Beliefs on Realism of Word Problems: A Case of Indonesian Prospective Mathematics Teachers

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Abstract: This study aims to investigate prospective teachers' beliefs toward the realism of mathematics word problems. The study employed both quantitative and qualitative analysis. A 36-item survey was distributed to 28 prospective mathematics teachers in one of the private universities in Indonesia. The survey was developed using the framework proposed by Palm (2006). The survey data was analyzed quantitatively by giving a score to each item's scale responded by participants. Then, interviews were conducted with six participants who have realistic, non-realistic, and neutral beliefs toward word problems. The interviews excerpts were analyzed qualitatively through excerpt coding. It was found that 20 participants possessed realistic beliefs toward word problems, while the rest possessed non-realistic and neutral beliefs (7 and 1 participant consecutively). Prospective teachers with realistic beliefs emphasized that any information presented in the word problem should simulate real life as accurately as possible. In contrast, those who have non-realistic beliefs stated that it was acceptable if it can be imagined. Neutral prospective teachers believe that word problems' realism is relative to cultural setting and students' background.

INTRODUCTION

In today's Mathematics teaching and learning context, it is never denied that one of the essential learning goals is to apply the knowledge obtained in the class into real life. The demand to contextualize teaching into students' lives increases as the need for real-life problem-solving skills increases. One of the ways used by teachers to introduce this contextual problem is by giving word problems.

A word problem is a written text containing a real-life situation completed by quantitative information and question. The answer can be obtained from the provided or inferred information given (Leder, et al. 2002). Despite the role as a tool to apply mathematical knowledge into a real-life situation, word problems in the classroom are sometimes considered "superficial" as they do not depict the realistic view of life. It hinders students from making sense of their learning back to a real-life context. For example, in the study by Palm (2008) on 161 fifth graders, the use of superficial solution strategies was found. Interestingly, this study revealed that the main reason for this was students' beliefs in word problem-solving. In this study, students believed that they did not have to consider what was not written in the word problem (i.e., considering how real the information is in the real situation). Although some students found the unrealistic information given in the problem, they preferred to ignore it as long as the problem was answered.

A similar case was also found in the study of undergraduate students by Inoue (2005). From the worked tests given, most students' answers were unrealistic. However, after being interviewed, the unrealistic view emerged from a realistic effort to adjust to schooling norm during Mathematics learning; this realistic effort leads to their beliefs on what a "realistic" situation is in word problem-solving. Further, the classroom practices showed how teachers supported students to make sense of the problem realistically, from what the teacher believed as "realistic", which in other words, restricted the ways students see the problems from their point of view of life.

Students' unrealistic answers may come from teachers' typical word problems. Greer (1997) investigated this issue in the study when he claimed that the word problems' structures are mostly unrealistic. The word problems were found to be unproblematic, i.e., they can always be solved, and all information will always be used. On the other hand, unproblematic word problems do not always happen in reality. A study by Krawitz (2018) revealed similar findings. Students' exposure to unproblematic word problems made them overlook irrelevant information and thus gave unrealistic answers. This exposure came from the learning experience provided by teachers possessing such beliefs.

Some studies on beliefs toward word problems had been conducted in Indonesia. Wijaya, et al. (2015) showed that teachers mainly offer plain word problems without irrelevant or redundant information linked to their beliefs. Another study by Siswono, et al. (2019) found teachers' beliefs that the problem solving should be done instantly, not inferring a realistic view. Some other studies discussed prospective teachers' beliefs, although not focusing on word problem-solving. For instance, the study by Muhtarom (2019) revealed that prospective teachers, who believed that mathematics is dynamic and a way to solve real-life problems, gave a problem close to students' life. In another study (Muhtarom et al., 2017), most prospective teachers' had a Platonist view, which saw mathematics as a static and structured knowledge rather than a dynamic one. Despite many studies discussing beliefs toward word problem solving, studies of prospective teachers, or even the specific beliefs toward word problem realism, are still limited in Indonesia. Taking the facts into account, studying prospective teachers' beliefs related to the realism of word problems

and, further, how the beliefs affect the decision to give it to students is necessary. Therefore, this study aims to investigate prospective teachers' beliefs toward the realism of mathematics word problems by answering a research question: "How are the beliefs possessed by prospective Mathematics teachers toward the realism of word problem?"

THEORETICAL FRAMEWORK

Word Problem and Reality

Word problem, in general, is a problem being put in a real-life situation (Verschaffel, et al. 2010). The existence of real-life situations embedded in the word problem makes it both unique and challenging. Rather than directly offer a question demanding an answer, the word problem is presented by three components: a set-up component, an information component, and a question (Gerofsky, 2014). Those components are formulated around a real-life situation while at the same time bringing a mathematical purpose. However, the situation in which word problem is formulated may not be that realistic.

There had been numerous studies exploring the notion of "reality" in mathematics education, and indirectly in the case of word problems. One of the most referred one was that of Freudenthal who regarded reality as rich source to be utilized in teaching and that mathematizing, a process of organizing reality with mathematical meaning, should be learnt by students (Freudenthal, 1973). He explained the reality to be experientially real to students, or in other words directly correlated to students' daily life (Gravemeijer & Terwel, 2000). However, he also admitted that mathematicians seemed to weigh logical connection more than reality as soon as the former brings faster progress. In this case, the practice of incorporating reality into the classroom may not be that apparent or dropped once the logical connection of mathematical matter seemed more promising. In the case of word problems, Boaler (1993) questioned explicitly, "how real is real". She stated that for some word problems, the reality presented in the word problem sometimes should be perceived as real although some aspects of that reality were ignored in the "real life version". This again showed that what is considered real in the word problems may not be students' reality and allowed for some variations of what is considered as realistic (or unrealistic) word problems.

Galbraith and Stillman (2001) distinguish four types of word problems based on how the real world is integrated into the word problem. An "injudicious problem" does not use the real world into the word problem or even violate it. A "context-separable problem" uses a real-world context, but the answer is context-independent. A "standard application problem" expects realistic consideration for the situation, and the procedure cannot be directly presented. The last type, a "genuine modeling problem", requires personal real-life knowledge and considers it to solve problems. In another study by Greer, Verschaffel, and De Corte (2002), the first two characteristics are considered that of non-problematic problems.

Types of word problems showed to what extent the real world is incorporated into the problems. Interestingly, various studies promoted a more realistic word problem, which considered the context embedded in it, to support students' mathematical learning (Greer et al., 2002; Krawitz et al., 2018; Tarim & Öktem, 2014; Van Dooren et al., 2019). The thinking process in considering the reality presented in the problems will benefit students in exercising transformation from real-world information into the mathematical model. This process imposes a more meaningful role of mathematics toward students. However, this is not always the case. For instance, a study by Sumarwati et al. (2014) showed more non-problematic and simple word problems in elementary school textbooks. The word problems presented there are the ones whose context is not considered for solving them.

Beliefs Towards Word Problem

In the context of mathematics teaching and learning, belief is defined as subjective conceptions that are held to be true, which may be shown explicitly or implicitly, and affect mathematical learning and problem solving (Op' T Eynde, et al. 2002). The practices in utilizing realistic word problems in the classroom are highlighted by several studies to be affected by teachers' beliefs (Beswick, 2005; Luz & Antoni, 2019). Despite being useful in promoting students' reasoning and meaningfulness of mathematics in their life (e.g., Boaler, 1993), there is a spectrum of beliefs on the realism of word problems.

In discussing the spectrum of beliefs on the realism of word problems, studies generally explain it in two complementary modes: realistic and unrealistic (Depaepe, et al. 2010; Luz & Antoni, 2019; Tarim & Öktem, 2014; Verschaffel, et al. 1997). When one believes that the real world can be totally or partly excluded from the word problem solution or modeling process, it can be inferred that an unrealistic mode is held. While realistic mode portraits beliefs of putting all real-world consideration into the solution and modeling process of the word problem.

In the case of word problems and reality, Palm (2006) offered a comprehensive framework on essential aspects in real life to be considered in word problems (Table 1). An individual's conception of how each aspect should be considered determines his/her beliefs about word problems (WP). Palm mentioned an example of the 'event' aspect. An event of some people wants to go up in a lift in the morning (as shown in the Lift Problem in Figure 1) can be considered to have a fair chance of happening. On the other hand, picking marbles from an urn and noting their colors is not something people do and therefore cannot be considered a realistic event. When a person agrees that a word problem should contain a realistic event, it shows his/her realistic beliefs on word problems; on the other hand, if he/she thinks it is okay to include non-realistic events, he/she is considered to have non-realistic beliefs.

Another example is a consideration for the question aspect. In the question aspect, there needs to be a consideration of whether the word problem's question is in accordance with the possible question posed by a real-life situation. There may be no real-life question like the question given

by the Lift problem in Figure 2. People may want to know when it may be their turn to use the lift instead. So, when a person believes that a word problem should pose a real-life question, he/she has realistic beliefs. To summarize, the closer each aspect presented in the word problem to the real situation, the more realistic it becomes. Thus, a person who believes that each aspect should be as realistic as possible is considered to have realistic beliefs toward WP, while on contrary, the one who does not, is considered to have non-realistic beliefs.

Aspect	Sub-aspects	Description
A. Event	A1. Event	The event described in the school task has taken place or has a fair chance of taking place
B. Question	B1. Question	The question being one that might be posed in the real-life event described is a pre-requisite for a corresponding real-life situation to exist. (The question would be asked in the described event
C. Information/ data	C1. Existence	The information given should be available in the real-life context.
	C2. Realism	The value presented in the WP should be reasonable or should be very close to the value in a real-life context
	C3. Specificity	The context given in the WP should be specific with regards to students' life
D. Presentation	D1. Mode	The WP presented written, visually, or orally should be considered
	D2. Language	Difficult terms, sentence structure, and amount of text should be considered
E. Solution Strategies	E1. Availability	The role of students in the story presented in WP should be clear
	E2. Experienced Plausibility	Strategy experienced is plausible for both school situation and real-life situation
F. Circumstances	F1. Availability of external tools	Availability of external tools (for example, calculator or software) should be considered
	F2. Guidance	Implicit or explicit hints in solving WP should be considered

	F3. Consultation and collaboration	Whether WP should be solved individually or collaboratively should be considered
	F4. Discussion opportunities	Opportunity to discuss the meaning of WP should be considered
	F5. Time	Time restrictions should be made such that there is no significant difference between solving the WP in school and real life context.
	F6. Consequences	Pressure and motivation created by how close the real-life context presented in the WP to the solver should be considered.
G. Solution requirements		The validity of the student's answer should be interpreted broadly as close as possible to the real-life context
H. Purpose	H1. Purpose in the figurative context	The purpose of finding the answer in WP should be made as clear as in the real-life context.
	H2. Purpose in the social context	Whether the WP serves as a real social purpose in a real-life context should be considered (not merely math problem 'dressed up' in real-life context')

Table 1: The framework of word problem reality aspects by Palm (2006)

METHOD

The study employed both quantitative and qualitative analysis. The quantitative analysis was done descriptively, while qualitative analysis was done through excerpt coding. The following sections explain the instruments, data collection, and data analysis of the study.

Instruments

The study used two instruments for data collection, i.e., a survey of beliefs toward WP realism (English translation is provided in Annex 1) and interview guidelines. The survey was developed to measure prospective teachers' beliefs. On the survey, participants were asked to complete each item on a four-point-scale: strongly agree, agree, disagree, and strongly disagree. The 36-item-survey was constructed based on Palm's framework, as shown in Table 1.

Each sub-aspect was manifested into two survey items, a positive statement, and a negative statement. Each sub-aspect's positive item contained a statement showing positive beliefs toward

the sub-aspect while the negative item contained negative beliefs. For example, the positive item for the sub-aspect 'Event' is "The event described in the word problem should have taken place or have a fair chance of taking place in real life." In contrast, the corresponding negative item is "The event described in the word problem does not necessarily happen in real life." On positive items, the greater scale shows greater beliefs in word problems' realism, while on negative items, the greater scale shows greater beliefs in non-realism. For instance, participant responding 'strongly agree' in positive items is most likely having realistic beliefs toward word problems.

Lift Problem

This is the sign in a lift at an office block:

This lift can carry up to 14 people.

In the morning rush, 269 people want to go up in this lift. How many times must it go up?

Bakery Problem

In a bakery, you see a 20 cm long cylinder-shaped Swiss roll. A dissection straight through the cake produces a circular shape with a diameter of 7 cm. The points of time in a day when the Swiss rolls are all sold are normally distributed with mean of 5.30 p.m. and a standard deviation of 15 minutes.

What is the volume of the Swiss roll?

What is the probability that the Swiss rolls are all sold before 6.00 p.m. when the bakery closes?

Figure 1: Problems for the interview, adapted from Palm (2006)

The content validity of the survey was checked and revised by two mathematics education experts. The construct validity was checked by conducting a statistical test correlating each item score to the total score (all items are valid with value < .05). The reliability of the survey revealed the Alpha-Cronbach coefficient of 0.773, which is considered reliable.

The interview guideline was divided into two parts. The first one is the two-word problems adapted from Palm (2006), as given in Figure 1. The second one is the list of questions about those problems and were constructed for the study needs. The guiding questions were (but not limited to): (1) describe your thinking about the problem, (2) have you made realistic considerations about the problem? Explain your reason, (3) if you were to modify the problem, how would that be? (4) what are the things you consider in giving or creating a word problem, and why do you believe so?

Data Collection

An online survey was distributed to 28 Mathematics prospective teachers of the first to the fourth year in one university in Jakarta, Indonesia. The survey was created using an online survey platform, and its link was distributed through university email. The survey link was made available for two months and only received one response for each participant. Participants' responses were recorded automatically by the platform feature.

To further clarify participants' stance toward word problems, a semi-structured interview was conducted. Six prospective teachers were being interviewed, two from each beliefs category, i.e., realistic, neutral, and non-realistic. Each participant was neither informed about the word problems (as shown in Figure 2) nor about the list of questions prior to the interview to allow for original responses. Each participant was also notified separately upon the interview's schedule and was interviewed individually to minimize the possibility of sharing information. The interview process was started by giving the participants time to read the first problem and solve it if they wanted to. After that, participants were asked the guideline questions related to their beliefs in the problem. This process continued similarly for the second word problem. All interview processes were recorded and transcribed for analysis.

Data Analysis

Prospective teachers' beliefs about the realism of word problems in Mathematics teaching was analyzed from Mathematics education students' responses to the survey and interviews. Participants' responses to the survey were converted to numerical values. For positive items, respond of "strongly disagree", "disagree", "agree" and "strongly agree" were converted to a score of 1, 2, 3, and 4 consecutively. On the other hand, for negative items, respond of those were converted to a score of 4, 3, 2, and 1 consecutively. The score of 3 and 4 indicated a tendency to realistic beliefs toward word problems. For each participant, the number of responses scoring 3 or 4 was counted. Then, the proportion of responses to all items was calculated by dividing the number of responses scoring 3 or 4 by 36. The illustration of this process was given in Figure 2.

A proportion greater than 0.5 inferred realistic beliefs and a proportion smaller than 0.5 inferred non-realistic beliefs. The exact proportion of 0.5 was also considered, which later was inferred as neutral beliefs. The same calculation process was done to responses in each aspect of the framework in Table 1.

Participant A	Response	Score
Item 1 (positive item)	Agree	3

Item 2 (negative item)	Strongly disagree	4	Conversion to score
Item 3 (positive item)	Disagree	2	
and so on	…	…	Proportion calculation
Number of responses scoring 3 or 4		14	
Proportion		$\dfrac{14}{36} = 0.39$	

Figure 2: Illustration of response conversion

Interview transcripts were coded based on the aspect being identified and analyzed further to explain each type of prospective teachers' beliefs. Statements indicating opinions on a certain aspect were coded as given in Table 2. The opinion was coded referring to the description of each aspect (Table 1) inferred and then written into memos. For instance, an agreement or disagreement to numerical value presented in the word problem would be included to "information" aspect (code C), especially in "realism" sub-aspect; while an opinion to modify the question due to its impossible event to happen would be included to "event" aspect (code A). The code-based statements were collected and then categorized based on similarity of beliefs presented within them. The analysis from the interviews was to enrich the findings obtained from survey.

Aspect	Code	Aspect	Code
Event	A	Solution Strategies	E
Question	B	Circumstances	F
Information/ data	C	Solution requirements	G
Presentation	D	Purpose	H

Table 1: Coding for analyzing excerpts

RESULTS

Indicated by the proportion of realistic answers given by all prospective teachers answers on the survey, there was an overall trend of realistic beliefs toward word problems ($n = 19$), while the rest of them showed non-realistic ($n = 7$) and neutral beliefs ($n = 2$). For example, 78.5% of prospective teachers responded "agree" or "strongly agree" to the item "The event described in the word problem should possibly take place or have a fair chance of taking place in real life." Similar

responses also emerged from the item "The value presented in the WP should be reasonable or should be very close to the value in real-life context," which yielded 96% of prospective teachers.

Aspect	% of participants		
	Realism	Non-Realism	Neutral
Event	60,7	14,3	25,0
Question	32,1	32,1	35,7
Information	46,4	25,0	28,6
Presentation	25,0	25,0	50,0
Solution Strategies	64,3	14,3	21,4
Circumstances	21,4	53,6	25,0
Solution Requirements	57,1	10,7	32,1
Purpose	21,4	39,3	39,3

Table 2: Beliefs in each aspect of WP realism

Prospective teachers' responses were broken down into each aspect, and it was revealed that realism beliefs were not that prevalent (as shown in Table 2). More prospective teachers responded with realism beliefs in 'event', 'information', 'solution strategies', and 'solution requirements' aspects, while more of them held non-realism beliefs in the 'circumstances' aspect. More prospective teachers gave neutral beliefs in the' question' aspect, but with more even distribution with other categories of beliefs. In the 'purpose' and 'presentation' aspect, non-realism and neutral beliefs were held most. To further understand prospective teachers' beliefs, an interview was conducted with two prospective teachers in each category of realistic, non-realistic, and neutral beliefs inferred from the survey.

Realistic Beliefs

Realistic beliefs revealed from prospective teachers' responses during interviews related to their views on the two-word problems. Their responses in the interview confirmed their stance inferred from the survey. Two prospective teachers with realistic beliefs tended to see word problems as close as possible to reality. One of the notable responses came from participant A. Her response related to the realism aspect of value presented in the lift problem was shown below.

I think it is not realistic in the real world for two hundred sixty-nine people for one lift. You know, like there are, in the (our campus), there are six lifts for not more than 100 people, I think in one time.

This response showed that she believed the value presented in the WP should be as close as possible to the value that existed in real life. After being asked her opinion regarding the possibility that the "real value" may make students struggle with the numbers instead of the problem itself, she emphasized that the case would actually be a good thing as it would lead to a fruitful discussion involving analytical thinking.

Another prospective teacher, participant B, stated an interesting view on how close a value should be to the real value with realistic beliefs. Participant B stated that she would choose the number of people for a struggling student that will lead to an integer number for the answer as it will have a more straightforward calculation. However, she stated the following related to a more advanced student.

For students who have achieved the standard or more, I will give this problem so they will be more challenged. Yes, the activity in it is the reflection of daily life, and students can use their learning to do a real calculation that is more complex.

Participant B acknowledged that the real value might lead to a more complex calculation, making students feel more challenged. Regarding the mode and language used in presenting the problem, the two prospective teachers with realistic beliefs showed a preference for a more familiar word than a technical one for the bakery problem. The following is the excerpt of participant A's opinion on the word "dissection" in the bakery problem.

The "dissection straight." I think "dissection straight" is not a familiar word. I think just that.

Participant A found the word to be confusing as it was not a familiar word for students. She further stated that the word might hamper students from imagining what was being asked in the problem. In this case, participant A showed a tendency to consider students' preference for words. Although not mentioning the same word, Participant B suggested removing the word "cylinder-shaped" and preferred giving students the picture of Swiss roll instead, as written in the excerpt below.

But I will delete the word 'cylinder-shaped. The reason is that this word makes the problem wordy, and maybe students who actually can solve the problem but not familiar with the word may be distracted. Besides, if the picture of Swiss roll is provided, there is no need for the additional word.

Participant B, like participant A, consider the efficiency of words presented in the problem. The highlight of both participants' opinions is their reason for making the word portrays the real situation and help students 'imagine' the situation and 'not distracted' by the word instead.

The prospective teacher also stated their opinions on the question of the bakery problem. Participant A thought that there is no real intention of asking the volume of bread. Her suggestion to modify the question is given in the following excerpt.

What about giving information on how many productions made each day? Without giving diameter information. So, how much flour will be needed for one Swiss roll? After this, give the information of diameter, then it can be connected to asking the weight of Swiss roll.

Her suggestion showed a preference for putting the question as close as possible to real context. She wondered about the purpose of asking the volume of bread and thus asking the amount of flour needed will be more relevant. Participant B gave an opinion that it is not common to use bread as the context of finding volume. She stated as follows.

Interesting. It is my first time seeing a question like this. Usually, the concept of volume is introduced using water, sand, or other objects whose shape can change, adjusting its container.

Both prospective teachers did not agree to give the original questions to students. Participant B still deliberately allowed the question to be given to upper-level school but preferred to modify them. Participant A emphasized this case quite often, as illustrated in the interview record below.

Interviewer   : if this one is an unrealistic problem, do you think it is okay to give students unrealistic problems?

Participant A   : I think no

Interviewer   : Why give students an unrealistic problem is not okay?

Participant A   : Because I have been there as a student, and I was really confused about the questions because I just like this question, I mean like who give one lift for 269 people, I was like Oh My God, I think it would be beneficial for the students if they see the real world and of course, okay, real-world is there, but mathematics is also there. So, for example, NASA makes a rocket plane to outer space, so we can imagine the real making of a rocket plane is like this, so everything is not only an imagination.

Participant A shows her beliefs by stating her personal reflection as a student, and she emphasized the importance of seeing Mathematics not separated from the world at all. In another part of the interview, she repeated her stance as follows.

In my opinion, if we can give the realistic (problem), why not giving it?

Both prospective teachers with realistic beliefs showed their realistic view toward word problems by suggesting a more realistic version of the given word problems by modifying the information, question, or language and mode of presenting the problem. Their further stance also includes avoiding giving an unrealistic problem to students whenever possible.

Non-realistic Beliefs

Non-realistic beliefs were shown through two prospective teachers' opinions on the realism of value presented in the Lift problem. Participant C preferred to give a value resulting in an integer

answer after being divided by 14. Her consideration on whether a problem with a non-integer answer could be given was due to students' level. Her opinion is given in the following interview record.

Participant C   : After calculating, 269 divided by 14 is 18 point something, while 266/14 is 19. But again, I think where this problem context is located? If it is in senior high school, I think the first one is okay because they (students) will know decimal, but for junior high school, I think I will choose the second value.

Interviewer   : Do you think the value of 266 is realistic?

Participant C   : I think it is not. We don't really know how many people will be in the lift, right? I think 269 is more realistic, but 266 is more comfortable to solve.

The interview noted that Participant C chose a word problem based on its relevance to the students' grade level. Her consideration was heavier on calculation skills needed to solve the problem rather than how realistic the value presented there. Related to the Bakery problem, Participant A chose to have the volume question modified into the weight question in which the weight in each volume unit is known. However, her reason was not that the weight was more reasonable to be asked, but due to its relevance with high school students' skills. She also mentioned her view on the picture given in the question, as shown below.

If needed, we can give images based on the context of the problem. I have once found an image in a problem, but they are not related. The image is just there but not related to the problem.

Further, her general view on whether a realistic problem should be given to students was written below.

Interviewer   : So, do you agree that word problems should be realistic?

Participant C   : I don't really agree that word problems should always be realistic.

Interviewer   : if the word problem is not realistic, do you think you will allow it to be given to students?

Participant C   : Yes, as long as appropriate with the grade and learning objective.

Participant C believed that word problem does not have to be realistic as long as it is appropriate with students' level and the learning objective. She further emphasized that her priority in choosing word problems is their alignment with the learning objective. A similar belief was presented by participant D, who stated that some things cannot always be realistic. Her interesting thought is written below.

Interviewer   : When we create a word problem, should it be realistic?

Participant D   : Hmm, but don't you think some things cannot be made realistic? It is good if it is realistic, but some things cannot be made realistic. For example, in geometry, the perfect square is the idea in our mind. In real life, we rarely find it.

She said that a realistic word problem is good, but she acknowledged that some things could not be made realistic. She thought that it would be helpful if the word problem were realistic, but due to the reason she mentioned before, she stated that as long as the problem can be imagined, that will be okay. Regarding the standard deviation information in the Bakery problem, participant D thought there is a rare possibility of its availability. However, despite thinking that, she stated as follows.

I think it is okay, but the teacher must explain it first, maybe if the students ask. For example, students read (the question) maybe they understand some words, but not understand the others. Maybe if the students get confused, the teacher can give clues.

Participant D showed affirmation of giving a word problem despite it being unrealistic in terms of its information. She further stated that the teacher might give students clues if they did not understand the information.

Neutral Beliefs

Some exciting opinions came from participants with neutral beliefs (with an answer proportion of 0.5). The interesting opinions differ by what they believe as 'acceptable' real-life situations. Participant E stated that what is considered real by some students may not be real for others. His opinion is stated below.

First, because this is a problem, I will see the students' grades. Is this for primary or secondary school students? This is about how realistic, right? The environments surrounding primary and secondary students must be different.

His thought revealed that what is considered realistic by a certain level of students might be unrealistic to others, because they might have different environments. He further explained that some WPs might be realistic for scientists but not for students.

Word problem is used to make students understand mathematics, but we know that many word problems are outside our daily lives, and I think closer to scientists' or academicians' lives. We may understand the context, but we don't find it in our real-life.

Participant E thought that the concept of realism in life might be different from it in the context of science, including mathematics. On the other hand, participant F had an opinion on what is important in deciding if WP is okay to be given to students or not. She highlighted that if the problem could help students learn doing the procedure, it was fine. She exemplified this by telling her experience below.

Like a problem given to my sister yesterday, 'there is a kid who buys three buckets of fish, and there are 35 fish in each bucket. Then, it was asked how many fish the kid had. That is not realistic, right? If we think about it, it is not realistic for a kid to buy three buckets of fish, and moreover, there are 35 fish in each. But it is okay; I think because the purpose is to learn multiplication.

In her example, she mentioned that because the problem helped students learn multiplication, it was acceptable to be given, even with unrealistic values. Like Participant E, she showed beliefs on two versions of "realistic", realistic in life and mathematics. She explained that what is not realistic in life may be acceptable in the context of mathematics learning.

Because if we bring realism to real-life, for example, we go to a bakery and are asked to determine the cake's weight, that is not realistic. But, in the context of mathematics, I think that is (short break) that is acceptable. Yes, that is calculating.

The notion of neutral beliefs shows the relativity of what participants perceived as "realistic". They may not have the exact definition of a realistic word problem, but they would consider the learning's cultural and developmental setting.

DISCUSSION

The findings of this research revealed some distinctions among the three types of beliefs possessed by prospective teachers. The main points are mainly about their definition of realistic WP and whether they will give realistic WP to students. Participants with realistic beliefs show a strong emphasis on making the word problem as closely as possible to students' lives. They believe that a realistic word problem should be manifested into the WP elements: the information, questions, and mode of presentation. They believe that the information should possess data, value, or facts available in real life with an accurate value as possible.

The question given in the WP should also be a question possible to be asked in real life. The mode of presentation, they believe, should avoid any ambiguity, and portray the real-life situation as straightforward as possible. This finding is similar to the beliefs found in the study by Chen et al. (2011). In the study, participants thought it is not realistic to measure the ability to run 1 kilometer far using an ability to run 100 meters. The participants thought that the problem should be formulated in an exact, unambiguous way.

A strong objection in giving unrealistic problems came from the participants with realistic beliefs. They believe that the consequence of making calculation more complicated by having close-to-real-life information will positively contribute to the discussion and thinking process during the learning process. This view has been inferred in a study stating that word problems should support students to acknowledge that mathematics learning is closely related to their life (Verschaffel, et al. 2001).

Participants with non-realistic beliefs have more practical consideration on whether a particular WP can be given to students. They mainly consider students' level of skill to do the needed calculation or procedure. Participants with these beliefs are found to prioritize less, if not disregard, the relevance of WP questions to real life. Most importantly, their beliefs show the stance of a good WP is the one that can be imagined; it does not have to fully simulate real life, if it is mathematically "acceptable". The beliefs are pretty interesting, as it resonates with what Galbraith and Stillman (2001) stated as a 'context-separable problem', in which a real-world context exists but is not used to solve the problem. Participants with these beliefs see the context-separable problem as an acceptable problem to be given to students.

Participants with neutral beliefs tend to question to whom the reality matters. Their opinion lies on a relative degree of what is called real. They consider several aspects of WP's realism, such as students' level of understanding, age, experience, and cultural setting. This "grey" beliefs have been noted by De Lange (1995), who stated that there might not be an exact way of determining the proximity of real-life to WP. This may be due to students' varied experiences and backgrounds, so, understandably, some prospective teachers believe it that way.

CONCLUSIONS

This study has described several points differentiating three types of beliefs possessed by prospective mathematics teachers toward WP: realistic, non-realistic, and neutral. By understanding the characteristics of each beliefs type, it is hoped that more effort can be made in promoting meaningful use of word problems to prospective teachers. Each beliefs' characteristics can also contribute to the broader understanding of teachers' beliefs, not only in word problem posing but also in the teaching practices in general. Besides, beliefs have been one of the essential factors in driving teachers' practices; thus, this study will provide ample insights into addressing the issue from an affective factor.

Due to the small sample used in this study, the findings cannot be generalized. Still, it provides valuable descriptions of prospective teachers' s toward WP's realism and its types of distinction. Further research can be done by investigating whether prospective teachers' beliefs are affected by their lecturers or whether the possessed beliefs are put into practice or compromised instead.

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Annex 1. The items of questionnaire of beliefs towards word problem
(This is the English translation. The original questionnaire was given in Indonesian language.)

1. The event described in the word problem has taken place or has a fair chance of taking place.
2. Value presented in the word problem should make sense or close to the actual value in real life.
3. Difficult terms, sentence structure, and amount of text should not be avoided for the word problem creator.
4. The use of external tools (such as calculator and software) in solving word problems should be allowed.
5. Teachers should consider whether solving a particular WP will need a discussion based on the real-life context.
6. Teachers should accept answers that use real assumptions of the context presented in WP (for example, not impossible for someone to carry 80 watermelons, etc.).
7. The events described in WP don't have to be really possible in the real world.
8. The values used in WP do not necessarily correspond to real-world values.
9. WP makers as much as possible simplify or reduce terms, complex sentence structures, and sentence length in WP.
10. Cannot complete WP with tools (such as calculators or software).
11. Each WP should be able to be done without any opportunity for discussion about the different meanings.
12. When there is a question with the sentence 'Ani can make 100 statues in 1 hour', we can ignore the assumption that this sentence cannot happen in real life.
13. Questions like, "What is the volume of the loaf?" should not be given to students because in real life no one really needs to know the volume of a loaf.
14. The context presented in the WP should be specific based on the real life of the students who were given question.
15. Students should be able to position their roles in the context of the WP, not merely solving problems as people outside the story.
16. WP solution instructions (such as 'start by calculating the total cost') should not be provided as such guidance would not be available in real-life problems.
17. The WP completion deadline should consider the time to solve similar real-life problems.
18. The purpose of finding answers to a WP should be made as clear as possible like finding answers to everyday problems.
19. It is okay for a WP to ask things that you don't really need to know in the real world, for example, questions about the volume of a cylindrical cake.

20. The context in WP does not have to be based on the real life of students who are given questions.
21. Solving story problems with a point of view outside the context of the problem is fine.
22. It's okay to give hints (hints, clues) on how to work on WP.
23. The WP completion time limit can be determined without considering whether the same problem in the real world can be solved within that time limit.
24. The purpose of finding WP answers is okay only to solve math problems, even though it has little relevance to real life.
25. The information given to WP should actually exist in real life. Information about the standard deviation, for example, should not be given but calculated first.
26. The word problem, as the name suggests, must be given in the form of words.
27. Strategies for completing WP should be possible for students both in school situations and in their daily lives.
28. Whether a WP can be done individually or in groups based on its real-life context should be considered.
29. Consequences of completing WP should be made as close as possible to the real-life of students (eg by presenting WP answers about a policy to local officials, etc.).
30. WP is a daily life problem that requires mathematics to solve.
31. In WP, it's okay to give information about a value that doesn't actually exist in the real world, for example, information about the average deviation.
32. WP does not have to be given in words.
33. The strategies needed to solve story problems may only be completed at school, not in everyday life, because at school students receive the support of assistive devices.
34. Each WP should be done individually by students, regardless of the actual context.
35. WP's answer is quite related to the real context by discussing in class.
36. WP is a math problem wrapped in the context of everyday life.