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An investigation of the variables affecting the problem-solving achievements of pre-service secondary school mathematics teachers*

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Abstract

In this study, whether the variables of pre-service teachers' reflective thinking related to problem solving, attitude towards mathematical problem solving, metacognitive awareness, mathematical literacy self-efficacy, belief related to mathematical problem solving and problem solving achievements differ according to class level and the prediction condition of these variables for problem solving achievement were investigated. The predictive correlational research method, among the quantitative research methods, was employed in the research. 226 pre-service secondary school mathematics teachers, from the Department of Secondary School Mathematics Teaching in the Faculty of Education in Eastern Anatolia Region, participated in the research. Data were collected with the "Reflective Thinking Skill Scale towards Problem Solving", "Attitudes towards Mathematical Problem Solving Scale", "Metacognitive Awareness Inventory", "Mathematical Literacy Self-efficacy Scale", " Belief Scale Related to Mathematical Problem Solving", and " Problem Solving Achievement Test". The One Way MANOVA and Structural Equation Model were applied in data analysis according to the aims. At the end of the analyses, it was observed that reflective thinking, attitude, metacognitive awareness and mathematics literacy levels of the pre-service teachers were higher in the first class compared to the second class, but increased again in the third and fourth classes respectively. The belief related to problem solving and problem solving achievement levels increased as the class level increased. In addition, it was concluded that the mathematical literacy self-efficacy levels predict the problem solving achievements of the pre-service teachers directly; on the other hand, reflective thinking, attitude, metacognitive awareness and belief variables predict problem solving achievement through the variable of mathematics literacy.

Keywords: Problem solving; reflective thinking; attitude and beliefs towards problem solving; metacognitive awareness; mathematics literacy

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1. Introduction

Due to the rapid developments in the scientific and technological life, the needs of individuals and society develop and change as well. The development and change of the needs also bring some changes in the expectations of the society from the individual. These expectations require individuals with many qualities such as using the information actively and adapting it to life, solving problems, thinking critically, empathizing, researching, questioning, with an entrepreneurial spirit, reflective thinking (The Ministry of National Education [MoNE], 2018). As a result of the changes and developments, the changes in the qualities expected from the individual demonstrated that some changes should be made in our education system (Yeşilova, 2013). For this reason, The MoNE mathematics curriculum was renewed three times in 2009, 2013 and 2017, one in 10 years, to meet the changing needs of the age and society (Şen, 2017), the latest renovation was performed in 2018.

According to the general purposes of the mathematics course curriculum, it is aimed for students to have the ability to understand mathematical concepts, develop their mathematical literacy skills, solve problems, use mathematical language accurately use their metacognitive skills well, comprehend that mathematics is a common value of humanity, and show a positive attitude towards mathematics (MoNE, 2018). Problem-solving, which is one of these purposes, is a way to apply mathematics in real life and mathematical problem-solving is a crucial part of mathematics education (Turhan and Güven, 2014). According to Baykul (1999), problem-solving takes an important place in the secondary school mathematics teaching goals. Problem-solving is the aim of learning mathematics, but it is also the meaning of doing mathematics and is considered as an inseparable part of mathematics learning (Işık and Kar, 2011). In the present research, problem-solving, which is among the skills stated in the curriculum, was focused.

In addition that problem-solving is one of the significant concepts of the education system in itself, there are several variables associated with problem-solving. As the current literature relavant to this in our country was reviewed, the variables, that are related to the problem-solving, can be stated as: reflective thinking (Alp and Taşkın, 2008; Kızılkaya and Aşkar, 2010), epistemological belief (Aksan and Sözer, 2007), critical thinking (Kantek, Öztürk and Gezer, 2010; Türnüklü and Yeşildere, 2014), attitude towards mathematics (Cantürk-Günhan and Başer, 2008; Uslu, 2006; Özgen and Pesen, 2008), metacognitive awareness (Bakioğlu et al., 2015; Kanadlı and Sağlam, 2013; Kaplan, Duran and Baş, 2016; Karakelle, 2012; Özçakır-Sümen and Çalışıcı, 2016; Sperling, Howard, Miller and Murphy, 2004), mathematical literacy (Akyüz and Pala, 2010; Birbiri, 2014; Özçakır-Sümen and Çalışıcı, 2016), belief related to problem-solving (Uğurluoğlu, 2008). However, the problem-solving variable used in these studies is generally considered as problem-solving skill. Problem-solving skill is the ability to recognize the nature of a problem, develop strategies for solution, and interpret the

results after applying these strategies (Özsoy, 2007). Problem-solving skill is not only about mathematics but also about the ability to cope with problems encountered in all sides of life. This skill was measured mostly with the problem-solving inventory in relevant researches. In this study, the basic factor handled related to the problem-solving is problem-solving achievement. Problem-solving achievement was determined as the score obtained by the participants' answers to the problem-solving test. The variables associated with the achievement of problem-solving in the literature are as follows: problem-solving attitude (Abdullah, Halim, Zakaria, 2014; Kasap, 1997), metacognitive awareness (Baş, 2016; Özsoy and Ataman, 2009; Çelik, 2012; Everson and Tobias, 2001; Howard, McGee, Shia and Hong, 2001; Pennequin, Sorel, Nanty and Fontaine, 2010; Yeşiller, 2013;), mathematical literacy (Akyüz and Pala, 2010; McLeod, 1992; Fennema-Sherman, 1977; Ma, 1997; Papanastasiou, 2000), self-regulation strategies and motivational beliefs (Kılıç and Tanrıseven, 2007; Kramarski and Revach, 2009; Marcou and Philippou, 2005; Perels, Gürtler and Schmitzi 2005), reading comprehension skills (Özcan, 2016), belief towards problem-solving (Bal, 2015; Güven and Özüm-Çabakçor, 2013). As it is noticed in the literature, problem-solving is related to many affective and cognitive psychometric variables. As all these variables were handled within the scope of this study, some of the affective and cognitive psychometric variables were selected considering the covering status of the variables associated with problem-solving and the intensity of their implementation. The attitude towards mathematical problem-solving was selected as it consisted of the attitude towards mathematics among the affective variables, and belief related to problem-solving was chosen as it consisted of the epistemological belief and motivational belief. On the other hand, among the cognitive variables, the reflective thinking towards problem-solving was selected as it consisted of reflective thinking and critical thinking and the mathematical literacy self-efficacy as it consisted of mathematical literacy and reading comprehension skill; and finally, the variables of metacognitive awareness. In this present study, it is aimed to determine the predictive status of pre-service teachers' reflective thinking towards problem-solving, attitude towards mathematical problem-solving, metacognitive awareness, mathematics literacy self-efficacy, belief levels towards problem-solving achievements on the problem-solving achievements. Existing studies do not give us a clear picture of the effect of all these variables on problem solving success. It is thought that considering the variables - predicting problem-solving achievement as a whole - will contribute to the literature. Accordingly, the research problem was determined as "What is the status of predicting pre-service secondary school mathematics teachers' reflective thinking towards problem-solving, attitude towards mathematical problem-solving, metacognitive awareness, mathematics literacy self-efficacy, beliefs towards problem-solving achievement problem and do these variables differ by class level?" In order to find answers to this problem, these sub-questions were asked.

1) Do pre-service secondary school mathematics teachers' reflective thinking skills for problem-solving, their attitudes towards mathematical problem-solving, metacognitive awareness, mathematics literacy self-efficacy, belief levels towards mathematical problem-solving and problem-solving achievements change according to class level?

2) What is the predictive status of pre-service secondary school mathematics teachers' reflective thinking skills for problem-solving, their attitudes towards mathematical problem-solving, their metacognitive awareness, mathematical literacy self-efficacy, and belief levels towards mathematical problem-solving and problem-solving achievements?

2. Method

This research was designed with predictive correlational research method that tries to predict the other depending on one of the variables by investigating the relationships between variables (Büyüköztürk, Kılıç-Çakmak, Akgün, Karadeniz and Demirel, 2016). As the structural equation model to be created according to the aim of the research is based on high-level correlational analysis, correlational research pattern is used in such studies.

2.1. Participants

The sample of the research consisted of 226 participants, who were the students at the Secondary School Mathematics Teaching Department of the Faculty of Education in the Eastern Anatolia Region. In creating the sample, the typical case sampling method, which is one of the non-random sampling methods, that necessitates determining an average situation from several situations in the universe and collecting information from this example related to the research problem (Büyüköztürk et al., 2016), was applied. 23% of the study group consisted of the first-class level, 25% second-class, 21% third, 31% fourth class. Besides, 25 of the participants had taken the mathematical modelling course and succeeded it. Similarly, 103 participants - included in the sample - took mathematics teaching through a problem-solving course and 92 of them achieved this course. To measure whether the scales applied to the participants make a difference between students taking and not taking mathematics teaching and mathematical modelling course through problem-solving, orderly the independent group's t-test and Mann-Whitney U test were applied. Nevertheless, as a result of the analysis, except for the problem-solving achievement, the fact that the participants took these courses did not constitute a significant difference in terms of reflective thoughts, attitudes, metacognitive awareness, mathematics literacy, and belief scales.

2.2. Instrument

As the data collection tool in the research, the "Reflective Thinking Skill Scale for Problem-solving (RTSSPS)", "Attitudes Towards Mathematical Problem-solving Scale (ATMPSS)", "Metacognitive Awareness Inventory (MAI)", "Mathematical Literacy Self-efficacy Scale (MLSS), "Belief Scale for Mathematical Problem-solving (BSMPS)" and "Problem-solving Achievement Test (PSAT)". Data related to the data collection tools, RTSSPS, ATMPSS, MAI, MLSS, BSMPS, and PSAT, which are in a 5-point Likert type, are presented in Table 1 in detail.

Table 1. Data collection tools

Scale	Developer	Number of Items	Minimum and Maksimum Score from the Scale	Cronbach Alpha Coefficient in the Main Study	Cronbach Alpha Coefficient in this Study
Reflective Thinking Skill Scale for Problem-solving	Kızılkaya and Aşkar(2009)	14	14-70	0.83	0.87
Attitudes Towards Mathematical Problem-solving Scale	Developed by Whitaker(1982), Adapted into Turkish by Çokçalışkan(2012)	35	35-175	0.78	0.89
Metacognitive Awareness Inventory	Developed by Schraw and Dennison (1994); Adapted into Turkish by Akın, Abacı, and Çetin(2007).	52	52-260	0.95	0.94
Mathematical Literacy Self-efficacy Scale	Özgen and Bindak (2008)	25	25-125	0.94	0.91
Belief Scale for Mathematical Problem-solving	Developed by Kloosterman and Stage (1992), adapted into Turkish by Hacıömeroğlu(2011).	24	24-120	0.76	0.74

As it is observed in Table 1, as the Cronbach Alpha coefficient gathered from the data collection tools used in the study are bigger than 0,7, the situation of scales is considered good (Kılıç, 2016). PSAT was created by the researcher to measure the achievements of the pre-service teachers related to the problem-solving by getting the literature support. As a result of the literature review, 10 questions were handled benefitting various resources (Altun, 2016; Krulik and Posamentier, 2008; Altun, Bintaş, Yazgan, and Arslan, 2004). To measure the applicability of prepared questions to the pre-service teachers, 10 problem-solving questions were asked to a group consisting of 49 secondary school pre-service mathematics teachers. The questions in this study were determined by

calculating the discrimination and difficulty indexes. The discrimination and difficulty indexes of the questions selected for this study were given in Table 2.

Table 2. The discrimination and difficulty indexes of gathered at the end of the pilot study

Questions	Discrimination(r_{jx})	Difficulty(P_j)
1. question	0,68	0,40
2. question	0,62	0,24
3. question	0,50	0,20
4. question	0,43	0,40
Average	0,56	0,31

In item selection, priority is given to the item discrimination index. Those with a discrimination power of 0.30 and above can be put into the test correctly (Turgut and Baykul, 2012). For this reason, as it is seen in Table 2, all the problems that will be used in the main study have a high discrimination index (rjx>0,30).

The answers given to the PSAT were scored considering the steps recommended by Polya (1957). The Problem-solving steps of Polya (1957) consist of four steps the understanding problem, planning, practicing of the plan, and evaluating the solution.

1- Understanding the problem: It is the stage that requires finding the conditions, the desired result, and unknown, finding the missing or excessive information, and dividing the problem into its lower steps.

2- Planning: It emerges after the realization of the step of understanding the problem. At this stage, relationships are established between the data in the problem and the unknown. This established relationship is transformed into mathematical expressions and expressed with a mathematical equation.

3- Practicing the plan: At this stage, by solving the mathematical equation or equations formed as a result of the planning phase, unknown and desired data are tried to be obtained.

4- Evaluating the solution: At this stage, the accuracy of the results and the logic carried out in the solution are checked, if any, the solution is sought in other ways, the problem is expressed in different ways, and in this case, the solution will be considered.

The answers given to the open-ended questions stated in the Problem-solving Achievement Test were scored changing between 0 and 20 values according to the degree of realization of the problem-solving steps specified by Polya. The test consisted of 4 questions. While the highest score that can be taken from the achievement test is 80, the lowest score is 0. The high score that is taken from the scale demonstrates that the problem-solving achievements of the pre-service teachers are high. The answers of the participants to each question by the participants were scored equally according to the problem-solving steps and the total score was calculated for each question. Two expert

opinions were applied to ensure the reliability of the results. Then, the total score gathered from the 4 questions were calculated and recorded. The Problem-solving Achievement Test is given in Appendix A1 and The Problem-solving Achievement Test Scoring Directive in Appendix A2.

2.3. Data Collection Procedure

The data of the research were collected at the end of the fall term of the 2018-2019 academic year. The collection of the research data was carried out in four stages. As the surveys that the pre-service teachers' answers would be associated with the achievement test, the student numbers of the participants were recorded but personal information of the students was not shared by no means. In the first step, the scales of RTSSPS and ATMPSS were applied to the pre-service teachers. The MAI in the second step and the MLSS and BSMPS in the third step and PSAT in the last step were applied to the participants. Each step of the application was carried out in one-week intervals and 20 minutes of the lesson. To ensure the reliability of the research, the interaction of the pre-service teachers with each other during the application of the scales was prevented as much as possible.

2.4. Data Analysis

The SPSS package program was applied for the first sub-problem of the data analysis. Firstly, whether the data demonstrate normal distribution or not was analysed in Table 3 considering the skewness and kurtosis values related to the distribution.

Table 3. Skewness and kurtosis values of the data

	Skewness Statistic	Kurtosis Statistic
Reflective thinking total	-,006	,090
Attitude total	-,302	,462
Metacognition total	,231	-,066
Literacy total	-,044	,278
Belief total	-,443	,335

As seen in Table 3, considering that the calculated values are between +2 and -2, the distribution is accepted as normal (DeCarlo, 1997). To analyse the change of the surveys and achievement tests applied for the pre-service teachers according to class level, a One-way MANOVA test was used. In situations in which more than one variable is associated with a single variable, the One-way MANOVA test is applied (Kalaycı, 2009). The homogeneity of variance-covariance matrices is determined by the "Box's M" test. The fact that the variance-covariance matrices of the Box's M test are not statistically significant indicates that the assumption of homogeneity of the variance-covariance matrices is met (Kalaycı, 2009); however, in the cases, in which the number of

participants is great, the Box's M test can be significant (Tabachnick and Fidell, 2007). For this reason, Pallant (2005) suggests that the significance criterion for these tests should be taken as,001. In this research, the significance criterion for the Box's M test was taken as,001. Levene test results, which are carried out to test the homogeneity of variances, are given in Table 4.

Table 4. The Levene test results of all scale scores according to class level variable

	F	df1	df2	p
Reflective thinking	,276	3	222	,842
Attitude	1,440	3	222	,232
Metacognition	6,532	3	222	,000
Literacy	4,992	3	222	,002
Belief	2,061	3	222	,106
Problem-solving Achievement Test	2,350	3	222	,073

p<,05

According to Table 4, the variance of the reflective thinking scores ($F_{(3,222)}$=,27; p=,84), the scores of the attitude scales ($F_{(3,222)}$=1,44; p=,23), belief scale scores ($F_{(3,222)}$=2,06; p=,10) and problem-solving achievement test scores ($F_{(3,222)}$=2,35; p=,07) are homogenous; on the other hand, the variance of the metacognitive awareness scores ($F_{(3,222)}$=6,53; p=,001) and literacy self-efficacy scores ($F_{(3,222)}$=4,99; p=,001) are not homogenous.

Scheffe and Tamhane tests, among the Post-Hoc tests, were used to reveal the existed classes among the class levels. Scheffe tests can be preferred when variances are equal but group sample numbers are not equal (Kayri, 2009). In the data obtained from the reflective thinking, attitude, belief, and problem-solving achievement test, the Scheffe test were preferred as the variances of the groups were equal and the group sample numbers were not equal. Tamhane test can be used when variances and group sample numbers are not equal (Kayri, 2009). According to the findings of the research, the Tamhane test was preferred as the variances and group sample numbers were not equal in the data of the metacognition awareness and literacy self-efficacy scale.

The data related to the second sub-problem of the research were analysed using the structural equation model (Çokluk, Şekercioğlu & Büyüköztürk, 2010), which was made to determine whether a previously designed structure was verified as a model and in what direction and level of relationship between the structures. In the analysis of the structural equation model, the AMOS 23.0 program was used.

3. Findings

In this section, the findings related to the first and second sub-problems of the research are presented in order.

3.1. Findings Related to the First Sub-problem

In this section, there are findings of how the pre-service teachers' scores obtained from RTSSPS, ATMPSS, MAI, MLSS, BSMPS, and PSAT scales changed according to class level. The descriptive findings of each variable are presented in tables, in order.

Table 5. Distribution of reflective thinking skill scores of participants for problem-solving by class level

	Class	n	$\bar{x}$	ss
	1	53	53,004	7,5865
	2	57	48,096	7,1461
Reflective Thinking	3	47	50,989	7,2984
	4	69	53,316	7,2172
Total		226	51,442	7,5613

When the change of the scores taken from the RTSSPS according to the class level is analysed, as it is indicated in Table 5, the reflective thinking skills scores are close to

each other as for the first classes ($\bar{x}$ =53,00; ss=7.58) and the fourth classes ($\bar{x}$ =53,31; ss=7,56). Nevertheless, a gradual increase is observed in the reflective thinking skills for the second class ($\bar{x}$ =48,09; ss=7,14) and third class ($\bar{x}$ =50,98; ss=7,29).

Table 6. Distribution of participants' attitude towards mathematical problem-solving scores by class level

	Class	n	$\bar{x}$	ss
Attitude	1	53	128,272	17,4644
	2	57	122,579	13,8922
	3	47	130,387	13,3731
	4	69	131,687	15,1838
Total		226	128,319	15,4105

As the change of the scores from the ATMPSS scale, seen in Table 6, is analysed, while the attitude score is high in the first classes ($\bar{x}$ =128,27; ss=17,46) a decrease is observed in the second classes ($\bar{x}$ =122,57; SS=13,89). A gradual increase is observed in the third classes ($\bar{x}$ =30,38; ss=13,37) and fourth classes ($\bar{x}$ =131,68; ss=15,18).

Table 7. *Distribution of participants' metacognitive awareness scores by class level*

	Class	n	$\bar{x}$	ss
Metacognitive Awareness	1	53	184,392	28,2301
	2	57	178,474	16,1160
	3	47	183,274	23,0583
	4	69	197,414	24,9809
Total		226	186,643	24,5393

As the scores from the MAI according to the class level, seen in Table 7 are analysed, while the scores from the scale are high in first classes ($\bar{x}$ =184.39; ss=28,23); a decrease is observed in the second classes ($\bar{x}$ =178,47; ss=16,11). A gradual increase is noticed in the scores of the third classes ($\bar{x}$ = 183,27; ss=23,05) and fourth classes ($\bar{x}$ =197,4; ss=24,98).

Table 8. Distribution of participants' mathematical literacy self-efficacy scores by class level

	Class	n	$\bar{x}$	ss
Literacy Self-Efficacy	1	53	87,725	12,5517
	2	57	84,325	10,6875
	3	47	89,870	7,1180
	4	69	96,180	13,3084
Total		226	89,895	12,2396

As the change of the scores from MLSS according to the class, the level is analysed, seen in Table 8, while the score from the scale is high in the first classes ($\bar{x}$ =87,72;

ss=12,55); a decrease is observed in the second classes ($\bar{x}$ =84,32; ss=10,68). A gradual increase is observed in the scores of the third classes ($\bar{x}$ = 89,87; ss=7,11) and fourth classes ($\bar{x}$ =96,18; ss=13,30).

Table 9. Distribution of participants' belief scores for mathematical problem-solving by class level

	Class	n	$\bar{x}$	ss
Belief	1	53	93,774	9,7382
	2	57	95,518	8,5129
	3	47	96,660	6,7928
	4	69	99,145	7,3310
Total		226	96,454	8,3531

As the change of the scores from the BSMPS according to the class level variable, seen in Table 9 is analysed, the scores from the scale demonstrate a gradual increase between the first and fourth classes orderly ($\bar{x}$ =93,77; ss=9,73; $\bar{x}$ =95,51; ss=8,51; $\bar{x}$ =96,66; ss=6,79; $\bar{x}$ =99,14; ss=7,33).

Table 10. *Distribution of participants' problem-solving achievement scores by class level*

	Class	n	$\bar{x}$	ss
Problem-solving Achievement Test	1	53	33,472	10,6583
	2	57	37,632	12,7302
	3	47	42,660	10,1195
	4	69	47,841	9,6522
Total		226	40,819	12,1160

As the change of the scores from the PSAT according to the class level, seen in Table 10 is analysed, the scores from the scale demonstrated a gradual increase between the first and fourth classes ($\bar{x}$ =33,47; ss=10,65; $\bar{x}$ =37,63; ss=12,73; $\bar{x}$ =42,66; ss=10,11; $\bar{x}$ =47,84; ss=9,65).

Since each variable differs according to the grade level, the results obtained from the MANOVA test to determine whether this difference is significant are given in the table below.

Table 11. MANOVA results of all scale scores by class level variable

	Wilk`s Lambda	F	Hypothesis sd	Error sd	p
Class level	,619	6,310	18000	614,254	,001

As it is seen from Table 11, the class levels make a significant difference in terms of pre-service teachers' scores from all scales (p = ,001< ,05),

Table 12. Variance analysis results of all scale scores according to class level variable

Variance Resource	Dependent Variable	Total of Squares	sd	Average of Squares	F	p
Class	Reflective thinking	1019,177	3	339,726	6,367	,000
	Attitude	2861,889	3	953,963	4,188	,007
	Metacognition	12611,531	3	4203,844	7,595	,000
	Literacy	4743,740	3	1581,247	12,120	,000
	Belief	932,393	3	310,798	4,672	,003
	Problem-solving Achievement Test	7001,292	3	2333,764	19,905	,000
Error	Reflective thinking	11844,756	222	53,355		
	Attitude	50571,833	222	227,801		
	Metacognition	14766,649	222	66,516		
	Literacy	26028,270	222	117,244		
	Belief	122878,182	222	553,505		
	Problem-solving Achievement Test	28963,214	222	130,465		
Total	Reflective thinking	610934,180	226			
	Attitude	3774672,660	226			
	Metacognition	2118241,530	226			
	Literacy	409581,000	226			
	Belief	8008330,730	226			
	Problem-solving Achievement Test	1860025,460	226			

p<,05

As it is seen in Table 12, there is a significant difference from all the scales according to the class variable in terms of the reflective thinking ($F_{(3,222)}$=6,36; p=,001); attitude ($F_{(3,222)}$=4,18; p=007); metacognitive awareness ($F_{(3,222)}$=7,59; p=,001); literacy self-efficacy ($F_{(3,222)}$=12,12; p=,001); belief ($F_{(3,222)}$=6,36;p=,003) and problem-solving achievement test ($F_{(3,222)}$=19,90; p=,001) for the pre-service teachers. Scheffe and Tamhane tests, which are among the Post-Hoc tests, were used to reveal between which classes the significant difference was found.

Table 13. *Comparison of significant differences between groups by class level*

		(I) Class	(J) Class	Average Difference (I-J)	Standard Error	p
Reflective thinking	Scheffe	1	2	4,907*	1,3938	,007
		2	1	-4,907*	1,3938	,007
			4	-5,219*	1,3074	,001
		4	2	5,219*	1,3074	,001
					1,3815	
Attitude	Scheffe	2	4	-9,108*	2,7015	,011

		4	2	9,108*	2,7015	,011
Metacognition	Tamhane	2	4	-18,941*	3,6879	,000
		3	4	-14,140*	4,5118	,013
		4	2	18,941*	3,6879	,000
			3	14,140*	4,5118	,013
		1	4	-8,455*	2,3536	,003
		2	3	-5,546*	1,7555	,013
Literacy	Tamhane		4	-11,855*	2,1379	,000
		3	2	5,546*	1,7555	,013
			4	-6,309*	1,9092	,008
		4	1	8,455*	2,3536	,003
			2	11,855*	2,1379	,000
			3	6,309*	1,9092	,008
Belief	Scheffe	1	4	-5,371*	1,4896	,005
		4	1	5,371*	1,4896	,005
		1	3	-9,188*	2,1695	,001
			4	-14,369*	1,9777	,000
Problem-solving Achievement Test	Scheffe	2	4	-10,209*	1,9381	,000
		3	1	9,188*	2,1695	,001
		4	1	14,369*	1,9777	,000
			2	10,209*	1,9381	,000

*p<,05 (Considering the length of the table, only the lines that express significant differences are included.)

As it is seen in Table 13, there is a significant difference in terms of the reflective thinking levels related to problem-solving between the first and second classes in favour of first classes ($F_{(3,222)}$=6,36; p=,007), in favour of fourth classes between the second and fourth classes ($F_{(3,222)}$=6,36; p=,001). As the attitude levels related to problem-solving were analysed, a significant difference was encountered in favour of fourth classes ($F_{(3,222)}$=4,18; p=,011) between the second and fourth classes. The significant difference was encountered in favour of fourth classes ($F_{(3,222)}$=7,59; p=,001) between the second and fourth classes according to the metacognitive awareness levels and even in favour of fourth classes ($F_{(3,222)}$=7,59; p=,013) between the third and fourth classes. Besides, a significant difference was found in favour of fourth classes ($F_{(3,222)}$=12,12; p=,003; ,001; ,008) among the first, second, third and fourth classes according to the literacy self-efficacy levels; in favour of third classes ($F_{(3,222)}$=12,12; p=,013) between the second and third classes. As the belief levels related to the problem-solving were analysed, a significant difference was encountered in favour of fourth classes ($F_{(3,222)}$=6,36; p=,005)

between the first and fourth classes. Finally, a significant difference was found in favour of third classes ($F_{(3,222)}$=19,90; p=,001) between the first and third classes according to the scores handled from the problem-solving achievement test; in favour of the fourth classes ($F_{(3,222)}$=19,90; p=,001) between the first and fourth classes; even in favour of the fourth classes ($F_{(3,222)}$=19,90; p=,001) between the second and fourth classes.

3.2. Findings Related to the Second Sub-problem

In the second sub-problem of the research, the findings of the prediction situation of the reflective thinking skills of the pre-service teachers related to the problem-solving, their attitudes towards mathematical problem-solving, metacognitive awareness, mathematical literacy self-efficacy, belief levels related to the mathematical problem-solving and problem-solving achievements each other are presented with the structural equation model. Before the analysis of the structural equation model, the correlational analysis and descriptive statistics were performed to observe the correlations between the variables.

Table 14. Descriptive statistics and correlations

		1	2	3	4	5	6
1	Reflective thinking total	-					
2	Attitude total	,43**	-				
3	Metacognition total	,65**	,44**	-			
4	Literacy total	,57**	,52**	,57**	-		
5	Belief total	,38**	,44**	,41**	,49**	-	
6	Problem-solving achievement	,13*	,20**	,15*	,27**	,15*	-

As it is seen from Table 14, all the variables are statistically significant with each other. The highest positive correlation relationship is between reflective thinking and metacognition (r=65). The lowest positive correlation relationship is between problem-solving achievement and reflective thinking. (r=,13). The correlation values change between -1 and 1. As this value approaches 1, the relationship between the two variables desired to be measured increases positively. When the value approaches -1, it increases negatively. When this value approaches 0, the correlational relationship decreases. Interpretation of the correlation coefficient value 0.00 - 0.25 is defined as very weak relationship; 0.26 to 0.49 weak relationships; 0,50 - 0,69 medium relationship; 0.70 - 0.89 high relationship; 0.90 - 1.0 a very high relationship (Köse, 2008).

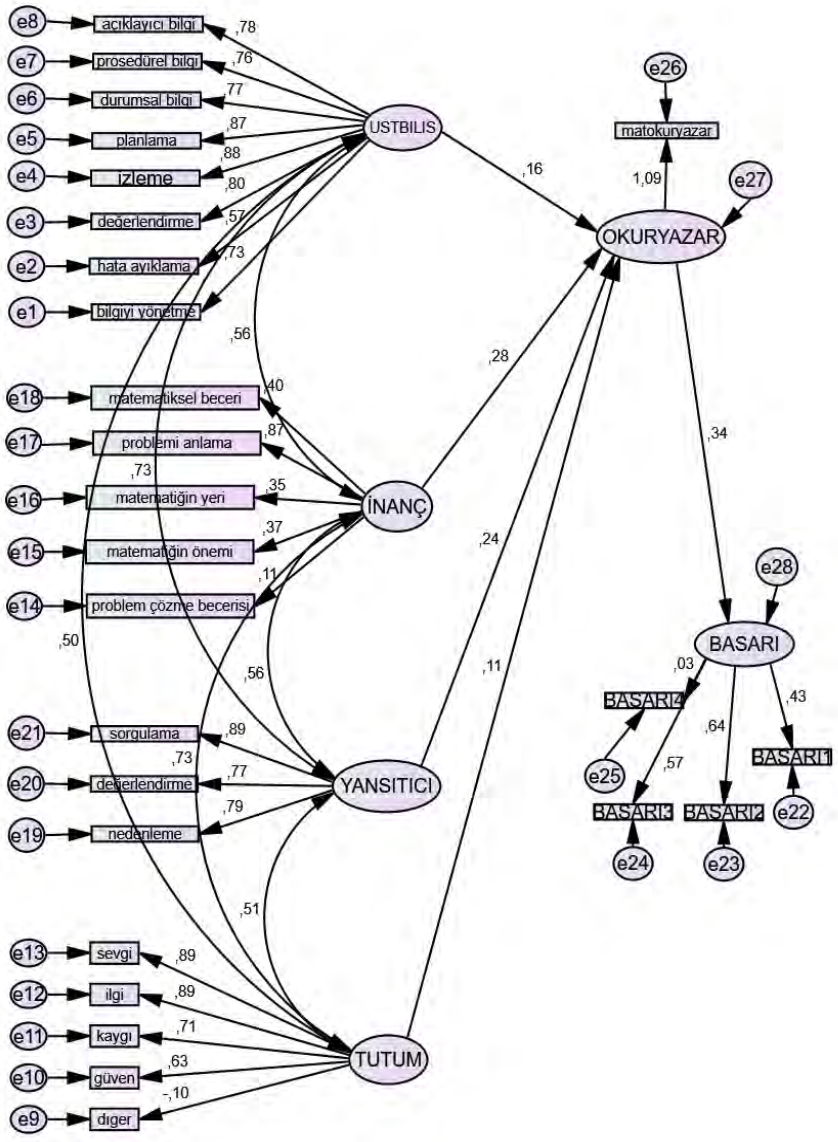

Figure 1. Structural equation model of variables predicting problem-solving achievement

*Note: N=226;**p<.01; (English translations of the words used in Figure 1 are as follows.)*

METACOGNITION (USTBILIS): declarative knowledge (açıklayıcı bilgi), procedural knowledge (prosedürel bilgi), conditional knowledge (durumsal bilgi), planning (planlama), monitoring (izleme), evaluation (değerlendirme), debugging (hata ayıklama), information management (bilgiyi yönetme) BELIEF (INANÇ): mathematical skill (matematiksel beceri), understanding the problem (problemi anlama), place of mathematics (matematiğin yeri), importance of mathematics (matematiğin önemi), problem solving skills (problem çözme becerisi) REFLECTIVE THINKING (YANSITICI): questioning (sorgulama), reasoning (nedenleme) and evaluation (değerlendirme) ATTITUDE (TUTUM): like (sevgi), interest (ilgi), anxiety (kaygı), confidence (güven), miscellaneous (diğer) LITERACY (OKURYAZAR): mathematical literacy (matokuryazar) ACHIEVEMENT (BASARI)

As it is seen from the Figure 1, the factor loads for the metacognition variable change between 0,57-0,88; factor loads for belief variable between 0,11-0,87; factor loads for reflective variable between 0,77-0,89; factor loads for attitude variable between -0,10-0,89; factor loads for achievement variable between 0,03-0,64. The correlation in same direction (γ = 0.56) was found between metacognition and belief, between metacognition and reflective in the same direction (γ = 0.73), between metacognition and attitude the same direction (γ = 0.50), between belief and reflector in the same direction (γ = 0.56), a same direction (γ = 0.73) between belief and attitude, and a same direction (γ = 0.51) between reflective and attitude. Positive correlation was found between the literacy and metacognition, belief, reflective and attitude orderly as (γ=0,16; γ=0,28; γ=0,24; γ=0,11). A positive correlation was found between the literacy- achievement (γ=0,34).

Table 15. Measurement model fit indexes.

Fit Criteria	Perfect Fit	Acceptable Fit	Results of Measurement Model
X^2/sd (CMIN/DF)	0≤X^2/sd≤2	2≤ X^2/sd ≤3	1,499
GFI	,95≤GFI≤1,00	,80≤GFI≤,95	,877
AGFI	,90≤AGFI≤1,00	,85≤AGFI≤,90	,850
CFI	,95≤CFI≤1,00	,90≤CFI≤,95	,946
IFI	,95≤IFI≤1,00	,90≤IFI≤,95	,947
RMSEA	,00≤RMSEA≤,05	,05≤RMSEA≤,08	,047

As the fit indexes of the measurement model of Table 15 are analysed, as the X2/ sd value is below 2, it shows that there is a perfect fit. That the GFI value is 0,877; the AGFI value is 0,850; the CFI value is 0,946 and the IFI value is 0,947 show that there is an acceptable fit. Finally, the RMSEA value is 0,047 shows that there is a perfect fit (Çokluk et al., 2010). Consequently, these fit indexes demonstrate that the model has a good fit. According to the findings gathered from the model, the metacognitive awarenesses of the pre-service teachers, their beliefs related to the mathematical problem-solving, reflective thinking skills related to the problem-solving, attitudes towards mathematical problem-solving predict the problem-solving achievement through the mathematical literacy. On the other hand, mathematical literacy directly predicts problem-solving achievement.

4. Discussion, Conclusion & Recommendations

In the first sub-problem of the research, how the pre-service secondary school mathematics teachers' reflective thinking skills related to the problem-solving, their attitudes towards mathematical problem-solving, metacognitive awareness, mathematical literacy self-efficacy, belief levels related to the mathematical problem-solving and problem-solving achievements change according to the class level were investigated. Within the scope of this sub-problem;

- It was found that the reflective thinking skills of the pre-service teachers related to the problem-solving were high in the first class level, lower in the second class level; however, increased in the third and fourth classes and at the highest level in the fourth class level. In parallel with these results, it was observed in the study by Tuncer ve Özeren (2012) that the reflective thinking skill levels of the pre-service teachers related to the problem-solving changed according to the class level.
- It was found that while the attitude levels of the pre-service teachers related to the mathematical problem-solving were high at the first-class level, it was low at the second class level; however, it increased in the third and fourth classes gradually and reached the highest level at the fourth class level. In parallel with the results gathered from the research, in the studies conducted by Çanakçı (2008), Özgen et al. (2017), Uğurluoğlu (2008), Çokçalışkan (2012), it was found that the attitude levels of the secondary school students related to the mathematical problem-solving changed according to the class level.
- It was determined that while the metacognitive awareness levels of the pre-service teachers were high at the first-class level, lower at the second class; however, it increased again in the at third and fourth class levels, and reached at the highest level at the fourth class. Even as a result of the studies conducted by Akçam (2012); Alcı and Altun (2007); Alcı and Yüksel (2012); Demir and Kaymak-Özmen (2011); Evran (2013); Mert (2018); Sezgin-Memnun and Akkaya (2012); Tüysüz, Karakuyu and Bilgin (2008), Sperling, Howard, Miller and Murphy (2004), it was observed that the metacognitive awareness levels differed according to class level and this demonstrated similarity with the results of the present study. Nevertheless, Akyüzlüer (2014); Deniz, Küçük, Cansız, Akgün, and İşleyen (2014); Gürefe (2015); Kacar and Sarıçam (2015); İflazoğlu-Saban and Saban (2008); Tunca and Alkın-Şahin (2014); Tuncer and Bahadır (2017); Baykara (2011) found that the metacognitive awareness did not demonstrate difference according to the class level.
- As how the mathematical literacy and self-efficacy levels of the pre-service teachers differed according to class level was analysed, it was observed that the mathematical literacy and self-efficacy levels of the pre-service teachers were high

at the first class and lower at the second class; however, increased again at the third and fourth class levels and reached at the highest level at the fourth class level. As a result of the studies conducted by Akkaya and Sezgin-Memnun (2012); Altıntaş, Özdemir, and Kerpiç (2012); Dinçer, Akarsu, and Yılmaz (2016); Özsoy-Güneş et al. (2013); Zehir and Zehir (2016), Schulz (2005), it was found that the mathematical literacy self-efficacy levels demonstrated difference according to class level and this was parallel with the result of the present study. Nevertheless, in the studies conducted by Kırmalı (2015); Soytürk (2011), Tang, Fouad, and Smith (1999), Schaub and Tokar (2005), it was found that the class variable did not create a significant difference on the mathematical literacy level.

- When the pre-service teachers' belief levels related to mathematical problem-solving differed according to the class level were analysed, it was seen that there was a gradual increase from the first class to the fourth class each year and reached the highest level at the fourth class. Problem-solving, modelling, etc. that the pre-service teachers had received during their undergraduate education may have led to an increase in the belief levels of lessons to solve problems. This result shows that as the class levels of the pre-service teachers increase, their belief levels related to the mathematical problem-solving increase too. According to the results of the studies conducted by Çokçalışkan (2015); Deringöl (2018); Sağlam and Dost (2014); Soytürk (2011), it was observed that the belief levels related to mathematical problem-solving differed according to the class level, and this demonstrates similarity with the results of the present study. Nevertheless, at the end of the studies by Başpınar (2015); Duran (2018), they claimed that the class level factor did not demonstrate any significant difference in the belief of the mathematical problem-solving.
- As how the problem-solving achievement scores of pre-service teachers change according to the class level was analysed, it was observed that it increased gradually through the years from the first class to the fourth class and reached to the highest level at the fourth class level. Courses such as problem-solving and modelling taken by pre-service teachers during their undergraduate education may have increased their problem-solving achievement. This result shows that the pre-service teachers' problem-solving achievement levels increase as the class level increases. In the studies conducted by Dündar, Akgün and Gündüz (2015); Işık and Kar (2011); Tarım and Öktem (2014), it was found that the problem-solving achievement differs according to the class level and this demonstrates similarity with the results of the present study.

In the second sub-problem of the research, the reflective thinking skills, attitudes towards mathematical problem-solving, metacognitive awareness, mathematical literacy self-efficacy, and belief level towards mathematical problem-solving of the pre-service

secondary school mathematics teachers were investigated. Consequently, a direct relationship was found between pre-service teachers' metacognitive awareness, beliefs about mathematical problem-solving, reflective thinking skills for problem-solving, attitudes towards mathematical problem-solving, and mathematical literacy. While these variables indirectly predict problem-solving achievements over mathematics literacy, mathematics literacy directly predicts problem-solving achievement. This shows that these variables directly or indirectly predict problem-solving achievement. The two variables with the highest factor load among them are mathematics literacy and problem-solving achievement. Then the relationship between mathematical literacy and belief in solving mathematical problems follow them. These are orderly followed by mathematical literacy and reflective thinking related to problem-solving; mathematical literacy and metacognitive awareness. The two variables determined to have the lowest level of relationship between them are attitudes towards problem-solving with mathematics literacy. As the relevant literature was analysed, while Kasap (1997) and Abdullah et al. (2014) suggested in the study that there was a correlation between the problem-solving achievement and attitude towards problem-solving, Baş (2016), Pennequi et al. (2010), Howard et al. (2001), Everson and Tobias (2001) stressed in their studies that there was a correlation between the problem-solving achievement and metacognitive awareness. On the other hand, Akyüz and Pala (2010); Birbiri (2014), McLeod (1992); Fennema-Sherman (1977); Ma (1997); Papanastasiou (2000) emphasized the correlation between the problem-solving achievement and mathematical literacy in their studies. Bal (2015) mentioned the significant relationship between problem solving success and belief in problem solving in her study. Finally, Güven and Özüm-Çabakçor (2013) referred in their study that there was a significant relationship between problem-solving attitude, belief in problem-solving and self-efficacy perceptions of mathematics, and problem-solving achievements. These studies support the results of the present study. With this study, the predictive status of the reflective thinking towards problem-solving, attitude towards mathematical problem-solving, cognitive awareness, mathematical literacy self-efficacy, and beliefs towards mathematical problem-solving related to the problem-solving achievement was put forth and it is thought that these results will contribute to the literature for future research.

According to the findings of the research, it was found that;

- In the undergraduate education, studies, which aim to increase reflective thinking skills, attitude levels towards problem-solving, levels of metacognitive awareness, and mathematics literacy self-efficacy levels of pre-service teachers, can be conducted more.
- The reason why pre-service teachers' reflective thinking skills, attitude levels towards problem-solving, levels of informatics awareness, and mathematics

literacy self-efficacy levels are lower compared to the first classes can be investigated with a qualitative study.

- In the study, the variables associated with problem-solving were limited to reflective thinking, attitude, metacognitive awareness, mathematics literacy, and belief. Similar studies with other variables can be conducted and the predictive status of problem-solving can be observed.
- A qualitative study in which the variables that predict the problem-solving achievements of the pre-service teachers are investigated in depth can be conducted.
- The sample of the study can be expanded and pre-service teachers from different departments can be included in the sample. Thus, whether there is a difference in problem-solving achievement according to the department variable or not can be observed.
- A similar study can be conducted with mathematics teachers or secondary and primary school students.

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Appendix A.

A.1. Problem Solving Test

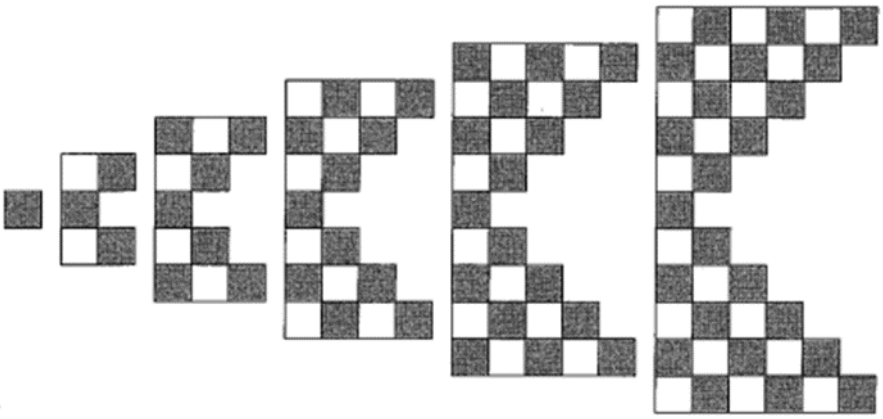
Some questions to predict your problem solving skills are asked below. We ask you to express your each thought/opinion related to your solving process of these problems in written. Everything you write for the solution are valuable for our research. Thank you for your participation.

1) The burning time of a candle of 3h length is 3 hours, the burning time of a candle of length h is 4 hours. How long do they reach the same length after they begin burning together?

2) How many squares are inside a large square including 64 small squares?

3) There are 3 jugs on a table. The largest jug is full of 8 litres of milk. Other two jugs are empty and can be filled with 3 or 5 litres. How can you divide the milk in equal litres by only using these jugs (without using any other measuring tools)?

4) The first 6 terms in an array are shown in the figure below. If the array continues like this, how many frames will there be in the 10th term?



A.2. Problem Solving Achievement Test Scoring Directive

	CRITERIA				SCORE
	Understanding the Problem	Planning	Applying the Plan	Evaluating the Solution	Total Score
1st Question	- The sizes and burning times of the candles that determined and the ones given and asked are written.	-Necessary strategy for the solution of the problem is determined. -The mathematical expression to determine when the candles will reach the same length is written -The mathematical model to visualize the length and burning times of the candles is created	-The mathematical model created is assessed correctly. -The prepared mathematical expression is solved without mistakes and the result t=8/3 is reached.	-The accuracy of the achieved result is checked. -The result is checked with different solutions.	
Score	5	5	5	5	20
2nd Question	-Those given and asked for the problem are determined.	-A big square consisting of 64 small squares is drawn and the required mathematical model is created. -The strategy required for the solution of the problem is determined. -The required pattern is reached with the selected strategy and a mathematical expression is written.	-The mathematical expression that is established is solved without any mistakes and the number of squares in a large square consisting of 64 small squares is calculated using the reached pattern. -That it consists of totally 204 squares is reached.	-The accuracy of the achieved result is checked. -The result is checked with different solutions.	
Score	5	5	5	5	20

3rd Question	-Those given and asked for the problem are determined.	-Necessary strategy for the solution of the problem is selected.	- Using logical reasoning and elimination methods, to have 4 litres of milk both in the jugs of 8 litres and 5 litres in the end, necessary fill-and-empty tests are expressed in written. -The result is reached as a result of necessary trials.	-The accuracy of the achieved result is checked. -The result is checked with different solutions.	
Score	5	5	5	5	20
4th Question	-Those given and asked for the problem are determined.	-The total squares number in the first 6 terms in the array and the number of shaded frames are determined. - Necessary strategy for the solution of the problem is determined and, the existing pattern is reached based on the first 6 terms is written with the proper mathematical expression.	-The established mathematical expression is solved without mistakes and the total number of squares in the 10th term is calculated using the pattern reached. - It is found that totally 109 squares are the squares.	-The accuracy of the achieved result is checked. -The result is checked with different solutions.	
Score	5	5	5	5	20