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Investigating the Geometric Habits of Mind and the Hemispheric Dominance Status of Mathematics Teachers

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Abstract: The geometric habits of mind are a process that enables the adoption of ways of thinking about the problems encountered and that works for filling the gap between the results and the thoughts behind these results. It is also important that which cerebral hemisphere activates the geometric habits of mind, so geometric habits of mind and hemispheric dominance of mathematics teachers were investigated in that regard. It was observed that most of the mathematics teachers dominantly used their left brain and some of them who used their right brain dominantly or who used their right and left brain equally. It was concluded that the geometric habits of mind of both the teachers who dominated the left brain and the right brain and used both sides equally were at the desired level and mathematics teachers who have different hemispheric dominance were sufficient in terms of characteristics of geometric habits of mind.

Key words: Habits of mind, geometric habits of mind, hemispheric dominance status, mathematics teachers.

1. Introduction

Thinking is a planned mental process consisting of activities such as classifying, separating and evaluating the information obtained in various ways in order to achieve the ultimate goal which is to interpret current situation (Dinçer and Saracaloğlu, 2011; Bono, 2007). Healthy functioning of this process is possible with the use of habits of mind (Köse and Tanışlı, 2014). Habits of mind manifest themselves in decision-making processes in uncertain situations (Leikin, 2007). Habits of mind involving both thinking and habit skills; is the ability of the individual to select and apply the most appropriate one of various high-level behaviours such as questioning and creativity in solving an encountered problem (Köse and Tanışlı, 2014; Lim and Selden, 2009). Moreover, studies have been carried out to show that solving problems with different methods as a habit of mind is both a supporter and a necessity of advanced mathematical thinking. (Leikin, 2007). Habits of mind are examined in two groups as general and mathematical. While general habits of mind include fundamental thinking processes such as describing, discovery, and visualization; mathematical habits, on the other hand, is a continuous questioning ability that performs actions of thinking in unusual situations and these habits make progress according to the learning steps (Köse and Tanışlı, 2014; Gürbüz, Ağsu and Güler, 2018).

Several opinions about habits of mind are discussed in the literature. Harel (2007) emphasizes that one's way of thinking determines his/her thinking aspect of habits of mind, in other words, he/she defines habits of mind as adopted ways of thinking (Lim and Selden, 2009). Harel (2008), who divides mathematical habits into two parts, defines the first part as definitions, problems, solutions and evidence accepted by the society, in short, the institutionalized ways of understanding where there is everything about mathematics, and defines the second part as the ways of thinking that include conceptual tools that facilitate generalization (Köse and Tanışlı, 2014; Lim and Selden, 2009). The distinction between these two paths emphasizes the importance of habits of mind that are not frequently included in mathematics programs, and therefore, Harel (2007) argues that ways of thinking and ways of understanding should be included in the learning objectives (Lim and Selden, 2009, p : 1576). Lim and Selen (2009) define the concept of habits of mind as a self-developing cognitive action taken by the person in the first approach that comes to his mind in the face of a problem. However,

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this spontaneous waiting is not seen as a desirable situation. Because the action to be taken in the face of the problem involves only certain solutions to specific situations and this action is needed to be internalized (Lim and Selden, 2009). Leiken (2007) defines habits of mind that support mathematical thinking as one's ability to choose active thinking. According to Goldenberg (1996), habits of mind are habits that an individual acquires by internalizing and then incorporates them into his/her thinking system. According to Mason and Spence (1999), the term of habits of mind is expressed as knowing behaviours that develop suddenly. Lim and Selden (2009) conceptualizes habits of mind as conscious behavioural schemes that prompt immediately, and defines the mathematical habits of mind, which is often excluded from the curriculum, as being productive about mathematics (Lim and Selden, 2009). Bass (2005), on the other hand, mentioned that mathematical habits of mind, which he emphasized to be critical in many respects, are not only related to mathematics education, but also essential for mathematical integrity. Besides, it is stated that the concepts that come to mind as the mathematical habits of mind also serve the ability of mathematics to be interdisciplinary (Leikin, 2007). Attention is drawn to the mathematical habits of mind in the field of geometry and it is expressed as the geometric habits of the mind (Köse and Tanışlı, 2014; Gürbüz, Ağsu and Güler, 2018). The fact that Driscoll (1999) defines the habits of mind as successful ways of learning algebra is proof of this (Lim and Selden, 2009).

Geometric habits of mind are defined in the literature as investigating all the relations in geometry that contain unique qualities and the generation and generalization of new geometric ideas through these relations (Driscoll, DiMatteo, Nikula and Egan, 2007). It is also seen as a way of creative thinking that supports the learning of geometry (Köse and Tanışlı, 2014; Gürbüz, Ağsu and Güler, 2018; Özen and Köse, 2013). The geometric habits of mind are processes that enable the adoption of ways of thinking about the problems and that works for filling the gap between the results and the studies behind these results (Cuoco, Goldenberg and Mark, 1996). Driscoll, DiMatteo, Nikula and Egan (2007) defined the framework of geometric habits of mind in their study; it was mentioned that the framework can be used as a teaching tool for geometric thinking and within this framework it was stated that the geometric habits of mind have four fundamental characteristics which are reasoning with relationships, generalizing geometric ideas, investigating invariants, and balancing exploration and reflection. It was also stated that these features were at the center of the geometry studies. The contents of these four characteristics are shown in Table 1 (Köse and Tanışlı, 2014, Gürbüz, Ağsu and Güler, 2018; Wiles, 2013; Driscoll, DiMatteo, Nikula and Egan, 2007).

Table 1. *Characteristics of Geometric Habits of Mind*

Hemispheric Dominance Group	f	%	Hemispheric Dominance Level	f	%
Left	22	63	Left light	19	54
			Left moderate	2	6
			Left dominant	1	3
			Left strong	0	0
Equal	2	6	Equal	2	6
Right	11	31	Right light	9	26
			Right moderate	2	6
			Right dominant	0	0
			Right strong	0	0

The characteristics of the geometric habits of mind, in a sense, represents the ways of thinking and it is stated that the development of these habits will increase the success level of both geometry and other courses (Köse and Tanışlı, 2014). Thus, studies have been conducted in the literature stating that these habits should be one of the key components of the curriculum and that learners and teachers should adopt these habits in order to reach higher level thinking skills (Köse and Tanışlı, 2014; Lim and Selden, 2009). Since the development of these desired habits will have a positive effect on the success of learners in all areas; it is very important for teachers to constitute learning environments that allow the individual to live experiences, which develop the geometric habits of mind, being aware of his/her own brainpower and to visualize geometrical concepts in his/her mind (Köse and Tanışlı, 2014; Toptaş, 2008; Bozkurt and Koc, 2012). It is emphasized that rather than the results, these learning environments should be aimed at gaining the ability to notice the habits of mind used by the people who created these results (Cuoco, Goldenberg and Mark, 1996). In this regard, Rasmussen, 2009 expressed the importance of encouraging the expected habits of minds and making the necessary environment sustainable for this purpose (Lim and Selden, 2009). Additionally, it is stated by Cobb and Yackel (1996) that the development process in the social life of the individual should be examined carefully for the development of the geometric habits of mind (Lim and Selden, 2009).

Focusing on the geometric habits of mind should not be seen only as ways of reviewing or thinking about ideas. At the same time, the existence of many other hidden features such as making assumptions, developing new question styles, and feeling the excitement of revealing geometric relationships should not be denied (Wiles, 2013). The geometry developed in this century and generally mathematics will be the basis of the scientific developments in the next century and the mathematical habits of mind used by scientists will be reflected in systems that will influence almost every aspect of our daily lives. In this context, it becomes very substantial to establish a program that enables the recognition of mathematical habits used for raising individuals who have a vision about a technology that does not exist yet and pave the way for its use (Cuoco, Goldenberg and Mark, 1996). At this point, for the success of the programs to be implemented; it is important to know which cerebral hemisphere can be used more effectively while acquiring the geometric habits of the mind. In fact, one of the most important factors affecting the geometric habits of mind is to know which one of the cerebral hemispheres is dominant.

Considering the fact that the need for individuals who know meaningful learning is increased and the brain manages all behaviors; behavioral changes acquired as a result of learning also change in a related manner with the structural differences of the brain (İlkörücü, and Arslan, 2017; Kurtuluş and Akay, 2017). The study of Herrmann (1982) can be shown as a supporting evidence for this idea. Herrmann (1982) stated that one of the cerebral hemispheres is more active in the learning process of individuals. Many studies have focused on the relationship between the dominant cerebral hemisphere and the planning of teaching, and the factors associated with the dominant cerebral hemisphere (İlkörücü, and Arslan, 2017; Şenel-Çoruhlu, Er-Nas and Keleş, 2016; Battro, Calero, Goldin, Holper, Pezzatti, Shalóm and Sigman, 2013). The left cerebral hemisphere is more active in mathematical, analytical and logical processes; while the right hemisphere is more active in spatial, holistic, intuitive processes (Herrmann, 1982). Therefore, it is seen that some people are more detailed, realistic, disciplined and competitive and organized while others are more creative, intuitive, emotional and unplanned.

In society, individuals are divided into two groups: ones who use one hemisphere of their brain dominantly and those who use both hemispheres equally. Therefore, the dominance of the cerebral hemisphere varies from person to person. Knowing the dominant hemisphere of the person ensures that learning and understanding are long-term and qualified (İlkörücü, and Arslan, 2017). However, the fact that one hemisphere is dominant does not mean that the other hemisphere remains passive. One can work with one hemisphere more effectively when using both hemispheres together (Herrmann, 1982). Rather than using one of the hemispheres more actively, using both hemispheres in collaboration with each other can help being more productive and capable (Kurtuluş and Akay, 2017; Ornstein and Haden, 2008).

Researchers have divided individuals into two categories in mathematics teaching. While people with left Hemispheric dominance solve problems with a single solution method using pen and paper and do not check their solutions; those with right hemispheric dominance solve problems by performing various solutions, checks their solutions, and exhibit a guaranteeing attitude (Dickson, Brown and Gibson, 1984). In this sense, starting from the individuality of education and training; it is important that at what rate the geometric habits of mind used by individuals are related to which hemisphere of the brain is dominant. In fact, organizing the educational environments established by the teachers according to hemispheric dominance can make geometry teaching more qualified.

In this study, it is aimed to determine the relationship between the geometric habits of mind and the hemispheric dominance of mathematics teachers. For this purpose, the following questions were determined as sub-problems:

- Which hemisphere of the brain of mathematics teachers is more dominant?
- Which of the geometric habits of mind of mathematics teachers use?
- Are hemispheric dominance of mathematics teachers related to the geometric habits of mind they use?

2. Method

The qualitative research method was adopted in the data collection, analysis and interpretation processes of the study which aimed to investigate the geometric habits of mind of mathematics teachers in terms of hemispheric dominance. 35 mathematics teachers working in different cities were included in this study. The teachers were interviewed face to face and data collection tools were applied one by one.

Two open-ended questions, which revealed the four geometrical habits of mind of participants which are reasoning with relationships, generalizing geometric ideas, investigating invariants, and balancing exploration and reflection, were used as a means of data collection in the process of data collection. These questions were selected from the questions developed by Driscoll et al. These questions are two math problems that require the calculation of the perimeter and the area of geometric shapes. The first question is to determine all possible states for the coordinates of the third corner of a triangle whose perimeter and the coordinates of its two corners are given. The second question is to find all possible states for the coordinates of the third corner of a triangle whose area and the coordinates of its two corners are given.

These open-ended questions were analyzed by applying descriptive analysis steps in accordance with the components of the geometric habits of mind. Firstly the theoretical framework of Driscoll et al. (2007) was used for the analysis of the data. Within this framework, the components they create for each geometric habit of mind (reasoning with relationships, generalizing geometric ideas, investigating invariants, and balancing exploration and reflection) are considered as themes and the indicators of each component are considered as sub-themes and categories. Themes, sub-themes and categories are listed in Table 2 (Driscoll et al., 2007; Köse & Tanışlı, 2014, p: 1207).

It is arranged by reading the data obtained in the light of this theoretical framework. The data of the participants were interpreted by visualizing the themes and sub-themes with figures in order to determine the cause and effect relationship between the findings obtained from the research. Direct comments were made from the responses of the participants and were presented.

'Hemispheric Dominance Inventory' was used to determine the hemispheric dominance of the participant teachers. The inventory was created by Davis, Nur and Ruru (1994). The scale, which consists of thirty-nine items, is multiple choice in the form of A, B and C and all items must be answered in order to be evaluated. The total number of B's is subtracted from the total number of A's to find the B-A difference in the scoring stage. If the total number of C's is equal to 17 or more than 17; then B-A score is divided into three and final score is obtained. If the total number of C's is or between 10-16; then B-A score is divided into two and the final score is obtained. If the total number of C's is less than 10; then B-A score is taken as the final score.

$$\text{Final Score} = \begin{cases} (B - A) / 3; & C \geq 17 \\ (B - A) / 2; & 10 \leq C \leq 16 \\ B - A; & C < 10 \end{cases}$$

Table 2: *Themes, Sub-Themes, and Categories of the Geometric Habits of Mind*

Themes	Sub-Themes and Categories		
Reasoning with Relations	***Focusing on Multiple Shapes*** Comparison of two geometric shapes by specifying some common properties The comparison of two geometric shapes by specifying the common properties and the reason	***Focusing on Parts in a Single Shape*** Recognizing and relating layouts-structures in a geometric shape Constructing configurations in a geometric shape Relating two geometric shapes by noticing that they can be seen as parts of a single geometric shape	***Using Special Reasoning Skills*** Reasoning about two or more geometric shapes Using symmetry to associate geometric shapes
Generalizing Geometric Ideas	***Underdeveloped*** Considering exceptions Finding other examples that match exceptions Trying to generalize by changing properties	***Transition*** Works for an infinite set, but only considers the discrete set in the example case. Sees infinity, but limits the set or reaches the wrong conclusion about the set, represents with the wrong geometric shapes.	***Advanced*** Sees all sets of solutions and explains why there is no more. Defines a rule that is universally correct for a class of geometric shapes. Places rules or problem states into a wide content.
Investigating Invariants	***Dynamic Thinking*** Thinking dynamically about a static situation. Thinking about the effect of continuous motion of one point or shape and predicting the formation between one point and the other Considering limited and extreme cases under transformations	***Checking the Evidences of Impacts*** Feeling that everything has not changed in an implemented transformation Recognizing the invariants in an implemented transformation and explaining that the things that cause the change are invariant	
Balancing Exploration and Reflection	***Remarkable Discovery*** Drawing, playing and discovering intuitively or predictably Attempting familiar strategies	***Remarkable End Goals*** Regularly returning to the big picture as a paving stone of progress Identifying intermediate steps that can help achieve the goal Being able to define what the final situation looks like	

If the final score is; 0 then it was interpreted as right and left brain were equal, between -1 to -3 then it was interpreted as left brain was light, between -4 to -6 then it was interpreted as left brain was moderate, between -7 to -9 then it was interpreted as left brain was dominant, between -10 to -11 then it was interpreted as left brain was strong, between +1 and +3 then it was interpreted as right brain was light, between +4 and +6 then it was interpreted as right brain was moderate, between +7 and +9 then it was interpreted as right brain was dominant, between +10 and +11 then it was interpreted as right brain was strong (Davis, Nur and Ruru, 1994).

Mathematics teachers' geometric habits of mind and Hemispheric dominance were examined and interpreted by handling together the comments obtained through open-ended questions used to determine the geometric habits of the mind and final scores of inventory applied to determine Hemispheric dominance. The teachers who participated in the research were coded as Ö1, Ö2, in the sample teacher opinions.

3. Finding

The findings obtained from the research conducted in order to reveal the relationship between the geometric habits of minds of mathematics teachers and the hemispheric dominance are given in three headings as the findings obtained from the hemispheric dominance inventory, the geometric habits of the mind, and the relationships between the geometric habits of the mind and hemispheric dominance.

3.1. Findings of hemispheric dominance inventory

Findings obtained from hemispheric dominance inventory are given in Table 3 as the frequency and percentage distribution according to participants' three hemispheric dominance groups, right, left and equal, and according to the hemispheric dominance levels of the participants.

Table 3: *Frequency and Percentage Distribution by Hemispheric Dominance Groups and Levels*

Hemispheric Dominance Group	f	%	Hemispheric Dominance Level	f	%
Left	22	63	Left light	19	54
			Left moderate	2	6
			Left dominant	1	3
			Left strong	0	0
Equal	2	6	Equal	2	6
Right	11	31	Right light	9	26
			Right moderate	2	6
			Right dominant	0	0
			Right strong	0	0

Table 3 shows that the teachers generally use left brain (63%) when hemispheric dominance is divided into three groups as left, right and equal. When the levels of hemispheric dominance are examined in Table 3, it is observed that the teachers slightly dominate the left (54%).

3.2. Findings from geometric habits of mind

The findings regarding the geometric habits of minds of mathematics teachers, obtained through the problems are presented separately for each problem applied.

3.2.1. Geometric habits of mind of mathematics teachers in perimeter problems.

Problem 1: Perimeter Problem

The applied first problem is a perimeter problem as “The two sides of the triangle, which are 12 units in circumference, are located at points (4,0) and (8,0). What are all possible positions for the third corner point? How do you know that you have all the possibilities?”.

Through this problem which includes all the components of geometric habits of minds, mathematics teachers are expected to realize that the coordinates of the possible third point are on an approximate ellipse that includes the points (4,0) and (8,0), take into account of the dynamism of these points, and consider the relationship between the edges of triangle shape.

Regarding researching the invariants as seen in Figure 1, it was determined that 10 teachers did not consider dynamism because they could not visualize the continuous movement of the third point in their minds. Contrary to this result, all 24 teachers who are aware of dynamism stated that the dynamism is related to the continuous movement of the third coordinate on the ellipse. In order to express the dynamism, they used expressions such as 'set of points', 'point moving continuously on the ellipse' and 2 teachers used expresion such as 'point obtained by moving a third point connected to these nails by placing a nail on two points given the coordinate'.

The geometric habits of mind for the results obtained are shown in Figure 1.

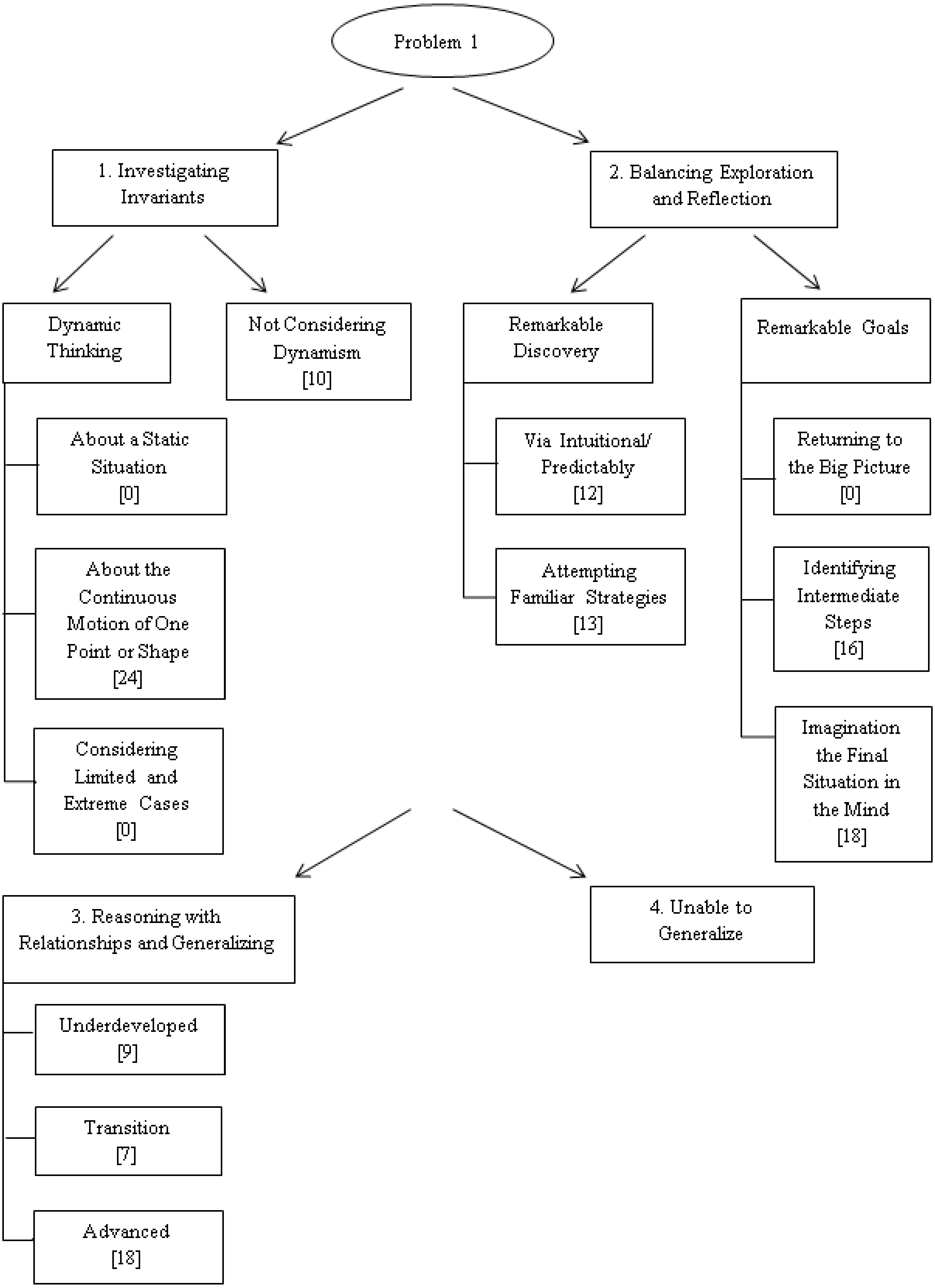

Figure 1. *Geometric habits of mind of mathematics teachers in perimeter problems*

In the context of remarkable discovery within the scope of balancing exploration and reflection; 12 of the teachers created intuitively random triangles through prediction. (See Figure 1). 13 teachers stated that an ellipse would be formed with all triangles to be obtained by using familiar geometric concepts that they knew before.

In the scope of the remarkable end goals, 18 teachers reached the conclusion by imagining what the final situation looked like (see Figure 1). 16 teachers are at the level of identifying intermediate steps that can help to achieve the goal. 11 of these were seen to reach the result intuitively, and 1 of them reached the result through familiar strategies.

Regarding the reasoning with relationships, and generalization of geometric ideas, 1 participant did not answer the question. Among the teachers answering the question, it was observed that 9 participants were underdeveloped, 7 participants were transitional and 18 participants were at advanced generalization level (See Figure 1).

Some of the participants at the underdeveloped level of generalization found some special cases and other examples that match the special cases. The solutions of these participants are presented in Figure 2.

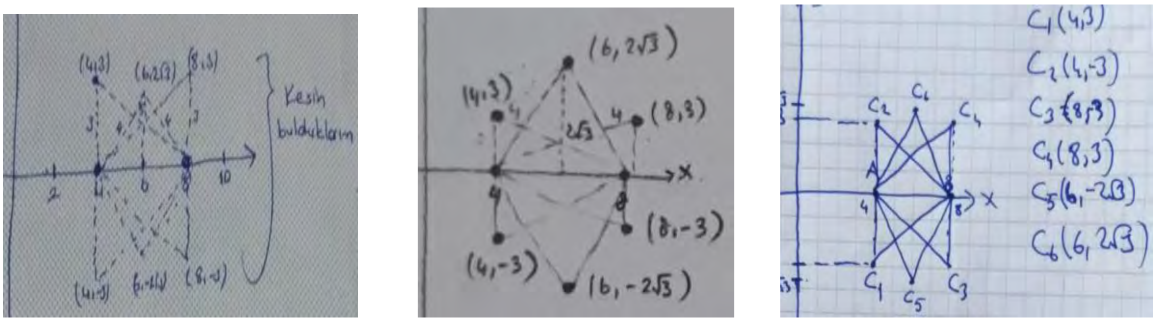

Figure 2. *Generalization examples compatible with underdeveloped exceptions (Ö2, Ö1, Ö3)*

In all three examples, teachers gave the third points of the triangle limited to six specific points, with a circumference of 12 br. Special sample point coordinates that match the special cases are given but not all possible cases have been identified for the third corner. Among the underdeveloped participants, Ö1 participant had right and left equal, Ö2 participant had left dominant and Ö3 participant had left light hemispheric dominance.

Figure 3 presents examples of the solutions, which underdeveloped participants applied to the generalization test by changing the characteristics.

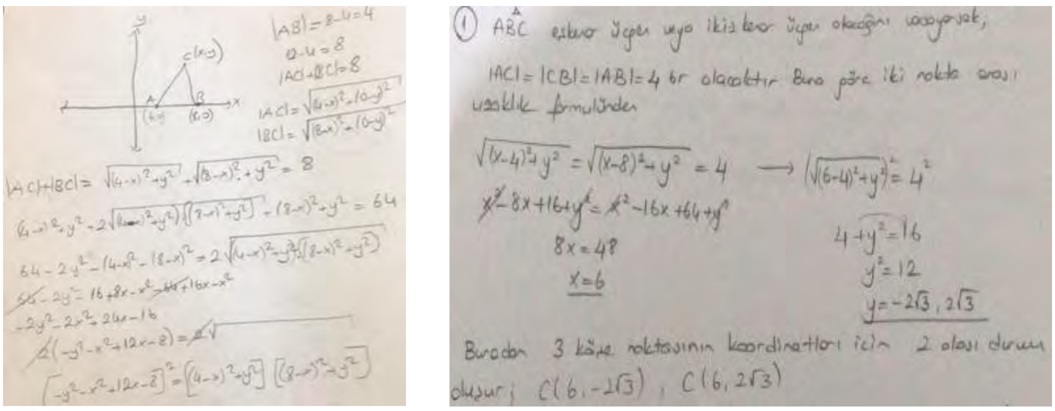

Figure 3. *Examples of underdeveloped generalization (Ö4, Ö5)*

In the sample solution, the teacher assumed that the given triangle would be an equilateral or isosceles triangle. By taking the equilateral triangle sample, he stated that $|AC| = |BC| = |AB| = 4$ br. The coordinates of the C point sought from these equations were found as $(6,2\sqrt{3})$ and $(6,-2\sqrt{3})$. As he made the solution by experiment on the special sample from the generalization by making assumptions, it is evaluated at the underdeveloped level of generalization. Among the underdeveloped participants, the Ö4 participant had right moderate and the Ö5 participant had left light hemispheric dominance.

As some of the candidates at the transition level of generalization found that the total length of the two sides must be 8 and the difference must be less than 4, they have tried a variety of different solutions. An example of these solutions is given in Figure 4.

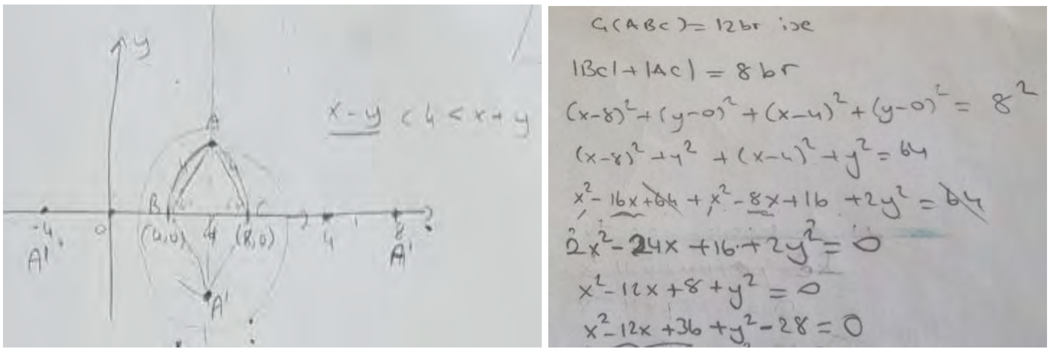

Figure 4. *Examples of transition-level solution trials (Ö6, Ö7)*

In the Figure 4, Ö6 models the situation in the coordinate system geometrically from the interpretation of x – y < 4x + y. Ö7 obtained an ellipse equation by performing the necessary algebraic operations by selecting a special example with a total distance of 8 unit to the points. Both the Ö6 and Ö7 participants of the exemplified transition-level participants had right light hemispheric dominance.

Some of the participants at the transition level of generalization found infinity but reached the wrong conclusion about the cluster. An example of these solutions is given in Figure 5.

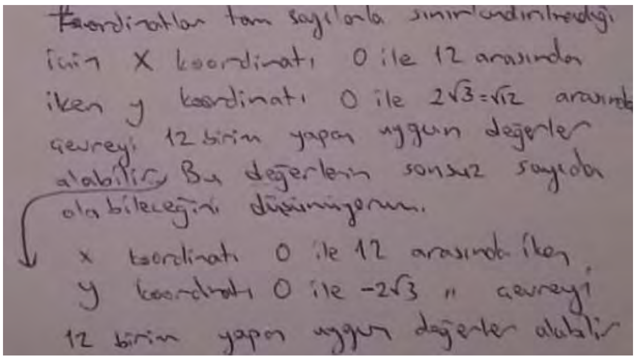

Figure 5. *Solution of Ö8 that sees infinity at transition level but concludes false*

In the example solution, the teacher mentiones that "since the coordinates are not limited to whole numbers, while x ordinaton is between 0 and 12, y apsis can take appropriate values that make the perimeter 12 units between 0 and $2\sqrt{3}=\sqrt{12}$ and these values can be an infinite number". The exemplified Ö8 participant, who saw the infinity at the transitional level but concluded incorrectly, had left light hemispheric dominance.

The solutions belonging to the participants at the advanced level of generalization who stated that ellipse will be formed by seeing all the solution sets are presented in Figure 6.

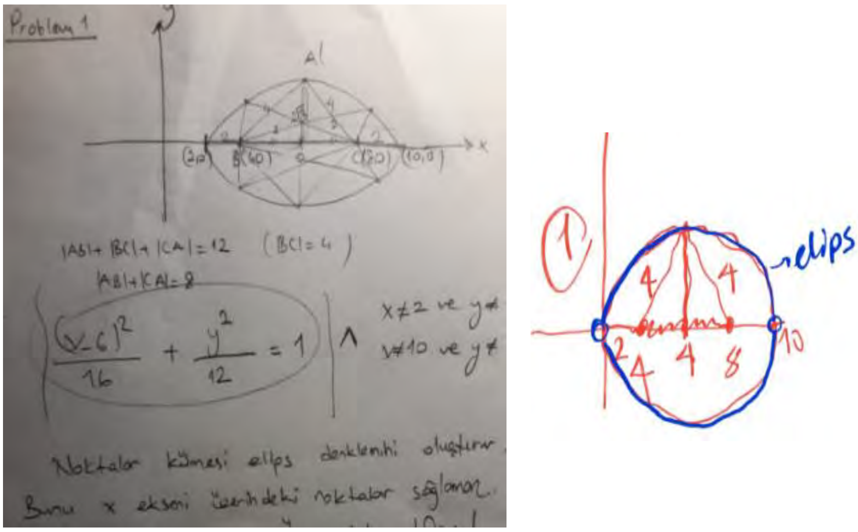

Figure 6. *Advanced level solution examples (Ö9, Ö10)*

In the Figure 6, in the exemplary solution, it is seen that the teacher reached the generalization that the third corner of triangle are on the set of points which form (6,0) centered $\frac{(x-6)^2}{16}+\frac{y^2}{12}=1$ ellipse. The exemplified advanced level Ö9 participant had right light and Ö10 participant had right moderate hemispheric dominance.

3.2.2. Geometric habits of mind of mathematics teachers in area problems.

Problem 2: Area Problem

The applied second problem is an area problem as "The triangle, whose area is 12 units square, has two corners (0,4), (0,10). What are all possible positions for the third corner point? Explain how you find these situations". Participants are expected to find the coordinates of the desired third corner point.

One of the 35 teachers did not answer the question. The geometric habits of minds of 34 teachers towards the second problem results are shown in Figure 7.

Within the scope of reasoning with relationships, the participants were expected to realize that the points required were above the x = 4 and x = - 4 lines which are symmetrical to each other and associate this with the area. While 34 participants focused on single parts of relationships in reasoning, 2 of them could not make association with the area although they realized the structure in reasoning with relationships. In this context, in order to determine the desired corner point coordinate of the given triangle by focusing on the parts of a single shape, 32 of the 34 participants who answered the question stated that the height of the given triangle should be 4 units and the coordinate of the desired point should be on the x=4 line. For the solution of the area problem, the examples of teacher solutions that focus on a single figure and associate them with the area are given in Figure 8.

In Figure 8, in the example solution, the teacher stated that the required points should be on the x=4 and x = - 4 lines by focusing on the two shapes, whose bases have the same heights and which have an area of 12 unit square, in the coordinate system that he draws geometrically. Of the participants illustrated in Figure 8, the Ö8 participant had left light hemispheric dominance.

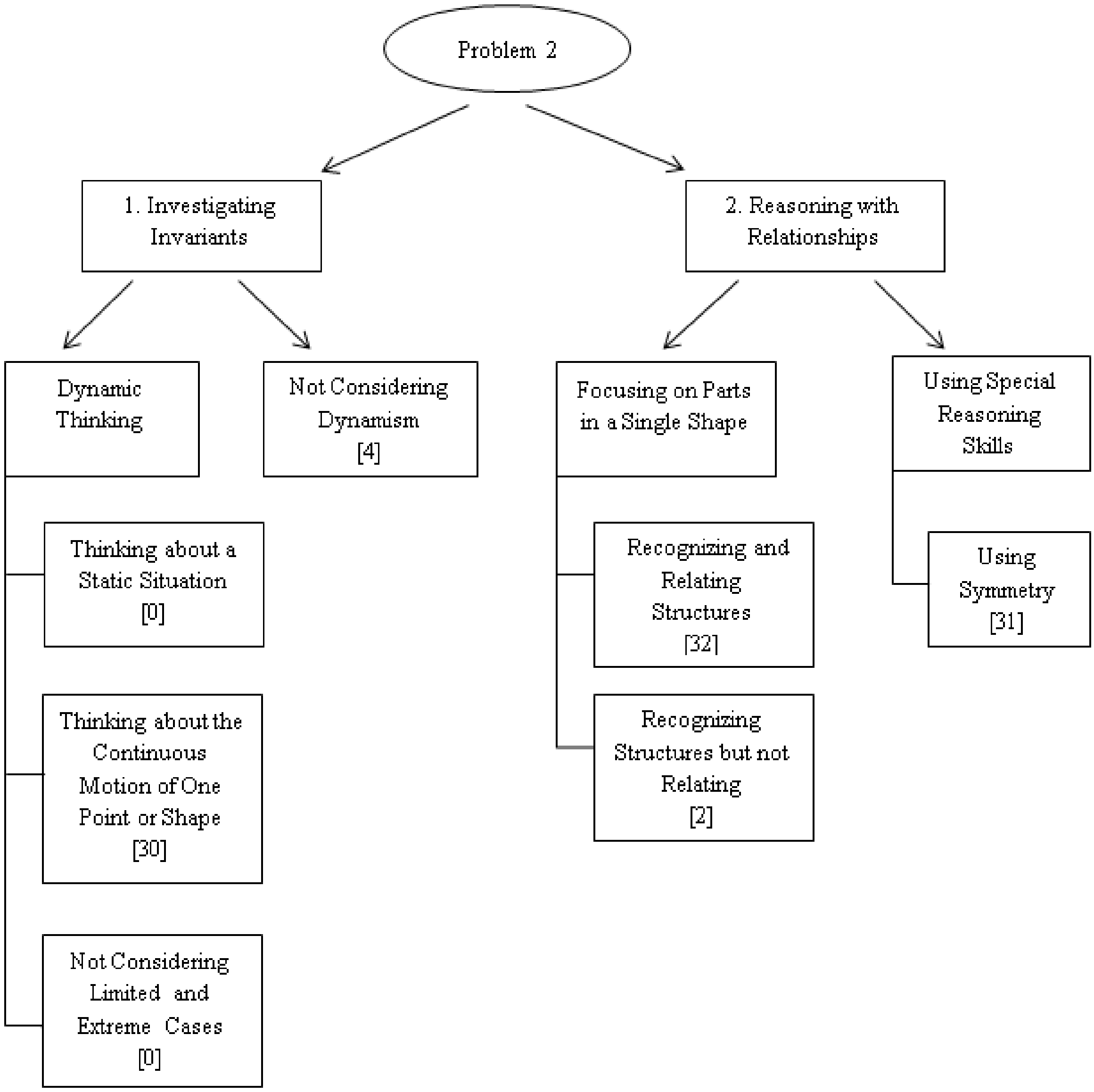

Figure 7. *Geometric habits of minds of mathematics teachers in area problems*

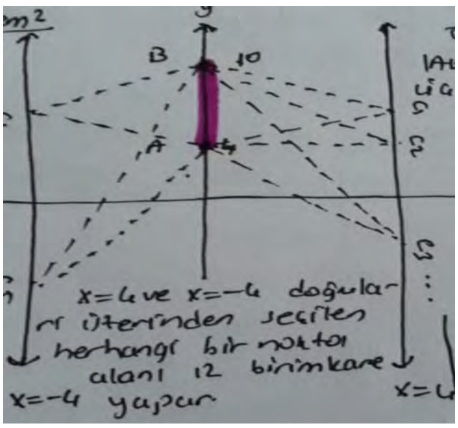

Figure 8. *Solution examples associating the structure with the area by distinguishing the structure (Ö11)*

In reasoning with relationships, 3 out of 31 participants using symmetric relationships in the context of special reasoning skills focused on single parts and used symmetry while 26 participants stated that the third corner point would be on x = - 4 line after finding out the x=4 line. In the context of special reasoning skills, they found the area of the triangle differently from the other participants by using the determinant method and realized the x = 4 and x = - 4 lines according to their absolute value properties. The solution examples of the participants who use the determinant path to find the desired solution and reach the correct result are also presented in Figure 9.

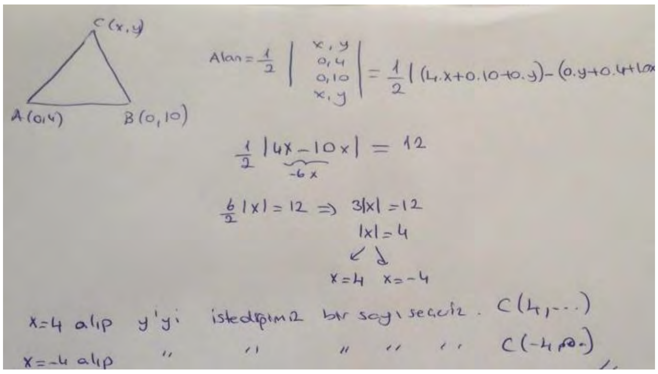

Figure 9. *Example of solution using determinant method for the desired result* (Ö7)

In Figure 9, in the example solution, the teacher (Ö7) calculated the area of the triangle with the help of determinant and found the coordinates of point C=(x,y) as absolute value with x = 4 and x = - 4. Among the exemplified participants above, Ö7 participant had right light.

Within the scope of investigating invariants, the teachers participating in the study were expected to realize that the third corner point changed on the lines found and that the triangular areas formed in this way did not change (12 square units). As can be seen in Figure 7, while 30 teachers took into account that the third point coordinate moves continuously on two lines determined by them, 4 participants did not consider dynamism. The solution examples of the participants who reached the result by using the third point coordinate to move continuously on the two lines determined to find the desired results in the problem are also presented in Figure 10.

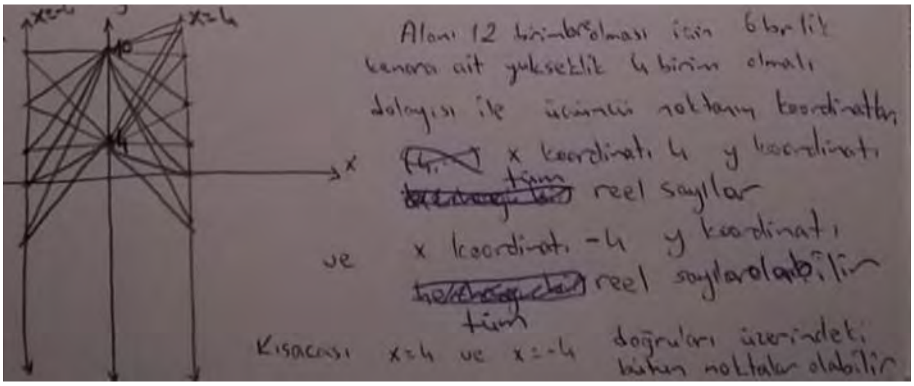

Figure 10. *Example of solution using point dynamism* (Ö8)

In Figure 10, in the example solution, the teacher (Ö11) reached the result by using the third point coordinate to move continuously on the two lines determined to find the desired results in the problem. Of the participants illustrated in Figure 10, the Ö11 participant had right light hemispheric dominance.

In the light of the results obtained with the two open-ended problems, it is seen that they generally focus on a single form within the context of reasoning relationships for the concepts of perimeter and area, and the vast majority (18) are able to generalize at an advanced level.

It was observed that the skills of balancing exploration and reflection, which is the last component of geometric habits of mind that support these results, are sufficient. So much so that most of the candidates imagined the final situation using familiar strategies. In addition, for both questions, participants noticed the dynamism.

3.2. Evaluation of Mathematics Teachers' Geometric Habits of Minds and Hemispheric Dominance

In this study, which aims to determine the geometric habits of minds of mathematics teachers and the status of Hemispheric dominance, the results evaluated according to the components of the geometric habits of the mind by means of these two problems and which brain hemisphere the participants use are evaluated as a whole. The findings are presented in Table 4 and Table 5.

In Table 4, geometric habits of the minds were determined with the perimeter problem and the frequency distributions were classified according to Hemispheric dominance and given.

Table 4: *Frequency Distribution of the Components of the Geometric Habits of the Mind According to Hemispheric Dominance by Using Perimeter Problem*

Problem 1: Perimeter Problem		**Left Brain(f)**	**Right Brain (f)**	**Equal (f)**
Reasoning with Relationships and Generalizing	Underdeveloped	6	2	1
	Transition	4	3	0
	Advanced	12	6	1
Investigation of Invariants	Dynamic thinking	17	6	1
	Disregarding Dynamism	5	4	1
Balance of Exploration and Reflection	Visulation of Final Status	12	5	1
	Defining Intermediate Steps	10	5	1
	With Familiar Strategies	10	3	0
	Intuitively	9	3	0

Considering the perimeter problem according to Table 4, it was seen that the majority of the participants (12 people) who were generally successful in searching for a relationship between the parts and the whole of the concepts related to the problem and reaching a generalization by reasoning use their left brain dominantly. It was seen that the participants (12 people) who used the dynamism in the investigation of invariants and the participants (17 people) who described and visualized the final situation in the balancing component of exploration and reflection use their left brain dominantly. In support of this result, the participants in general realized that the final status was an ellipse with the help of familiar strategies. They expressed the dynamism, ie the mobility of the points on the ellipse, as 'the set of points (infinitely many points) whose sum of the distances from an ellipse to a point is constant.' or 'the third point on the ellipse moves continuously.' Here the set of points indicate dynamism. Unlike these expressions, an advanced participant has used the expression of 'ellipse is obtained by placing a nail on the two given points and moving the third point connected with a rope to these nails' for dynamism.

In Table 5, geometric habits of the minds were determined with the area problem and the frequency distributions were classified according to Hemispheric dominance and given.

Table 5: *Frequency Distribution of the Components of the Geometric Habits of the Mind According to Hemispheric Dominance by Using Area Problem*

Problem 2: Area Problem		Left Brain(f)	Right Brain(f)	Equal(f)
Reasoning with Relationships	Realizing and associating the structure	20	10	2
	Recognizing and not associating the structure	2	1	0

	Symmetry	20	9	2
Investigation of Invariants	Dynamic thinking	20	9	1
	Disregarding Dynamism	2	2	0

When the area problem is considered according to Table 5, 20 of the participants who actively use the left brain, 10 of those using the right brain and 2 of the users using the brain equally have realized the x = 4 line by making a correlation on the figure. However, the majority of those who express x = - 4 using symmetry (20 people) use dominantly their left brain, but 10 of the right brain users and 2 of the users who use the brain equally are also judging the relationships using symmetry. The majority of the participants noticed the dynamism by stating that each point on the x = 4 and x = - 4 lines they found would give the desired. While 20 of the teachers stating the dynamism are dominant in the left brain, 9 are teachers with dominant right brain. It is observed that two teachers from both left and right brain active users did not take into account dynamism.

The majority of mathematics teachers who participated in the study were found to be at the level of reasoning with relationships, exploration, investigating the invariants, generalization, balancing exploration end reflection under the framework of geometric habits of mind related to these problems. According to the findings obtained from the Hemispheric dominance inventory, 22 of the 35 teachers used the left brain and 11 of them used the right brain and 2 of them were equal. It has been concluded that the teachers who uses right brain and equal as well as left brain also have sufficient level of thinking in the context of geometric habits of mind as shown in the Table 4 and 5. This result also shows that the geometric habits of the mind of mathematics teachers with different Hemispheric dominance are at the level of reasoning with relationships, exploration, investigating the invariants, generalization, balancing exploration end reflection.

4. Conclusion and Discussion

In this study, it is aimed to determine the status of Hemispheric dominance and the effect of this dominance on the geometric habits of mind of mathematics teachers. When the findings are examined, it is seen that mathematics teachers predominantly use their left brain, but there are teachers who also uses right brains and equal dominantly. In terms of left Hemispheric dominance levels, it was found that the number of teachers who had left light Hemispheric dominance was the majority. The findings do not coincide with the results of İlkörücü and Arslan (2017) study on mathematics and science teacher candidates. They found that the left brain hemisphere was less preferred in teacher candidates.

Another result is that geometric habits of minds of mathematics teachers are at the desired level in individuals who use the right brain and equal as well as the dominant individuals in the left brain. According to the findings, it was concluded that mathematics teachers with different Hemispheric dominance were sufficient in terms of the components of geometric habits of minds. Although this result seems to contradict with the results of the geometric habits of minds of the teacher candidates of Köse and Tanışlı (2014) regarding reasoning with relationships, investigating invariants, research and exploration components, and generalization at a less developed level, it can be said that the study groups consisted of teachers in different fields.

In the Kurtuluş and Akay's (2017) study, it is stated that providing a suitable learning environment for Hemispheric dominance would facilitate learning. And the study investigated geometric thinking level and Hemispheric dominance; but it could not find any relationship between them. In this sense, it can be said that mathematics teachers who have different Hemispheric dominance status can gain the geometric habits of mind and reach higher levels in geometry learning with appropriate learning environments. Moreover, it can also be concluded that knowledge of Hemispheric dominance would be beneficial for the development of geometric habits of mind. As the brain learns as a whole, it is clear that mathematics teachers need to carry out studies that activate both hemispheres of the brain. Activities that support the development of both teachers and students in this direction should be given more space.

A more valid study can be created by increasing the number of participants. A study can be conducted by comparing the fields and the teachers in different fields to see if there is a significant difference.

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