"Math Is So Much More"

The Design, Implementation, and Outcomes of an Elective Mathematics Methods Course

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Abstract

There is a growing need for teacher preparation courses that support teacher candidates (TCs) in meeting requirements to obtain a supplemental teaching credential. In particular, given the demands for secondary mathematics teachers, developing effective cross-disciplinary coursework within preparation programs is timely and important work. Our study examines the development and evolution of TCs' experiences in a mathematics teaching methods course designed for TCs seeking a supplemental credential. We describe the design principles and practices

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used to create, and modify, a course to meet (a) state licensure requirements, (b) the goals of the teacher education program, and (c) the individual needs of the TCs. Using survey and interview data, we report on TCs' experiences with the course and their future plans for obtaining a supplemental teaching credential in mathematics. Our study contributes to discussions on teacher preparation for interdisciplinary teaching, licensure requirements, conceptual and practical tools for teaching, and effective course design.

Introduction

Secondary teacher preparation often happens within discipline-specific silos. These silos can occur when curriculum and instruction (C&I) courses—the hallmark teaching methodology courses for acquiring pedagogical knowledge and skills—focus on a single disciplinary domain. Although some teacher education programs provide opportunities for teachers to work across disciplines (Davis et al., 2019; Ryu et al., 2019), the majority focus on the discipline-specific knowledge required to become a single-subject secondary teacher. Yet, many factors push these programs to break out of silos and embrace opportunities for teacher candidates (TCs) to cross disciplinary lines.

One factor influencing this shift is TCs' desire to become credentialed in multiple subjects (e.g., obtain a single-subject credential in both science and mathematics). Secondary TCs are finding supplemental credentials valuable on the job market. As teacher shortages in mathematics increase and the pipeline for qualified TCs shrinks (Carver-Thomas & Darling-Hammond, 2017), secondary teachers from other disciplines often seek a mathematics credential to increase their employment prospects. To ensure that TCs who pursue a supplemental credential are well prepared, some states have instituted coursework requirements. For example, California requires secondary TCs seeking a supplemental credential to show subject matter proficiency via coursework or examination and to complete a C&I course in the subject area (44225 California Education Code § 80499). The C&I course requirement has motivated universities to provide access to these courses for teachers seeking a supplemental credential. Whereas some universities opened enrollment in existing C&I courses to supplemental credential seekers (see California State University, Long Beach, n.d.), other universities created new courses specifically designed for supplemental credential seekers (Stanford University, 2021). Depending on the setting, these courses can include current classroom teachers who already have one credential or preservice TCs.

Another factor that may encourage the removal of silos within teacher preparation programs is the necessity to educate students for a knowledge economy (Darling-Hammond et al., 2019), which requires that students think flexibly about current problems. Recent initiatives in K–12 schools include STEM or STEAM programs, which integrate practices from multiple disciplines. Such integration is not a new pursuit; more than 100 years ago, Dewey (1915) argued that designing

education around experiences in which students inquire about the world allows them to synthesize different subjects naturally. Although this call has been taken up by numerous educational reform movements across the decades, such as project-based learning (Savery, 2015) and cored teaching teams (math and science, English and history; Applebee et al., 2007), the vast majority of schools—and teacher education programs—offer courses that are organized within silos along conventional disciplinary lines (Beane, 2016). Despite the interdisciplinary demands of the 21st century, teacher education programs rarely see secondary teachers as interdisciplinarians. Even for teachers who do not formally enact interdisciplinary instruction, the opportunity to analyze curriculum and pedagogy from a different disciplinary perspective can provide opportunities for more flexible thinking and expand teachers' pedagogical repertoires.

We report on the efforts of one teacher education program to provide an elective methods course that meets the C&I requirement for supplementary credential seekers while also offering TCs an opportunity for additional professional growth. Stanford University introduced a suite of courses that allow prospective teachers to take a C&I course in an elective discipline in addition to their primary discipline. In this initiative, elective C&I (eC&I) courses are offered in mathematics, science, history, and English in the final quarter of the 12-month program. All TCs had the option to take a course in a discipline of their choosing.

In this article, we report on the mathematics eC&I course, which included TCs with primary specializations in science, history, English, and world languages. First, we describe how the mathematics eC&I course was initially implemented. Then, we explore how the mathematics eC&I course evolved over 4 years of implementation. Finally, we examine the TCs' outcomes and experiences of participating in the course. Specifically, we address the following research questions:

RQ1. How did the eC&I course evolve with different cohorts?

RQ2. How did participation in the eC&I course influence teacher candidates' professional trajectories?

RQ3. What did the teacher candidates report they learned in the course?

In this way, our study contributes to the growing knowledge base of experiences that may help teacher preparation programs design courses for supplemental credential–seeking candidates. In doing so, we highlight modifications made by instructors and candidate-reported experiences within a course. We also look to increase the field's understanding of which pedagogical tools translate across disciplines. Finally, this work highlights an experience that provides TCs opportunities to prepare as interdisciplinary practitioners.

The Design of an Elective Mathematics C&I Course

The mathematics eC&I course was a culminating experience within the Stanford Teacher Education Program (STEP). Because this course occurred in the final quarter of the preparation program, assumptions were made about the candidates' conceptions of teaching based on program-level pedagogical strategies. The design of the course was also heavily influenced by the conceptual and practical considerations that informed the design of the mathematics C&I course sequence. Figure 1 is a visual representation of the design considerations as the eC&I course was built on the foundations of the C&I courses and the philosophical and pedagogical principles of STEP in general. The following sections describe the design principles that were leveraged from each level.

Stanford Teacher Education Program

To create an experience that helped TCs develop into confident new teachers, STEP drew on the following teacher education pedagogical strategies: analyzing beliefs and forming new visions of instruction (Feiman-Nemser, 2001; Hammerness, 2006), artifacts and representations of practice (Ball & Cohen, 1999; Little, 2003), rehearsals (Lampert et al., 2013), and the principle of overcorrection (Grossman, 1991). The principle of overcorrection occurs when TCs participate in idealized learning experiences supported by research-based pedagogical practices with the goal of counteracting the effects of the "apprenticeship of observation" (Lortie, 1975, p. 62)—the 13 plus years students spend observing their own education. STEP coursework incorporated multiple artifacts and representations of practices, such as video cases, samples of student work, and planning documents. As TCs

Figure 1
Theoretical Foundations of the Design of the eC&I Course

experienced and analyzed teaching across the courses within the program, they had opportunities to practice teaching through rehearsals. These experiences exposed TCs to ambitious teaching through experiencing, interrogating, and enacting high-quality teaching practices.

STEP's C&I course sequences shared common universal course design aspects and differed by content and grade band-specific design considerations. Complex instruction (Cohen & Lotan, 2014) is a common design principle across STEP. Complex instruction supports student-centered learning experiences within heterogeneous small groups. Its key tenets include collaborative learning, broad opportunities to show competence, and mitigation of academic status issues (e.g., who is considered "smart" in relation to content). Complex instruction was taught in cross-cohort sections, allowing for content-focused units via small-group activities.

Mathematics C&I Courses

The mathematics C&I (referred to as C&I in this study) course sequence included three consecutive 10-week classes. C&I augmented complex instruction via additional pedagogical theory, which supported the tenets of: (a) overcorrection, via norm setting, culture building, doing mathematics together, and status mitigations (if necessary); (b) rehearsals, both with TC colleagues wearing "student hats" and with actual students in placement settings; and (c) infusion of the theory of growth mindset—the belief that everyone is capable of doing mathematics and that someone's mathematical capability is not fixed—into all interactions within the course (Boaler, 2015). Key considerations included how the TCs positioned themselves, and each other, in terms of mathematical status. While every TC had obtained passing scores on mathematical content exams, they differed in prior experiences learning mathematics. Thus every cohort brought the potential of status challenges, with some students feeling, or being positioned as, more or less capable than others. To mitigate any academic status differences, instructors positioned each student as mathematically competent through the complex instruction practice of status treatments (Cohen & Lotan, 2014). With an understanding that learning to teach is an ongoing, lifelong process, C&I aimed to equip TCs with a solid foundation of practical and conceptual tools for teaching.

Within the C&I courses, the TCs build on their knowledge for teaching. Specifically, the TCs participated in a learning experience that provided them opportunities to acquire discipline-specific knowledge for teaching mathematics. We draw on the work of Grossman et al. (1999) and their definitions of conceptual and practical tools when conceptualizing the specific knowledge that is available within the mathematics C&I course. Conceptual tools include the theories, principles, and frameworks that teachers use to think about, reflect on, and critique their own practice. Practical tools are the instructional practice and material resources that teachers employ within their classrooms.

Mathematics eC&I Course

The eC&I course was organized on the same principles as the C&I sequence, but it faced three logistical challenges. These included compressing the most essential course content from the three-quarter C&I course into one quarter, reckoning with the lack of mathematics-specific teaching placements, and maintaining high expectations for course engagement while acknowledging the competing demands and burnout many students experienced in the fourth and final quarter in their program. The eC&I course also differed from C&I in terms of the students' general prior experience with mathematics, their access to secondary mathematics students, and the degree of commitment to teaching mathematics.

The design for this course began with the question of how to include the most important elements of the three-quarter math C&I course. In practice, this meant examining the C&I course sequence syllabi and lesson plans, choosing key readings and activities, and modifying where necessary to account for lack of access to secondary mathematics students and classrooms. The modifications included substituting eC&I students in "student hats" for actual secondary mathematics students and relying on video records of practice to take the place of immersive observations in mathematics classrooms. Special consideration was paid to identifying and mitigating potential status differences and (re)framing what it means to know and do mathematics. In general, the eC&I course aimed to equip students with the same starter kit of practical and conceptual tools as C&I candidates while acknowledging the challenges that might impact the depth and fluency of acquisition. Because the instructors had no knowledge of which students would go on to attain secondary certification, the default goal was preparing all students to teach secondary mathematics effectively. With the assumption that eC&I candidates pursuing the additional credential would be required to show mathematical content proficiency through coursework or exam, the eC&I course was designed to meet the methods requirement of the credentialing process.

With the constraints described, the eC&I course included conceptual and practical tools. Conceptual tools included (re)framing what counts as mathematics and rigor (Boaler, 2015), (re)framing mathematical competence (Horn, 2012), and content organization through big ideas (Charles & Carmel, 2005). Practical tools included norm setting (Boaler, 2015); task choice, design, and launch (Jackson et al., 2012); productive discourse (Kazemi, 1998); number talks (Humphreys & Parker, 2018); the five practices for orchestrating classroom discussion (Smith & Stein, 2018); assessment for learning (Black & Wiliam, 1998); examining video recordings of mathematics pedagogy (Boaler et al., 2003); and diagnostic questioning and feedback (Boaler & Brodie, 2004).

Positionality

Robin, Jennifer, Rosa, and Anthony were instructors of the math eC&I course.

Emily taught the science methods courses in the eC&I program, which meant that she worked with many of the eC&I participants in other contexts. Peter was the director of STEP and led the research, design, and implementation of the elective program. We consider our close involvement with the program as an advantage in this work; we have a strong understanding of the context, goals, participants, and structure of the course and broader program. At the same time, we have taken steps to mitigate bias through strategies like participating in formal interviews, questioning each others' interpretations during analysis, and engaging in collective conversations that involved both course instructors and outside observers. We elaborate on these efforts throughout the methods section of the study.

Methods

This study draws on data from a larger study of the eC&I program over the first 3 years of implementation. The larger study included surveys and interviews of course participants (TCs) and interviews of course instructors. Results of the larger study were used to provide recommendations to instructors for strengthening the elective program.

Participants

Over the first 3 years of the program, 38 students participated in the math eC&I course. Of the participants, 20 responded to a survey, and eight continued to participate in an interview. Of the survey respondents, 10 were currently participating in the elective course, two had participated in the previous year, and eight had participated two years earlier. These students were distributed across four primary subjects: science, world languages, English, and history/social sciences (see Table 1).

In addition, four instructors participated in the research. The instructors of the first two iterations of the course shared their reflections on the course implementation through formal interviews. Additionally, the instructors of the third and fourth iterations of the course participated in later analysis to highlight similarities and differences between courses. Students from the fourth iteration of the course are

Table 1
Distributions of Participants Across Primary Subject Areas

Subject area	*Math eC&I course participants*	*Survey respondents*	*Interview respondents*
Science	30	16	7
History/social sciences	4	2	1
World languages	3	2	0
English	1	0	0
Total	38	20	8

not included in the student data set as they had not taken the course at the time of data collection.

Data Sources

The following data were drawn on in the analysis of the design and TCs' experiences in the mathematics eC&I course.

TC Surveys. Surveys ($n = 20$) consisted of selected-response and constructed-response items. Selected-response items asked TCs to report on their motivations for taking the course, plans to pursue a credential, confidence/preparedness to teach mathematics, other outcomes of participating in the course, and professional trajectories. Open-ended items asked students to report on what they learned in the course, including *conceptual tools*, which were framed as "principles, frameworks, and ideas," and *practical tools*, which were framed as "practices, strategies, and resources." In addition, TCs were asked to report on how they currently draw from or plan to draw from what they learned in the elective course in their teaching. Surveys were emailed to both current students and alumni through the program's Listserv. Responses were anonymous, unless students added their contact information expressing willingness to participate in interviews.

TC Interviews. In semistructured interviews ($n = 8$), TCs elaborated on the topics from the surveys. In addition, TCs contextualized their experiences in the math elective course by talking about how they came into the field of teaching and what they hoped to accomplish in their future classrooms and schools. Interviews were conducted by Robin and Emily.

Instructor Interviews and Reflections. In semistructured interviews ($n = 2$), instructors shared instructional goals, instructional designs, successes, challenges, and perceived outcomes of the course. Interviews were conducted by Robin and Emily.

Data Analysis

The following analysis methods were used for each type of data.

Surveys. We drew from survey data to characterize different professional trajectories of the TCs in regard to math teaching (no intention to pursue a math credential/position, originally intended to pursue a math credential/position but did not pursue this path, pursuing a math credential/position). We conducted a content analysis using a priori codes (Grossman et al., 1999). Finally, we report overall descriptive statistics on TCs' motivation to take the course and confidence and preparedness to teach math after participating in the course.

Student Interviews. We drew on grounded theory (Charmaz, 2014) to identify patterns within the interview data. For the first pass of data, we used an iterative

open-coding technique. These codes were then refined to represent broader themes (Miles et al., 2019).

Instructor Interviews. During the writing process, instructors reviewed their interviews and discussed their reflections to come to common understandings about the goals, successes, and challenges of the course and to identify similarities and differences in their experiences and approaches.

Case Studies. We purposively selected cases ($n = 3$) of TCs who represented three different professional trajectories. We drew from their interviews to report their specific experiences related to each of the three broad themes that were identified across the corpus of interviews. Pseudonyms were selected to represent race and gender. Gender was assigned on the basis of the participant's preferred pronouns.

Findings

Findings from the data analysis are presented in answer to the following research questions:

RQ1. How did the eC&I course evolve with different cohorts?

RQ2. How did participation in the eC&I course influence teacher candidates' professional trajectories?

RQ3. What did the teacher candidates report they learned in the course?

We summarized general themes from instructor interviews to answer RQ1. RQ2 and RQ3 were answered using student surveys and interviews. After a summary of general themes for RQ2 and RQ3, detailed case studies of eC&I participants are presented.

Instructor Experiences

To answer RQ1, instructor interviews were analyzed to identify modifications made to the course. First we report individual instructor modifications to the syllabus, then we summarize general modifications that impacted all implementation years.

Year 1. Jennifer faced the additional challenges of designing and launching the course and adapting it in real time. Jennifer regularly used exit tickets to inform modifications of the course and elicit questions and challenges for whole-class discussion. That first cohort included 1 history and 13 science TCs. The course heavily featured doing mathematical problems together, so Jennifer brought in tasks that both met the standards of C&I (e.g., low floor–high ceiling, multiple entry points) and related to science constructs to bridge content areas.

Year 2. The second cohort consisted of eight science, one English, and four social science TCs. Anthony attended to perceived ability and status differences due

to variations in TCs' math ability. Additionally, based on Jennifer's feedback from the first iteration of the course, Anthony limited out-of-class assignments to allow candidates more time to gain access to students and classrooms. He also chose mathematics tasks that were more related to middle school mathematics concepts than Jennifer's tasks, which focused primarily on science.

Year 3. The third cohort consisted of 10 science, two world languages, and two history TCs. As the instructor of the third iteration of the course, Robin leveraged the modifications previously implemented and was able to make fewer modifications during the implementation of the course. Just as Anthony attended to status, Robin found that status issues surfaced early and combated this by attending to co-created group norms that fostered equal status. In response to one history candidate's desire to focus on social justice issues within mathematics classrooms, Robin modified readings to include more work by Rochelle Gutiérrez and Rico Gutstein.

Year 4. The fourth cohort consisted of six science and two world languages TCs. One major modification to the delivery of the course was the shift to use three online modules to engage students on the topic of teaching mathematics with technology. Other modifications included centering discussions of how the candidates could apply teaching principles to their primary subjects and drawing on tasks that could be interdisciplinary in nature by connecting mathematics with their primary subjects.

Across the Years. Offering the course in the final quarter provided unique opportunities. Instructors reported that students had well-developed understandings of multiple pedagogical tools. Instructors leveraged these tools while making modifications to the course, including knowledge of group work strategies, general principles of equitable classrooms, and argumentation within disciplines. Robin reported that history candidates drew on their training within history C&I to use primary sources to construct arguments, which supports the Common Core State Standards Mathematical Practice Standard 3: Constructing Viable Arguments and Critiquing the Reasoning of Others (National Governors Association, 2010). Situating the elective course at the end of the school year allowed for instructors to build from, and connect, learning in other STEP courses.

Instructors also shared that offering the course at the end of a yearlong program was difficult. Instructors reported that students felt pressed for time to accomplish course requirements because of the additional demands of credentialing requirements and job searches. Because the instructors explicitly planned the course to be flexible to reflect the needs of the students, modifications were made to the syllabus to reduce readings and focus more on experiencing mathematics learning. The timing of the course also led to classroom culture considerations. Instructors reported that because the course was offered at the end of the preparation program, the TCs had an established cohort culture. Instructors reported that this was both challeng-

ing, in terms of establishing classroom norms, and beneficial, as the cohort had already established productive working relationships in previous courses. Taking this into consideration, instructors made space for previously established norms to be discussed during early norm-setting activities.

Overall, instructors reported that attending to individual student needs guided their modifications of the course. Having a clear understanding of candidates' background knowledge of both the mathematics content and pedagogical tools informed how instructors chose tasks, modeled pedagogical strategies, and created practice-based activities. In the next section, we report on how the students' experiences in the course affected their professional trajectories and overall learning to become a teacher.

Student Experiences

Candidates had several shared experiences and takeaways from the elective course. We begin by presenting descriptions of the themes that emerge from the interview and survey data. These themes are reported because they occurred in a majority of the candidates' data. We then provide contextual descriptions of these themes through the presentation of three case studies. These case studies were developed as rich descriptions of individual experiences that exemplify the different ways the course influenced candidates' professional trajectories.

Professional Trajectories. The primary intention of the creation and design of the course was to prepare participants to become mathematics teachers. One instructor reported, "So when I think about the goals for any class, especially a methods class like this, what I'm really thinking about is when would I feel comfortable signing off saying this person is prepared to be an excellent first-year math instructor?" The desire to become a mathematics teacher was shared by a majority of the participants in the course (see Table 2). Yet, after completing the course, only two candidates had obtained the credential, and two others were in the process of

Table 2
Candidates' Desire to Pursue Teaching Mathematics Credential Before and After eC&I

Primary subject area	*Survey respondents*	*Desire to teach math before taking eC&I*	*Obtained, or in the process of obtaining, the math credential after the course*
Science	16	10	4
History/social sciences	2	2	0
World languages	2	2	0
English	0	0	0
Total	20	14	4

pursuing it. Candidates attributed their decrease in desire to obtain the credential to how different mathematics teaching was from what they remembered, the abbreviated nature of the 10-week course, and lack of authentic practice-based learning experiences. Overall, after completing the course, candidates were less likely to pursue the mathematics supplemental credential.

Acquiring Pedagogical Tools. When asked on the survey to describe their learning from the course, TCs reported acquiring both conceptual and practical tools. Conceptual tools that TCs reported included growth mindset in mathematics, focusing on ideas rather than the right answer, valuing mistakes, and teaching through open-ended tasks (Boaler, 2015). The practical tools candidates described included number talks (Humphreys & Parker, 2018), launching tasks (Jackson et al., 2012), and using strategies from *5 Practices for Orchestrating Productive Mathematics Discussions* (Smith & Stein, 2018). Although participating in the eC&I course decreased TCs' overall desire to teach mathematics, 17 of 20 candidates surveyed reported that taking the eC&I course increased their confidence in teaching in their primary subjects. The TCs shared that the conceptual and practical tools learned in the eC&I complemented or augmented pedagogical tools acquired in their primary subjects' C&I courses. One tool that most TCs drew on in their justification was the concept of growth mindset. The TCs reported that after taking eC&I, they used growth mindset ideas when working individually with students. A majority of science TCs reported that when they approach mathematical content in their classes, they engage their students in productive discourse (Kazemi, 1998), a practical tool from the elective course.

Shifting Perception of Mathematics. With a primary focus on what mathematics learning looks like at the classroom level, TCs were immersed in learning experiences that often positioned them as mathematics learners. The immersive experience within the course was mentioned when TCs shared how their views of mathematics teaching and learning changed through taking the course. The TCs reported that, prior to the course, their vision of teaching and learning mathematics was grounded in their previous experiences as students or tutors. Transmission-based forms of mathematical instruction were foregrounded in the TCs own experiences as students, such as attributing success in mathematics to speed and accuracy (Ruef, 2013, 2021). But after taking the elective course, the TCs described mathematics as a complex subject with pedagogical tools that were previously invisible to them.

Interdisciplinary Experiences. The TCs reported two different interdisciplinary advantages to taking the mathematics eC&I course: one connected to their own learning experiences and another connected to the learning experiences they create within their primary-subject classrooms. First, TCs reported that learning from instructors outside their primary discipline, and with students from other disciplines, was an advantage of the course. Learning from a mathematics teacher educator provided candidates with another expert teacher to observe and draw on when developing

their individual teacher identities. Although the majority of the TCs who took the elective course were science candidates, a few came from other disciplines. TCs reported the diversity in classmates as an advantage of the course and noted that learning mathematics within a heterogeneous group provided interesting discussion opportunities as they filtered their learning through the lens of their primary discipline. Although many TCs abandoned obtaining a mathematics credential, they did report the desire to incorporate mathematics into their primary subjects. Some candidates reported using mathematics within their content to interpret the world, while others grappled with how to support mathematics conceptually within their subject areas.

Case Studies

The following case studies report on individual experiences within the elective course. The case study teachers represent three different professional trajectories as a result of the course. The three professional trajectories can be described as (a) planned to pursue credential before course but did not pursue credential after the course (Margaret); (b) planned to pursue credential before the course and obtained credential after the course (Minh); and (c) did not plan to pursue credential before the course and did not pursue credential after the course (Deborah).

The Case of Margaret: Revelatory Experience.

Professional Trajectory. Margaret's primary subject was science. Prior to taking the course, she was pursuing different types of teaching positions, some in science and others in mathematics. At the conclusion of the course, she reported that she was offered a mathematics position that she did not take, instead accepting a high school physics position. She attributed rejecting the mathematics position to her experiences in the elective course.

Reported Learning Outcomes. Margaret came to STEP after a short career in the animal rescue industry. To prepare for her pursuit of a teaching credential, Margaret took a part-time tutoring job the year before entering the program. During the tutoring experience, Margaret felt successful in her ability to support students in their mathematics learning. This experience inspired Margaret both to take the eC&I course and to pursue the mathematics credential. At the same time as she was taking the eC&I course, Margaret was pursuing jobs for the following school year. Her experience tutoring inspired her to apply to both science and mathematics positions. Taking the eC&I course drastically changed Margaret's professional trajectory, overhauled her perception of mathematics learning, and provided her with new pedagogical tools to support mathematics learning within her physics classroom.

Within 2 weeks of starting the eC&I course, Margaret received a job offer to teach mathematics. She shared in her interview that she initially leveraged taking the eC&I course to pursue the mathematics position but quickly realized that the course was drastically changing her perception of mathematics teaching and learn-

ing. Even before finishing the elective course, Margaret had decided that she was not ready to teach mathematics:

> So, I ended up not taking that position because of the math [e]C&I, actually. I went into it thinking, "I know how to teach math, because I know how to tutor kids on math procedures." And what I've learned in math C&I is that math is so much more than procedures. It's not just about numeracy and number fluency stuff. It's about being able to excite students about big ideas. I actually told the people who offered me the math job, "I can't, because I just learned that math is so much more."

Margaret's experiences within the course exposed her to mathematics pedagogy that was different from what she felt comfortable enacting and from what she had used as a tutor. Although Margaret felt confident in supporting students as they worked on procedures and fact fluency, she did not feel confident in supporting the mathematics learning experiences that were taught within the eC&I course. Through her experiences in the eC&I course, Margaret's perception of the teaching and learning of mathematics changed. She learned that mathematics teaching and learning went beyond learning procedures and facts, and because of this revelation, she did not accept the mathematics teaching position.

While her experience in the eC&I course discouraged her from becoming a mathematics teacher, Margaret reported that taking the course inspired her to change the way she worked with students on mathematics. Her previous tutoring experience provided her multiple interactions with students who were averse to doing mathematics. Prior to the course, Margaret tutored students in mathematics using a cheerleader approach. She encouraged students through the procedural content, not attempting to make connections between the conceptual and procedural aspects of the curriculum:

> Working last year with students who were animated and joyous until the conversation shifted to numbers and then they shut down. I knew that people had this emotional fit. I thought last year, well the thing that I can do is instill in my students about my belief in them and hopefully they will internalize that. Then we'll learn some math procedures together. What I've learned through [e]C&I is it's not just we learn some math procedures, it's we think about how do the big ideas of math give us tools to ask and answer exciting questions.

Throughout her interview, Margaret attributed her shifting perception toward mathematics to teaching through "big ideas," the conceptual frames that undergird discrete content standards (Charles & Carmel, 2005). The evolution of her understanding of teaching mathematics, from focusing on procedures and facts to teaching for conceptual understanding through big ideas, inspired her to change the way she worked with students. While her experience dissuaded her from taking on a full-time mathematics position, Margaret noted that the course changed the way she supported mathematics content in her physics class. She shared that while physics can often become a "sneaky math class," she was inspired by the elective course to keep the discussion of the mathematics concepts at a conceptual level and not get "bogged down in the procedures."

While pursuing the credential was a key reason for Margaret's initial course taking, her experience in the course shifted her perception of her readiness to teach mathematics. The course changed the way she viewed mathematics as a subject. She shifted from seeing mathematics teachers as experts who build learning experiences to develop student procedural fluency to seeing them as experts who help students build understandings of the nature of mathematics through conceptually connected big ideas.

The Case of Minh: Science and Mathematics Connections and Disconnections

Professional Trajectory. Minh's primary subject was science, and her student teaching placement was in a sixth-grade science and mathematics classroom. At the start of the elective course, she had already accepted a teaching position as a sixth-grade science and mathematics teacher. Her experience in the course did not affect her intention to get the mathematics credential, as it was a requirement for her teaching position in the fall.

Impact of the Elective Course. Minh came to STEP after completing her PhD in science. Minh shared that she decided to become a teacher to make an impact in her local community. She completed her student teaching and subsequently accepted a teaching position at the middle school in the community where she had grown up. Knowing that her position for the following school year relied on her obtaining a mathematics credential, Minh's primary intention for taking the eC&I course was to experience a condensed mathematics methods course and fulfill the state's course requirement to apply for the supplemental credential.

Minh often filtered her new learnings and experiences through what she had learned while becoming a science teacher. To interpret her experiences in the elective course, Minh reported connections and disconnections between the teaching of mathematics and of science. Minh found connections between teaching mathematics and teaching science using classroom discussion strategies presented in *5 Practices for Orchestrating Productive Mathematics Discussions* (Smith & Stein, 2018). Her experiences in the eC&I course also revealed disconnections between the two subjects, specifically around grounding learning experiences in context. As Minh reported on how the eC&I course impacted her understanding of teaching mathematics, she constantly drew on her previous experiences as a developing science teacher.

Leading a class discussion surfaced as an important learning from Minh's experience in the eC&I course. She reported that using *5 Practices* to systematically develop a learning experience that culminated in a whole-class discussion was a skill that she could use in both her mathematics and science classes in the future. Although discussion was a frequently supported practical teaching tool in her science C&I courses, Minh saw the explicit structure taught in the mathematics elective course as helpful:

I definitely think you could apply those practices to science instruction because it's

> the same idea, basically. You could do some anticipatory work to think ahead about where students are going to have difficulties and then based on those, structure your lesson and the conversations you're going to have about the work.

The eC&I course provided both exposure to *5 Practices*, which detailed the steps from planning to orchestrating a discussion, and also time to rehearse the execution of the practices. Through the elective experience, Minh developed a more practical approach to planning and leading whole-class discussions that impacted her pedagogical approaches to teaching both mathematics and science.

Minh also reported that some topics were in direct contrast with her experiences within the science C&I courses. The attention to context in mathematics was different from science. Through many of the activities in the mathematics eC&I course, Minh noticed that the context within the problems was either stripped away or backgrounded. She contrasted this to her understanding of teaching science through context-rich problems:

> I think the ultimate message I got from the math [e]C&I was that you can put context to the side. It's not the most important thing for you to ground whatever concepts you're learning in real-life stuff, whereas I feel like in science C&I, that was something that was emphasized to us throughout, was that you have to constantly be connecting what you're doing to the real world. That is an area where I saw some disjunction between the two subjects.

This experience challenged Minh's perception of teaching and learning generally but also specifically challenged what she thought good teaching should be in a mathematics classroom.

Minh reported feeling hesitant to bring context-free mathematics to her future sixth-grade students. Although she spent 10 weeks in the eC&I course predominantly doing abstract mathematics problems without context, Minh hoped to draw on her training as a science teacher to develop mathematics problems that helped students see the relevance of mathematics:

> I find it very hard for me to try to convince students to do math just for math's sake. . . . We just kept doing these contextless problems, and for me, that was a big point of frustration. Like, how do we actually ground this in something the students will actually care about? I will be teaching math next year; I want to make the problems as context-grounded as possible.

The disconnect between her training as a science and as a mathematics teacher inspired Minh to reflect on her ability to teach mathematics. Her training as a science teacher dominated her understanding of context within teaching, and she planned to subvert the pedagogical approaches foregrounded in the mathematics eC&I course and use contextually rich problems.

As a TC who entered the elective course knowing she was going to teach mathematics in the following fall, Minh reported both connections and disconnections between preparing to be a mathematics and preparing to be a science teacher.

Although Minh saw the pedagogical tool of discussion as a beneficial practice to bring to both her mathematics and science teaching, she also reported as a disconnect how the different subjects attended to context. The elective course provided a condensed experience in training to be a mathematics teacher that enhanced Minh's pedagogical tools while challenging her perception of the role of disciplinary teaching and learning.

The Case of Deborah: Never Going to Be a Mathematics Teacher

Professional Trajectory. Deborah's primary subject was history. She did not plan to obtain the supplemental credential in mathematics. Deborah reported that her main goals for taking the course were to improve her own relationship with mathematics and to learn better ways of supporting marginalized students in mathematics.

Impact of the Elective Course. Deborah came to STEP to pursue a teaching credential in history. She had earned an undergraduate degree in government and spent time working in a political research firm after college. Deborah changed career directions, shifting away from politics and toward education, with the goal of supporting students of color. She saw taking the eC&I course as part of her mission to increase access by developing ways to support mathematics learning. Although she liked math in high school, Deborah shared, "[because of a] failure of growth mindset, I didn't pursue it in college." One of her goals in taking the eC&I was to "continue learning math and inspire students to continue math, especially from minority backgrounds." Deborah's experiences in the eC&I course allowed her to engage in, and learn the advantages of, heterogeneous learning opportunities that leverage differences among students. She also acquired practical pedagogical tools that she planned to implement in her history classroom.

Deborah reported that experiencing mathematics learning within a heterogeneous environment broadened her perception of who could do mathematics. Prior to taking the elective course, Deborah's perception of mathematics ability was grounded in a fixed mindset, often associated with performance assessments. Within the elective course, Deborah experienced the value of multiple perspectives:

> I came from a history background, and the other students were from science backgrounds; we just approached problems in such different ways, and each way was unique and amazing to see. So, it was collaborative in the sense that we learned how to give praise to each other and to respect each others' lenses in solving problems. And we were always together. There was sometimes individual work to just think in the beginning, and then it was, "Hey, let's find different methods to solve this problem."

Because the eC&I course drew students from different subjects together to do mathematics, Deborah was able to experience collaborative learning that leveraged diverse perspectives. The opportunity to learn in this heterogeneous environment shifted Deborah's perspective on mathematics learning. Having previously experienced the valuation of one way of doing mathematics, eC&I provided her

an environment where multiple ways of knowing and doing created collaborative learning experiences.

The eC&I course also introduced Deborah to pedagogical tools that she planned to incorporate into her future history teaching. She discussed the sequencing strategy from *5 Practices* as a tool she would leverage to bring multiple student voices to a class discussion. Building from her newfound perspective of heterogeneous collaborative mathematics learning, Deborah explained an activity in which student work samples were unpacked and candidates discussed how to sequence the work meaningfully:

> I just felt certain strategies I learned were really great. Such as one lesson in which we unpacked student work in a series, so take five pieces of student work and then develop a story around them. What does this student do well, how does another student elaborate on that answer. How does a student's work illuminate what another student did wrong. So, that was a really good skill I've learned, that you can do it in math, like in problem solving, but you can also do that with history and writing, and also any kind of assignment.

The pedagogical tool of sequencing student work provided Deborah with diverse student ideas that supported building a common understanding of the content. Deborah, just like Minh, was able to envision using pedagogical tools introduced through *5 Practices* within her primary subject.

Deborah's original goal was to help marginalized students in mathematics; she never planned to pursue the mathematics credential. Therefore, when asked, Deborah reported that through changing her perception of mathematics, she would be able to support her students also to change their perceptions of mathematics. Deborah's new appreciation of mathematics as a collaborative subject with multiple ways of knowing and doing impacted how she positioned the discipline of mathematics to her students. Instead of framing mathematics as a rote subject with only one way of being correct, Deborah now encourages her students to be "more open about mistakes" and to share their thinking through "open-minded dialogue" when interacting in mathematics class. Deborah's experience in the eC&I course positioned her as an ally with mathematics teachers by amplifying mathematics as a collaborative subject.

Case Study Summary. The case studies presented herein provide contextual details of how the eC&I course affected the TCs differently. The three case studies exemplify different professional trajectories. Minh entered the course knowing she was going to teach mathematics in the fall, and taking the course did not affect her plans for the future. On the other hand, taking the eC&I course dissuaded Margaret from pursuing mathematics teaching positions. Deborah never intended to teach mathematics, and the course did not change her plans. Although their professional trajectories were different, these teachers all reported acquiring pedagogical tools that could help them across content areas. Finally, the course provided these teach-

ers time to reflect on mathematics teaching and learning: Margaret changed her perception of mathematics teaching, Minh challenged the disconnect between the role of context within mathematics teaching, and Deborah experienced mathematics learning that leveraged diversity of perspectives.

Discussion

As a contribution to teacher education literature, this study demonstrates how the development of an eC&I course, in response to local credentialing policies, can affect professional trajectories, combat siloed teacher education by providing an interdisciplinary experience, and diversify pedagogical tools across disciplines.

Changing TCs' Beliefs About Mathematics Teaching and Learning

The eC&I course was originally created to meet the requirement of a methods course to obtain a secondary single-subject credential in California. Based on the findings of RQ2, a majority of TCs (10 of 14) changed their trajectories after taking the course. Although we cannot say it was solely the course that changed their plans, the majority attributed their decrease in desire to obtain the credential to how different mathematics learning was from what they remembered, the abbreviated nature of the 10-week course, and the lack of authentic practice-based learning experiences. The TCs were influenced by the collaborative nature of the course, often noting that their perception of math learning changed as they participated in doing mathematics in heterogeneous groups. These types of interactions are critical for students to experience as they challenge and inform their beliefs about teaching (Hammerness, 2006) and allow them to participate in a learning experience grounded in the principle of overcorrection (Grossman, 1991). We argue that these experiences must take place in settings as close to real classrooms as possible. We see the TCs' desire for practice-based learning as a request for placements in classrooms with real students learning mathematics. As teacher education programs develop methods courses to meet requirements for credentialing, we recommend face-to-face or online synchronous courses that provide genuine experiences that might challenge candidates' previous ideas of mathematics teaching and learning. Without these experiences, candidates may continue to perceive mathematics teaching and learning as they learned them during their apprenticeship of observation as students (Lortie, 1975).

Breaking Down Silos

The eC&I program also challenges the current siloed nature of some teacher education programs. Although programs are providing times for candidates to interact across disciplines in courses on equity, classroom management, and assessment, rarely are candidates interacting around C&I. Some might also argue that the siloed C&I sequences produce robust disciplinary teaching, but we argue through our

findings that programs should provide opportunities where heterogeneous groups of candidates learn about subject specific pedagogical tools for teaching. Our findings from RQ3 illuminate the potential for cross-content transfer of pedagogical tools. Candidates reported that they learned new ways of preparing and leading whole-class discussions. The concept of discussion has been identified as a "core practice" (McDonald et al., 2013) and is often described and enacted differently across disciplines (Grossman & Dean, 2019). Our findings suggest that candidates are transferring the pedagogical tool of leading a mathematics discussion into other subjects. This supports the loosening of the siloed model of preparing TCs and challenges the discipline-specific ideas of core practices.

The opportunity to learn across disciplines also provided candidates opportunities to reflect on the pedagogical practices and epistemological assumptions of their primary disciplines. As candidates learned about pedagogical tools used within mathematics, they often juxtaposed them with similar tools within their primary disciplines. The opportunity to reflect on and analyze pedagogical practices (Ball & Cohen, 1999) across disciplines provided time to interrogate and question deeply held assumptions with the disciplines. Minh, the science candidate, grappled with the attendance to context within mathematics and science. The experiences in the eC&I course provided her an opportunity to again justify why context is important in science and question its absence in mathematics. The ability to interrogate the teaching and learning within both mathematics and a candidate's primary discipline helped the TCs develop as practitioners through reflective experiences that are absent when disciplines are siloed for C&I courses.

Informing Teacher Education Program Coursework and Practice

Our findings may also inform coursework and practices within teacher education programs that currently provide one C&I course, or one sequence of courses, to support multiple single-subject TCs. These courses are often labeled as STEM methods courses or general teaching methods courses. Within these courses, TCs across multiple disciplines are learning pedagogical tools together. Through the intentional design of the eC&I course, we found that instructors created learning experiences that allowed candidates outside of mathematics to see the application of mathematics to their disciplines differently. Specifically, science candidates reported that they developed ways of supporting mathematics learning within their content that went beyond just supporting the procedural nature of using mathematics as a tool in science. As teacher education programs look to support candidates from across disciplines to develop pedagogical tools, the findings of RQ3, specifically around which tools candidates were acquiring within eC&I, can help inform development of these multidiscipline methods courses.

The findings from this study also helped STEP refine the eC&I course. As reported in RQ2, the instructors created yearly changes that informed future itera-

tions of the course. These changes were made to enhance the experience for all students, especially those pursuing the supplemental credential. The findings from RQ3 also provided insight into changes needed to make the course more contextually relevant. As Minh shared tensions around lack of focus on context within the eC&I course, these findings highlighted the need to add more context-dependent mathematics to the eC&I coursework.

Flexibility in Course Design to Meet Students' Needs

Studying the design of the course was not an initial goal of our study. Yet, we do report on modifications made by instructors during the enactment of the course through RQ1. These findings inform how other programs might design and initiate an eC&I course in mathematics. Although the course was created to meet credentialing requirements, many students did not pursue a credential. The lack of credential-seeking candidates led the instructors to grapple with developing mathematics teachers in addition to developing nonmathematics teachers to integrate mathematics into their subjects. Although instructors did make modifications to meet the needs of candidates (more participation quizzes in Year 3) and requests of candidates (more science-specific applications in Year 1), they also valued mathematics-specific experiences (more math talks in Year 2). These findings highlight the importance of being strategic and flexible when selecting the goals of the eC&I course. The importance of knowing the candidates was clear as instructors reported on the modifications they made to the course.

Conclusions

Limitations

Our study has limitations that need to be addressed. First, our study findings are influenced by selection bias as TCs self-selected into the research study. Of the 38 TCs who took the course, only 20 answered the call to complete a survey. Of those 20, 8 volunteered to be interviewed. Another limitation is that a majority of participants were science TCs, and though this does reflect the course-taking trends, we are still missing important insights from non-STEM TCs. Finally, our findings are limited to self-reports by TCs. While they do report on changes in their perceptions and teaching intentions, we do not know about enactment of pedagogical tools in their teaching practices. To address some of these limitations, we propose the following future research.

Future Research

This study suggests that when teacher preparation programs unsilo C&I courses, TCs participate in learning opportunities that enhance their understanding of teaching and learning. To measure whether these experiences changed candidates'

pedagogical approaches to teaching, we propose extending our project into the classrooms of eC&I graduates. Following TCs into their classrooms would allow observation of which aspects of the eC&I course are enacted. We also propose a follow-up survey to capture the continued needs of the eC&I students. As the course is rarely preparing candidates to become mathematics teachers, we would aim to ask eC&I participants about their continued challenges in supporting mathematics learning in their classrooms.

In closing, we realize that not all programs are able to offer an additional elective C&I course. Therefore we propose that teacher educators work within their institutions to understand how interdisciplinary teacher preparation can best be cultivated within different programmatic constraints. We also seek to expand our research to look within programs that offer only general C&I courses or STEM C&I courses. These courses are natural spaces where interdisciplinary teacher training is occurring, unsiloed and prime for exchanging pedagogical tools across disciplines.

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