

The Skills of Mathematics Teachers with **Different Professional Experiences to** Notice the Evidence of Student Thinking

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To cite this article:

Ozdemir Baki, G. & Kilicoglu, E., (2021). The skills of mathematics teachers with different professional experiences to notice the evidence of student thinking. International Journal of Research in Education and Science (IJRES), 7(4), 1226-1244. https://doi.org/10.46328/ijres.2404

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2021, Vol. 7, No. 4, 1226-1244

https://doi.org/10.46328/ijres.2404

The Skills of Mathematics Teachers with Different Professional Experiences to Notice the Evidence of Student Thinking

Gulsah Ozdemir Baki, Elif Kilicoğlu

Article Info	Abstract							
Article History	There are different types of evidence that reflect students' thinking in classroom							
Received: 23 March 2021 Accepted: 07 September 2021	interactions. Student discourse, gestures, actions can be shown among these. The aim of the current study is to reveal the skills of secondary mathematics teachers with different professional experiences to notice different types of evidence of student thinking. For this aim, the study was designed within the context of a case study, one of the qualitative research methods. The data of the study were							
<i>Keywords</i> Teacher noticing Student thinking Mathematics teacher Video	obtained through video-based interviews with five secondary mathematics teachers. Six video episodes containing different types of evidence for video-based interviews were shown to teachers. In the interviews, targeted questions were asked to reveal what types of evidence the participating teachers took into account in the videos they watched, how they interpret these types of evidence, and what kind of instructional decisions they suggested. The types of evidence that teachers noticed in the videos and how they made sense of the evidence were analyzed qualitatively. Various findings have been revealed depending on the different professional experiences of the teachers. First, experienced (5 years and above) teachers paid more attention to the types of verbal evidence of student thinking. Student statements and questions were more visible to experienced teachers, especially among oral evidence. Second, the evidence-based comments described by the experienced teachers who make inferences based on student's cognitive thinking also attend postgraduate education or professional experience courses. Third, the instructional decisions of experienced teachers who attended postgraduate education and professional experience courses, based on student thinking, were mostly related to specific mathematical subjects.							

Introduction

Teacher and student roles are the basis of classroom interaction. Because, in instructional activities, knowledge is acquired both through teachers' discourse and students' interactions (Chin, 2006). Examination of teacher and student behaviors in the classroom has been deemed important for ensuring the permanence of knowledge, that is, meaningful learning, and even theoretical frameworks related to this issue have been proposed (Mortimer &

Scott, 2003). The focus of these theoretical frameworks is the patterns created by classroom interactions through teacher discourse and student thinking. The formation of patterns in a healthy way is necessary for a strong theoretical framework. In this case, how to determine student thinking becomes important.

Student thinking can be presented in different ways. Heritage (2012) states that determining what students say and what they do or write can be valuable for this. As a matter of fact, revealing student thinking is effective in the realization of teaching goals (Hiebert, Morris, Berk, & Jansen, 2007). However, it is not an easy situation to determine student thinking, on the contrary, it requires expertise. Teachers need specialization in classroom interactions to identify student ideas and to make sense of these ideas (Jacobs, Lamb, & Philipp, 2010). One of the main components of teaching expertise is teacher noticing (Sherin, Jacobs, Philipp, 2011). What, when and how the teacher focuses on what, when and how in the classroom is really complex and requires expertise. In teaching mathematics, teachers' noticing brings great responsibilities (Sherin & Dyer, 2017). The mathematics teacher is expected to be able to shape the teaching within the framework of events in the classroom and to adapt the course of the lesson to the new situation. This situation, which expresses the effective management of the lesson, is related to the expertise of the mathematics teacher (Dyer & Sherin, 2016).

The number of studies on teacher noticing has increased over the past decades. Relevant literature has examined teachers' noticing skills in various contexts and different focal points based on different conceptualizations. Some of these studies dealt with the nature of noticing skill (see Goldsmith & Seago, 2011; van Es & Sherin, 2020), while others dealt with how the teacher handles student thinking (see Jacobs et al., 2010; Sherin & Han, 2004; Walkoe, 2015). At the same time, different methodological approaches were used to examine teacher noticing in the previous studies. These methodological approaches have been classified under three headings by Dindyal, Schack, Choy and Sherin (2021). These; (1) using teaching-specific activities, (2) using technology, and (3) using tools to assess teacher noticing. Most of the studies conducted have examined teacher noticing with video-based approaches in the use of technology. In such studies, video clips selected from the participants' own classes (e.g., Özdemir Baki & Kılıçoğlu, 2020; van Es, 2011, van Es & Sherin, 2010) or video archives (eg, Him Lam & Kam Ho Chan, 2020; Roth McDuffie et al., 2014) were used. In this study, targeted video clips were used to examine mathematics teachers' noticing ability to recognize student thinking. Through video-based interviews, the types of evidence teachers identified in the videos they watched as a result of their different professional experiences, how they made sense of these evidences and what kind of instructional decisions they suggested were examined. In the videos containing episodes of classroom interactions, readers are presented with rich examples of how and how teachers notice students' thinking. Although studies on teacher noticing have recently gained momentum, it is thought that taking a sample with different professional experiences (teaching experience, graduate education, professional experience course) will contribute to the studies in the relevant literature with a different perspective.

In line with this perspective, the following research questions guided the study.

- 1. What are the skills of secondary mathematics teachers to notice the different types of evidence of student thinking?
- a. What are the skills of teachers to pay attention to the different types of evidence of student thinking?

- b. What are the skills of teachers to interpret the different types of evidence of student thinking?
- c. What are the skills of teachers to make instructional decisions based on different types of evidence of student thinking?

Related Literature

Evidence of Student Thinking

Studies on teacher noticing in mathematics education have mostly concerned with students' mathematical thinking (e.g., Jacobs et al., 2010; Sherin & van Es, 2009; Schifter, 2011; Stockero, Rupnow, Pascoe, 2017; Walkoe, 2015). The teacher noticing student's thinking is not only paying attention to particular student ideas but also making sense of these ideas (Sherin, Russ, & Colestock, 2011). However, in order to understand and interpret students' ideas, it is necessary to access student thinking first. As a matter of fact, it is not possible to directly access student thinking, it is necessary to determine observation evidence that can reveal student thinking. Teachers can improve their teaching practices by identifying and using evidence of students' mathematical thinking (National Council of Teachers of Mathematics [NCTM], 2014).

Many studies on noticing in mathematics education have highlighted the importance of using evidence of student thinking (e.g., Jacobs et al., 2010; Schack et al., 2013; van es, 2011). Schack et al. (2013) showed evidence of student thinking interpretation as prominent features of students' actions and words. They also emphasized the relationship between interpretation and evidence, arguing that evidence-based productive interpretations would support instructional decisions. Beattie et al. (2017), on the other hand, revealed types of evidence that show improved noticing levels. For this, van Es (2011) revised its theoretical framework. The researchers demonstrated the importance of interpretation and evidence in noticing by noting that teachers use worksheets, pictures, dialogue) to interpret student thinking. Similarly, Krupa et al. (2017) emphasized the relationship between evidence of student thinking. Studies have been conducted to determine the evidence of student thinking in science teaching as well as in mathematics education. For example, Lam and Chan (2020) examined the evidence-based noticing skills of student thinking by identifying different types of verbal and nonverbal evidence in their study with prospective teachers.

Related studies show that generative interpretations based on evidence based on student thinking will support instructional decisions. In the present study, the skills of secondary mathematics teachers to determine the types of evidence based on student thinking was examined based on the types of evidence they used in the studies of Lam and Chan (2020). By making arrangements specific to mathematics teaching, evidence types were expanded with mathematical notation.

Noticing Based on Professional Experience

It is not easy to make interpretations based on evidence of student thinking, on the contrary, it is a skill that requires expertise. Many researchers have tackled expertise in different ways. For example, Hattie (2012) drew

attention to the differences between the expert teacher and the experienced teacher, stating that his expertise is not only dependent on the duration of professional experience, but is a great skill that shapes teaching practices. Berliner (2004), on the other hand, emphasized that expert teachers form meaningful patterns in the field they experience by stating that novice teachers are more opportunistic and flexible. Wolff, Jarodzka and Boshuizen (2021) argued that expert teachers have episodic information about classrooms and that this information helps them make analytical comments about classroom events.

Studies on noticing have shown that there are differences between experienced teachers and novice teachers' noticing skills (Berliner, 2001; Jacobs et al., 2010; Star & Strickland, 2008; Yang, König & Kaiser, 2021). These studies reveal that novice teachers tend to pay attention to the superficial characteristics of the classroom, and experienced teachers make more detailed comments by determining the important characteristics of classroom interactions. Erickson (2011) suggests that differences in teachers 'comments are influenced by teachers' professional experiences and pedagogical approaches. Some studies have suggested that there is an almost linear relationship between professional experience and noticing ability (e.g., Jacobs et al., 2010; Yang et al., 2021).

In contrast, Bonaiuti, Santagata, and Vivanet (2020) found no statistically significant difference between experienced and novice teachers' noticing skills in their studies. A holistic view of the researches shows that experienced teachers need to develop noticing skills in the context of making productive comments based on student thinking and presenting instructional suggestions. As documented by a number of studies, the skill of noticing is not a skill that can only be developed through teaching experience, it is a skill that can be learned with support (e.g, Amador, 2021; Jacobs et al., 2020; van Es & Sherin, 2010). The current study was conducted with mathematics teachers with different professional experiences. Different professional experience means both teaching experience, postgraduate experience, and professional development course participation experience. Based on the relevant literature, the skill of teachers with different professional experiences to consider the evidence of student thinking, to interpret and to make decisions was examined.

Method

Research Approach

This study was conducted with five secondary school mathematics teachers who worked in the eastern region of Turkey in the fall semester of 2020 with different professional experiences (teaching experience, educational level and professional development course). The current study was designed as qualitative research to identify the types of evidence teachers use in noticing student thinking and how they make sense of those types of evidence. The participants were five secondary school math teachers who voluntarily participated in the study and had between one and fifteen years of professional experience (SMTs). Teachers participating in the study were defined as experienced teachers with at least five years of experience, based on their teaching experience period (Berliner, 1991). Participating teachers were given pseudonyms and their descriptive features were presented in Table 1.

Teacher	Gender	Teaching	Education Level		
		Experience			
Hale	Female	1	Master		
Gizem	Female	5	Master		
Eda	Female	9	PhD		
Ege	Male	12	Undergraduate		
Sude*	Female	15	Undergraduate		

Table 1. Participating Teachers' Information

*Sude had attended a video-based professional development course.

Data Collection

The data of the study were obtained from video-based interviews with five secondary mathematics teachers. The teachers answered the questions posed by the researcher by watching a total of six video episodes, including one video episode each week. These video episodes included sections from the lessons of four secondary mathematics teachers who participated in the lesson study professional development process. The researchers took into account some criteria to determine the video episodes suitable for the purpose of the study. First of all, it was important to have different types of evidence to reflect students' thinking in the video episodes selected (Lam & Chan, 2020). Thus, teachers could comment on the video episodes they watched. Another was that the selected video episodes contained evidence from the whole class, individual students, and small groups of students. In addition, the video episodes were arranged for approximately 3-5-minutes in order to prevent the teachers from getting bored while watching the video. Finally, the video episodes provided an opportunity for teachers to provide instructional suggestions based on student thinking.

The data were obtained through interviews using the zoom program to allow time and place flexibility during the pandemic period. Each week, the researcher and the teacher came together in a virtual environment at the scheduled time and watched a video episode. The researcher paused the video for moments related to student thinking and asked the teacher the question as "What did you notice?" In the meantime, to get more information, the question as "Have you noticed any other situations?" was posed. Then the researcher asked the following questions to reveal how each participating teacher interpreted the situations he considered: "Why did you think that?" "On what basis did you say that?" Thus, it was tried to determine the types of evidence that the teacher took into consideration and how they made sense. Finally, the researcher asked the following questions to the participating teachers at the end of the video episode: "If you were the teacher in the video, how would you conduct the lesson?" In addition, participating teachers were asked to explain why they gave such a response. It was important for teachers to explain the reasons for their responses, to determine what kind of instructional decisions they made and to what extent they were based on student thinking. We cannot ignore some limitations of the method. First of all, because the videos contain sections from different teachers' classes, participating teachers may not have enough information about the students in the video. In addition, while videos reveal the richness of classroom interactions, they may not reflect all the features of the classroom.

Data Analysis

Data analysis is based on data from video-based interviews with five SMTs. The interviews with SMTs were written down and separate voice transcripts of the teachers were created. Then, the transcripts were carefully read and divided into idea units, which are defined as shifts in the focus of the teacher's speech or changes in the topic (Jacobs & Morita, 2002). In accordance with the purpose of the study, 252 idea units were determined in which teachers made comments and inferences about evidence of student thinking.

The theoretical framework developed by Lam and Chan (2020) was used to examine SMTs' skill to notice different types of evidence of student thinking. At this stage, for RQ 1 (a), the types of evidence of student thinking that teachers take into account in the video episodes were determined as verbal evidence or non-verbal evidence. During the analyzes, we developed the "mathematical notation" code to evaluate the explanations of SMTs regarding students' mathematical operations and representations and accepted it as a non-verbal form of evidence. Then, we examined the size of the subject (specific student / student group or whole class) and the level of detail (general or specific) of the evidence they pay attention to in order to determine the nature of the observations made by SMTs on the types of evidence of student thinking. We analyzed this for the observation evidence of each SMT.

We dealt with the teachers' evidence-based approaches to student thinking as descriptive and inferential for RQ 1 (b). While the descriptive approach included situations where teachers simply described the evidence they used, the inferential approach included making conclusions based on the evidence they used. We detailed the analyzes and examined the approaches of each SMT regarding the observation evidence descriptively or inferentially. Then, we determined whether the situations where each SMT inferred from the evidence were related to students' cognitive thinking. Finally, for RQ 1 (c), we analyzed teachers' instructional decisions in two categories: (a) not connected to students' cognitive thinking, (b) connected to students' cognitive thinking. Then, we tried to determine to what extent SMTs' instructional decisions were based on student thinking. For this, we examined whether the instructional decisions associated with students' cognitive thinking. For this, we examined whether the instructional decisions associated with students' cognitive thinking. For this, we examined whether the instructional decisions associated with students' cognitive thinking were at a content-specific level (content-general, content-specific). The coding process was carried out by two researchers simultaneously but independently of each other. The first researcher coded all of the data. The second researcher encoded the data obtained from two video segments (2nd video and 5th video) that he randomly selected. The percentage of agreement between the two encoders was 91% for the 2nd video and 96% for the 5th video. Coding differences were discussed among researchers and consensus was reached.

Results

In this section, we present the types of evidence that the participant teachers participated in the videos they watched, how they made sense of these types of evidence, and what kind of instructional decisions they suggested, depending on the sub-problems of the research. The sub-problems of the study were given under three headings.

SMTs' Skills to Pay Attention to Different Types of Evidence of Student Thinking

From the analysis of the video episodes watched by the SMTs, 277 pieces of evidence based on student thinking were determined in 252 idea units. It was found that made interpretations without based on any evidence in 4 of the identified idea units, teachers. In addition, it was determined that 21 idea units contain more than one type of evidence. The types of evidence used by SMTs in six video episodes and the meaningful situation episodes are shown in Table 2.

	51					-					
	Video episodes										
Form of evidence	1^{st}	2 nd	3 rd	4 th	5 th	6 th	Total				
	(n=29)	(n=47)	(n=50)	(n=42)	(n=47)	(n=61)	n=277				
Verbal Evidence	16	25	30	22	28	38	159				
Explanation	7	9	11	8	15	17	67				
General responses	5	5	8	4	12	6	40				
Question	0	5	5	7	1	1	19				
Short responses	4	6	6	3	0	14	33				
Non-verbal evidence	13	22	20	21	19	23	118				
Action	5	5	4	14	0	4	32				
Artefacts	0	4	0	0	0	0	4				
General responses	4	5	6	4	6	5	30				
Gesture	2	3	5	3	5	8	26				
Voting	2	0	5	0	0	0	7				
Mathematical notation	0	5	0	0	8	6	19				

Table 2. The Types of Evidence that SMTs Identify in Six Video Episodes

When Table 2 is examined, it is seen that SMTs take into account the evidence of student thinking at least in the first video episode and at the most in the sixth video episode. When the types of evidence were examined, it was found that SMTs paid more attention to verbal evidence types (n=159) than non-verbal types of evidence (n=118). In addition, analysis shows that this is the case for every video episode. It is seen that SMTs focus more on student statements among verbal evidence types, and student actions in non-verbal types of evidence. In this direction, the analyzes were detailed to determine which types of evidence teachers with different professional experiences primarily focus on. Table 3 shows the types of evidence attended by teachers with different experience.

Data analysis revealed different situations specific to each video episode. For example, while watching the fifth video, SMTs, tended towards student explanations (n = 15) without paying attention to students' short answers (n=0). However, in the fifth video, student groups had short answers about the problem situation. On the other hand, the short answers (n=14) of the students about planar shapes in the sixth video were easily determined by the SMTs. In addition, although the voting method was used in both the first and third videos, the voting in the third video was more visible by SMTs.

	SMTs							
	Hale	Gizem	Eda	Ege	Sude			
Verbal Evidence	18	37	39	23	42			
General responses	6	9	9	6	10			
Short responses	4	8	8	2	11			
Explanation	7	14	19	12	15			
Question	1	6	3	3	6			
Non-verbal evidence	22	27	25	18	26			
Action	5	8	8	5	6			
Artefacts	0	1	1	1	1			
General responses	7	9	2	6	6			
Gesture	7	3	7	2	7			
Voting	1	2	2	1	1			
Mathematical notation	2	4	5	3	5			

Table 3. Different Types of Evidence for Student Thinking Defined by SMTs

All of the SMTs were able to easily identify non-verbal forms of evidence regarding student thinking. However, the consideration of these types of evidence by SMTs has differed. For example, while Hale focused more on students 'general non-verbal responses and gestures, Eda focused more on students' actions. In addition, SMTs used the mathematical representation evidence by paying attention to the mathematical operations and representations of the students among the non-verbal evidence. Although SMTs are interested in students' actions and gesture, their main focus has been on student discourse. Among the SMTs, only Hale considered more non-verbal types of evidence (n=22) than verbal evidence (n=18). So much so that in the second video part, where the students cut colored paper and formed triangles, they talked about these actions of the students, but did not mention the product of the student showing the relationship between the triangles. Although Ege focused more on student discourse (n=23) in the video episodes he watched, this was not the case for every video episode, and he easily identified the non-verbal evidence of the student in some videos (for example, 1st and 2nd video). In some videos, Ege focused more on student discourse (for example, 5th and 6th video). The other three SMTs (Gizem, Eda, Sude) focused more on student discourse in each video episode.

Among the verbal evidence of student thinking, SMTs paid more attention to student descriptions (see Table 3). It is an important finding of the research that this type of evidence regarding student thinking is considered more than other types of evidence. Eda focused the most on this type of evidence, while Hale paid the least attention. Analyzes revealed that Eda focused more on student explanations in other video episodes, except for the second video (focused on short answer). However, although there are questions asked by the students in the video episodes, these were not easily visible by SMTs. Only two teachers (Gizem and Sude) considered almost every student question in the video they watched. Sude not only dealt with student questions, but also focused on the student's general answers, short answers and explanations. Therefore, it was the teacher who considered the verbal evidence of student thinking (n=42) the most.

Table 4 shows the nature of the evidence observed by SMTs. SMTs identified evidence of student thinking based on both the whole class (n=136) and specific student / groups (n=141). They showed a similar interest in verbal (n=67) and non-verbal evidence (n=69) in their observations for the whole class. On the other hand, in their observations of particular students/groups, they focused more on verbal evidence than non-verbal evidence. However, SMTs took into account specific details (n=86) rather than general (n=73) in their explanations of oral evidence. Especially among the types of verbal evidence, they took into account the level of detail of the evidence by focusing more on specific students/groups in student statements. As a matter of fact, they tended to use more general details (n=69) in their explanations regarding non-verbal evidence.

	Subject size	of observation	Level	Total		
Form of evidence	Whole class	Particular	General	Specific		
		students/groups				
Verbal Evidence	67	92	73	86	159	
Explanation	24	43	21	46	67	
General responses	28	12	37	3	40	
Question	4	15	4	15	19	
Short responses	11	22	11	22	33	
Non-verbal evidence	69	49	69	49	118	
Action	14	18	9	23	32	
Artefacts	0	4	0	4	4	
General responses	29	1	28	2	30	
Gesture	18	8	25	1	26	
Voting	6	1	3	4	7	
Mathematical notation	2	17	4	15	19	

Table 4. The Nature of SMTs' Observations on Different Types of Evidence

The analyzes were detailed to determine the nature of the observation evidence of SMTs with different professional experiences (see Table 5). In their observations, SMTs focused on particular students/groups rather than the whole class. However, on the contrary, Hale focused more on the whole class (n=27) rather than particular students/groups (n=13) and gave general explanations. In her observations based on oral evidence, her explanations tended to be more general although she paid attention to both the whole class (n=10) and particular students (n=8). A representative example reflecting this situation is as follows:

The groups explained how they solved the problem one by one. Students must have understood the solution so they told their teacher. (Hale, 5th video)

The other four SMTs focused on specific students/groups rather than the whole class in their observations and made more detailed explanations. Especially Eda and Sude focused more on particular students/groups, and they established relationships between subject contexts and student ideas based on evidence in their explanations. So much so that this situation became more visible in student expressions such as short responses, explanations and questions of the student. On the other hand, although Ege focused on particular students (n=24) in his observations based on oral evidence, the level of detail showed a similar interest to general (n=10) and specific

(n=13) explanations. Representative examples including the explanations of SMTs about student discourse are presented below:

The student defined height as "the line passing from one corner to the opposite edge". (Gizem, 4th video) *At the end of the lesson, a student asked how large areas such as the ocean could be measured*. (Sude, 3rd video)

The student said that all the shapes they cut from colored paper were triangles as a common relationship, but could not establish a relationship between their areas. (Eda, 2nd video)

The students' estimates were very high. Especially, the estimation of the student who said 50 square meters was quite different from the real value. (Ege, 1st video)

Although the observations of the SMTs based on non-verbal evidence were mostly focused on the whole class, this situation differed depending on the experiences of the teachers. In this context, non-verbal evidences determined by Hale were mostly related to the whole class, while Gizem and Ege were related to both the whole class and particular students/groups. Unlike the others, Eda and Sude also made specific statements about student actions, products and mathematical representations by addressing particular students/groups in non-verbal evidence. In addition, SMTs made general explanations focused on the whole class in their evidences based on students' general non-verbal responses and gestures. For example, in the fifth video episode, Hale focused on groups and commented on the group students' use of movement, explaining: *"Student groups seemed content to work collaboratively. The number of students who raised their finger on the questions was quite high."*

		Н	ale			Gi	zem			E	la			E	ge			Sı	ıde	
	Siz	ze	De	tail	Size De		Detail S		Size Detail	Size	Detail		Size		Detail					
	W	Р	G	S	W	Р	G	S	W	Р	G	S	W	Р	G	S	W	Р	G	S
Verbal Evidence	10	8	14	4	11	26	14	23	15	24	16	23	10	13	10	13	13	29	14	28
General responses	5	1	6	0	5	4	8	1	6	3	9	0	4	2	6	0	7	3	8	2
Short responses	1	3	3	1	2	6	3	5	3	5	1	7	2	0	1	1	1	10	2	9
Explanation	4	3	5	2	4	10	3	11	6	13	6	13	3	9	2	10	4	11	3	12
Question	0	1	0	1	0	6	0	6	0	3	0	3	1	2	1	2	1	5	1	5
Non-verbal evidence	17	5	16	6	14	13	15	12	9	16	8	17	8	10	11	7	11	15	12	14
Action	3	2	1	4	3	5	2	6	3	5	2	6	1	4	2	3	0	6	0	6
Artefacts	0	0	0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
General responses	7	0	7	0	7	2	9	0	2	0	1	1	5	1	6	0	6	0	6	0
Gesture	5	2	7	0	1	2	2	1	2	5	4	3	1	1	2	0	5	2	5	2
Voting	1	0	0	1	2	0	0	2	2	0	0	2	1	0	0	1	0	1	0	1
Mathematical notation	1	1	1	1	1	3	2	2	0	5	1	4	0	3	1	2	0	5	1	4

Table 5. The Nature of Each SMT's Observations of the Types of Evidence

SMTs' Skills to Interpret Different Types of Evidence of Student Thinking

Approaches of SMTs based on evidence of student thinking have been examined in two parts as descriptive and inferential. Table 6 reveals how SMTs make sense of observation evidence regarding student thinking in the videos they watch. The analyzes reveal a few striking findings. First, SMTs' approaches based on observation evidence tend to be inferential (n=164) rather than descriptive (n=113). Especially, the comments of the students based on the explanations were aimed to draw more conclusions among the verbal evidence types of SMTs. On the other hand, the comments of the students regarding their short responses mostly included simple definitions. Among the SMTs, the teacher who showed the most inferential approach to student discourse was Sude. In addition to student explanations, Sude made inferences based on both the student's responses and questions about the student's short responses. In this direction, Gizem made simple definitions of the student's short responses in some video parts, while in others she tended to draw conclusions based on evidence. Hale, on the other hand, took a questioning stance regarding student explanations, but made more descriptive statements based on verbal evidence of student thinking. Representative examples showing what kind of inferences SMTs make from oral evidence are presented below:

Students in the group explained the solution one by one. Every student must have understood the solution so that they could explain it to their teacher. (Hale, 5th video)

The student said that as we went up [floors of a square meter] we delete zero, while going down [floors of a square meter] we add zero. But we can't always implement this. When they need to convert with decimals, when they cannot find a zero to throw, or when they add a zero, the number does not change, students experience confusion. (Gizem, 3rd video)

The student said that the shapes are all triangles as a common relationship. Another said that the two triangles in the figure are right triangles. I think they gave these answers just because they think shape-oriented. I guess they didn't understand that they needed to make a connection between the fields of the shapes. (Eda, 6th video)

"What is desired? "It took students a long time to give the correct answer to this question. Students who raised a finger and asked to speak gave very interesting answers. For example, a student said, "the upright space of the tree and flower is requested." This shows that the student does not understand the problem. (Sude, 6th video)

On the other hand, SMTs easily made inferences based on non-verbal evidence of student thinking. In this direction, it is seen that Ege exhibits an inferential (n=13) approach rather than descriptive (n = 5) based on student actions, gestures, and mathematical representations. (See Table 6). An example of a situation where Ege makes inferences based on the student's mathematical representation is as follows:

Since the student did not write down the digits one under the other, he miscalculated the result while performing the addition. Actually, he said the result correctly, but he wrote the units digit below the hundreds digit. His friend noticed his mistake and corrected it by typing the digits one under the other. I don't think it's a mistake that can be a momentary mistake. (Ege, 5th video)

Among the SMTs, only Hale took a descriptive approach based on both verbal and non-verbal evidence. So much so that it did not make any inferences based on the actions of the students, but was limited to only defining the action. For example, regarding the student's height drawing in the 4th video, the Teacher asked the student to draw the height. The student tried to draw the height alone, but could not use the ruler correctly. "She explained that the student could not use the ruler correctly, but did not make any inferences about it. In contrast, other SMTs have tended to draw more conclusions based on student actions. Especially, it is seen that Eda makes inferences (n=7) about student actions rather than simple definitions (n=1). For example, Eda tended to draw the following conclusion based on the student's height drawing in the same video episode: "The students probably didn't know how to use a set square. They constantly sought help from their teachers, as they did not understand how to hold the square when drawing the altitude. "However, there have been cases where SMTs made inferences based on students' products and gestures. Reflective examples are as follows:

They [students] seemed pleased with the collaborative work; because all the students were interested in the worksheets and raised their fingers to answer the questions asked by their teachers. (Hale, 5th video) The teacher asked the students to cut the shape they drew, but the students did not understand what to do with the triangles they cut. They did not understand what kind of relationship they had to establish between the areas of the shapes they cut. Only one student must have understood that he was showing two right triangles glued on top of the big triangle. (Gizem, 2nd video)

					SN	ЛТs				
	На	le	Gizem			Eda		ge	Su	ıde
	D	Ι	D	Ι	D	Ι	D	Ι	D	Ι
Verbal Evidence	11	7	13	24	15	24	10	13	11	31
General responses	4	2	4	5	5	4	3	3	1	9
Short responses	4	0	4	4	5	3	2	0	7	4
Explanation	2	5	3	11	3	16	3	9	1	14
Question	1	0	2	4	2	1	2	1	2	4
Non-verbal evidence	16	6	12	15	11	14	5	13	9	17
Action	5	0	3	5	1	7	1	4	1	5
Artefacts	0	0	0	1	0	1	0	1	0	1
General responses	5	2	3	6	2	0	3	3	4	2
Gesture	4	3	2	1	3	4	0	2	2	5
Voting	1	0	2	0	2	0	0	1	1	0
Mathematical notation	1	1	2	2	3	2	1	2	1	4

Table 6. SMTs' Interpretation of Student Thinking on Different Types of Evidence

* Descriptive (D), Inferential (I)

SMTs' Skills to Make Instructional Decisions Based on Student Thinking

The instructional decisions of SMTs based on student thinking have been examined according to whether they are related to student's cognitive thinking. The decisions related to the cognitive thinking of the student were

evaluated in terms of whether they are content-specific or not. Table 7 shows the nature of the instructional decisions of teachers with different professional experiences based on students' cognitive thinking. The SMTs based 26 of their instructional decisions regarding the video segments they watch (5 SMTs, 30 units in total since there are 6 video episodes) on the cognitive thinking of the students in the video. When Table 7 is examined, it is seen that 10 of the suggested decisions are not specific to the content (content-general), and the other 16 are based on the student's thinking based on specific mathematical subjects (content-specific). Among the SMTs, Hale presented three suggestions based on student cognitive thinking, but only one of them (5th video) was based on content-specific student thinking. As a matter of fact, this proposal of Hale includes a teacher-centered strategy to deal with the situations where students have difficulties while converting land measurement units and area measurement units into each other. The representative example is as follows:

Students often confuse the relationship between area measurements and land measurements. For this, I would explain the connection between field measurements and land measurements through encryption. Passwords will be easier to remember because they are familiar words for students.

Although Ege suggested actions based on students' cognitive thinking in the video episodes he watched; only two of his suggestions (1st and 5th videos) were related to content-specific student thinking. Ege noticed that the group students tended to explain the operations rather than the strategy they developed while explaining the solution of the problem related to land measurements and made a suggestion regarding this situation:

When the groups explained their solutions for the land surveying problem, they only mentioned the procedures. They explained, "We transformed the square meter by are, we added and subtracted it," but they did not say why they added those two numbers or why they eventually subtract the sum from the other number. There may be these questions on the worksheet, but I still wish they could talk about why they needed those procedures when you asked for their explanation briefly, it would be better. So I would try to understand the way the students were thinking. (Ege, 5th video)

Other SMTs (Sude, Gizem and Eda) determined student thinking specific to particular mathematical subjects and suggested student-centered applications in this direction. For example, Eda stated that students tend to multiply two numbers while calculating the area (6th video) constantly and stated that they plan to do student-focused activities for this. Sude suggested instructional actions based on the question asked by a student at the end of the lesson (How can we measure large areas like the ocean?) In this context, Gizem's instructional decisions for students to accurately draw the heights of the triangle are as follows:

The students had difficulty drawing height. If I were you, I would first watch educational videos about drawing height, I would show you how to draw the height with a miter and then I would wait for the students to draw? Although students have difficulties in drawing the first height since they do not use the miter frequently, they can understand how to use it later and draw height accurately. In this way, after drawing the heights of the different triangles on the worksheet, I would ask the students what they noticed about the heights of the triangles. From here, I would have asked questions such as how many heights a triangle has, whether the heights are always in the inner part of the triangle, in which case it is in the outer part of the triangle, and in what case, I would realize that the positions of the heights of the triangles to their angles. (Gizem, 4th video)

Video	SMTs											
Video episodes	Нε	ıle	Giz	Gizem		Eda		Ege		de		
	GT	ST	GT	ST	GT	ST	GT	ST	GT	ST		
1 st video	1	0	1	0	0	1	0	1	0	1		
2 nd video	0	0	1	0	0	1	1	0	1	0		
3 rd video	1	0	0	1	1	0	1	0	0	1		
4 th video	0	0	0	1	0	1	0	0	0	1		
5 th video	0	1	0	1	1	0	0	1	0	1		
6 th video	0	0	0	1	0	1	1	0	0	1		

Table 7. Instructional Decisions of SMTs Based on Students' Cognitive Thinking

Discussion

The current study examined the ability of SMTs with different professional experiences to notice the types of evidence of student thinking. In the video episodes of our study, which contain different types of evidence related to student thinking, we explain in detail what evidence SMTS participate in, how they make sense of this evidence, and what kind of educational decisions they make. Our findings show that SMTS also take into account more types of oral evidence than non-verbal evidence of student thinking in 6 video episodes. In addition, SMTS can be said to be more sensitive to student explanations among oral evidence types, and to student actions in non-verbal evidence types. Although the main focus of SMTs in the video sections they watch is student discourse, this has changed depending on the duration of professional experience. A teacher with a year of professional experience in this direction tended to be more involved in non-verbal evidence of student thinking. However, among these types of evidence, it can also be said that student products and mathematical representations that provide information about the content of student thinking are less considered. In addition, when the nature of the observations made by SMTs based on student thinking is examined, it is understood that they show a similar interest to the evidence based on both the whole class and particular students/groups (see Table 4). In this context, experienced teachers focused on particular students/groups rather than the whole class and made context-specific explanations in their classroom observations. On the other hand, the inexperienced teacher tended to make more general explanations by focusing on the whole class. This was especially evident in the explanations of the teachers who had doctorate education and who were included in the professional experience course. Studies conducted in the relevant literature are generally carried out with prospective teachers and reveal that prospective teachers tend to make general explanations about the whole class rather than specific students (e.g., Barnhart & van Es, 2015; Lam & Chan, 2020). Therefore, the data obtained from the inexperienced teacher support the results of this study on noticing.

The data obtained in the study reveal that the visibility levels of different video segments containing the same types of evidence of student thinking by SMTs are not the same. For this case, SMTs' not paying attention to student short answers in the fifth video and focusing on student explanations, but showing a similar interest in both student short responses and explanations in the sixth video episode can be given as examples. The same is

true for video episodes that include voting. These findings are consistent with the result of Lam and Chan (2020) that paying attention to students' verbal explanations varies in different classroom situations and contexts. The same form of evidence being more visible to SMTs in different video episodes may also be related to the nature of selective attention for teaching (van Es & Sherin, 2021). Miller (2011) explains this process, which is defined as "cognitive tunneling", as individuals paying attention to a subset of their existing knowledge while remaining indifferent to other fields.

Our data analysis reveals that SMTs make more inferential interpretations to make sense of observation evidence regarding student thinking (see Table 6). So much so that all of the SMTs made the most inferences about student statements among the oral evidence. This situation can be explained by the fact that SMTs focus more on student descriptions and become aware of context-specific details. Indeed, when they do not notice particular details, they are less likely to make sense of the content of student thinking. (Lam & Chan, 2020). In contrast, it is noteworthy that the SMTs' comments on student short responses are more descriptive. Therefore, SMTs tended to draw conclusions based primarily on which type of evidence they were more involved in. In this respect, an example is the inexperienced teacher making inferences to understand what the students mean because she focuses more on student explanations, although she takes a descriptive approach based on verbal evidence. The data also reveal that the teacher, who has the most professional experience time among SMTs, and who participates in the video-based professional development course aimed at improving the noticing skills, makes inferences based on the student's cognitive thinking rather than describing the verbal evidence of the student. This finding is in line with the results of Erdik (2014) and Santagata (2011) research. Erdik (2014) revealed that experienced teachers analyze events from multiple perspectives on the basis of their noticing and make richer interpretations of the situations they noticed. In this context, Erickson (2011) argues that teachers' interpretations differ from teachers' professional experiences and pedagogical approaches.

Another finding reveals that SMTs' instructional decisions based on student thinking are mostly related to students' cognitive thinking. Moreover, most of the proposed instructional decisions are based on specific (content-specific) student thinking on specific mathematical topics (see. Table 7). As a matter of fact, this situation has changed depending on the professional experiences of SMTs. Accordingly, it was seen that the teacher with one year of professional experience offered suggestions based on the student's cognitive thinking but not related to specific mathematical issues. Experienced SMTs, on the other hand, have been determined to suggest more content-specific instructional applications based on student thinking. Teachers' focus on interpretation and evidence provides a basis for making instructional decisions (Amador, 2021). In addition, data analysis reveals that experienced teachers tend to make student-centered instructional decisions when they make content-specific inferences about the observation evidence of SMTs, especially those who have graduate education and attend professional development courses. This finding shows that not only the duration of professional experience alone is sufficient in making quality instructional decisions based on student thinking (Sherin et al., 2011) but also that teachers' expertise and professional development are also effective. This perspective is consistent with research results suggesting that teachers who are experts in their field can reason more effectively using contextual knowledge (e.g., Berliner, 2001). This finding also supports studies showing that teachers' instructional decisions are related to the nature of their observations and the information they use

to make sense of student thinking (e.g., Amador, Males, Earnest & Dietiker, 2017; Jacobs et al., 2010; Lam & Chan, 2020; Sherin & van Es, 2005; van Es, 2011; van Es & Sherin, 2021; Ulusoy, 2020). Therefore, if SMTs attend to the details of the types of evidence for student thinking and make inferences to make sense of these evidence, they can suggest mathematically high-quality instructional decisions.

Implications

The current study examined the ability of SMTs with different experiences to notice evidence of student thinking. It is believed that the results of the study will give a deeper understanding of the field of mathematics education. These findings have important implications for the nature of the teacher noticing. First, our study shows that experienced SMTs are more sensitive to particular types of evidence, such as student explanations and questions that reflect the content of student thinking. This highlights the need for inexperienced teachers to develop skills to consider the types of evidence that reflect the content of student thinking. We believe that identifying such evidence is the first step towards improving the ability to notice. Second, our findings suggest that experienced SMTs tend to draw conclusions by using details of evidence to make sense of student thinking. The fact that experienced SMTs make deductive comments based on observational evidence is an important indicator of their improved awareness skills. It may also be due to the academic orientation of experienced graduate teachers to make productive comments based on student thinking. Thirdly, our findings show the need to support SMTs' suggestions based on content-specific student thinking in the dimension of making instructional decisions. Obviously, it can be stated that experienced teachers who make inferential interpretations by focusing on the evidence of student thinking make more complex instructional decisions specific to the content. Finally, the findings reveal that professional experience alone is not sufficient for teachers' advanced noticing skills, in addition, postgraduate education and professional development courses are necessary and important.

Viewing video episodes containing different evidences of student thinking provided a deep insight in the study for SMTs to interpret student thinking about different subject contexts. A detailed analysis of the skills of teachers with different experiences to participate in the types of evidence of student thinking, to interpret and to make instructional suggestions will provide a direction for research in the related field. In addition, there is a need for additional studies to support noticing skills of teachers with different experiences based on mathematical evidence of student thinking.

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