Flow Through Regression: A Guide to Interpreting and Communicating Regression Models for Data Analytics Students

Katarzyna Toskin
toskink1@southernct.edu
Business Information Systems
Southern Connecticut State University
New Haven, 06515, USA

Niki Kunene
kunenek@easternct.edu
Accounting and Business Information Systems
Eastern Connecticut State University
Willimantic, 06226, USA

Abstract

Faculty teaching data analytics at undergraduate level are often faced with the tension created by student under-preparedness, the demands of the course, and time constraints. How do faculty close this gap? In this paper, we propose the use of flow diagramming as an accessible method for interpreting regression analyses, in ways that are time efficient and not alienating to the student. Our study shows that the use of such flow diagrams has a positive effect on student understanding without additional remedial instruction. Time saved can be directed at core learning objectives of the analytics.

Keywords: Data Analytics, Regression Modeling, Flow Diagram, Flow Chart, Teaching Aid

1. INTRODUCTION

The demand for analytics knowledge has led to the incorporation of data analytics into business curricula at both graduate and undergraduate levels. Though these programs variously cater to different levels of user expertise, from the casual user to business analyst and data scientist (Watson, 2013, 2015), what they all have in common is an expectation for a foundational understanding of statistical concepts. As a result, data analytics courses typically stipulate an introductory statistics course as a minimal prerequisite because statistical understanding is foundational to explanatory statistical modeling, inferential testing and predictive analytics (Shmueli & Koppius, 2011). Yet statistics are and have often been one of the most difficult subjects for undergraduate and even graduate students to grasp. For many higher education institutions, at undergraduate level particularly, the challenge is compounded for the thousands of students who graduate high school academically underprepared for college.

Moreover, due to the large overlap between business analytics, data analytics, and information systems programs (Ceccucci, Jones, Toskin, & Leonard, 2020) many analytics courses are delivered from within Information Systems (IS) programs and therefore are taught by IS faculty. In environments where the student population is diverse, faculty teaching these courses have to manage conflicting forces such as meeting course objectives and analytics content coverage, against balancing the needs and foundational knowledge gaps of underprepared students or students intimidated by the statistical content. Faced with these constraints, it seems to us, that faculty and/or programs have one of two options. First, to accept that they have to reteach statistical foundations and yield on some of the depth in analytics content. While understandable, the downside of this approach is arguably watering down standards and increasing the cost of the program. Another approach would be for faculty to develop innovative methods and approaches that readily open access to the essential content even for underprepared and/or students with low confidence about the material. Such approaches would strategically and efficiently assist in reviewing core concepts to bring students up to speed while leaving time for coverage of analytics course objectives.

Hence, the purpose of this study is to propose an effective pedagogical, visual artifact that increases student efficacy in "relearning" important fundamental statistical concepts for analytics without explicit remedial instruction. In this pilot study, we chose the topic of regression analysis: namely, interpreting and communicating one of the most prominent and commonly used statistical modeling techniques, simple and multiple linear regression. More specifically, we use flow diagramming, an easily understandable and widely used pedagogical aid, to graphically depict the steps required to successfully interpret and communicate linear regression models. In addition, the proposed artifacts are platform independent and can be applied with a wide range of tools (i.e., SPSS, R, Excel, or Python).

2. BACKGROUND

In the experience of the authors, students taking an introductory analytics course often arrive, notwithstanding the typical statistics prerequisite, with little or no understanding of foundational statistical concepts. This lack of understanding extends both theoretically and in an applied sense. Specifically, our analysis from two data analytics undergraduate courses taught in the Fall 2019, at two regional universities, revealed the following key challenges: 80% of students struggled with interpreting and articulating the regression coefficients; 60% of students had a hard time explaining the role of R-squared; 30% of students made incorrect conclusions about the model fit.

These difficulties were not limited to regression analysis, but also extended to statistical inference. Undergraduate data analytics students, in our experience, have difficulty interpreting and communicating the results of performed analyses. In reviewing prior literature in statistics education, it shows that there are three types of reasons for these difficulties: affective (Ashaari, Judi, Mohamed, & Wook, 2011; Reid & Petocz, 2002), cognitive (Chiesi & Primi, 2010), and pedagogical (Ramsey, 1999) reasons. Weinberg & Abramowitz (2000, p. 1), researchers in statistics education, concluded that "our challenge is to find ways of presenting information to our students so that it is accessible, relevant, applicable and even vital to their own areas of interest".

Additionally, the introduction of technology in statistics education shifted the approach to teaching statistics in ways that are instructional to data analytics. In particular, technology encouraged a shift away from emphasis on computations, formulas, and procedures to an emphasis on "statistical reasoning and the ability to *interpret*, evaluate, and *flexibly apply* statistical ideas" (Ben-Zvi, 2000, p. 130) [emphasis added]. Arguably, this shift presaged the widespread adoption and use of data analytics where ability to interpret, evaluate and apply to a variety of contexts is essential for analytics students.

3. FLOW DIAGRAMMING USE FOR PEDAGOGY

Flow diagrams, artifacts in computer science and information systems were first introduced into computing by John von Neuman in the 1940s; they were introduced as a visual representation of the logical structure of a computer program (Ensmenger, 2016). At the time of the ENIAC project, it is understood that flow diagramming was chosen as a form of representation that was readily accessible to the diverse members of the team with different levels of prior knowledge. Flow diagramming was also seen as "superior to introducing a more radical departure in (logical) notation" that some members would have been familiar with (Arawjo, 2020). Priestley's (2018) historical treatment of Von Neuman's work retells how flow diagramming was used in the planning and coding reports of the project to broaden access to understanding of the work by a diverse team:

> *"[W]e have acquired a conviction that this programming is best accomplished with the help of some graphical representation of the problem. We have attempted ... to*

> *standardize upon a graphical notation ... in the hope that [it] would be sufficiently explicit to make quite clear to a relatively unskilled operator the general outline of the procedure. We further hope that from such a block-diagram the operator will be able with ease to carry out a complete coding of a problem" (p. 59).*

Flow diagramming use increased, primarily because of Von Neuman's fame and it eventually grew to have a commanding influence on software engineering and programming for decades to come (Arawjo, 2020). Flow diagrams have since been used in a variety of contexts, namely: in modeling production processes; in aviation for training and process management (Yazgan & Yilmaz, 2018); in the accounting field to teach CPAs to communicate complex plans (Lehman, 2000); as a quality improvement tool for documenting, understanding, analyzing, and improving business processes (Nesbitt, 1993); to aid reading comprehension in the teaching of the law (Zacharias, 1986). Flow diagram use endures in aiding the teaching of introductory programming and systems analysis and design courses. Although, they have been criticized specifically for their modeling accuracy in programming (Hosch, 1977), they have lasted as both process documentation and teaching aids that make complexity readily accessible to a novice. For information systems and computer science students, they are familiar and useful aids.

4. METHOD

To illustrate the use of flow diagramming in analytics, we designed two flow diagrams: one for simple linear regression and another flow diagram for multiple linear regression. We emphasize that the focus of these artifacts is on aiding and strengthening students' capacity for interpretation and communication of analyses (implying understanding).

Description of Flow diagramming Artifacts

In this section we describe the proposed flow diagramming artifacts: to support simple linear regression interpretation (Appendix A), and to support multiple linear regression interpretation (Appendix E). We refer here to regression as a statistical method that seeks to estimate the relationship between an outcome variable and a predictor variable or set of predictors.

Simple Linear Regression

This flow diagram (Appendix A) focuses on four key elements as well as the need to articulate the regression equation: the significance F or *p-value for the F statistic*, the *intercept* or constant; the *slope* or coefficient of the independent variable and its p-value; and the interpretation of *R-squared* for model fit. We note that for simple linear regression, the instructor may want to remind students that the significance of F statistic (p-value) is equal to the p-value of the coefficient of the independent variable (IV) or slope.

Multiple Linear Regression

This flow diagram (Appendix E) focuses on five key elements as well as the need to articulate the multiple regression equation. The process is similar to simple linear regression. However, with multiple regression we assume students start with all hypothesized independent variables included in the regression model. The five key elements are: significance F (p-value for the F statistic), the *intercept* or constant; *coefficients* of the hypothesized independent variables and their respective p-values (we also assume students can, in stepwise fashion, remove non-significant variables, then re-run the model, i.e. return to Step 1 of the flow diagram); the interpretation of *R-squared* for model fit; and *adjusted R-squared*. The final step is intended to help nudge students to use *adjusted R-squared* to reinforce understanding that adjusted R-squared penalizes the addition of independent variables that do not aid in predicting the dependent variable where R-squared increases with every additional variable regardless of its effect on the dependent variable.

We note that to use both flow diagrams, we make rudimentary assumptions related to prior instruction. For instance, for multiple regression (Appendix E − Step 5), we assume that prior instruction already covered that adjusted R-squared is based on R-squared adjusted for the number of predictors and sample size. We use regression equations, in both cases, without the error term. We also assume students are previously instructed on the fundamentals underlying regression analysis including the checking of regression assumptions: linearity, normality of errors, homoscedasticity, independence of errors, and the role of residuals in assessing regression assumptions.

Lastly, the proposed pedagogical flow diagrams are intended to be used over time, with other related and ideally concept repeating assignments. In other words, they can be used again for predictive analytics based on regression models. Below we provide sample assignments and their rubrics (Appendix B, C, and D for simple linear regression; and Appendix F, G, and H for multiple linear regression). It is not our intention that these particular flow diagrams be used for instructing regression analysis from scratch per se. Rather we propose that instructors use them as a remedial mechanism, to close the gap of the forgotten or previously misunderstood concepts, and for review. The purpose is to aid students in gaining proficiency on how to *interpret* and *communicate* regression analysis results by focusing on essential information.

5. EXPERIMENTAL CONTEXT

To test the efficacy of the proposed pedagogical aids, students enrolled in a business analytics undergraduate course, in the Fall of 2020, at a regional public university were given an opportunity to use the flow diagrams as a mechanism to review assumed prior knowledge and provide feedback for this study. All students admitted to the course were required to have previously completed an introductory statistics course as well as an introductory information systems course, which covered introduction to databases, data analysis and Excel.

Study Design

Our experimental design used a pre-test/post-test approach (Campbell & Stanley, 2015) to test the effect of using the above-mentioned pedagogical aids on student understanding with respect to interpreting and communicating regression analysis results.

Students were presented with four different problems: two for simple linear regression and two for multiple linear regression. Each problem included a hypothetical scenario describing the student's role and the problem being investigated, model output (generated using Microsoft Excel data analysis tools), and seven different questions about the model.

<u>Step 1:</u> students received a pre-test for simple regression model (Appendix B), followed by a post-test (Appendix C). Although both simple linear regression models used the same data set, the variables used in each model were different. During the post-test, students were asked to use the simple linear regression flow diagram (Appendix A) as an aid to formulate responses to the assigned problem questions.

<u>Step 2</u>: for multiple linear regression, the process was similar to Step 1 above except a new data set was used to generate the models. Students were presented with a pre-test (Appendix F), followed by a post-test (Appendix G). Variables in the post-test model were changed; and only the post-test included the flow diagram (Appendix E).

To consistently evaluate students' responses to the assigned pre-test and post-test problems, grading rubrics were designed using the same criteria and scoring schema (see Appendix D for simple linear regression rubric and Appendix H for multiple linear regression rubric).

Data Collection – Participants and Procedure

Each student in the course was asked to complete the four problems. Responses were recorded using a Qualtrics survey where each problem was presented in a single screen and students were not permitted to go back to a previous screen. This ensured students could not change or correct their answers in the pre-test while completing the flow-diagram aided post-test. No review of linear regression was conducted in class, students had to rely on (assumed) prior knowledge. Out of nineteen students invited to complete the survey, fourteen students participated in the study (73% response rate).

6. RESULTS

The data was analyzed using paired sample t-tests. The results presented in Table 1 show statistically significant differences in the mean test scores between the pre-test and post-test for both simple and multiple linear regression. These results indicate that both flow diagrams had a positive effect on student understanding and interpretation of the statistical models presented in the problems. More specifically, in the simple linear regression assignment, we observed the biggest improvements in responses relating to the interpretation of the model significance, explanation of the model fit i.e., R-squared, and articulation of the findings. In the multiple linear regression assignment, the improvements were even more prominent and widespread. The biggest differences were evident in the interpretation of the intercept and model fit including adjusted R-squared, formulation and interpretation of the regression equation, calculation of a predicted value of Y as well as articulation of the overall findings. In addition, when asked "How useful did you find the flowchart aid in interpreting the data?", almost 90% of students responded that they found the flow diagram useful ranging from slightly to extremely useful. Furthermore, 93% of students said, they were somewhat likely to extremely likely to use similar flow diagramming aids for other topics in data analytics field.

	Pre-test		Post-test		
Outcome	Min	Max	Min	Max	
SLR	0	79	0	95	
MLR	0	77	0	96	
	Pre-test		Post-test		
Outcome	M	SD	M	SD	t
SLR	36	27	45	34	-3.20**
MLR	34	29	45	34	-3.94**

Note. SLR – Simple Linear Regression. *MLR* – Multiple Linear Regression, ** $p < .01$, $n = 14$

Table 1. Descriptive Statistics & t-test Results

7. DISCUSSION AND CONCLUSION

In this study, we proposed the use of strategically designed flow diagrams focused on specific knowledge gap areas in the most prominent and commonly used statistical modeling techniques, simple and multiple linear regression. We selected flow diagrams because they have been proven to foster conceptual understanding; are good alternative to lecturing; and are both time effective and time efficient. Using such flow diagrams can assist faculty in reviewing core concepts to bring students up to speed while leaving time to focus on new analytics content.

From a student perspective, flow diagrams are easy to understand and perhaps even familiar for students in information systems and computer science; ease of use and familiarity are precursors to favorable affective evaluation. We believe creating mechanism for underprepared students to quickly feel more comfortable or less intimidated by the demands and prior knowledge assumptions of analytics courses can increase student retention and avert conditions where students struggle or prematurely drop out of the course.

Additionally, our study shows that the test scores were statistically significantly higher when using flow diagrams. Similar methods may not only help student understanding of individual concepts but also may serve as important tools for managing remedial work in analytics classes.

Finally, information systems students, in particular, could be encouraged to create their own flow diagrams for other analytical processes they find individually challenging, thus unlocking complexity for themselves. e.g., systematically checking regression assumptions, hypothesis testing from problem statement, data analysis to drawing correct conclusion.

8. LIMITATIONS AND FUTURE RESEARCH

The proposed pedagogical flow diagrams do not make a claim to comprehensively cover all issues related to regression analysis. For instance, an iterative part of the analysis includes examining linear regression assumptions inclusive of examining and interpreting residuals; these flow diagram aids do not incorporate that. To minimize the complexity and maintain the effectiveness of the aid, we believe it would require a different but similar flow diagraming aid. Such an aid could, "walk" a student through how to use/interpret the diagnostic features and charts for assessing residuals generated by most statistical tools like R, SPSS and Excel. For example, another flow diagram could be used to aid a student needing remedial activity on how to run the output to examine residuals, remove outliers, or log-transform the data and re-run the regression model before interpretation. Likewise, we do not explicitly model or review steps for performing stepwise regression for multiple regression. For remedial activities, instructors can design such aids.

Finally, this study is limited by a small sample size. However, we reported qualitative observations about students results and specifically which questions to the problems were addressed with more success while using the aids, to supplement our quantitative analysis and increase the validity of this study.

9. REFERENCES

Arawjo, I. (2020). *To Write Code: The Cultural Fabrication of Programming Notation and Practice.* Paper presented at the Proceedings of the 2020 CHI Conference on Human Factors in Computing Systems.

Ashaari, N. S., Judi, H. M., Mohamed, H., & Wook, M. T. (2011). Student's attitude towards statistics course. *Procedia-Social and Behavioral Sciences, 18*, 287-294.

Bahr, P. R. (2008). Does mathematics remediation work?: A comparative analysis of academic attainment among community college students. *Research in Higher Education, 49*(5), 420-450.

Bahr, P. R. (2010). Preparing the underprepared: An analysis of racial disparities in postsecondary mathematics remediation. *The Journal of Higher Education, 81*(2), 209-237.

Ben-Zvi, D. (2000). Toward understanding the role of technological tools in statistical learning. *Mathematical thinking and learning, 2*(1-2), 127-155.

Campbell, D. T., & Stanley, J. C. (2015). Experimental and quasi-experimental designs for research. Ravenio Books.

Ceccucci, W., Jones, K., Toskin, K., & Leonard, L. (2020). Undergraduate Business Analytics and the Overlap with Information Systems Programs. *Information Systems Education Journal, 18*(4), 22-32.

Chiesi, F., & Primi, C. (2010). Cognitive and non-cognitive factors related to students' statistics achievement. *Statistics Education Research Journal, 9*(1), 6-26.

Ensmenger, N. (2016). The multiple meanings of a flowchart. *Information & Culture, 51*(3), 321-351.

Hosch, F. A. (1977). Whither flowcharting? *ACM SIGCSE Bulletin, 9*(3), 66-73.

Lehman, M. W. (2000). Flowcharting made simple. *Journal of Accountancy, 190*(4), 77.

Nesbitt, T. E. (1993). Flowcharting business processes. *Quality, 32*(3), 34.

Priestley, M. (2018). *Routines of Substitution: John Von Neumann's Work on Software Development, 1945–1948*: Springer.

Ramsey, J. B. (1999). Why do students find statistics so difficult. *Proccedings of the 52th Session of the ISI. Helsinki*, 10-18.

Reid, A., & Petocz, P. (2002). Students' conceptions of statistics: A phenomenographic study. *Journal of Statistics Education, 10*(2).

Shmueli, G., & Koppius, O. R. (2011). Predictive analytics in information systems research. *MIS quarterly*, 553-572.

Watson, H. J. (2013). The business case for analytics. *BizEd, 12*(3), 48-54.

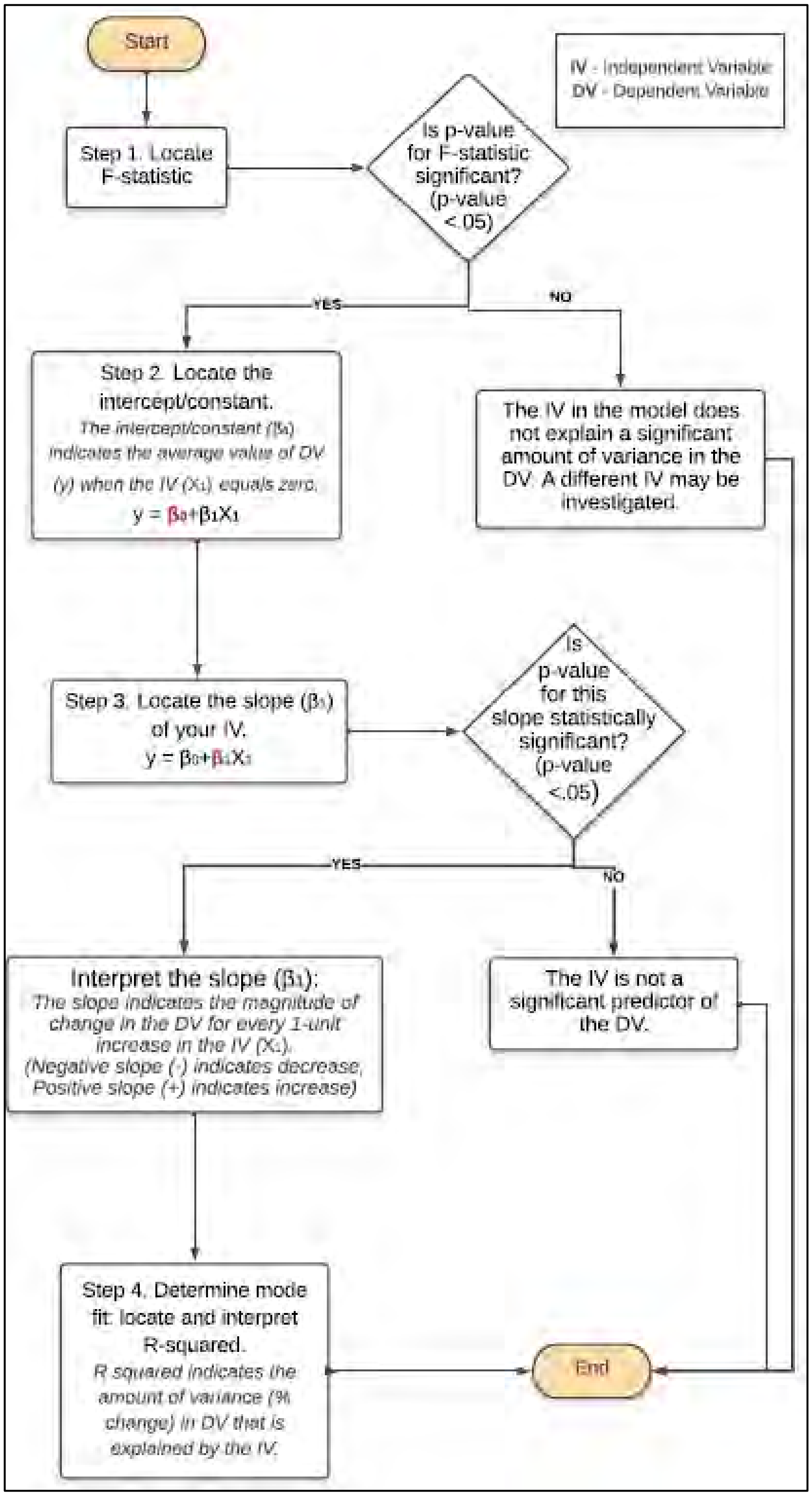
Watson, H. J. (2015). Should you pursue a career in BI/analytics. *Student Edition of the Business Intelligence Journal*.

Weinberg, S., & Abramowitz, S. (2000). Making general principles come alive in the classroom through an active case studies approach. *Journal of Statistics Education, 8*(2), 15.

Yazgan, E., & Yilmaz, A. K. (2018). Designing flow chart for aircraft type training in aviation training process management. *Aircraft Engineering and Aerospace Technology*.

Zacharias, F. C. (1986). Flowcharting the First Amendment. *Cornell L. Rev., 72*, 936.

Appendix A:
Simple Linear Regression Flow Diagram



Appendix B:
Simple Linear Regression Assignment – Pre-test

Instructions: You are a research analyst for an automotive industry. Your boss has asked you to investigate if the engine displacement size (in liters) affects the highway fuel economy (miles per gallon) based on a sample of 234 vehicles. Given the following input from a regression analysis, please interpret the results by answering the questions listed below.

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.766020021
R Square	0.586786672
Adjusted R Square	0.585006518
Standard Error	3.835588408
Observations	234

ANOVA

	df	SS	MS	F	Significance F
Regression	1	4847.833384	4847.833384	329.4533229	2.0389E-46
Residual	232	3413.829009	14.71478021		
Total	233	8261.662393			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%
Intercept	35.69765106	0.720467619	49.55476915	2.1225E-125	34.27835254	37.11694954	34.27835254	37.11694954
Displacement	-3.530588680	0.194513697	-18.15084917	2.0389E-46	-3.913827848	-3.147349764	-3.913827848	-3.147349764

Questions:

1. Interpret the overall significance of the regression model. Report the statistic you used to answer this question and its value. (10 points)
2. Report and Interpret the intercept/constant. (10 points)
3. Report and Interpret the slope/regression coefficient of the independent variable (10 points)
4. Report and Interpret the model fit (10 points)
5. Formulate and Interpret the regression equation for this model (10 points)
6. Use the model to predict the value of Y given X1=3.7. (10 points)
7. Write a precise summary of your findings to your boss. Remember that your boss is not a data analyst so make sure to articulate your findings in an easy to understand language. (40 points)

Notes: Please see below, the description of the variables and a preview of the first 10 records of the data set.

Manufacturer	Model	Displacement	Year	Cylinders	Transmission	Drive	City	Highway	Fuel	Class
audi	a4	1.8	1999	4	auto(l5)	f	18	29	p	compact
audi	a4	1.8	1999	4	manual(m5)	f	21	29	p	compact
audi	a4	2	2008	4	manual(m6)	f	20	31	p	compact
audi	a4	2	2008	4	auto(av)	f	21	30	p	compact
audi	a4	2.8	1999	6	auto(l5)	f	16	26	p	compact
audi	a4	2.8	1999	6	manual(m5)	f	18	26	p	compact
audi	a4	3.1	2008	6	auto(av)	f	18	27	p	compact
audi	a4 quattro	1.8	1999	4	manual(m5)	4	18	26	p	compact
audi	a4 quattro	1.8	1999	4	auto(l5)	4	16	25	p	compact
audi	a4 quattro	2	2008	4	manual(m6)	4	20	28	p	compact

Manufacturer – name of the manufacturer of the car
Model – model name
Displacement – engine displacement (size) in liters
Year – year of car manufacture
Cylinders – number of cylinders
Transmission – type of transmission such as automatic or manual
Drive – type of drive such as f = front wheel drive, r = rear wheel drive, 4 = four-wheel drive
City – city miles per gallon
Highway – highway miles per gallon
Fuel – fuel type such as p = premium, r = regular
Class – type of car

Appendix C:
Simple Linear Regression Assignments – Post-test

Instructions: You are a research analyst for an automotive industry. Your boss has asked you to investigate if the number of cylinders affects the city fuel economy (miles per gallon) based on a sample of 234 vehicles. Given the following input from a regression analysis, **as well as using the flow chart included at the end of this assignment**, please interpret the results by answering the questions listed below.

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.805771408
R Square	0.649287792
Adjusted R Square	0.647775280
Standard Error	2.525918897
Observations	234

ANOVA

	df	SS	MS	F	Significance F
Regression	1	2740.13386	2740.13386	429.4729925	1.0E-54
Residual	232	1480.212294	6.380225406		
Total	233	4220.346154			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%
Intercept	29.39043478	0.626431425	46.88750217	2.4988E-120	28.1554752	30.62544412	28.1554752	30.62544412
Cylinders	-2.137981843	0.103086449	-20.72372377	1.0E-54	-2.340095079	-1.935472604	-2.340095079	-1.935472604

Questions:

1. Interpret the overall significance of the regression model. Report the statistic you used to answer this question and its value. (10 points)
2. Report and Interpret the intercept/constant. (10 points)
3. Report and Interpret the slope/regression coefficient of the independent variable (10 points)
4. Report and Interpret the model fit (10 points)
5. Formulate and Interpret the regression equation for this model (10 points)
6. Use the model to predict the value of Y given X1=8. (10 points)
7. Write a precise summary of your findings to your boss. Remember that your boss is not a data analyst so make sure to articulate your findings in an easy to understand language. (40 points)

Notes: Please see below, the description of the variables and a preview of the first 10 records of the data set.

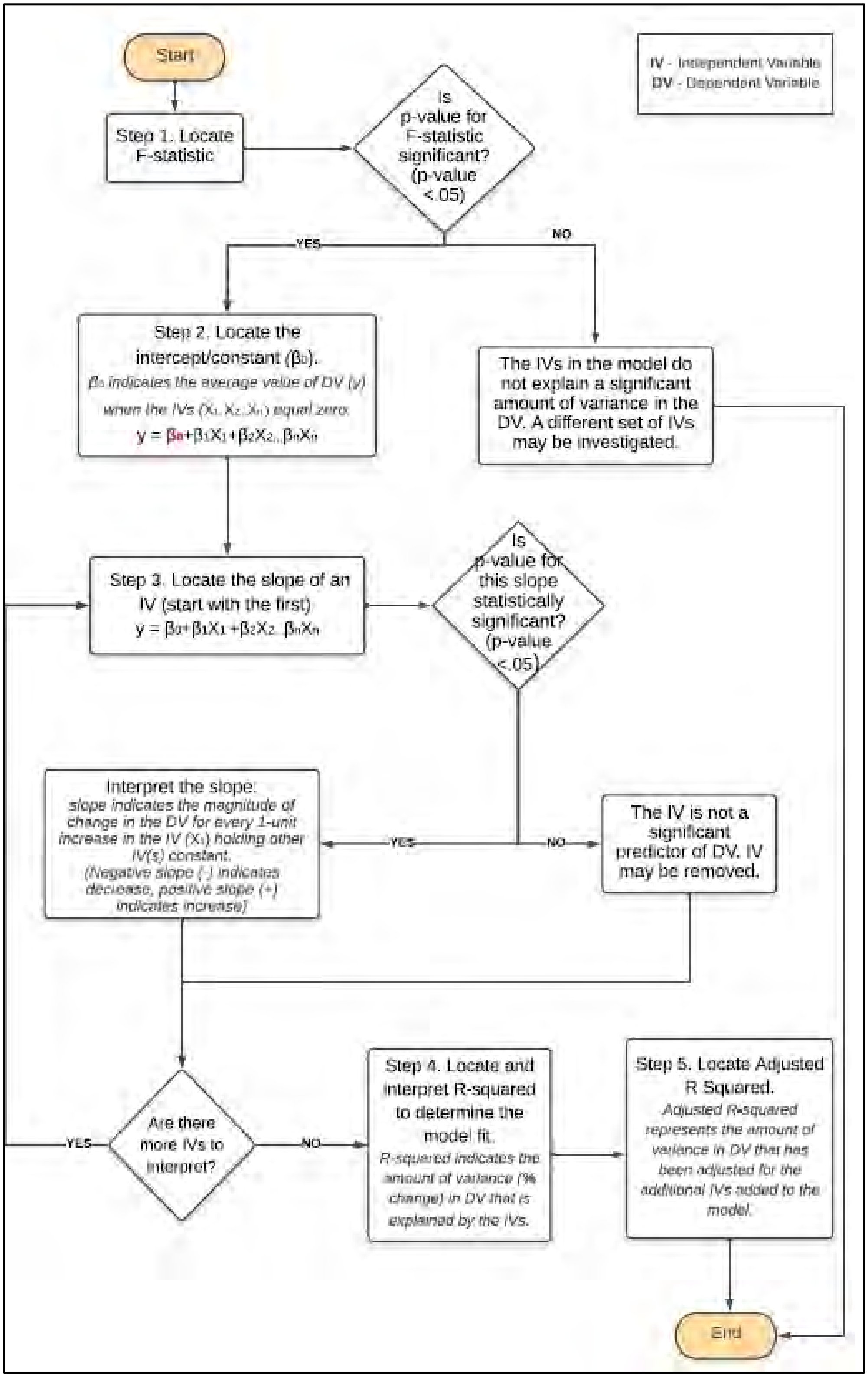
Manufacturer	Model	Displacement	Year	Cylinders	Transmission	Drive	City	Highway	Fuel	Class
audi	a4	1.8	1999	4	auto(l5)	f	18	29	p	compact
audi	a4	1.8	1999	4	manual(m5)	f	21	29	p	compact
audi	a4	2	2008	4	manual(m6)	f	20	31	p	compact
audi	a4	2	2008	4	auto(av)	f	21	30	p	compact
audi	a4	2.8	1999	6	auto(l5)	f	16	26	p	compact
audi	a4	2.8	1999	6	manual(m5)	f	18	26	p	compact
audi	a4	3.1	2008	6	auto(av)	f	18	27	p	compact
audi	a4 quattro	1.8	1999	4	manual(m5)	4	18	26	p	compact
audi	a4 quattro	1.8	1999	4	auto(l5)	4	16	25	p	compact
audi	a4 quattro	2	2008	4	manual(m6)	4	20	28	p	compact

Manufacturer – name of the manufacturer of the car
Model – model name
Displacement – engine displacement (size) in liters
Year – year of car manufacture
Cylinders – number of cylinders
Transmission – type of transmission such as automatic or manual
Drive – type of drive such as f = front wheel drive, r = rear wheel drive, 4 = four-wheel drive
City – city miles per gallon
Highway – highway miles per gallon
Fuel – fuel type such as p = premium, r = regular
Class – type of car

Appendix D:
Rubric for Grading Simple Linear Regression Pre-test and Post-test

Grading Rubric: Simple Linear Regression Assignment							
Student Name:							
	Needs Improvement	Satisfactory	Excellent				
	0-1 points	2-3 points	4-5 points	Weight	Points Possible	Points Earned	Weighted Points
Interpretation of overall significance of the model: F statistic/Null Hypothesis	The significance of the model is not reported using the correct statistic. The interpretation is missing or completely incorrect.	The significance of the model is reported correctly and interpretation is partially correct.	The significance of the regression model is reported correctly and interpreted accurately.	0.1	5		0
Interpretation of the intercept/constant: B0	Intercept/constant of regression line is not reported correctly. The interpretation is missing or completely incorrect for the constant/slope.	Intercept/constant is reported correctly and interpretation is partially correct.	Intercept/constant is reported and interpreted accurately.	0.1	5		0
Interpretation of the slope/regression coefficient: B1	Slope/regression coefficient is not reported correctly. The interpretation is missing or completely incorrect for the slope/regression coefficient.	Slope/regression coefficient is reported correctly and interpretation is partially correct.	Slope/regression coefficient is reported and interpreted accurately.	0.1	5		0
Interpretation of model fit: R-squared	R-squared is not reported correctly. The interpretation is missing or completely incorrect for R squared.	R-squared is reported correctly and interpretation is partially correct.	R-squared is reported and interpreted accurately.	0.1	5		0
Formulation and Interpretation of regression equation Y hat = B0+B1X1	The equation is not formulated correctly. The interpretation of the equation is missing or completely incorrect.	The equation is formulated correctly and interpretation is partially correct.	The equation is formulated and interpreted accurately.	0.1	5		0
Calculate the predicted value of Y using a specific value of X1	The predicted value of Y is not calculated correctly.	The predicted value of Y is partially correct.	The predicted value of Y is calculated accurately.	0.1	5		0
Articulate your findings: Write a summary of your findings.	The articulation is incorrect or missing.	The articulation is partially complete and partially correct.	The articuation is complete and accurate.	0.4	5		0
						Total Score	**0**

Appendix E:
Multiple Linear Regression Flow diagram



Appendix F: Multiple Linear Regression Assignments – Pre-test

Instructions: Department of Public Safety (DOS) has hired you as a consultant to help them explore historical data collected from 50 states of the United States of America. Their first priority is to study if Population size (in millions) and Income per capita (in thousands) affect murder rate (%). Given the following output from a regression analysis, please interpret the results by answering the questions listed below.

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.461732393
R Square	0.213196803
Adjusted R Square	0.179735816
Standard Error	3.343410573
Observations	50

ANOVA

	df	SS	MS	F	Significance F
Regression	2	142.3612699	71.18063494	6.370497618	0.003571521
Residual	47	525.1845301	11.17409426		
Total	49	667.7458			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%
Intercept	14.34337892	3.490050075	4.109073707	0.000157786	7.321077034	21.36568081	7.321077034	21.36568081
Population	0.338434824	0.109381813	3.09405451	0.00332034	0.118389999	0.558482648	0.118389999	0.558482648
Income	-0.189425027	0.079472426	-2.38351648	0.021239504	-0.349302926	-0.029547127	-0.349302926	-0.029547127

Questions:

1. Interpret the overall significance of the regression model. Report the statistic you used to answer this question and its value. (10 points)
2. Report and Interpret the intercept/constant. (10 points)
3. Report and Interpret the slopes/regression coefficients of the independent variables (10 points)
4. Report and Interpret the model fit (10 points)
5. Formulate and Interpret the regression equation for this model (10 points)
6. Use the model to predict the value of Y given X1=10, X2=50. (10 points)
7. Write a precise summary of your findings to DOS. Remember that your boss is not a data analyst so make sure to articulate your findings in an easy to understand language. (40 points)

Notes: Please see below, the description of the variables and a preview of the first 10 records of the data set.

	Population	Income	Illiteracy	LifeExp	Murder	HSGrad	Frost	Area
Alabama	3.615	36.24	2.1	69.05	15.1	41.3	20	50708
Alaska	0.365	63.15	1.5	69.31	11.3	66.7	152	566432
Arizona	2.212	45.3	1.8	70.55	7.8	58.1	15	113417
Arkansas	2.11	33.78	1.9	70.66	10.1	39.9	65	51945
California	21.198	51.14	1.1	71.71	10.3	62.6	20	156361
Colorado	2.541	48.84	0.7	72.06	6.8	63.9	166	103766
Connecticut	3.1	53.48	1.1	72.48	3.1	56	139	4862
Delaware	0.579	48.09	0.9	70.06	6.2	54.6	103	1982
Florida	8.277	48.15	1.3	70.66	10.7	52.6	11	54090
Georgia	4.931	40.91	2	68.54	13.9	40.6	60	58073

Population – population estimate in millions as of July 1, 1975
Income – income per capital as of 1974, in thousands
Illiteracy – percent of population who is illiterate as of 1970
LifeExp – life expectancy in years from 1969-1971
Murder – murder and non-negligent manslaughter rate per 100,000 population as of 1970
HSGrad – Percent of high-school graduates as of 1970
Frost – average days with minimum temperature below freezing between 1930-1960
Area – land area in square miles

©2021 ISCAP (Information Systems and Computing Academic Professionals)

Appendix G: Multiple Linear Regression Assignments – Post-test

Instructions: Department of Public Safety (DOS) has hired you as a consultant to help them explore historical data collected from 50 states of the United States of America. They would like to investigate if Murder rate (%) and High School graduation rate (%) affect Life expectancy (number of years). Given the following output from a regression analysis, **as well as using the flow chart attached on the second page of this assignment**, please interpret the results by answering the questions listed below.

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.81415359
R Square	0.662840008
Adjusted R Square	0.648409002
Standard Error	0.795871738
Observations	50

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	58.52864627	29.26432314	46.20110018	8.0161E-12
Residual	47	29.77035573	0.633411824		
Total	49	88.299002			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	70.29708402	1.015667862	69.21266811	5.90043E-49	68.25382379	72.34034424	68.25382379	72.34034424
Murder	-0.237090095	0.035285233	-6.719244068	2.14051E-08	-0.308074828	-0.166105361	-0.308074828	-0.166105361
HSGrad	0.043887209	0.016126888	2.721374355	0.009088366	0.011444185	0.076330413	0.011444185	0.076330413

Questions:

1. Interpret the overall significance of the regression model. Report the statistic you used to answer this question and its value. (10 points)
2. Report and Interpret the intercept/constant. (10 points)
3. Report and Interpret the slopes/regression coefficients of the independent variables (10 points)
4. Report and Interpret the model fit (10 points)
5. Formulate and Interpret the regression equation for this model (10 points)
6. Use the model to predict the value of Y given X1=8, X2=70. (10 points)
7. Write a precise summary of your findings to DOS. Remember that your boss is not a data analyst so make sure to articulate your findings in an easy to understand language. (40 points)

Notes: Please see below, the description of the variables and a preview of the first 10 records of the data set.

	Population	Income	Illiteracy	LifeExp	Murder	HSGrad	Frost	Area
Alabama	3.615	36.24	2.1	69.05	15.1	41.3	20	50708
Alaska	0.365	63.15	1.5	69.31	11.3	66.7	152	566432
Arizona	2.212	45.3	1.8	70.55	7.8	58.1	15	113417
Arkansas	2.11	33.78	1.9	70.66	10.1	39.9	65	51945
California	21.198	51.14	1.1	71.71	10.3	62.6	20	156361
Colorado	2.541	48.84	0.7	72.06	6.8	63.9	166	103766
Connecticut	3.1	53.48	1.1	72.48	3.1	56	139	4862
Delaware	0.579	48.09	0.9	70.06	6.2	54.6	103	1982
Florida	8.277	48.15	1.3	70.66	10.7	52.6	11	54090
Georgia	4.931	40.91	2	68.54	13.9	40.6	60	58073

Population – population estimate in millions as of July 1, 1975
Income – income per capital as of 1974, in thousands
Illiteracy – percent of population who is illiterate as of 1970
LifeExp – life expectancy in years from 1969-1971
Murder – murder and non-negligent manslaughter rate per 100,000 population as of 1970
HSGrad – Percent of high-school graduates as of 1970
Frost – average days with minimum temperature below freezing between 1930-1960
Area – land area in square miles

Appendix H:
Rubric for Grading Multiple Linear Regression Pre-test and Post-test

Grading Rubric: Multiple Linear Regression Assignment							
Student Name:							
	Needs Improvement	Satisfactory	Excellent				
	0-1 points	2-3 points	4-5 points	Weight	Points Possible	Points Earned	Weighted Points
Interpretation of overall significance of the model: F statistic/Null Hypothesis	The significance of the model is not reported using the correct statistic. The interpretation is missing or completely incorrect.	The significance of the model is reported correctly and interpretation is partially correct.	The significance of the regression model is reported correctly and interpreted accurately.	0.1	5		0
Interpretation of the intercept/constant: B0	Intercept/constant of regression line is not reported correctly. The interpretation is missing or completely incorrect for the constant/slope.	Intercept/constant is reported correctly and interpretation is partially correct.	Intercept/constant is reported and interpreted accurately.	0.1	5		0
Interpretation of the slope/regression coefficient: B1 & B2	Slope/regression coefficients are not reported correctly. The interpretation is missing or completely incorrect for the slope/regression coefficients.	Slope/regression coefficients are reported correctly and interpretation is partially correct.	Slope/regression coefficients are reported and interpreted accurately.	0.1	5		0
Interpretation of model fit: R-squared and Adjusted R-squared	R-squared and Adjusted R-squared are not reported correctly. The interpretation is missing or completely incorrect for R squared.	R-squared and Adjusted R-squared are reported correctly and interpretation is partially correct.	R-squared and Adjusted R-squared are reported and interpreted accurately.	0.1	5		0
Formulation and Interpretation of regression equation Y hat = B0+B1X1+B2X2	The equation is not formulated correctly. The interpretation of the equation is missing or completely incorrect.	The equation is formulated correctly and interpretation is partially correct.	The equation is formulated and interpreted accurately.	0.1	5		0
Calculate the predicted value of Y using a specific value of X1 & X2	The predicted value of Y is not calculated correctly.	The predicted value of Y is partially correct.	The predicted value of Y is calculated accurately.	0.1	5		0
Articulate your findings: Write a summary of your findings.	The articulation is incorrect or missing.	The articulation is partially complete and partially correct.	The articuation is complete and accurate.	0.4	5		0
						Total Score	**0**