,2,3,4,5,6,7,8,9. T1 wrote it down in Powerpoint and presented it to students. The two teachers also misread the decimals of two digits behind the point, for example 3.92 they read three point ninety-two; it should be three point nine two. Based on the reflection after teaching, the teacher realized their misunderstanding about decimals, that it is different to the decimal system.

DISCUSSION

Teachers' knowledge regarding teaching strategy focused more on reality principle, activity principle, interactivity principle, and guidance principle of RME. Both teacher T1 and T2 asked the students to measure items and write the length using the decimals symbol before they knew the decimals symbol from the scale. Both teachers did not follow the sequence of activities in video lesson. In the video lesson, the sequence of activities was measuring the length without the decimals symbol, then measuring the weight of items and write them in decimals symbol. The last was measuring the length of items using decimals symbol. Referred to the level principle, teacher ask a student to solve a problem in an informal rather than formal way (Gravemeijer, 1994; Treffers, 1987; van den Heuvel-Panhuizen & Drijvers, 2014). Not implementing the level principle impacted the intertwinement principle. This research suggests teachers make connections between measuring the length and weight; therefore, teachers should implement the intertwinement principle.

During the lesson, the teacher seemed busy with demonstrating the media and sometimes ignored the students' incorrect answers. For instance, students wrote the length of all items in the symbol of decimals when it should be in integers. For instance, the length of "*pelok air*" (tumbler) was 2.1 m rather than 21 cm. Another example, some students wrote 3.4 between 3 and 4, and 5.6 between 5 and 6. Star and Strickland (in Amador, 2014) stated that teachers are rather weak in observing events in the classroom and interpreting students' understanding. Chick and Baker (2005) explained that teachers give varied responses to students' misconceptions. Their responses are influenced by the type or nature of the task given, and their PCK is related to the emphasis on procedural and conceptual aspects.

This research begins by providing training to teachers and providing video lessons, lesson plans, and student worksheets. Teachers were asked to understand all the materials provided and may revise them following the characteristics of students and the available facilities. However, the teachers did not fully follow the important activities. One of the activities in the video lessons that the teachers did not implement was weighing 10 packs of green beans, each weighing 100 grams or 0.1 kg. The teacher and students take 3 of 10 packages of green beans and then read the scales (0.3). Students are directed to find the relationship between $\frac{3}{10}$ and 0.3. This is in line with the findings of Ahmad and Sultana (2013) reporting that some teachers teach according to what they believe to be true and even teachers fail to carry out activities based on what has been written in the lesson plan. Tanudjaya and Doorman (2020) added that the lack of information also caused this weakness.

Teachers' knowledge of decimals is limited to recognizing symbols, namely point and numbers, unrelated to various representations, such as measuring length and weight. This finding is in line with Widjaja (2008), revealing that the models for learning decimals presented in the Indonesian textbooks are more symbolic, such as emphasizing positions of points rather than lengths of lines. When sorting decimals, the teacher only provided reasons based on the digits after the point, so that students experience misconceptions in learning. Shulman (1986) asserted that "lack of content knowledge is likely to be as useless pedagogically as content-free skill". There is a correlation between PCK and CK (Krauss, 2008). Ball et al. (2009) also added that the mastery of the subject that will be delivered to students is a must for teachers. The very basic in teaching is teacher competency. The research results of Turnuklu and Yesilderes (2007) stated that as a teacher, it is impossible to teach mathematics without knowledge of how to convey the mathematical concept. This finding is important to the next teacher professional development program in developing teachers' PCK through RME. The level principle and intertwinement principle need to be explained and practiced more.

CONCLUSIONS

Teachers' PCK through PMRI workshops was measured on three aspects: learning strategies, identifying students' ways of thinking, and understanding of content were unsatisfactory. Teachers only implemented only four of the six RME principles observed, namely: reality principle, activity principle, and interactivity principle, and guidance principle. They paid less attention to the other RME principle: level principle and intertwinement principle. In addition, teachers PCK in

designing teaching strategies were more likely to prepare learning resources, worksheets, and classroom management. Teachers were less experienced in making conjectures to students' possible answers and preparing strategies to anticipate them. In addition, the teacher's knowledge about the content, third aspect of PCK measured in this study, was also unsatisfactory.

It is suggested that to help students be engaged in learning mathematics, the teacher must design learning related to the Realistic Mathematics Education approach, using appropriate teaching aids that are easy to understand by students and preparing questions, to lead students to a higher understanding of mathematics. Teachers are expected to improve their content knowledge as it is the main requirement in designing activities and helping students learn mathematics. Future researchers are expected to design training to help teachers improve their content knowledge mastery and ask teachers to conduct peer teaching to enhance their PCK.

To improve teacher PCK, it is recommended that teacher professional development workshops be carried out by modeling based on lesson plans and worksheets. This means that the researcher as trainer in the workshop acted as teacher and teacher as participant acted as student. So that participants can follow the sequence of activities in the lesson plan and understand how to complete the activities on the students' worksheet. Another advantage of this modeling is that trainers can analyze participants' content knowledge. After the modeling process, researchers discussed the six RME principles one by one, and put more emphasis on the principal level and intertwinement principle.

Teacher content knowledge is the main requirement in teaching. Therefore, during the workshop, there is a special session focus on analysis of students' misconceptions. The results of the students' misconception analysis were discussed as well as adding to the teacher's content knowledge. Mentoring to improve teachers' PCK needs to be carried out continuously so that RME principles can be implemented by teachers during teaching.

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