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Unpacking The Relation Between Spatial Abilities and Creativity in Geometry

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Abstract: This study aims to examine the relation between spatial ability and creativity in Geometry. Data was collected from 94 ninth graders. Three spatial abilities were investigated: spatial visualization, spatial relations and closure flexibility. As for students' creativity, it was examined through a multiple solution problem in Geometry focusing on three components of creativity: fluency, flexibility, and originality. The results revealed that spatial visualization predicted flexibility and originality while closure flexibility predicted all creativity components. Additionally, it was deduced that auxiliary constructions played an essential role in the problem-solution process. Finally, further study opportunities for the teaching and learning of Geometry are discussed.

Keywords: *Creativity; Geometry; Geometrical Figure Apprehension; Multiple-solution Tasks; Spatial Abilities.*

Introduction

One of the most important goals of education today is to liberate thought and develop student creativity. Creative thinking is an invaluable human skill. Its importance has been underlined by international governing and educational institutions, such as the European Parliament and Council (2006) and the National Council of Teachers of Mathematics (2000). The ability of students to think creatively and flexibly about mathematical concepts and ideas is highlighted for students of all educational levels.

A significant causal relationship was found between mathematical creativity and performance in solving mathematical problems (Tyagi, 2016). In fact, several researchers (e.g. Ervynck, 1991; Silver, 1997; Stupel & Ben-Chaim, 2017) suggest that making connections among different mathematical ideas in multiple ways in order to either solve problems or prove statements is essential for the development of mathematical reasoning. Additionally, it fosters deeper understanding and increased creativity in Mathematics. To this end, recent studies (e.g. Levav-Waynberg & Leikin, 2012a; 2012b) used multiple-solution tasks (MSTs) as a dual tool; as an instructional tool to help develop students' knowledge and creativity and also, as a tool to evaluate creativity in Geometry. Due to the fact that geometry is a domain that creates many problems for students, independently of their age and their culture, teaching and learning of geometry is an area of great interest among mathematics education and psychology researchers. Thus, several research studies have focused on teaching and learning geometry to young children (Elia, Van den Heuvel-Panhuizen, & Gagatsis, 2018) and also to primary and secondary school students (Gagatsis, 2012; Gagatsis; 2015). Geometry provides students with opportunities to investigate and practice mathematics in ways that are similar to the work of mathematicians (Herbst, 2002). MSTs in Geometry encourage students to work on a variety of solutions by using concepts and properties which have been taught entirely through the school Geometry curriculum. For the solutions of a MST, extracurricular knowledge is not required (Levav-Waynberg & Leikin, 2012a). It is worth noting here that, most, if not all, problems included in the standard mathematical textbooks have the capacity to be converted to a MST.

It is well known that Geometry is related to various spatial skills, such as identifying the characteristics of shapes, angles and lines and their properties (Ünlü & Ertekin, 2017). Moreover, it is quite common that geometrical problems are accompanied by figures. Taking into consideration the fact that Geometry refers to a model of space (Soury-Lavergne & Maschietto, 2015) and also that visuospatial modality is the main factor affecting geometric ability (Xie et al., 2020), spatial visualization and reasoning are prevalent in geometrical thinking (Dindyal, 2015; Van den Heuvel-Panhuizen & Buys, 2008).

Kell et al., (2013) agree that spatial ability plays a unique role in integrating and applying prior knowledge but they also underline that it plays a crucial role in developing new knowledge. As they point out, without spatial ability the cognitive structure supporting creative and innovative thought is incomplete. In Mathematics, Pitta-Pantazi et al. (2013), investigated the relationship between the creative process in mathematics and spatial, object and verbal cognitive styles. The study explored 96 prospective primary school teachers' cognitive style through a translation of the Object-Spatial Imagery and Verbal Questionnaire (Blazhenkova & Kozhevnikov, 2009) and their mathematical creativity through a self-made mathematical creativity test. Mathematical creativity was measured in terms of fluency, flexibility and originality. The results of the study suggested that only spatial imagery positively predicted mathematical creativity, whereas both object and verbal cognitive styles were negatively related to mathematical creativity and mathematical flexibility respectively. The focus of this study is to investigate the relationship between creativity in Geometry and spatial ability as related international literature is sparse.

Theoretical Framework

Spatial Ability

Spatial ability is one of the most researched human cognitive processes (Carroll, 1993). Recently research in spatial ability has become even more popular as it seems to contribute to educational performance in science, technology, engineering, and mathematics through correlational evidence (Buckley et al., 2018).

Being a multidimensional concept (Yilmaz, 2009), spatial ability has been defined in so many different ways that one cannot be precise about the interpretation of the term (Eliot & Smith, 1983). There is however growing consensus that it is "the ability to generate, store, retrieve, and transform visual information" (McGrew, 2009). Spatial ability related factors are identified as mental abilities concerned specifically with visual stimuli; they have nothing to do with semantic knowledge (Buckley et al., 2018). As Hagarty and Waller (2005) state, John Carroll was the researcher who, in 1993, performed the broadest and most complete study of factors related to spatial ability. The present study adopts Carroll's (1993) meta-analysis that defines five major spatial abilities; visualization, spatial relations, flexibility of closure, perceptual speed and closure speed. However, it concentrates on the first three as, according to our opinion, they are more relevant to Geometry:

- Spatial visualization, which is described as the ability to imagine rotations of objects or their parts in 3-D space.
- Spatial relations, which reflect the ability to perceive an object from different positions. It is usually defined by time constrained tests involving rotations and/or reflections (Lohman, 1988).
- Flexibility of closure factor is the ability to find hidden patterns or figures in a bigger complex pattern, when the subjects are informed about what to look for. The flexibility of closure factor is alternatively called field independence or disembedding (Velez et al., 2005)

Mathematical Creativity, Geometrical Figure Apprehension and Multiple Solution Problem Solving

How could we define (mathematical) creativity? Some definitions of creativity are based on the properties of the creative act and product (e.g. Silver, 1997) while others concern the stages of the creative process (e.g. Ervynck, 1991). Torrance (1994) saw creativity through a multidimensional perspective; the different aspects are fluency, flexibility, novelty (or originality) and elaboration. Based on this definition, Silver (1997) claimed that the following three components need to be developed in order for creativity to be fostered through problem solving; a. Fluency is developed when an individual is encouraged to think of multiple ideas so as to easily arrive to different valid solutions. b. Flexibility has to do with the facility of an individual to move from one solution to another c. Novelty is nurtured when a new solution, different to those already known to an individual at the time is produced by him/her.

Leikin et al. (2006), and Leikin and Levav-Waynberg (2007) defined multiple solution tasks (MSTs) as tasks that explicitly require more than one solution to a specific mathematical problem. The differences among the solutions can be categorized according to the following criteria: "(a) different representations of a mathematical concept; (b) different properties (definitions or theorems) of mathematical concepts from a mathematical topic; or (c) different mathematical tools and theorems from different branches of mathematics" (Leikin, 2009). When solving a geometrical MST in particular, the different auxiliary constructions are included in category (b) (Leikin, 2009). According to Leikin and Elgrabli (2015) the level of complexity of the auxiliary construction is determined by two factors: 1) the location of the auxiliary construction; i.e. whether it is within or outside the given figure and 2) the number of constructions needed in order to identify a specific property.

We strongly believe that the findings of Roza Leikin and her colleagues' research are linked to Duval's analysis of cognitive apprehension problems related to the teaching and learning of geometry (Duval, 1999, 2005, 2006, 2014). Duval has identified four kinds of apprehension of a "geometrical figure": perceptual, sequential, discursive and operative. In order for any drawing to act as a geometrical figure, it must provoke perceptual apprehension and at least one of the other three kinds of apprehension. Perceptual apprehension is the ability to recognize a shape in a plane or depth. Figural organization as well as pictorial cues, which act as signals on the figure, are decisive factors on an individual's perception of a particular figure. To be more specific, the ability of an individual to name figures as well as to identify several sub-figures in a given figure is evidence of perceptual apprehension. Sequential apprehension is necessary for either the construction or the description of a construction of a figure. It is technical constraints and mathematical properties that define the organization of the fundamental figures and not perceptual laws and cues. Discursive apprehension is related to the fact that mathematical properties represented in a drawing cannot be determined through perceptual apprehension. The perceptual recognition of the geometrical properties of any representation must be constrained by statements (e.g., denomination, definition, primitive commands in a menu).

Operative apprehension gives an insight into a problem solution when looking at a figure. According to Duval, "operative apprehension depends on the various ways of modifying a given figure: the mereological, the optic, and the place way" (Duval, 1999; Michael et al., 2011, 2012, 2013, 2015). The mereological way refers to a reconfiguration: to split a figure into parts and to unite them in a new one. The optic way has to do with making the figure larger, narrower or slant, and the place way refers to its position or orientation variation. Modifications may be performed mentally or physically through various operations. These operations include specific figural processing which provides figures with a heuristic function (Duval, 1999, 2005, 2006, 2014).

This multi-dimensional approach to the apprehension of a geometrical figure makes its measurement possible. There is wide research investigating geometrical figure apprehension (e.g., Gagatsis et al., 2010; 2015, Gagatsis, 2011; 2012; 2015). These studies focused on primary and secondary school students. Despite the differences in the school level of students, statistical methods like factor analysis and equation structural modeling yielded similar results (Michael et al., 2011; 2012; Michael – Chrysanthou, Gagatsis, 2013; 2014; 2015). For example, Gagatsis et al. (2010) measured primary and secondary students' geometrical figure apprehension by using three of the types of cognitive apprehension (perceptual, discursive and operational). Furthermore, some studies by Michael et al. (2011, 2012, 2013, 2015) investigated primary and secondary school students' operative apprehension of geometrical figures.

Moreover, some studies have examined the relations between geometrical figure apprehension and spatial ability (Kalogirou, Elia, & Gagatsis, 2009; Kalogirou, & Gagatsis 2011:2012).

Recently, Gridos et al. (2021) showed primarily that the way through which students perceive the geometrical figure and their ability to process it, is an important factor in predicting their mathematical creativity. At a second level, it was found that perceptual apprehension of geometrical figures on its own is not a reliable predictor of creativity components. On the contrary, the same study showed that operative apprehension of geometrical figures can predict three of Torrance's components of creativity which are fluency, flexibility, and originality.

Methods

Research Questions

In order to investigate the relation between spatial ability and mathematical creativity in geometry through multiple solution problem solving, we will address the following research questions:

- How do students respond to spatial ability tasks and to a geometry problem with multiple solutions?
- What is the relationship between spatial ability and the creativity components (fluency, flexibility, originality) in solving multiple-solution geometry problems?

Population and Data Collection

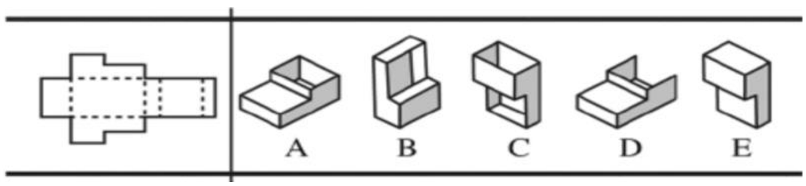
To examine the relation between spatial ability and mathematical creativity in geometry a two-part test was administered to 94 ninth graders (50 boys and 44 girls from four classes at a BLIND FOR REVIEW school) in usual classroom conditions. The first part which assessed students' spatial ability lasted 12 minutes. It included nine tasks that were developed and used in previous studies (e.g. Eliot & Smith, 1983; Ekstrom, French & Harman, 1976; Wai, Lubinski & Benbow, 2009). This part of the test was structured according to Carroll's model (1993) and reflects three spatial abilities that are more relevant to mathematical achievement; spatial visualization, spatial relations, and closure flexibility. In order to examine spatial visualization three tasks are used: "form board" (Fb), "paper folding" (Pf) and "surface development" (Sd). Spatial relation tasks include "cube comparison" task (Cc), "card rotation" task (Cr) and "hands" (Ha) task, while closure flexibility is assessed through the following tasks: "hidden figures" (Hf), "hidden patterns" (Hp) and "overlay figures" (Of). The false and correct responses were scored with 0 or 1, respectively. Table 1 represents a sample of spatial ability tasks.

In the second part of the test students' creativity is examined through a MST in Geometry that is presented in table 2. This problem is used by Leikin (e.g. 2011). Students are explicitly required to solve the problem in different ways. Students' responses were first evaluated for their appropriateness and then they were classified into solution spaces according to the auxiliary lines that students constructed. A solution space is a group of solutions to a mathematical problem. Solution spaces were used in order to analyze student problem-solving performance (Leikin, 2009). According to Leikin, "an expert solution space is the most complete collection of solutions for a problem known to a researcher or expert mathematician at a certain time" (Leikin, 2009). As new solutions to a problem are discovered, they are included in the expert solution space. An individual solution space is a collection of solutions to a problem found by one person. Individual solution spaces can be distinguished in two categories, according to someone's ability to solve a problem independently. To be more precise, a solution space can consist of solutions that the individual may present, either immediately or after some attempts, without help from others, while another potential solution space will consist of solutions that a student finds with support from other individuals. Collective solution spaces consist of solutions found by groups of individuals (Leikin, 2009).

Table 1

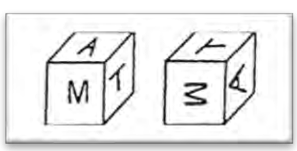
Sample of Spatial Ability Tasks

Surface development: There is a drawing of a flat piece of metal on the left. On the right five objects are shown, only one of which might be made by folding the flat piece of metal along the dotted line. You are to pick out the one of these five objects that shows how the piece of flat metal will look when it is folded along the dotted lines. When it is folded, no piece of metal overlaps any other piece or is enclosed inside the object.



Wai, Lubinski, & Benbow (2009)

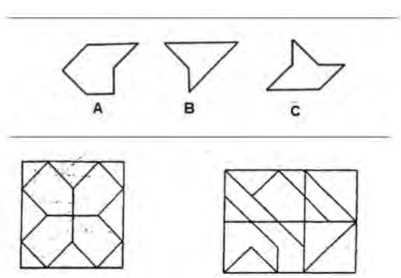
Cube comparison: The figure below presents two drawings of a cube. Assuming no cube can have two faces alike; the aim of the task is to indicate which items present drawings that can be of the same cube and which one's present drawings that are not of the same cube.



Same Different

(Eliot & Smith, 1983)

Hidden Figures: At the top of each page of this test there are three simple figures called A, B and C. Beneath each row of figures there is a page of patterns. Write the letter of the figure which you can find in the pattern.

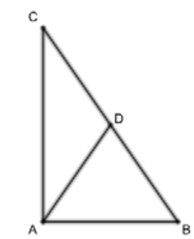


(Ekstrom, French & Harman, 1976)

The Cronbach's Alpha reliability coefficient for student responses across the spatial abilities (spatial visualization a = 0.713, spatial relations a = 0.668, closure flexibility a = 0.767) was found to be acceptable (Taber, 2018). This indicates that there is internal relevance and reliability in students' responses to the spatial ability test.

Table 2

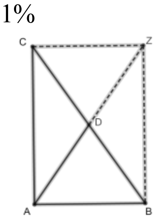
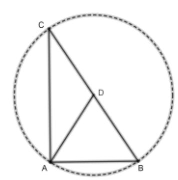
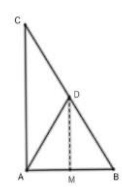
Geometry Multiple-Solution Problem

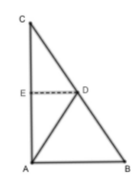
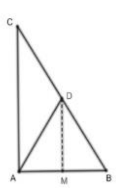
<table>
<tr><td>Prove in as many ways as possible, that the median (AD) of the right triangle BAC, equals half of the hypotenuse.</td><td>
</td></tr>
</table>

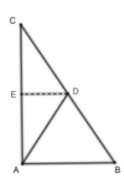
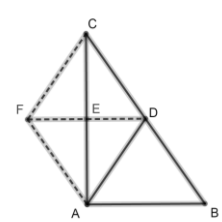
The five different collective solution spaces that we distinguished, based on students' responses and the percentages of students that use each solution of the problem, are presented in table 3. Students produced nine different solutions that correspond to five solution spaces. The fifth space consists of three alternative solutions because these solutions rarely appeared in the responses of students.

Table 3

Collective Solution Space for the MST in Geometry.

<table>
<tr><td>1. Double median - Double triangle
(SP1.1) (SP1.2)
11% 3%

</td><td>2. Circumscribe the triangle
(SP2)
13%

</td><td>3. Comparison of triangles
(SP3.1) (SP3.2)
40% 15%



</td></tr>
<tr><td>4. Property of perpendicular bisector
(SP4.1) (SP4.2)
10 % 4%



</td><td colspan="2">5. Other ways
a. Similarity b. Pythagorean theorem c. Rhomb
(SP5.1) (SP5.2) (SP5.3)
10% 2% 2%

</td></tr>
</table>

Student creativity was evaluated by first measuring each one of the creativity components (fluency, flexibility and originality) separately and then by combining those scores (e.g. Levav-Waynberg & Leikin, 2012a, Levav-Waynberg & Leikin, 2012b).

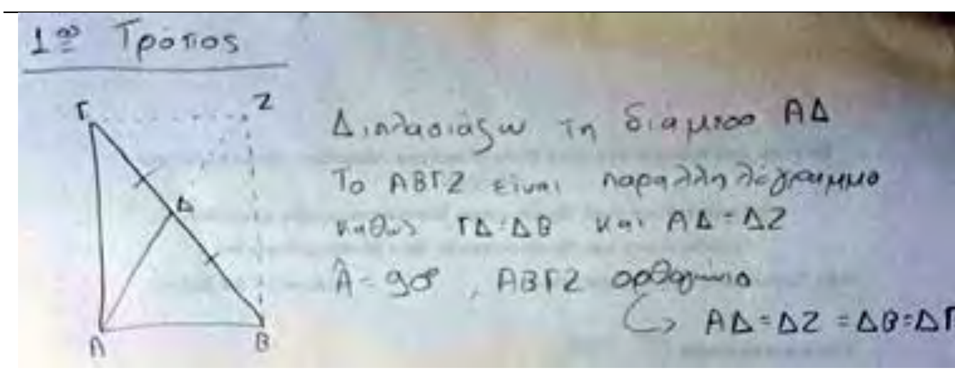
a. A student's fluency was measured by the number of appropriate solutions in his/her individual solution space. For instance, a student who proposed 3 appropriate solutions to a problem received a fluency score of 3.
b. A student's flexibility was measured by the differences among appropriate solutions in his/her individual solution space. In other words, the flexibility score represents the extent to which the student can shift from one solution to an entirely different one. The establishment of groups of solutions is needed to evaluate flexibility. Each solution from a different solution space was rated with a score of 0.2. The maximum score was 1 which corresponds to the 5 solutions from 5 different solution spaces.

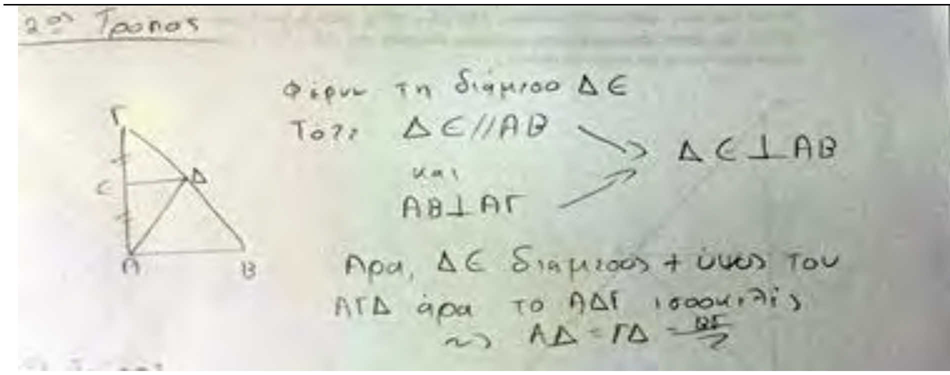
c. The originality of a student's specific solution was evaluated on the basis of how rare its solution group is in the population of this study. Solutions from a solution group that appeared in more than 40% of the individual solution spaces were given a score of 1 (SP 3.1-SP3.2). No solutions appeared between 15% and 40%. Solutions were most original if they belonged to groups with a frequency lower than 15% (SP1.1-SP1.2, SP2, SP4.1-SP4.2, SP5.1-SP5.2-SP5.3). The originality of these solutions was given a score of 10. The border values of 15% and 40% for different levels of originality were based on Leikin's experimentation (e.g. Leikin, 2009).

The notion of appropriateness is used in order to set apart the reasonable ways of solving a problem. Reasonable ways include those that lead to the correct solution outcome even if there are minor mistakes made in the process (Levav-Waynberg & Leikin, 2012a).

Table 4

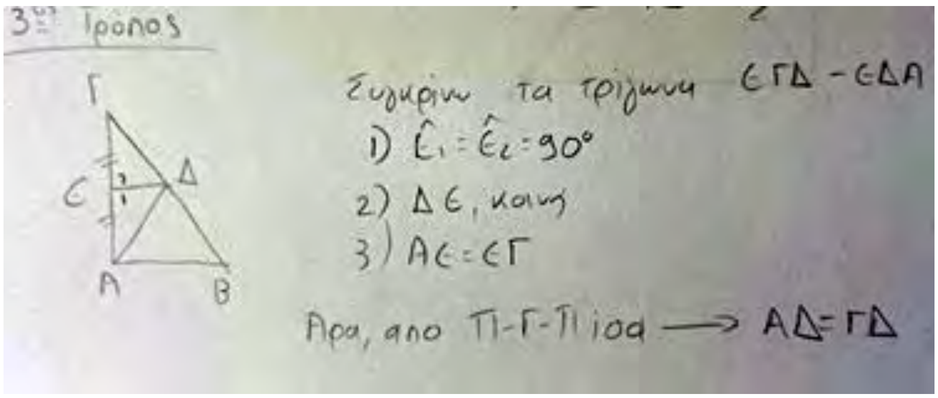
The Solutions Given by the Student with the Highest Creativity

	<u>1st solution</u> Extend ΑΔ to Z so that ΑΔ = ΔΖ. ΑΒΖΓ is a parallelogram because ΓΔ=ΔΒ and ΑΔ=ΔΖ. The angle ΒΑΓ=90°. Thus, ΑΒΓΖ is a rectangle. Thus, ΑΔ=ΔΖ=ΔΒ=ΔΓ=ΒΓ/2 (SP 1.1) <u>2nd solution</u>



The median ΔE is constructed.
Then, ΔE//AB and AB⊥AΓ.
So, ΔE⊥AΓ.
So, ΔE is the median and altitude of triangle AΓΔ, thus the triangle AΔΓ is an isosceles triangle.
Therefore, AΔ=ΓΔ=BΓ/2

(SP4.2)

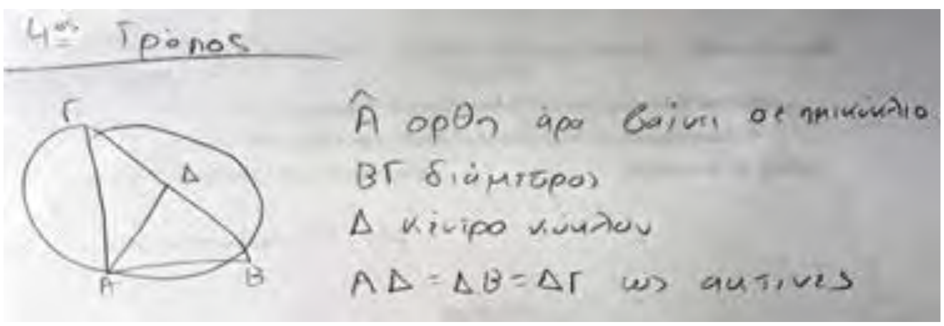


<u>3rd solution</u>
I compare triangles EΓΔ and EΔA:

1. Angle E_1 = Angle E_2 = 90º
2. ΔE is a common side
3. AE=EΓ

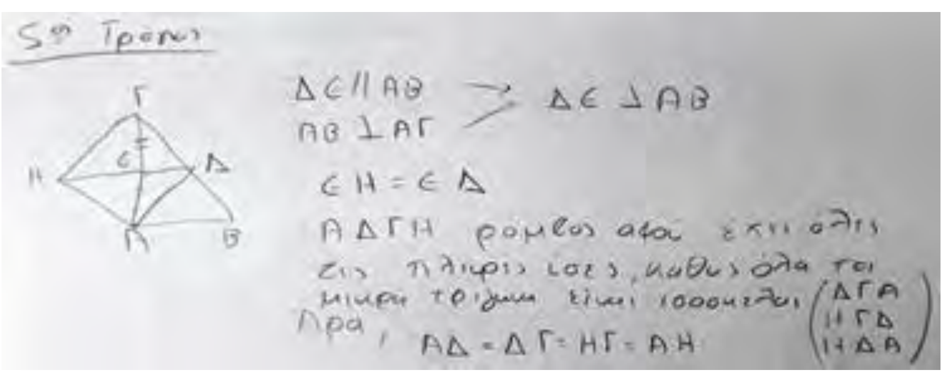
According to the Side-Angle-Side (SAS) rule the two triangles are congruent.
Therefore, AΔ=ΓΔ=BΓ/2

(SP3.2)



<u>4th solution</u>
The angle BAΓ = 90°
The right-angle triangle is inscribed in a circle.
BΓ is a diameter of the circle and Δ its centre.
Therefore,
AΔ=ΔB=DΓ=BΓ/2 as radii

(SP2)



<u>5th solution</u>
ΔE//AB and AB ⊥ AΓ therefore ΔE ⊥ AB
EH = EΔ
AΔΓH is a rhombus because all sides are equal (since all small triangles ΔΓA, HΓΔ, HΔA are isosceles)
Therefore, AΔ = ΔΓ = HΓ = AH

(SP5.3)

Table 5

A Student's Creativity Components Ccore.

Solutions	Fluency	Flexibility	Originality Ori
SP1.1	1	0.2	10
SP2	1	0.2	10
SP3.2	1	0.2	1
SP4.2	1	0.2	10
SP5.3	1	0.2	10
Total	5	1	41

Table 4 presents how a student of the population solved the problem. It is worth mentioning that this student achieved the highest creativity score. This student solved the problem in five different ways: SP1.1, SP2, SP3.2, SP4.2, SP5.3. Table 5 shows how the fluency, flexibility, and originality scores were measured for this student. The total fluency score was 5 since the student solved the problem in five different ways. As these solutions belonged to five different solution spaces, the student's score for flexibility was 1, i.e. 0.2 for each solution from a different solution space. Four of the solutions belonged to groups with a frequency lower than 15%. The originality for these solutions (SP1.1, SP2, SP4.2, SP5.3) was scored 10. The solution SP3.2 was scored 1, as it belongs to a group with a frequency of more than 40%. Thus, the student's originality score was 41. The Cronbach's Alpha reliability coefficient for students' responses to all mathematical creativity components was found to be acceptable (fluency a=0.834, flexibility a= 0.754, originality a=0.732), indicating that there is an internal relevance and reliability in students' responses to the multiple-solution problem.

Data Analysis

Descriptive statistics and multiple linear regression analysis with computer software SPSS, were used to answer the research questions.

Results

How Do Students Respond to Spatial Ability Tasks and a Geometry Problem with Multiple Solutions?

Table 6 presents the success rates in spatial visualization, spatial relations, and flexibility of closure tasks. According to the results, the percentage of success is different according to the category to which the task belongs. The highest success rates appeared in "card rotation" (93%) and "cube comparison" tasks (80%). However, students faced difficulties in the "form board" (28%), "hidden patterns" (15%) and "hidden figures" (41%) tasks. The percentages of students who were able to solve each task correctly are shown below.

Table 6

Success Rates in Spatial Ability Tasks.

Category	Variables	Success Rate (%)
Spatial visualization (Vz)	Form board (Fb)	28%
	Paper folding (Pf)	70%
	Surface development (Sd)	53%
Spatial relations (Sr)	Cube comparison (Cc)	80%
	Card Rotation (Cr)	93%
	Hands (Ha)	70%
Flexibility of closure (Cf)	Hidden figures (Hf)	41%
	Hidden patterns (Hp)	15%
	Overlay figures (Of)	73%

According to Figure 3 above, the solutions provided by students were based on one (PS2, PS3.1-PS3.2, PS4.1-PS4.2, PS5.1, PS5.2) or more (PS1.1-PS1.2, PS5.3) auxiliary constructions. In the solutions in which comparison of triangles, the property of perpendicular bisector, similarity and Pythagorean theorem were used, the auxiliary constructions created sub-figures within the given figure. In the solutions in which the students doubled the triangle (around hypotenuse) or the median, the auxiliary construction was outside the given figure. In the solution in which the students constructed a rhomb to solve the problem, a part of the auxiliary construction was inside and a part of it outside the given figure.

Table 7 presents the results of the students' responses in terms of fluency, flexibility and originality in their solutions to the geometrical problem. Almost half of the students (48%) solved the problem in 2 or 3 ways, while the percentage of students who solved the problem in 4 or 5 ways is quite high (18%) as well. Even though students' fluency was at a satisfactory level, a large percentage (34%) of students either found one or did not provide any solution at all. It is worth mentioning that the corresponding rates of fluency and flexibility are close enough, with the flexibility of student solutions being lower. On the contrary, we observe that the originality of solutions that emerge based on the frequency of the solution across all students is at lower levels. More specifically, 52% of students show low originality, 32% show moderate originality while only 9% show high originality in their solutions.

Table 7

Creativity Components' Rates.

Fluency	Rate (%)	Flexibility	Rate (%)	Originality	Rate (%)
0	10%	0	10%	0-20	59%
1	24%	0.2	34%	20-40	32%
2	32%	0.4	32%	40^{+}	9%
3	16%	0.6	16%		
4	10%	0.8	5%		
5	8%	1	3%		

What is the Relationship Between Spatial Ability and the Creativity Components in Solving Multiple Solution Problems?

Table 8 presents students' success rates in spatial visualization, spatial relations, and closure flexibility according to their score of fluency, flexibility, and originality. According to the results, students with low fluency and flexibility scores solved correctly only some spatial relation tasks. Their percentage of success in spatial visualization and flexibility of closure tasks was low. In contrast, students with a high score of fluency, flexibility, and originality solved correctly not only spatial relation tasks but almost all spatial visualization and flexibility tasks, as well. Students' success in spatial abilities tasks increased, as the group score of fluency, flexibility, and originality increased.

Table 8

Students' Success Rates in Spatial Abilities According to Fluency, Flexibility, and Originality Score.

Component	Score	N	Spatial visualization (%)	Spatial relations (%)	Flexibility of closure (%)
Fluency	0	9	22.2	51.8	11.1
	1	23	27.2	76.7	30.1
	2	30	46.4	78.4	39.6
	3	15	66.2	86.5	41.8
	4	9	85.2	100	81.4
	5	8	95.8	100	87.5
Flexibility	0	9	22.2	51.8	11.1
	0.2	32	34.5	76.5	29.3
	0.4	30	38.5	80.3	38.4
	0.6	15	74.8	93.1	49.5
	0.8	5	93.3	100	100
	1	3	100	100	100
Originality	0-20	41	47.7	69.1	35.6
	20-40	50	51.3	76	47.3
	40+	3	100	100	100

To investigate further the relationship between spatial ability and mathematical creativity, multiple regression analysis was performed using the spatial abilities (spatial visualization, spatial relations and closure flexibility) as independent variables and the components of mathematical creativity (fluency, flexibility, originality) as dependent variables. Using the enter method, a significant model emerged for each of the characteristics of creativity (Fluency: F = 11.017, p = 0.001; Flexibility: F = 9.571, p = 0.003; Originality: F = 10.352, p = 0.012). Table 9 presents the results of the regression analysis and indicates the coefficients and percentage of variance.

Table 9

Multiple Regression Analysis Which Explores the Relationship Between Spatial Abilities and Mathematical Creativity omponents.

		Spatial Visualization		Spatial Relations		Closure Flexibility	
		B (SE)	*β*	*B* (SE)	*β*	*B* (SE)	*β*
Mathematical Creativity	Fluency	0.06 (0.01)	0.189	-0.02 (0.04)	-0.040	0.12 (0.01)	0.327**
	Flexibility	0.04 (0.03)	0.115*	0.01 (0.07)	0.032	0.09 (0.02)	0.304**
	Originality	0.05 (0.03)	0.166*	0.03 (0.09)	0.102	0.11 (0.02)	0.298**

R^2 = 0.107 for fluency, R^2 = 0.094 for flexibility, R^2 = 0.102 for originality, * p < 0.05, ** p < 0.01.

According to the findings, spatial visualization can be a statistically significant predictor of flexibility (β = 0.115, p = 0.26) and originality (β = 0.166, p= 0.029), proving it has a positive effect on them. In fact, it explains 10% of the variance in the total performance of the flexibility and originality, respectively. As for the second component of spatial ability, that is spatial relation ability, it appears that it is not a statistically significant predictor of creativity components (p > 0.05). Multiple regression analysis also reveals that closure flexibility can be a statistically significant predictor of all mathematical creativity components, having a positive effect on them: fluency (β = 0.327, p = 0.001), flexibility (β = 0.304, p = 0.003) and originality (β = 0.298, p = 0.001). Closure flexibility explains 11%, 10% and 10% of the variance in the total performance of the fluency, flexibility, and originality, respectively.

Discussion

This paper aims at enriching the knowledge regarding mathematical creativity by solving MSTs in Geometry through empirical data. For this purpose, the relationship between creativity and spatial abilities was examined.

For our first research question we examined how students respond to spatial ability tasks and to a geometrical MST. Regarding the spatial ability tasks, our results revealed that students have difficulties with tasks that require the ability to mentally fold or assemble parts of an object. Findings can be explained if we consider that spatial visualization involves complicated, multistep manipulations of spatially presented information (Linn & Petersen, 1985). Furthermore, disembedding, which is the spatial skill necessary for an individual to distinguish a simple object within a more complex figure, requires high perceptual apprehension. Additionally, students had to be able to identify a pattern within a given simple configuration in order to solve the task. The aforementioned processes all together required students to assimilate a given set of visual-spatial relations and then modify them into a new set of relations in a constrained way (Sanchez & Wiley, 2017). On the contrary, students achieved high scores in spatial relation tasks. This is explained by the fact that these tasks involved simpler mental operations, such as a single-step mental rotation of a two-dimensional object (Kozhevnikov & Hegarty, 2001).

Regarding the MST, students produced different solutions which were categorized in solution spaces according to the auxiliary lines they constructed. The need to transform the initial figure by constructing one or more auxiliary elements arose in order for students to provide solutions to the geometrical problem. Our findings indicated that the auxiliary constructions played a critical role in the problem-solution process. Previous research suggests that students introduce auxiliary lines to recall some known results or definitions and modify the given figure accordingly, as part of a learned procedure. By modifying the given figure, they anticipate to retrieve more information (Palatnik & Dreyfus, 2018). Some auxiliary elements unite previously unrelated components of the original figure while others split up a given complex figure into simpler ones (Palatnik & Sigler, 2019). In our study, the auxiliary construction(s) either created various sub-figures within the given figure or resulted in the construction of another figure outside the given one. We may assume that the ability to recognize either the sub-figures embedded within the given figure or the given figure embedded in a more complex one, was needed in order for the problem to be solved.

Our findings regarding the construction of auxiliary elements were in line with Duval's operative apprehension of a geometrical figure that depends on the various ways of figure modification (Duval, 2014; Kalogirou & Gagatsis, 2012; Michael-Chrysanthou & Gagatsis, 2015). Each of these different modifications can be performed mentally or physically through various operations such as figural processing, which aims to identify the heuristic function of figures. It is usually one or more of these operations that can lead to a modification of the figure which allows students to have an insight of the solution of a problem (Duval, 1999). Findings revealed that multiple solutions were produced by less than 5% of the participants. A possible explanation for the low frequency is Hsu's (2007) argument that the use of auxiliary lines is demanding for two reasons. First, because perceptual apprehension including auxiliary construction is a dynamic and demanding process and secondly because students need to apply transformational observation to visualize a solution. Furthermore, Fujita et al. (2020) pointed out that for the solutions of problems demanding more than one step of reasoning, spatial visualisation and property-based spatial analytic reasoning must be consistent with domain-specific knowledge. This way, the perceptual appearance of the given geometrical figure can be overcome. Based on our results, we believe that students who are successful at generating the most solutions, those who are the most creative, can perform transformations revealing the heuristic characteristics of the figure.

As far as the creativity components are concerned, our results indicated that students' fluency was of a satisfactory level. This finding was expected as the past few years, it is strongly advised that solving problems in multiple ways is beneficial for mathematics teaching and learning (e.g Bingolbali, 2020; Stupel & Ben-Chaim, 2017). Additional learning opportunities arise when students attempt to make sense of explanations given by others, compare solutions of others to their own, and make judgements about similarities (Yackel & Cobb, 1996). However, it is a time-consuming process and considering the requirements of the school curriculum, it is not always a feasible classroom routine. This is, perhaps, one of the reasons why many students found one solution only or did not provide any.

As expected, the corresponding rates of fluency and flexibility were similar. This is due to the fact that in most research studies the two concepts appear as a dipole (Leikin, 2009), with flexibility rates of student solutions usually being lower. As far as the originality scores are concerned, these were found to be quite low. This is in line with previous studies (e.g. Levav-Waynberg & Leikin, 2012a) in which originality appears to be a more inner characteristic compared to fluency and flexibility. It is worth noting here that students with deep knowledge of the mathematics content may also show originality (Van Harpen & Presmeg, 2013).

In order to address the second research question, we also focused on the relationship between creativity and spatial abilities. Our results implied that students with a high ability to imagine and mentally transform spatial information, were also able throughout the solution process to focus on different sub-figures at the same time, to use various properties and to construct new auxiliary elements in the figure. These students achieved high creativity scores. This study's results are in agreement with the results of a study by Gridos et al. (2018) investigating the relation between spatial ability and creativity through MSTs. The study focused on 9th grade students and measured students' spatial ability as well as two creativity components; their fluency and flexibility. The findings first revealed that spatial ability is a multi-dimensional concept. Secondly, it revealed that it is in fact the operational apprehension of the figure that enhances students' ability to perform geometrical proofs in more than one ways.

Our findings revealed that components of creativity were predicted by some spatial abilities. Spatial visualization in particular proved to have a positive effect on flexibility and originality while closure flexibility had a positive effect on all creativity components. In other words, students with a high ability to mentally fold or assemble parts of an object and imagine the result and students with the ability to recognize and quantify the orientation changes in a scene had a high level of flexibility and originality. Furthermore, students with high disembedding skills had a high level of fluency, flexibility and originality. Pitta-Pantazi et al. (2013) indicated similar findings by stating that spatial imagery cognitive style allowed individuals to be more creative and provide numerous, different, and unique solutions in mathematical tasks.

Our findings conclude that closure flexibility may contribute to the improvement of all the creativity aspects and that spatial visualization perhaps enhances the aspect of flexibility and originality. The implication of these findings is important for mathematics teaching and ought to be considered by educational policy makers, curriculum developers and mathematics textbook authors, so that spatial ability tasks are included in educational practice in a targeted manner and supported by teacher training. Additionally, geometrical MSTs are an efficient tool to encourage students to exercise their spatial ability skills and practice their mathematical creativity.

Limitations of the Study and Recommendations for Future Research

It is important to note that the results of this study are not immediately deduced to the general population. The interpretation of our results is limited by the sample and the specific tools we used. Even though reliability scores were acceptable, the number of tasks for each spatial ability was quite low. A future similar research could include more spatial ability tasks for students. Furthermore, the creativity components were measured based on one MST in geometry. Another limitation of this study is that we followed a research model used by previous studies in which participants solve MSTs accompanied by the relevant geometrical figure. However, in geometry three registers are used; the register of natural language, the register of symbolic language and the figurative register. Thus, there is a variation of the problem wording based on the semiotic registers that is used. When the relevant figure is not provided, students construct it to solve the problem. This distinction between problems that are accompanied by the relevant figure or not is justified by the well-known students' difficulty to construct a geometrical figure based on the wording of the problem. In problems that are not accompanied by the relevant figure, a two-step visualization is needed by the students to arrive to a solution; the first step has to do with the passage from the wording of the problem to the creation of a mental image of the described object. The second step concerns the passage from the created mental image to the construction of the geometrical figure. So, in the future, it would be interesting to examine MSTs that are not accompanied by the relevant figure.

Although our findings lead to a potential review of the teaching and learning of geometry, there is still room for further investigation. For example, Uttal et al. (2013) meta-analysis suggested that the pliable nature of spatial skills means that a wide variety of training procedures can be successful in strengthening spatial ability. Furthermore, several previous studies (e.g. Clements et al. 2011) have shown that spatial training can improve mathematical achievement. Mulligan et al. (2020) recently emphasized the need for learning opportunities that explicitly show the connection between mathematical and spatial concepts and processes. In order for this to happen, they further stress the need for the development of the relevant curriculum and teacher training support, i.e. resources and strategies.

The results of the studies presented previously concerning not only students' geometrical figure apprehension but also the relations between students' geometrical figure apprehension and their spatial abilities, together with the results of the present study, confirm the need for a composite model that could clearly interpret the nature and structure of students' creativity in geometry. The above-mentioned model is in line with the ideas of the article of A. Gagatsis and Z. Geitona (Gagatsis & Geitona, 2021) who propose a multidimensional approach to students' creativity in geometry which will take into consideration spatial ability, geometrical figure apprehension and multiple solutions. Their proposed research project will aim to identify students' levels of creativity in geometrical problem solving (Gagatsis & Geitona, 2021). According to the above proposed research project, a classification of students' creativity in geometrical problem solving simultaneously considering spatial abilities and geometrical figure apprehension will contribute to the learning and teaching of geometry. Finally, such a study can also provide insight into ways to strengthen and advance students' creativity in not only geometry but also, other areas of mathematics.

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