The State of Primary School Third-Grade Pupils' Making Sense of the Concepts of "0" and "1"

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Abstract

This paper focuses on determining the opinions of primary school third-grade pupils about the conceptual meaning and use of the numbers "0" and "1". The current study employed the case study design, which is one of the qualitative research methods. In the selection of the sample, the criterion sampling method, one of the purposive sampling methods, was used. The study was conducted with the participation of a total of 114 third-grade pupils (58 girls and 56 boys) attending two state primary schools located in the city of Ankara in the fall term of the 2019-2020 school year. As the data collection tool, a semi-structured interview form developed by the researchers in relation to the concepts of "0" and "1" was used. In the analysis of the data obtained in relation to the concepts of "0" and "1", content analysis was used. The primary school third-grade pupils made sense of the number "0" in association with the categories of four operations, ineffective element, absorbing element, number, natural number, meaningless, valueless, absence, even number, beginning, and letter. The categories in which most thoughts about the meaning of the number "1" are gathered are number, ineffective element, and uniqueness. It was determined that the pupils are more successful in the use of the numbers "0" and "1" in the addition and subtraction operations than in the multiplication and division operations.

Keywords: Zero, One, Primary School, Problem

Introduction

Various problem situations are encountered in daily life. This ranges from what time to wake up in the morning to the time spent in traffic after work. It can be said that numbers and mathematics are used to overcome many of these problem situations. For this reason, numbers make human life easier and constitute an important part of life. According to Connor (2019), numbers give shape and texture to everything that is felt, said, imagined, and done. This feature and prevalence of numbers have found a place in one of the many definitions of mathematics. Mathematics is expressed as the common name of the sciences that examine the properties of quantities such as arithmetic, algebra, and geometry on the basis of numbers and measures (Toptaş & Olkun, 2007). Based on the definition, it can be stated that numbers are the basis of mathematics. It affects mathematics achievement because they improve pupils' math skills, counting, and thinking strategies which contribute to pupils' problem-solving skills (Baroddy, 1987; Hughes, 1989). Failure to grasp numbers and the relationships between them causes the problem-solving skills not to develop, and pupils develop a negative attitude towards mathematics due to failure in

problem solving (Başar, Ünal & Yalçın, 2002). Due to their importance in the development of mathematical skills, the relationship between numbers and arithmetic operations are addressed at different densities in the mathematics curriculums of all grade levels from preschool to higher education. This is especially at the primary school level where the objectives related to numbers and operations occupy an important place (Ministry of National Education [MEB], 2018; National Council of Teachers of Mathematics [NCTM], 2000). Therefore, the basis of the concept of number is laid in primary school years.

Number is a concept that has an abstract structure and is a product of the mind as it is intellectually created (Baykul, 2016; Witzel & Little, 2018). It is important for children to be exposed to meaningful experiences about numbers through activities at early ages as numbers are not only for counting but also form a basis for learning through understanding at more advanced levels (Haylock & Cockburn, 2013). Mathematics subjects addressed in the first years of primary school generally consist of natural numbers and four operations performed with natural numbers. The mathematical concepts encountered in the set of natural numbers are also encountered in other number sets. In addition, the teaching of natural numbers and four operations performed with natural numbers constitute the basis for the teaching of other number sets (Olkun & Toluk Uçar, 2018).

Since number information is a necessity of counting skill, the development of the concept of number is associated with the acquisition of counting skill (Aktaş Arnas, 2002). Counting is also the basis of mathematical operations, and every mathematical operation means counting (Connor, 2019). This is because counting-based approaches are generally used to teach four operations (Sarama & Clements, 2003). The four basic operations, addition, subtraction, multiplication, and division, are the basic operations in arithmetic, the broadest and most well-known branch of mathematics (Akkan, Baki & Çakıroğlu, 2011). Arithmetic includes finding the unknown based on the known with the help of the four basic operations, numbers and the relationships between numbers, four operations with numbers, and all calculations based on these four operations (Akkan, 2009). Numbers and counting are the basis of arithmetic, and arithmetic is the basis of mathematics (Albayrak, 2010). Therefore, knowledge of arithmetic requires knowledge of numbers. In this case, it can be said that the numbers 0 (zero) and 1 (one) require knowledge of their properties according to the operations in arithmetic, but they pose difficulties for pupils. The multiplication properties of 0 (zero) and 1 (one) may be confused by some pupils with their properties of addition. For example, regarding 0 (zero), while the result does not change in the “6 + 0” operation, the result is always zero in the "6x0" operation. Similarly, regarding 1, in the “1 + 4” operation, one more of the number is obtained, while the number remains the same in the “1x4” operation (Van de Walle, Karp & Williams, 2016). Therefore, the ability to perform the four operations by pupils depends on the discovery of numbers (Ferah, 2006). The discovery of numbers and the development of the concepts behind the rules and properties of the four operations can be accomplished through word problems and verbalizing numerical expressions (Van de Walle, Karp & Williams, 2016).

The concepts of counting and number, which can be seen as the basis of arithmetic, are among the concepts in whose definitions mistakes are commonly made (Albayrak, 2010; Yenilmez & Demirhan, 2013). In terms of the concept of number, it is seen that pupils often make mistakes in the use of the numbers 0 (zero) and 1 (one) (Arslan & Ubuz, 2015; Dinç Artut & Tarım, 2006; Haylock & Cockburn, 2013; Hughes, 1986; Sadi, 2007; Tall & Razali, 1993). Studies show that 0 (zero) and 1 (one) are two important numbers in the number system. In addition, although the concept of number is one of the concepts in which the most identification errors are made, it also forms the basis of the learning area of algebra (Paydar, Doğan & Şahin, 2019; Yenilmez & Demirhan, 2013). In the primary school period, it is important to determine how the numbers 0 (zero) and 1 (one) are perceived by pupils due to their unique characteristics, how these numbers are used when performing operations, and how these numbers are used in problem-posing processes. Numbers are used in almost every aspect of daily life. For this reason, students should learn numbers accurately, especially in primary schools, which are the basis of education.

Therefore, it is thought that it is important to determine the problems experienced by the students in the nominal and ordinal meaning of the number 0. Additionally, in order to eliminate the difficulties experienced in the numbers 0 and 1 regarding the properties of four operations, it must be determined first. Therefore, the primary aim of the current study is to determine the opinions of primary school 3rd grade pupils on the concepts of 0 and 1. To this end, the sub-aims that the study sought to answer is as follows: "What are the opinions of the primary school 3rd grade pupils about the meaning of the number 0?", "What is the state of the primary school 3rd grade pupils' using the number 0 in the four operations?", "What is the state of the primary school 3rd grade pupils' posing and solving problems using the number 0?", "What are the opinions of the primary school 3rd grade pupils about the meaning of the number 1?", "What is the state of the primary school 3rd grade pupils' using the number 1 in the four operations?", "What is the state of the primary school 3rd grade pupils' posing and solving problems using the number 1?"

Literature Review

Numbers are not the names of objects or phenomena observed in nature, but are concepts formed in our minds during the counting process and are used to quantify the quantities (Yıldırım, 2010). In order to express the quantity of the multiplicities, the set reached by starting from the number "1" and continuing to infinity consecutively is called the set of counting number. The child uses counting numbers first. When 0 (zero) is added to the set of counting numbers, a set of natural numbers is obtained (Baykul, 2016).

When 0 (zero) is taken from the figural point of view, it results to the expressions "zero" as a word and "0" as a symbol in the language of mathematics (Haylock & Cockburn, 2013). Mathematical symbols (0, 1, 2, 3, +, -, <,>, %, etc.) are important tools used in conveying mathematical information (Olkun & Toluk Uçar, 2018). Sometimes, the 0 (zero) symbol is used to denote "nullity", "nothing" or "absence". If a child wins six stones while playing and then loses these six stones, then there is no stone left, 0 (zero) stone. If there is no seven-year-old child in a class, the seven-year-old group refers to the "empty set" (Haylock & Cockburn, 2013). While pupils do not experience difficulties in understanding this meaning of "0" denoting absence, they may have difficulties in comprehending the nominal and ordinal meanings that reveal the labelling and sorting features. According to Haylock and Cockburn (2013), the reason for this situation is that the counting feature of numbers is overemphasized. For example, representing a ground floor with 0 (zero) in the elevator or not using 0 (zero) in the sense of absence in the thermometer can cause difficulties for pupils (Güneş & Aydoğdu-İskenderoğlu, 2019). When its ordinal meaning in the ordering feature of numbers is considered, it is seen that 0 (zero) is as important as the other numbers because 0 (zero) can be the starting point on the number line, it represents the point before one, and it indicates the point that separates positive and negative numbers on the number line (Haylock & Cockburn, 2013). Another difficulty with zero is seen in multi-digit numbers. In multi-digit numbers, a false relationship is established between the word "hundred" and the symbol 0 (zero); for example, the placeholder property of the zero is difficult to grasp, as in the case of numbers "637" and "60037" (Arslan & Ubuz, 2015; Olkun & Toluk Uçar, 2018).

Another number concept that pupils have difficulty with is the number 1 (one). "One" is the nature of the bond that forms the common feature between one thing and another that can be counted as one. There are two forms of a number; the unity of the indivisibility whose part is the same and the first unit of a sequence when seen from the perspective of a situation referring to two or more than two (Connor, 2019). The difficulties experienced by pupils regarding the first meaning of the number "one" have been revealed by studies. Dinç Artut and Tarım (2006) asked primary school pupils to take number bars as many as the number "16" and then to show the number bars as many as the number in each digit. They concluded that number bars (10 pieces) equal to the number "1" in the tens place could not be modelled by the pupils. It is seen that in this error that emerged depending on the digit value, the pupils were unable to comprehend that ten

one numbers come together and form a ten in the sense of the unity of the indivisibility, each part of which is the same.

Understanding the concepts of “zero” and “one” can be difficult for children who are new to mathematics. As a result of this difficulty, mistakes can be made by children (Ashlock, 1994; Brown & Burton, 1978; Chick & Baker, 2005; Doğan, 2002; Engelhardt, 1977; Govindan & Ramaa, 2013; Kubanç, 2012; Önal, 2018; Radatz, 1980; Roberts, 1968; Sadi, 2007; Spooner, 2002; Tall & Razali, 1993; Thompson & Bramald, 2002; Varol & Kubanç, 2015; Yorulmaz & Önal, 2017; Young & O’Shea, 1981). However, learning numbers without making mistakes is important for mathematics education.

Method

In this section of the study, information about the research model, study group, data collection tool, data collection process, and data analysis is presented.

1. Research Model

Qualitative research is used to define and interpret the basic characteristics of individual and social events and phenomena by the researcher with an explanatory and interpretative perspective (Creswell, 2016; Neuman, 2010). The current study employed one of the qualitative research methods in the case study design. Case study is an approach involving the collection of in-depth information from multiple sources of information (observation, interview, audio-visual materials, documents and reports) about a real-life, current and constrained system (case) or multiple constrained systems in a given time and an in-depth and longitudinal examination of the analyzed data (Creswell, 2016; Flyvbjerg, 2006; Glesne, 2012). In this study, students' views on the numbers "0" and "1" and their usage in mathematics will be examined in depth. In this connection, this design was employed in the current study, as it was aimed to reveal and describe primary school third-graders’ level of understanding the concepts of “0” and “1”.

2. Study Group

The study group of the current research was determined by using criterion sampling, which is one of the purposive sampling methods. In the current study carried out in the fall term of the 2019-2020 school year, the following criteria were used in the selection of the participants: being a third grade pupil and attending a state primary school. The reason why the study was carried out in the third grade of primary school is that the development of the concepts of "0" and "1" ends in this grade. Correspondingly, a total of 114 third-grade pupils (58 girls and 56 boys) attending two different state primary schools in the city of Ankara participated in the study. A great care was taken to involve pupils from different socio-economic levels in the study and schools were selected based on these criteria.

3. Data Collection Tool

In the current study, a semi-structured interview form developed by the researchers was used to collect data. The semi-structured interview form was developed to determine primary school third-grade pupils’ level of understanding the concepts of “0” and “1” by reviewing the primary school math curriculum, teacher books, pupil textbooks, workbooks and related literature consisting of three parts. In the first part of the data collection tool, there are questions to determine how pupils make sense of the numbers “0” and “1”; in the second part, there are questions to determine their state of using the numbers “0” and “1”; in the four operations and in the third part, there are questions to determine their state of using the numbers of “0” and “1” in problem posing and problem solving. The semi-structured interview form prepared by the researchers was sent to two mathematics education experts and three classroom teachers, and also expert opinions were obtained. The semi-structured interview form was finalized by making arrangements regarding the appropriateness of the questions and procedures in line with the expert opinions and supporting with appropriate visuals. The form was piloted on 34 third-grade pupils who were studying at

another primary school that was not included in the study group. The semi-structured interview form, which is the final form that was given as a result of the pilot study, is presented below.

Number	Questions	Responses
1	What is/are the meaning(s) of this number?	
	Find the results of the operations given in the next column.	5 + 1 = 6 – 1 = 4 x 1 = 3 ÷ 1 =
	Pose a word problem by using this number and find its result.	

Number	Questions	Responses
0	What is/are the meaning(s) of this number?	
	Find the results of the operations given in the next column.	3 + 0 = 7 – 0 = 2 x 0 = 10 ÷ 2 =
	Pose a word problem by using this number and find its result.	

Figure 1. *Data collection tool regarding the numbers "0" and "1"*

4. Data Collection Process and Consent

Necessary permissions were obtained from the administrators and teachers in the schools where the study would be carried out. The pupils were informed about the purpose of the study in the mathematics lesson together with the classroom teacher. After the necessary information was given, the interview form prepared by the researchers was applied individually to the third-grade pupils in the study group. Each interview lasted for 10-15 minutes. The interviews were also tape-recorded. The data obtained from the semi-structured interviews were organized.

5. Data Analysis

The data obtained regarding the concepts of "0" and "1" were analyzed by using the content analysis technique. The data analysis process included the following stages: Firstly, preparing and organizing data for analysis; Secondly, coding the data and reducing them to categories by combining codes; and finally the presentation of the data in the forms of figures, tables, and discussions (Creswell, 2016). Content analysis is defined as a systematic, repeatable technique in which some words of a text are summarized into smaller content categories with coding based on certain rules (Büyüköztürk, Kılıç Çakmak, Akgün, Karadeniz & Demirel, 2012). Coding and analyzing data is an analytical step. Organizing coding hierarchically is part of the analysis process (Gibbs, 2007; Glesne, 2012).

In-depth data collection, data triangulation, detailed description and consistency are taken into account in establishing the validity and reliability of the data obtained in qualitative research (Yıldırım & Şimşek, 2018). Written documents obtained from interview forms and audio recordings from the 114 primary school third-grade pupils participating in the study show that in-depth data were collected, and the data collected from different schools and socio-economic environments show that data triangulation was ensured. The data obtained from the pupils participating in the study were coded as Ö1, Ö2,….. . With the analysis of the data as themes and sub-themes in line with the sub-problems and elicitation of the web of relationships, a detailed description was made. In order to increase the reliability, the coder reliability method was chosen, and a second researcher also carried out the coding of the data and the review of the coding. The data were re-coded by the second researcher and the coding consistency value was found to be

93.6%. As a result of the analysis, codes and categories for the concept of "0" and "1" were created, and the analyzed data were digitized and presented in tables. Sample statements from raw data and photographs from forms are included in the findings section in order to ensure the credibility of the results of the data analysis.

Results

Findings derived from the data are presented in tables in this part of the study. The data obtained in the context of research questions from the primary school third-grade pupils participating in the current study were collected under the following headings; the meaning of the numbers "0" and "1", the state of their use in the four operations, word problem posing and solving, and the formation of themes and codes.

The opinions of the primary school third-grade pupils regarding the meaning of the number "0" are presented in Table 1.

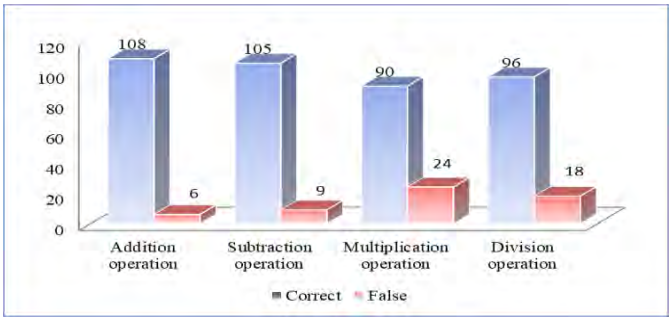
Category	Code	f
Four operations (24)	It is used in the addition, subtraction, multiplication, and division operations.	21
	It is used only in the multiplication operation.	2
	It cannot be found in the quotient of any number.	1
Ineffective element (45)	It means ineffective.	25
	It has no effect in the addition and subtraction operations.	20
Absorbing element (53)	It is the absorbing element in the multiplication operation.	47
	Multiply by whatever number the result will be 0.	6
Number (36)	It is the smallest number.	30
	It is a number.	4
	It is used to denote nothing as a number.	2
Natural number (6)	"0" is the natural number.	4
	It is the smallest of the natural numbers.	2
Meaningless (21)	It has no meaning.	19
	It alone doesn't mean anything.	2
Valueless (11)	It means valueless.	7
	It is a number that has no value on its own.	3
	If there is no number in front of it, it has no value.	1
Absence (73)	It means absence.	34
	It means nothingness.	34
	It represents the absence of an attribute.	4
	There is no digit value.	1
Even number (7)	"0" is an even number.	7
Beginning (6)	It is the beginning of numbers.	4
	It is the beginning of everything.	2
Letter (5)	It is a letter.	3
	It indicates the letter "O".	2
Others (7)	Generally, there is a new meaning.	2
	Blood group.	1
	It has the meaning of empty and plain.	1

	It means a circle.	1
	It means unqualified.	1
	The number I don't like.	1

Table 1. *The opinions of the primary school third-grade pupils regarding the meaning of the number "0"*

It can be seen in Table 1, from the responses of the primary school third grade pupils, the following categories of meaning for the number "0" were obtained: "absence (f=73)", "absorbing element (f=53)", "ineffective element (f=45)", "number (f=36)", "four operations (f=24)", "meaningless (f=21)", "valueless (f=11)", "even number (f=7)", "others (f=7)", "natural number (f=6)", "beginning (f=6)" and "letter (f=5)". The category most frequently emphasized by the primary school third grade pupils for the meaning of the number "0" is "absence" and in this category, the expressions used the most include "It means absence" and "It means nothingness". The expression used the most in the category of absorbing element is "It is the absorbing element in the multiplication operation". The expression used the most in the category of "ineffective element" is "It means ineffective". The expression used the most in the category of number is "It is the smallest number". The expression used in the category of "four operations" is "It is used in the addition, subtraction, multiplication and division operations". The expression used the most in the category of "meaningless" is "It has no meaning". The expression used the most in the category of "valueless" is "It means valueless". The expression used the most in the category of "even number" is "0 is an even number". The expression used the most in the "natural number" is "0 is a natural number". The expression used the most in the category of "beginning" is "It is the beginning of numbers" and the expression used the most in the category of "letter" is "It is a letter". In the category of others for the meaning of the number "0", the following expressions were stated; "Generally, there is a new meaning (f=2)", "blood group (f=1)", "It has the meaning of empty and plain (f=1)", "It means a circle (f=1)", "It means unqualified (f=1)", and "The number I don't like (f=1)".

The primary school third-grade pupils' state of using the number "0" in the four operations is given in Graph 1.



Graph 1. *Pupils' state of using the number "0" in the four operations*

When the primary school third-grade pupils' state of using the number "0" in the four operations was examined, it was seen that 108 of them used it correctly while 6 of them used it wrongly in the addition operation. In the subtraction operation, 105 of the pupils used it correctly while 9 of them used it wrongly. In the multiplication operation, 90 of the pupils used it correctly while 24 of them used it wrongly. In the division operation, 96 of the pupils used it correctly while 18 of them used it wrongly. The pupils made the largest number of mistakes in the multiplication operation while the smallest number of mistakes in the addition operation. The pupil coded as Ö23 used the number "0" correctly in all the operations and the pupil coded as Ö52 used it wrongly in the multiplication operation. Therefore, the photographs of the pupils showing the operations are given below.

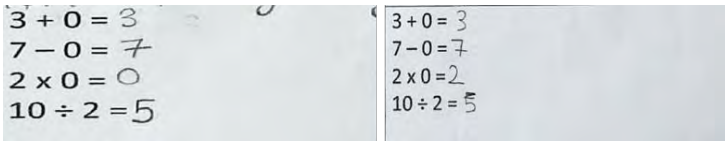
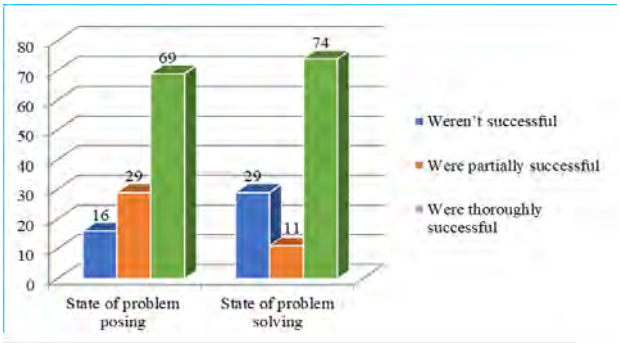


Figure 2. *Ö23 and Ö53's state of using the number "0" in the four operations*

Findings related to the primary school third-grade pupils' state of posing and solving word problems including the number "0" are given in Graph 2.



Graph 2. *Pupils' state of posing and solving a problem including the number "0"*

When the primary school third-grade pupils' state of posing a problem using the number "0" was examined, it was seen that 69 of them were thoroughly successful in posing a problem; 29 of them were partially successful in posing a problem while 16 of them weren't successful in posing a problem. When the pupils' state of solving a problem including the number "0" was examined, it was seen that 74 of them were thoroughly successful in solving the problem; 11 of them were partially successful in solving the problem while 29 of them weren't successful in solving the problem. It is seen that the pupils were more successful in posing a problem including the number "0" than solving a problem including the number "0". The photos showing the correct use of the number "0" in posing and solving a problem by the pupils coded as Ö39 and Ö65 are shown below.

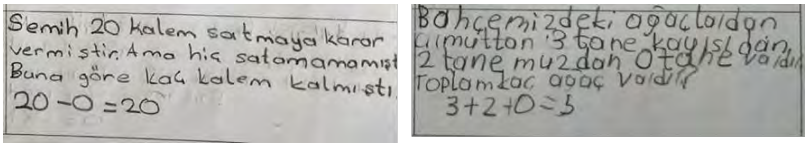


Figure 3. *The use of the number of "0" in problem posing and solving by Ö39 and Ö65*

The opinions of the primary school third-grade pupils regarding the meaning of the number "1" are presented in Table 2.

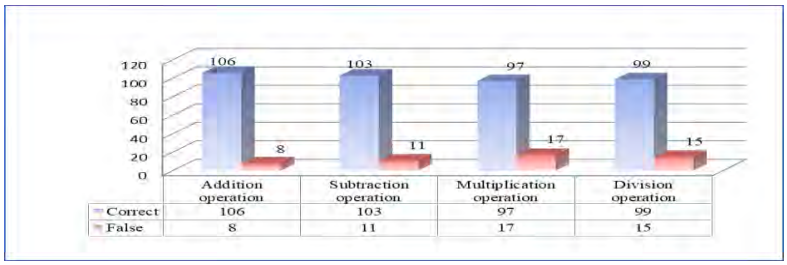
Category	Code	F
Four operations (25)	It is used in the addition, subtraction, multiplication, and division operations.	23
	We put it at the end of operations.	1
	It increases and decreases the number by one.	1
Ineffective element (43)	It is an ineffective number.	24
	If the number is multiplied by 1, the number does not change.	12
	It has no effect in the multiplication and division operations.	7
Uniqueness (32)	It is a unique number.	20
	It expresses the uniqueness of something.	10

	Something is unique.	1
	Uniqueness of God.	1
Number (44)	It is a number.	23
	It is the first of the numbers.	16
	It is the second of the numbers.	5
Beginning (29)	It means the beginning.	14
	It is the beginning of all numbers.	14
	It is the beginning of mathematics.	1
Unity (12)	It means unity.	9
	It can be used in the ones, tens, hundreds places.	3
First (14)	It means being the first.	9
	It means being the number 1.	4
	The number "1" always means being the first.	1
Counting (7)	While counting anything, it starts from 1.	5
	It comes after the number "0".	2
Number (23)	It is used to mean a number.	11
	It is a natural number.	8
	It is the smallest natural number.	3
	A two-digit number can be written with "1".	1
Being successful (6)	It means being a leader.	4
	It means being appreciated, being superior.	2
Priority (3)	It is the number that comes first.	2
	It means priority.	1
Others (3)	It is a number I like.	1
	It makes other numbers bigger.	1
	It doesn't evoke negative things.	1

Table 2. The opinions of the primary school third-grade pupils regarding the meaning of the number "1"

It can be seen in Table 2, from the responses of the primary school third grade pupils, the following categories of meaning for the number "1" were obtained: "number (f=44)", "ineffective element (f=43)", "uniqueness (f=32)", "beginning (f=29)", "four operations (f=25)", "number (f=23)", "first" (f=14)", "unity (f=12)", "counting (f=7)", "being successful (f=6)", "priority (f=3)" and "others (f=5)". The category most frequently emphasized by the primary school third grade pupils for the meaning of the number "1" is "number" and in this category, the expression used the most is "It is a number". The expression used the most in the category of "ineffective element" is "It is an ineffective number". The expression used the most in the category of "uniqueness" is "It is a unique number". The expressions used the most in the category of "beginning" are "It means beginning" and "It is the beginning of all numbers". The expression used the most in the category of "four operations" is "It is used in the addition, subtraction, multiplication, and division operations". The expression used the most in the category of "number" is "It is used to mean a number". The expression used the most in the category of "first" is "It means being the first". The expression used the most in the category of "counting" is "While counting anything, it starts from 1". The expression used the most in the category of "being successful" is "It means being a leader". The expression used the most in the category of "priority" is "It is the number that comes first". In the category of others, for the meaning of the number "1", the following expressions were stated; "It is a number I like (f=1)", "It makes other numbers bigger (f=1)" and "It doesn't evoke negative things (f=1)".

The primary school third-grade pupils' state of using the number "1" in the four operations is given in Graph 3.



Graph 3. *Pupils' state of using the number "1" in the four operations*

When the primary school third-grade pupils' state of using the number "1" in the four operations was examined, it was seen that 106 of them used it correctly while 8 of them used it wrongly in the addition operation. In the subtraction operation, 103 of the pupils used it correctly while 11 of them used it wrongly. In the multiplication operation, 97 of the pupils used it correctly while 17 of them used it wrongly. In the division operation, 99 of the pupils used it correctly while 15 of them used it wrongly. The pupils made the largest number of mistakes in the multiplication operation while the smallest number of mistakes in the addition operation. The pupil coded as Ö29 used the number "1" correctly in all the operations and the pupil coded as Ö78 used it wrongly in the division operation. Therefore, the photographs of the pupils showing the operations are given below.

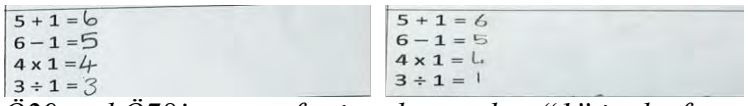


Figure 4. *Ö29 and Ö78's state of using the number "1" in the four operations*

Findings related to the primary school third-grade pupils' state of posing and solving word problems including the number "1" are given in Graph 4.

Graph 4. Pupils' state of posing and solving a problem including the number "1"

When the primary school third grade pupils' state of posing a problem using the number "1" was examined, it was seen that 78 of them were thoroughly successful in posing a problem; 21 of them were partially successful in posing a problem, while 15 of them weren't successful in posing a problem. When the pupils' state of solving a problem including the number "1" was examined, it was seen that 82 of them were thoroughly successful in solving a problem; 13 of them were partially successful in solving a problem, while 19 of them weren't successful in solving a problem. It was noted that the pupils were more successful in posing a problem including the number "1" than solving a problem including the number "1". The photos showing the correct use of the number "1" in posing and solving a problem by the pupil coded as Ö57, shows the correct use of the number "1" in posing a problem but is wrongly used in solving a problem by the pupil coded as Ö89. It also shows the wrong use of the number "1" in posing and solving a problem which is given below.

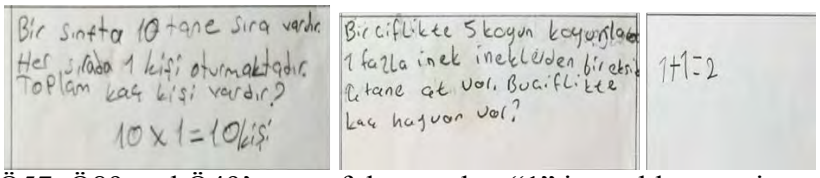


Figure 5. Ö57, Ö89 and Ö49's use of the number "1" in problem posing and solving

Discussion

The concepts of counting and numbering, which can be seen as the basis of arithmetic, are among the concepts in whose definitions mistakes are commonly made (Albayrak, 2010; Yenilmez & Demirhan, 2013). In terms of the concept of number, it is seen that students often make mistakes in the use of the numbers 0 (zero) and 1 (one) (Arslan & Ubuz, 2015; Dinç Artut & Tarım, 2006; Haylock & Cockburn, 2013; Hughes, 1986; Sadi, 2007; Tall & Razali, 1993). For this reason, this study was conducted to determine how the numbers 0 (zero) and 1 (one) are perceived by pupils, how these numbers were used in the four operations, and how these numbers were used in problem-posing processes.

In the current study, the category of "absence" was strongly emphasized, and the expressions used the most in this category are "It means absence" and "It means nothingness". When the meanings assigned to the number "0" by the pupils were examined, it was seen that the pupils generally had the perception of it as a cardinal number and that they did not emphasize its meaning related to place value. The reason why the cardinal meaning of 0 (zero) is emphasized intensely by the pupils can be shown because greater emphasis is placed on the multiplicity feature of the number in mathematics lessons. As a result of bringing the cardinal meaning of 0 (zero) to the fore, it is thought by the pupils that it has only the meaning of absence, causing them to have the perception that 0 (zero) in any place means absence and has no meaning. However, the presence of 0 (zero) in any digit is used instead of the absent digit and acts as a placeholder (Arslan & Ubuz, 2015; Olkun & Toluk Uçar, 2018). Chick and Baker (2005), in their study of primary school teachers' pedagogical content knowledge, examined how the teachers would react to some possible situations involving misconceptions and mistake by pupils. Consequently, they concluded that the placeholder property of 0 (zero) and the absorbing element property in the multiplication operation were given as operational information by the teachers and that it was emphasized by one teacher that 0 (zero) in a digit does not mean nothingness. The finding obtained from the study conducted by Chick and Baker (2005) with teachers shows the reason why the absence meaning of 0 (zero) was overemphasized in the current study.

In the study, it is seen that the primary school third-grade pupils have information about the two properties of 0 (zero); an ineffective element in the addition operation and an absorbing element in the multiplication operation. However, when the meanings that the pupils attributed to 0 (zero) are examined, it can be stated that they have a misconception. It was concluded that for the number 0 (zero), which is an effective element in the addition operation, the pupils also thought that it has no effect in the subtraction operation. However, the reason for their not being aware of the fact that the number 0 (zero) serves different functions depending on its being within the minuend or the subtrahend may be because of the generalization of the feature of 0 (zero) being an ineffective element in the addition operation to the subtraction operation.

On the other hand, the pupils were more successful in using 0 (zero) in the addition and subtraction operations than in the multiplication and division operations. This finding of the current study concurs with the finding reported by Sadi (2007). Sadi (2007) stated that many mistakes were made by pupils of all ages in the multiplication and division operations, and the use of zero in the multiplication and division operations was found to be the source of many mistakes and misunderstandings. On the other hand, in the current study, the pupils made the most mistakes in the multiplication operations and the fewest mistakes in the addition operations in the use of 0 (zero). In a study conducted on the use of 0 (zero) by pupils, it was concluded that the pupils found

a number other than zero as a result of multiplying a number by zero. Based on the reason for this situation, the pupils' difficulty in interpreting multiplication by zero was shown, and it was stated that zero represents nothingness for many pupils (Rees & Barr, 1984; as cited in Sadi, 2007). One of the mistakes made due to the use of zero in the subtraction operation is that when any number is subtracted from zero, the rest is zero and borrowing from zero (Brown & Burton, 1978; Hughes, 1989).

Another concept that is examined about pupils' opinions in the study is the number 1 (one). While it is difficult for children to learn the numerical meaning of "one" in the context of counting, "one" is more common in talking to children as a quantitative marker than being embedded in a counting routine. However, it is easy to grasp it in this way (Carey, 2009). The fact that the study was conducted on the primary school third-grade pupils and that the number of conversations in which these pupils encounter with the number of 1 is high can explain the meanings assigned to this number in the current study.

A pupil participating in the study said that the number 1 (one) "increases the number by one, decreases the number by one" and thus drew attention to a situation that allows the flexible use of the number and the better understanding of the number. According to the Peano axioms, each natural number has a successor and a predecessor, and it is possible to obtain this successor or predecessor number with 1 (one). Rips, Bloomfield and Asmuth (2008) expressed the ability to reach "n + 1", which is the successor of any "n" number in natural numbers, as forward counting and stated that this is the proof of the knowledge of the number system. It can be stated that one of the pupils participating in the study grasped that the number "1" is required to obtain the successor and predecessor of a number. In other words, he/she has the forward counting knowledge about numbers. In the current study, the following expressions were uttered by pupils within the category of "number"; "It is the first of the numbers" and "It is the second of the numbers".

These two expressions might show that the ordinal meaning of the number "1", which includes the ordering direction of this number, is not fully understood by the pupils. According to Rips, Bloomfield and Asmuth (2008), the succession-premise relationship between 0 and 1 is a relative situation. According to Rips et al., in many theories "1" is regarded as the first number term because of its position in the array of number terms and its role in counting. However, these assumptions do not fully explain that the "1" rules out "0" as the starting number. However, the objectives in the learning area of numbers and operations in primary schools in Turkey are limited to natural numbers and according to the Peano axiom, the number "0" is not the successor of any number but is the predecessor of the number "1" (Delil, 2010; MEB, 2018). In this case, it can be said that the primary school third-grade pupils participating in the current study have erroneous thoughts about the first element of the natural number set.

Conclusion

In the current study, it was concluded that the primary school third-grade pupils made sense of the number (zero) in association with the categories of four operations, ineffective element, absorbing element, number, natural number, meaningless, valueless, absence, even number, beginning, and letter. In the "absorbing element" category obtained for the meaning of the number 0 (zero), the following expressions were used by pupils to define it; "It is the absorbing element in the multiplication operation" and "Multiply by whatever number the result will be 0". In the category of "ineffective element", the following expressions were used by pupils to define it; "It means ineffective" and "It has no effect in the addition and subtraction operations". On the other hand, the pupils were more successful in using 0 (zero) in the addition and subtraction operations than in the multiplication and division operations. When the primary school third-grade pupils' state of posing a problem using the number "0" was examined, it was seen that 69 of them were thoroughly successful in posing a problem; 29 of them were partially successful in posing a

problem while 16 of them were not successful in posing a problem. Furthermore, the pupils were found to be more successful in posing problems including 0 (zero) than in solving such problems.

When the pupils' opinions about the meaning of 1 (one), which is another concept examined in the current study, were examined, a total of 12 categories were reached. The categories including the highest number of pupil opinions about the meaning of 1 (one) were found to be number, ineffective element and uniqueness, respectively. The expression used the most in the category of "number" is "It is a number"; the expression used the most in the category of "ineffective element" is "It is an ineffective number"; and the expression used the most in the category of "uniqueness" is "It is a unique number". Other expressions used in the category of "uniqueness" are "It expresses the uniqueness of something" and "Something is unique". When the pupils' state of using the number "1" in mathematical sentences including the four operations was examined, it was seen that they were more successful in the addition and subtraction operations than the multiplication and division operations as with the number "0". On the other hand, the pupils made the most mistakes in the multiplication operations and the fewest mistakes in the addition operations in the use of the number "1". When the primary school third-grade pupils' state of posing a problem using the number "1" was examined, it was seen that 78 of them were thoroughly successful in posing a problem; 21 of them were partially successful in posing a problem while 15 of them were not successful in posing a problem. The pupils were found to be more successful in solving problems including the number "1" than posing problems including the number "1".

Recommendations

Considering the findings of the current study, the following suggestions can be made. In the learning-teaching process, rich examples of the meanings of the numbers "0" and "1", its relationship with the properties of the four operations, and teaching about the use of the number "0" in multi-digit numbers can be given. In teaching the concepts of number and counting to primary school pupils, conversations and concrete objects including daily life situations can be used. Studies at other grade levels can also be conducted to determine the meanings attributed to the numbers "0" and "1" and the difficulties in using these numbers in arithmetic operations. It should be considered that a different learning style is required for each individual, and each student should be supported to attribute meaning to the numbers "0" and "1" in different ways. Studies that examine the relationship between students' interpretation of numbers "0" and "1" and their mathematics achievement can be conducted. Research can be conducted on students' understanding of numbers "0" and "1" and associating mathematics with daily life.

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