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Facilitating Preschool Children's Mathematics Development in China, Japan, and the United States: Is the Classroom Library Considered?

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Abstract: The two studies examined in this paper compare the different mathematical opportunities provided in preschool classrooms in China, Japan, and the United States, with an emphasis on mathematical-themed books in classroom libraries. Study one presents the results of an online survey to examining the content of preschool classroom libraries in China (N = 134), Japan (N = 168), and the United States (N = 291). Study two presents data obtained from semi-structured interviews of teachers in China (N = 8), Japan (N = 8), and the United States (N = 8). The interviews examined teacher perceptions of how they teach mathematics, the importance of teaching mathematics, and the use of the classroom library as a venue for mathematics. Study one results indicated that teachers from all three countries encourage classroom library use; however, teachers from China reported more mathematics storybooks than their Japanese or United States counterparts. Study two results indicated that teachers from all three countries viewed mathematics as important and provided various mathematics learning opportunities to children throughout the school day. Chinese teachers reported providing the most mathematics learning opportunities using whole group instruction, mathematics centers, and free play. Japanese teachers reported few whole group forms of instruction other than circle time but reported providing opportunities for using mathematics during free play and other embedded activities. United States teachers indicated that mathematics learning occurred using whole group instruction and mathematics centers.

Keywords: preschool children's mathematics development; mathematics development; classroom libraries

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Educators and researchers have known for years that children in the United States do not do as well in mathematics as children from East Asian countries. More specifically, children in the United States consistently score significantly lower on research-designed and standardized mathematics assessments than their peers in China and Japan [1–3]. Most of the research comparing math performance with children from different countries has been based on children in elementary school and older [2,3]. However, differences across countries are evident even before children enter kindergarten [4,5]. We know that children's mathematics skills in kindergarten predict their subsequent development [6]. Therefore, we need to consider their mathematics skills before the start of formal schooling. In the following sections we will use the term "preschool" to refer to school that occurs between the ages of three and five years. This includes kindergarten in China, *yōchien* in Japan, and preschool/pre-kindergarten in the United States.

The longitudinal impact of early math skills has been well-documented [6–10], although most of the studies did not include children in preschool. Previous research has

established the association between early-grade mathematics knowledge and later mathematics achievement; however, few studies have measured mathematical skills prior to school entry. One such study by Watts et al. (2014) [11] related children's mathematical skills at 54 months to their mathematics achievement during adolescence using data from the NICHD Study of Early Child Care and Youth Development. Children's mathematics skills assessed in preschool predicted their mathematics skills at age 15. Such findings, as well as those showing differences in the mathematics skills children from the United States and East Asian countries bring to the start of formal school, highlight the need to focus on mathematics instruction in preschool.

The current study explores how preschool teachers in China, Japan, and the United States view themselves teaching mathematics, with particular interest in the role of classroom libraries. The major theoretical framework for the study is Bronfenbrenner's (1979) [12] ecological model which posits that children's development occurs in several overlapping contexts (e.g., microsystems) and stresses that these contexts need to work well together (mesosystems) to optimize children's development. Development is further impacted by societal influences which may differ across countries (macrosystems). All three contexts or systems have an influence on children's mathematics development. A secondary influence is motivational theories such as those by Eccles and Pomerantz [13] who discuss how children's experiences with mathematics activities increase their interest in mathematics [14]. As we discuss later in the Introduction, children's interest may be spurred in situations where they have more agency or control over the selection of their activities (e.g., picking out reading materials in classroom libraries).

Consistent with Bronfenbrenner's (1979) [12] model, children's mathematics development is influenced by the experiences they have at home and their school-related experiences. We focus here on how mathematics is taught in children's preschools in China, Japan, and the United States. As Ni et al. (2010) [15] and others have noted, however, some of the differences in children's mathematics development in these three countries may reflect differences in the nature of language in the countries and parents' expectations and practices, as well as what takes place in school. That is, the Chinese and Japanese languages, unlike English, make explicit the base 10 substructure which underlies basic mathematical operations [15]. Chinese and Japanese societies also emphasize much more strongly than does the United States the importance of mathematics [16,17]. Within the home, Chinese and Japanese parents emphasize the equal importance of reading and mathematics [17] whereas in the United States, parents more strongly emphasize reading [18]. Although this paper focuses on preschools and mathematics instruction, consistent with Bronfenbrenner's (1979) [12] theory, we believe these three factors, the home and school microsystems and the societal macrosystem in each country, interact.

We turn next to considering educational philosophy and instruction in China, Japan, and the United States, followed by a section on how preschool children learn mathematics, and then the use of classroom libraries for fostering growth in mathematics skills. The final section in the Introduction is the present study.

1. Preschool Classrooms in China, Japan, and the United States

In China, preschool is provided to children between the ages of three and six years using primarily full-day programs. In the last 20 years, early childhood education in China has undergone what many consider an increase in westernization practices, such as developmentally appropriate practices [19]. These reforms include implementing traditionally Western theories of learning such as those of Piaget, Montessori, and Reggio Emelia [20]. However, while some Chinese educators have implemented child-directed learning and other developmentally appropriate practices, many do not view such practices as culturally appropriate [19] and continue to use teacher-directed learning [21,22]. Although Western approaches are becoming more evident in Chinese preschools, mathematics instruction in many or most preschool classes still relies heavily on direct teacher instruction and repeated practice [15,23].

In Japan, *yōchien* is the educational program for children ages three to six, similar to kindergarten in China and preschool/pre-kindergarten in the United States. Japanese preschool is based on the idea that children learn through play, while teachers structure the environment to observe their learning [24]. *Yōchien* is designed to focus on social/emotional development, building friendships, and good citizenship (http://www.usjp.org/jpeducation_en/jpEdPrimary_en.html (accessed on 15 October 2021). Preschool teachers in Japan engage in practices that embed mathematics into every part of the school day including routines, music, storybook reading, and attendance [25]. While some structured instruction does occur, almost 98% of the mathematics activities are embedded within general activities throughout the day [25].

Preschool classrooms in the United States are designed as venues for developmentally appropriate practice (DAP) and are, at least in theory, designed as places that provide "a strengths-based, play-based approach to joyful, engaged learning" [26] (National Association for Educating Young Children, NAEYC). NAEYC, the United States accrediting agency for early childhood, suggests that the classroom be centered around child strengths and embrace instruction that reflects the cultural, linguistical, and ability diversity of each child. While early academics are addressed in United States preschools, many teachers focus on learning through play. To the degree that there is academic-related instruction, it focuses more on reading than mathematics [27]. When academic instruction occurs, it is done in whole- and small-group settings in addition to child-directed time such as free choice and centers [27].

2. How Preschool Children Learn Mathematics

Children develop mathematical skills and understanding in a variety of ways. One is through the curriculum they receive in school [28]. In China, preschool children are taught numeracy skills including counting numbers in the correct order, identifying numbers, apprehending the relations between numbers, and performing simple addition and subtraction [29]. Unlike what occurs in Chinese preschools, in Japan mathematics instruction in preschool is embedded in everyday classroom activities [30,31]. According to recent observations in Japanese preschools, about 40% of teacher/child interaction involved mathematics, most commonly counting [25]. In the United States, there is relatively little mathematics instruction in preschool with children spending as little as 24 min a day on mathematics activities [32]. The content of mathematics instruction when it occurs, focuses on counting and cardinality skills [33].

In addition to the mathematics curriculum, children learn through exposure to mathematical opportunities that both draw upon their prior experiences and engage their interests [13]. Such interest in mathematics generally leads to the development of mathematics skills [34]. Young children develop mathematical skills by engaging in mathematics activities, including the reading of mathematical storybooks, that they find engaging and interesting as well as teacher-led instruction [4,35]. Teachers may engage in direct instruction as well as in-depth mathematical conversations where teacher-child dyads share their thinking. While direct instruction may be a part of their mathematical learning, peer and adult modeling of mathematical ideas and play-based exposure are also critical to development [18,36]. When children's interests are combined with more traditional instruction, children are more likely to engage in mathematical activities and therefore develop a stronger mathematical foundation [13].

One type of activity that could engage children's interest in mathematics is the use of classroom libraries. These could be considered either a teacher-directed activity or a free choice activity. Based on observations of 2061 children in 652 prekindergarten classrooms in 11 states in the United States, Early et al. (2010) [27] found that only six percent of free choice time was spent engaging in some form of mathematics. However, Early et al. (2010) [27] did not separately consider classroom library time as part of free choice time, leaving the question of how the classroom library is used to foster mathematics development.

3. Math Storybooks and the Classroom Library

The research reviewed in this section is based mainly on teachers and children from Western countries, as research on how storybook reading in preschool classrooms occurs in East Asian countries is limited. There is a well-established positive association between storybook reading and literacy development, at least within Western research [37]. While there is less research with mathematics storybooks, there is a positive association between mathematics storybook reading and subsequent skill development. For example, Sonnenschein and Dowling (2019) [13] reviewed research showing that reading mathematics storybooks, either with a parent or in a classroom setting led to higher levels of mathematical engagement. Reading mathematics storybooks also increases mathematical conversations [38–41], and mathematical skill development [42].

When children read mathematical storybooks, either in whole group classroom settings or one-on-one with an adult, their mathematical skills increase [43–45]. Even books that do not explicitly focus on mathematics have been shown to be effective. van den Heuvel-Panhuizen and colleagues (2016) [46] found that when teachers read books with implicit mathematical themes (e.g., *The Very Hungry Caterpillar*) to their classes and ask questions about mathematical concepts, skills increased [47]. Even without direct teacher prompting, mathematics storybooks have been shown to be effective. Children who explore storybooks that contain implicit mathematical themes and concepts and then discuss them with their teachers are more likely to build on the mathematical concepts they were exposed to in school and thus develop more mathematical skills [45].

Despite the known effectiveness of mathematics storybooks, teachers, at least those in the United States, do not very frequently read mathematical storybooks with their children. Pentimonti et al. (2011) [48] observed 13 United States preschool teachers and found that out of the 426 books read, only 13 were mathematically themed. Yopp and Yopp (2011) [49] examined 120 read-alouds of informational texts. Their analysis indicated that while 85% of the informational texts were science related, only 1% were mathematics related. Given what we know about the effectiveness of reading mathematical storybooks and the commonality of classroom libraries, using the classroom library may be an untapped resource to increase children's exposure to mathematics.

4. The Present Study

Use of the classroom library is positively associated with literacy development [50,51] and the use of mathematics storybooks, in particular, facilitates young children's mathematics skills [44,45]. Despite this, the classroom library, at least in the United States, is not seen as a venue for mathematics learning [52]. We know that children in China and Japan, on average, start kindergarten or formal schooling with higher levels of mathematics skills than their peers from the United States [5,16]. We also know that there are differences in how mathematics instruction, in terms of style and content, is approached in the three countries. What is unknown is whether preschool teachers in China and Japan use the classroom library as an opportunity for mathematical engagement and/or learning.

Understanding preschool mathematical pedagogy from countries where children start formal schooling with stronger mathematical foundations may drive positive change for children in lower achieving countries. While differences in societal beliefs need to be considered, understanding effective pedagogy in different societies may provide a starting point.

Study one examined the results of an online survey distributed in China, Japan, and the United States. The online survey was used to examine the content of the preschool classroom libraries and if/how it is used for mathematics. The data from the United States sample came from a reanalysis of data from Stites et al. (2021) [52]. Study two used semi-structured interviews with preschool teachers in China, Japan, and the United States [52] to further examine the use of mathematics storybooks in the preschool classroom library as well as how preschool mathematics is taught in these three countries. This paper addresses the following main research questions:

Study 1:

How is the classroom library used in China, Japan, and the United States? Are there differences? Given what we know about mathematics instruction in the three countries, we expect there will be differences.

Study 2:

How do preschool teachers in China, Japan, and the United States report teaching mathematics? Are there differences? Given the differences in mathematics achievement we expect there to be differences.

4.1. STUDY 1

Study 1 examined if and how the classroom library is used to teach mathematics in preschool classrooms in China, Japan, and the United States. This study used a mixed-methods approach to investigate the content of preschool teachers' classroom libraries. Of particular interest was the relative number of mathematics texts in the libraries and whether and what teachers said they did to encourage children to use them.

4.1.1. Method

Participants

The total sample consisted of 594 teachers from China (N = 134), Japan (N = 168), and the United States (N = 291). Almost all teachers (98%) were females. Participants were recruited through social media sites (e.g., Facebook in the US and WeChat in China) that are moderated by reliable preschool teachers in the three countries. Additionally, in Japan, researchers contacted local preschools to recruit teachers.

Chinese teachers were significantly younger ($Mean_{age}$ =29.99, SD = 8.13) than Japanese teachers ($Mean_{age}$ =32.55, SD =10.11, p =0.035) who were significantly younger than United States teachers ($Mean_{age}$= 44.73, SD = 11.63, p = 0.001), $F(2, 541) = 112.57$, $p < 0.001$, $\eta p^2 = 0.294$. Not surprisingly, the United States teachers ($Mean$= 11.60, SD = 8.85) had a significantly greater number of years' experience teaching than their counterparts from China ($Mean$ = 4.43, SD = 8.13, $p < 0.001$) and Japan ($Mean$= 6.33, SD = 6.01, $p < 0.001$), $F(2, 436) = 40.65$, $p < 0.001$, $\eta p^2 = 0.157$. However, the group differences for number of years teaching were not significant after controlling for age, $F(2, 428)= 1.06$, $p = 0.348$, $\eta p^2 = 0.005$.

Consistent with what others have found with elementary school children (Organization for Economic Cooperation and Development, [OECD]2014, [53]), the mean number of children in Chinese classrooms was significantly higher (M = 25.51, SD = 7.69) than classrooms in Japan (M= 21.39, SD = 6.63, $p < 0.001$) and the United States (M= 17.33, SD = 7.19, $p < 0.001$), $F(2, 514) = 88.98$, $p < 0.001$, $\eta p^2 = 0.167$. The number of children in classrooms in Japan and the United States did not differ significantly, $p > 0.05$.

Measures

Participants reported on the content and usage of their classroom libraries and demographic information using a survey created by the researchers. It consisted of 37 multiple-choice and open-ended questions along with questions about the participant's demographic characteristics. The survey was piloted with 10 preschool teachers in the United States and items were modified as appropriate. Subsequently, the survey was reviewed by the Chinese and Japanese investigators to see if the items were consistent with mores and procedures in the respective countries. Changes were made to ensure that the measures were appropriate for each country. All materials (e.g., consent forms and IRB approved forms) and instruments used in the study were translated from English to Chinese and Japanese and back-translated by another bilingual individual. The researchers from the United States, Japan, and China then compared back-translated versions for linguistic and conceptual equivalence.

The online survey included teachers' reports on whether they had a library in the classroom, how frequently students used the library, and the content/types of books (e.g., informational, mathematics, ABC, or picture/story books) available to students in the

classroom library (Table 1). Teachers also responded to open-ended questions (Table 2) on if and how they encouraged their students to use the classroom library. Sample items included "*What type of books are available in your classroom library?*" and "*Do you do anything to encourage children to use the classroom library?*".

Table 1. Sample Survey Questions.

Sample Question	Response Type
On a typical day, approximately how many children are in your classroom?	Open-ended
Do you have a library in your classroom that is accessible to children?	Yes/No
What type of books are available in your classroom library?	Check all that apply
(a) Informational texts	
(b) Fantasy books	
(c) Story/picture books about people	
(d) Story/picture books about people	
(e) Math story books (e.g., books focused on math topics like counting etc.)	
(f) Other	
Of the books available in your classroom library, approximately how many of these are informational texts?	Open-ended
Of the books available in your classroom library, approximately how many of these are story/picture books?	Open-ended
Of the books available in your classroom library, approximately how many of these are math storybooks?	Open-ended
Do you allow children to take books/games from the classroom library home?	Yes/No
Are there materials you want/need for your classroom library, but are unable to get some reason?	Open-ended

Table 2. Interview Questions.

Sample Question	Response Type
Can you provide examples of how math is incorporated throughout the day?	Open-ended
Do you set aside explicit instructional time for math?	Yes/No
For each one of the activities described above can you tell me if these activities occur:	Open-ended
One-on-one group	
Whole group	
In teacher-led small groups or other?	
Do you have a math center(s)?	Yes/No
Do you do anything to encourage children to use math centers?	Open-ended
What types of math materials are available during free playtime? (Probe if needed: For example, blocks)	Open-ended
Do you do anything to encourage children to use math materials during free play?	Open-ended
Does your classroom library have math storybooks?	Yes/No
Can you tell more about this? What types of books does your library have? Can you provide specific titles?	Open-ended
Does the library include any math games?	Yes/No
When you read traditional storybooks aloud to children, do you ever incorporate math themes?	Yes/No
Do you use math activities in any other ways during the day that we have not talked about?	Open-ended
How important do you think it is to teach math to preschool children?	Likert scale 1–5
How confident are you in your ability to support your students' math learning?	Likert scale 1–5
What, if any, additional assistance would be helpful to make you better able to support your students' math learning?	Open-ended
Do you have any other thoughts or opinions about fostering math in preschool that you would like to share with us?	Open-ended

Procedure

The study was approved by the Institutional Review Board at the United States authors' academic institution (Protocol Number: Y18SS20204). The United States IRB served as the IRB of record for China and Japan. Authors from each country ensured questions were culturally and linguistically appropriate and reviewed transcripts [54].

The participants were screened for their eligibility (i.e., currently teaching 3- to 5-years-old children) prior to data collection. After receiving their written consent to participate in the study, the Qualtrics online survey link was distributed via email in the United States, China, and Japan, and paper copies were distributed as appropriate

Coding and Scoring of Data

Qualitative responses to survey questions were downloaded directly from Qualtrics and analyzed. Responses to open-ended questions were reviewed independently by two of the authors. The authors then compared their codes and reconciled any discrepancies. Overall, coders demonstrated 99% interrater reliability. The single discrepancy in coding was resolved by discussion.

4.1.2. Results

How the Classroom Library Is Used China, Japan, and the United States

Most preschool teachers in China (95%), Japan (98%), and the United States (98%) reported having a classroom library that was used an average of 18 min per day. Teachers in all three countries (China 71%, Japan 58%, United States 58%) reported that they encouraged the children to use the classroom library. As noted below, there were commonalities and differences in how teachers in the three countries encouraged children to make use of the libraries. Teachers from China encouraged children to use the library by using the books as read-alouds (42%), providing rewards to the children for reading books from the library (17%), and allowing children to take books home (5%). In Japan, teachers reported that they encouraged use of the library by selecting these books for read-alouds (40%), making the library an inviting place using attractive seating and displays (24%), and frequently rotating the books contained in the library (16%). Teachers from the United States encouraged use of the library through read-alouds (25%), rotating available materials in the library (18%) and choosing materials based on the children's interests (14%).

As noted above, teachers in each of the countries used read-alouds. In fact, this was the most common way they encouraged classroom library use (China 42%; Japan 40%; United States 25%). For example, a Chinese teacher said, "I read before meals and share stories before sleeping to encourage children to read in the book area", while a United States teacher said that she "reads a story from the library every day". By engaging in read-alouds, teachers served as role models of appropriate reading activities as well as exposed children to interesting content and interactions.

In addition to using books from the library as read-alouds, teachers in Japan (16%) and the United States (18%) also reported that they rotated the available materials in the library. For example, a teacher from Japan noted that she liked to "place books that suit the season or event in an easily visible place". A United States teacher mentioned, "Book rotation, I have a huge flannel board and I change it out with holidays and seasons. So, the students love that", when asked how she encourages library usage. This was not a strategy reported by teachers from China (<1%). Interestingly, 17% of teachers from China noted that they used rewards to encourage children to use the classroom library. For example, one teacher from China stated that she "uses stars to encourage reading more story books". The use of rewards was not something that was reported by teachers from either Japan or the United States.

Differences in Classroom Library Usage

Preschool classroom libraries in China, Japan, and the United States contained mathematics storybooks, traditional storybooks, and informational texts (Table 3). We considered the number type of books in two ways.

One, within each country we compared the types of books the library contained. Ninety-seven percent of Chinese teachers reported that their classroom library contained mathematics storybooks and that there were more mathematics books (M = 19.10, SD = 21.18), t (98) = 8.97, $p < 0.001$, than traditional storybooks (M = 15.04, SD = 15.84), t (98) = 9.49, $p < 0.001$, or informational texts (M = 10.68, SD = 14.17), t (98) = 7.49, $p < 0.001$. Forty-six percent of Japanese teachers reported that their libraries contained mathematics storybooks. However, there were significantly fewer mathematics texts (M = 2.98, SD = 5.41), t (122) = 6.10, $p < 0.001$, in the classroom libraries than storybooks (M = 35.29, SD = 39.04,), t (146) = 10.96, $p < 0.001$, or informational texts (M = 10.23, SD = 17.06), t (141) = 7.14, $p < 0.001$.

In the United States, 94% of teachers with a classroom library reported that their libraries contained mathematics storybooks. However, there were fewer mathematics texts (M = 6.87, SD = 6.47), t (182) =14.37, $p < 0.001$ in the classroom libraries than storybooks (M = 25.22, SD = 22.60), t (189) = 15.38, $p < 0.001$, or informational texts (M = 15.38, SD = 21.18), t (190) = 9.37, $p < 0.001$.

Table 3. Proportion and Number of Each Type of Book in Three Countries.

	Mean Types of Books					
	Mathematics Books		Story Books		Informational Books	
	M	%	*M*	%	*M*	%
China	19.10	42.62	15.04	33.56	10.68	23.82
Japan	2.98	6.14	35.29	72.76	10.23	21.09
United States	6.87	14.47	25.22	53.13	15.38	32.39

Note. M presents the mean number of books; % is the percentage of each category.

Two, we compared the differences in the number of types of books across countries. First, however, we compared the overall number of books in the classroom libraries. The overall number of books in classroom libraries did not differ significantly across countries (China (M = 52.18, SD = 46.77, Japan (M = 41.34, SD = 38.33, United States (M = 54.13, SD = 72.28, F(2462) = 2.31, $p > 0.05$). Chinese teachers reported that their classroom libraries contained a significantly larger number of mathematics storybooks (M = 19.10, SD = 21.18) than United States teachers (M = 6.87, SD = 6.47, $p < 0.001$), who reported a significantly higher number than their Japanese counterparts (M = 2.98, SD = 5.41, $p < 0.005$), F(2402) = 55.96, $p < 0.001$, $\eta p^2 = 0.218$. In contrast to the pattern with mathematics storybooks, traditional storybooks were more commonly found in Japanese preschool classrooms. Japanese preschool teachers (M = 35.29, SD = 39.04) reported significantly more traditional storybooks than Chinese (M = 15.04, SD = 15.84, $p < 0.001$) and United States teachers (M = 25.22, SD = 22.60, p = 0.003), F(2434) = 15.63, $p < 0.001$, $\eta p^2 = 0.067$. Additionally, informational texts were a more common occurrence in United States preschool libraries. Teachers from the United States (M = 15.38, SD = 22.68) reported that their libraries contained significantly more informational texts than those from China (M = 10.68, SD = 14.17, $p < 0.05$) and Japan (M = 10.23, SD = 17.06, $p < 0.02$), F(2429) = 3.57, $p < 0.05$ $\eta p^2 = 0.016$.

Twenty-seven percent of the Chinese preschool teachers mentioned including mathematics games in their classroom library. For example, one Chinese preschool teacher said that she included "number and pictures matching games" while another said she included "materials, number, and pattern cards and other math materials for children". In contrast, fewer than 1% of Japanese and United States teachers indicated the presence of mathematics games in their classroom libraries. Interestingly, while United States teachers were less likely to report including mathematics games or books in their classroom libraries, 11% of United States preschool teachers spontaneously reported keeping mathematics materials, including games and even books, in a separate "math center" and not in the library. For example, one United States teacher said, "I have them (math artifacts) in a math center and place math related books in that center as well." Another United States teacher said, "I have a math area that is not in the library but there are math books in the math area." Neither teachers from China nor Japan made such comments.

The presence of a math center along with the lack of mathematics storybooks and games in United States classroom libraries may suggest that United States teachers are providing additional mathematics materials outside of the classroom library. It also suggests possible differences in how mathematics is facilitated in the three focal countries. This is something we consider in Study 2.

4.1.3. Discussion

Almost all teachers in China, Japan, and the United States reported having libraries containing a variety of books. However, consistent with our expectations, the nature of these books and how they were used differed somewhat across the countries. For example, Chinese teachers reported having significantly more mathematics texts than teachers in either Japan or the United States. Some Chinese teachers also reported having mathematics games in the library. Additionally, they rewarded children for using the library, something that no teachers in Japan or the United States mentioned.

The most common means of encouraging children to use the library was read-alouds. However, this was reported by fewer than half the teachers (China 42%, Japan 40%, United States 25%). There were other differences reported in how teachers encouraged children to use their libraries. Teachers in Japan and the United States also reported providing books that they thought the children would like as well as making the library a comfortable and inviting place. They did the latter by rotating the books based on the season or what topics were being covered in class and providing comfortable seating.

4.2. STUDY 2

This study builds upon the findings from Study 1 and aims to better understand how teachers conceptualize the nature of mathematics exposure and instruction in the preschool classroom. Of particular interest was similarities and differences in how preschool teachers in China, Japan, and the United States viewed opportunities for mathematics instruction.

4.2.1. Method

Participants

After obtaining IRB approval from the lead institution, researchers in three countries (China, Japan, and the United States) contacted local preschools' administrators to see whether they would agree to allow their teachers to participate. Participants included 24 preschool teachers, eight each from China, Japan, and the United States. All were female. Participating teachers from China had been teaching for almost 6 years (M = 5.87, SD = 9.79), from Japan about 13 years (M =13.12, SD = 10.78) and from the United States for an average of 14 years (M = 14, SD = 8.78). The difference in number of years teaching across countries was not statistically significant, $F(223) = 1.18$, $p > 0.05$.

Measures

The main focus of the interview questions was on how mathematics is taught and, relatedly, how mathematics skills are developed using teacher-directed centers, free-play, and the classroom library. Sample interview questions included "*Do you set aside explicit instruction time for math?*", "*Do you have a mathematics center?*", and "*What types of mathematics materials are available during free play*?" (See Table 2). Teachers also reported on whether and how they encouraged children to use mathematics books/materials and were asked if they kept other mathematics materials in the library. Additionally, teachers rated their beliefs about the importance of teaching mathematics in the preschool classroom and their confidence level in facilitating children's mathematics learning processes. For instance, "How important do you think it is to teach mathematics to preschool children?" and "How confident are you in your ability to support your students' mathematics learning?" Interview questions were originally created in English and then translated into Chinese and Japanese by native speakers and modified based on cultural norms.

Procedure

This was a mixed-methods design including a semi-structured interview and rating scales. Prior to conducting interviews, questions were piloted with five preschool teachers from the United States and were modified based on feedback. Semi-structured interviews were conducted by trained graduate/advanced undergraduate research assistants and university faculty and were conducted in a quiet room in the teacher's school. The interviews

were recorded by audio tape and interviewers took field notes. After data collection, all the interviews were transcribed, and the accuracy was checked by the interviewer as well as another research assistant. Interviews conducted in China and Japan were translated into English after they were transcribed in the original language by Chinese and Japanese speaking research assistants.

Coding and Scoring Approach

Initial coding took place by scoring all questions that did not require qualitative analysis (e.g., confidence level for teaching mathematics, importance of mathematics in preschool, presence of a library, etc.). When scoring was complete, all 24 transcripts were independently examined by the first two authors to establish initial codes using predetermined coding categories. After the initial examination, the two primary authors met to compare initial codes and reconcile discrepancies. There was 98% overall agreement between the authors. Discrepancies in codes were addressed and resolved through discussion. The authors also deliberated on the different mathematics activities reported by the teachers. For example, whole-group, small-group, and individual class activities were considered in relation to the different themes that emerged from the initial codes. While predetermined coding categories were established, the analysis allowed the richness of the data to reveal other themes.

4.2.2. Results

Teachers' Views of Mathematics and Their Confidence Teaching It

Teachers in all three countries reported that they believe teaching mathematics in preschool was important or very important (China, $M = 3.75/5$; $SD = 1.28$; Japan $M = 4.50/5$; $SD = 0.545$; United States $M = 4.38/5$, $SD = 0.74$). There was not a significant difference between countries when asked how important it was to teach mathematics in preschool, $F(2, 21) = 2.447$, $p = 0.111$, $\eta p^2 = 0.189$. Although most teachers reported feeling confident to teach mathematics to their preschool students, Japanese teachers ($M = 2.88$; $SD = 0.835$) reported being less confident than teachers from China ($M = 3.63$; SD = 0.916, $p = 0.08$) and the United States (M = 4.25, SD = 0.707, $p < 0.003$), $F(2, 21) = 5.588$, $p = 0.011$, $\eta p^2 = 0.347$.

How Mathematics Is Taught in China, Japan, and the United States

Teachers in the three countries reported teaching mathematics in a variety of ways. Teachers in both China ($n = 7$), and the United States ($n = 6$) reported setting aside time for explicit mathematics instruction during the day. Consistent with the philosophy of embedding mathematics instruction, the use of explicit instruction was less frequently reported in Japan ($n = 2$). In China explicit instruction occurred in whole group settings ($n = 4$) or a combination of whole group settings and centers ($n = 3$). In the United States, teachers reported that explicit mathematics instruction occurred in whole groups ($n = 3$), small groups ($n = 2$), and a synthesis of the two ($n = 3$). Teachers from Japan ($n = 2$) did not indicate the manner in which explicit instruction was used.

The Use of Whole Group Instruction

Teachers who indicated using whole group mathematics instruction used it for circle time, calendar time, or mathematics instructional time. Most often the objectives during whole group instruction focused on early number sense, such as recognizing numbers and the relationships between numbers, counting, and simple computation [8].

In China, seven teachers noted they engaged in counting activities during whole group instruction. However, a few teachers also mentioned doing sorting and computation as well as counting. For example, one teacher said her class engages in the "addition and subtraction of numbers, and will learn the multiplication of numbers". In addition to mathematics instruction occurring during explicit mathematics instructional time, all eight teachers from China reported that whole group mathematics instruction also occurred during circle time. The primary skill focused on by these teachers during circle time was

counting. One teacher said, "During the circle time, we will ask the child to sign in. After signing in, the child will count the total number of children".

While most Japanese teachers indicated they did not engage in explicit mathematics instruction, all eight teachers noted counting during circle time. One teacher stated that she has children "count the number of absentees or different kinds of things, etc." These teachers also indicated focusing on number identification using the calendar. No other mathematics skills during circle time were mentioned by Japanese teachers. Teachers from Japan were also more likely to report that they incorporated mathematics into other activities. For example, one teacher from Japan noted she has her children "count games, turns, number of people" while another said she has the children "read the numbers in the picture books and say the number when lining up".

Teachers from the United States reported that mathematics instruction frequently occurred during circle time. Four teachers reported engaging in counting activities during circle time with five of the teachers indicating a specific focus on number identification. One example of this was the teacher who shared, "Circle time always includes the calendar. The date of the current day of the week will not be up when circle time begins. Instead, they will start by asking how old the month (e.g., April) was yesterday and then asking how old it is today?" One teacher indicated engaging in basic addition during whole group instruction. Shape identification (geometry) was mentioned by one teacher as occurring during whole group instruction when reviewing the calendar. Seven teachers from the United States also reported embedding mathematics into other parts of the day. One teacher noted counting items from nature during outside time and embedding mathematics into transitions between activities (counting the number of children in line and counting to 10 while washing hands).

The Use of Centers and Free Play

Consistent with the philosophical underpinnings of the three societies, teachers from China and the United States were more likely to report than those in Japan that they had a mathematics center where either teacher-directed, small-group instruction or free-choice exploration occurred (China $n = 6$; Japan $n = 2$; United States $n = 8$) China. Teachers in all the three countries mentioned stocking their mathematics centers with artifacts such as counting cubes, measuring items, pattern blocks, traditional blocks, and number sticks.

Teachers from China indicated that their math centers focused on concepts such as patterning, sorting, and counting. For example, one teacher said that in her class's math center the children "divide beads into colors and classify according to colors" while another noted that "There are math cards, they can read and remember". The most common way teachers from China encouraged children to use the mathematics center was to give the children a choice of activities ($n = 5$). One teacher noted, "our [center] activities are actively selected according to the children's own interests".

United States teachers reported stocking their math centers with similar artifacts to Chinese teachers (e.g., blocks, counting cubes, etc.). For example, a United States teacher indicated having "bean bags with numbers" in her mathematics center. However, Chinese teachers also indicated that games were kept in their mathematics centers. United States teachers encouraged the children to use the mathematics center by populating it with activities related to the theme ($n = 3$) and having appealing items such as Play-Doh available ($n = 2$). Two teachers indicated they did not encourage use of the mathematics center.

While only two teachers from Japan reported having a mathematics center, all eight teachers indicated having mathematics materials (typically games and blocks) available for the children at free choice/playtime. One teacher mentioned that she has, "building blocks, skipping ropes, Sugoroku and the number of karuta" available for children during free choice time. Interestingly, only two teachers explicitly indicated that they encourage the children to use the mathematical items.

4.2.3. Discussion

The results of this study clarify and expand upon those from Study 1. Teachers from each of the countries viewed mathematics as important and offered their preschool children various learning opportunities. Consistent with our expectations, how they did so and what they did varied across the countries. Chinese children appear to have the most exposure to mathematics learning opportunities, both in their classroom libraries and through whole group instruction and mathematics centers and free play. In addition, Chinese teachers taught their children a greater range of concepts. The majority of Japanese children engaged in few whole group forms of instruction, except at circle time. They did not have math centers but had opportunities for using mathematics artifacts during free play. Children in the United States experienced whole group math instruction and mathematics centers. As noted in Study 1, a nontrivial number of United States teachers viewed mathematics centers as the locus of mathematics instruction. Here they reported on having a range of different mathematics artifacts in their centers.

Across the three countries teachers talked about the need to make things interesting and engaging for the children but how they did so varied by country. For example, in China teachers talked about varying activities at centers according to children's interests. In the United States, teachers talked about providing artifacts in centers consistent with the theme of the week or choosing items children would like. Teachers in Japan did not discuss using centers but did talk about how they made the library appealing (e.g., nice seating areas).

We discuss these findings more fully in the next section.

5. General Discussion

The two studies in this paper explored two questions about children's exposure to mathematics learning opportunities in their preschool classrooms in China, Japan, and the United States. Study 1 investigated how the preschool classroom library is used in China, Japan, and the United States, and whether there are differences (research question 1). Study two explored how preschool teachers in these countries report teaching mathematics (research question 2). The results of these two studies provide an interesting and important addition to our knowledge of how children's mathematics skills are fostered in three societies with different histories of academic success in mathematics.

Schooling in different countries reflects societal beliefs. For example, in China there is a move toward a more Western philosophy; however, teacher-directed learning is still evident [21,22]. In contrast, Japanese preschools are designed to foster social-emotional development; mathematics learning occurs as a part of the larger environment [24]. United States preschools are designed to focus on learning through play; however, academic instruction, particularly in reading, does occur [27]. Understanding what occurs in these systems may influence pedagogy in other countries, particularly those where mathematics achievement is low.

This study has six findings of particular interest. In general, preschool teachers in all three countries encouraged children to use the classroom library, although Chinese teachers did more of this than the other teachers. Teachers in the three countries were most likely to report using read-alouds. One notable difference was that Chinese preschool teachers rewarded children for using the library, and a few allowed the children to take home books. Japanese preschool teachers made the library inviting-looking and rotated the books in the library. United States preschool teachers also rotated the books and chose books based on children's interests.

Two, how much mathematics content was contained in the libraries differed by country. Chinese preschool teachers were most likely to report having mathematics books in their libraries as well as other mathematics artifacts (e.g., games).

Three, the content of mathematics instruction appeared to be broader in China than Japan or the United States. Most teachers in the three countries reported that they focused on counting activities. However, Chinese preschool teachers also mentioned that they

engaged in simple computational activities including addition, subtraction, and multiplication.

Four, the preschool teachers in all three countries talked about engaging children's interests in the classroom library. However, the means for doing so was not necessarily the same across countries. As noted previously, teachers in China offered children rewards for selecting certain books from the library. Teachers in Japan talked about making the library comfortable for children. In the United States, teachers noted rotating books and materials to avoid over-saturation. Teachers in all three countries discussed increasing children's interest by having artifacts consistent with the themes they were covering in class.

Five, children in China and the United States engaged in a broader array of mathematics instructional activities than those in Japan. Teachers in China and the United States used different types of activities to foster preschool children's mathematics development. Children engaged in whole group instruction, they worked at centers, and they had libraries containing mathematics books. Additionally, as noted previously, teachers from China reported that their classroom libraries contained more mathematics storybooks than the libraries in Japan or the United States.

Six, consistent with what has been reported in the Introduction to this paper, Japanese teachers did not explicitly focus on mathematics instruction. Recall that the purpose of preschool or *yōchien* in Japan is to foster children's social/emotional development through play [24]. Mathematics development was embedded in other activities. This is consistent with what the teachers in our study reported. It is also consistent with what Sakakibara (2014) [25] found in a review of her 2006 study based on observations of 207 three- and four-year-old children in their classrooms. She reported on many available opportunities for learning mathematics skills that occurred. She further argued that such an approach can be extremely successful given the strong math skills with which children from East Asia, particularly Japan, enter kindergarten [3]. Relatedly, Ginsburg [55,56] described how children in the United States frequently engage in informal activities that foster mathematical development.

Given the differences in how children's mathematics development is facilitated in preschool classrooms in China, Japan, and the United States, what accounts for the group-related differences in children's mathematics skills? As we noted in the Introduction to this paper, children's development is impacted by the contexts in which they reside [12]. We focused here on one microsystem, the school. However, we do not want to minimize the importance of the home microsystem or the societal macrosystem. Research shows that East Asian parents emphasize mathematics more than United States parents [3]. For example, Sonnenschein et al., (2018) [14] reported that Chinese immigrants to the United States were displeased with how the United States teachers taught their preschool and early elementary children mathematics. They therefore provided their children supplementary tutoring and experiences.

In addition to what goes on at home and school, the language in China and Japan, because of its closer connection to base 10, is more conducive to children's understanding mathematics operations and relations than is English. Additionally, East Asian countries emphasize the importance of mathematics more than the United States does [3].

6. Limitations and Future Research

There are two important limitations to these studies. One, our sample was not representative of the populations in the three countries. Therefore, we cannot infer causality or be sure that all possible subgroups within the respective countries would respond in a similar manner. However, as Sonnenschein et al. (2021) [57] have discussed, these sampling issues are common in psychology. We also think the importance of collecting data from different countries may outweigh the possible sampling bias. Two, the size of our sample in Study 2 was purposely small, as is common with interview data. However, we reached saturation, or agreement in responses among the different participants, which is considered the gold standard for this type of research [58].

In conclusion, this research was limited to interviews with preschool teachers in China, Japan, and the United States about how they used the classroom library and how, more broadly, they fostered their children's mathematics development. Future research should include observations of what is occurring in children's preschools as well as document how these forms of mathematics activities afford opportunities for acquiring mathematics skills. Future research also should consider preschools in other countries beyond the focal three included in this paper.

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