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Comparing the Mathematical Practices Pre-Service Teachers and Mathematics Teacher Educators Identified as Relevant to Problems and Tasks

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Abstract

In the U.S., state adopted or developed college- and career-ready mathematics standards, including the Common Core State Standards for Mathematics, not only impact districts, students, and their teachers, but also university teacher preparation programs. In order to attain and sustain Common Core's vision of developing mathematically competent citizens, teacher preparation programs must support pre-service teachers' development of practical conceptions of the Standards for Mathematical Practice. In this article, we examine the mathematical practices middle grades pre-service teachers (grades 4-9 licensure) and mathematics teacher educators identified as playing a role in attempts to make sense of and work toward solutions to mathematics problems. In addition, we compare the mathematical practices indicated both within and across pre-service teachers and mathematics teacher educators. Results identify pre-service teachers' potential difficulties operationalizing six specific mathematical habits of mind. Finally, we describe how such comparisons can guide the design of future teacher education and professional learning by describing a process for identifying problems and tasks with the greatest potential to support pre-service teachers' development of practical conceptions of mathematics or other content-specific habits of mind.

Introduction

In the U.S, state adopted or developed college- and career-ready mathematics standards, such as the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers [NGA Center & CCSSO], 2010b), have situated districts, schools, and their teachers in positions primed for long-term change and reform. Along with changes in content standards and their progressions come increased emphasis on mathematical processes and proficiencies—the Standards for Mathematical Practice (NGA Center & CCSSO, 2010b). Table 1 displays the alphanumeric identifier and title for each of the eight mathematical practices, frequently referred to as mathematical habits of mind and denoted by SMPs or MPs. In addition, Table 1 provides a brief summary of each practice for grades 6-8 (Jordan School District, 2012). (see the Common Core State Standards Initiative website http://www.corestandards.org/Math/Practice/ for a more comprehensive description for how students might engage in each mathematical practice.

Table 1. Summary of the Common Core Standards for Mathematical Practice

MP1 - Make sense of problems and preserve in solving them. • When presented with a problem, students can make a plan, carry out their plan, and evaluate its success.	MP2 - Reason abstractly and quantitatively. • Students can contextualize numbers, decontextualize words, and use reasoning habits to help them make sense of problems.
MP3 - Construct viable arguments and critique the reasoning of others. • Students can make conjectures and critique the mathematical thinking of others.	MP4 - Model with mathematics. • Students can recognize math in everyday life and use math they know to solve problems.
MP5 - Use appropriate tools strategically. • Students can use certain tools to help them explore and deepen their math understanding.	MP6 - Attend to precision. • Students can use precision when solving problems and communicating their ideas.
MP7 - Look for and make use of structure. • Students can see and understand how numbers and spaces are organized and put together as parts and wholes.	MP8 - Look for and express regularity in repeated reasoning. • Students can notice when calculations are repeated. Then, they can find more general methods and short cuts.

Several college- and career-ready standards and aligned documents, including the Common Core, indicate the need for the mathematical practices to connect to and be assessed along with mathematical content. For example, according to the Common Core State Standards for Mathematics (2010) ―Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction" (p. 8). Similarly, Florida Mathematics Item Specifications (Florida Department of Education, 2020) assert the ―Mathematical Practices are a part of each course description for Grades 3–8, Algebra 1, and Geometry. These practices are an important part of the curriculum. The Mathematical Practices will be assessed throughout" (p. 5).

Finally, supporting documents for South Carolina's College- and Career-Ready Standards for Mathematics (South Carolina Department of Education, 2020) indicate the ―process standards drive the pedagogical component of teaching and serve as the means by which students should demonstrate understanding of the content standards . . . [and] must be incorporated as an integral part of overall student expectations when assessing content understanding" (p. 3). Therefore, providing K-12 students with opportunities to not only engage in problems, tasks, and activities that coherently connect content with the mathematical practices, but also experiences at exhibiting evidence of such understandings, skills, and habits of mind in their oral and written work has become increasingly important as cognitively challenging Common Core and aligned assessments become the norm.

In this report, the mathematical practices a sample of pre-service mathematics teachers (referred to as PSTs) conceived as being associated with attempts to solve five content-specific problems are compared with the practices indicated by a sample of mathematics teacher educators (referred to as MTEs). In addition, we identify mathematics problems, and a process for identifying such problems and tasks, with the greatest potential to support PSTs' development of practical conceptions of mathematical habits of mind. Finally, we describe how such comparisons can guide the design of teacher education and professional learning not only in mathematics, but also in other areas incorporating general or content-specific processes, proficiencies, habits of mind, or competencies (e.g., Costa & Kallick, 2008; Niss & Højgaard, 2019; OECD, 2019).

Background

In addition to an increased focus on research involving the mathematical practices over the past decade (e.g., Bernander et al., 2020; Bleiler et al., 2015; Bostic & Matney, 2014; Davis et al., 2018), there exists extensive research pertaining to those processes (National Council of Teachers of Mathematics [NCTM], 2000) and proficiencies (National Research Council [NCR], 2001) that ground them. Furthermore, there is a growing body of research (e.g., Riordan & Noyce, 2001; Senk & Thompson, 2003) indicating that students in classrooms that utilize reform curricula (e.g., aligned to NCTM Standards) not only perform as well on standardized tests as do their counterparts in more traditional programs, but also outperform these same students on tests measuring conceptual understanding, applications, and problem-solving ability (Schoenfeld, 2007, p. 540). Such results suggest curricula that focus on the development of powerful understandings, ways of thinking, and mathematical habits of mind can positively impact student achievement. Problematically, existing research also highlight teachers' difficulties in operationalizing and providing students with opportunities to engage in these same mathematical processes and proficiencies (e.g., Jacobs, et al., 2006; Olsen et al., 2014; Stephen, 2014; Weiss, et al., 2003).

Implementation of the Common Core State Standards for Mathematics (NGA Center & CCSSO, 2010b) or aligned college- and career-ready mathematics standards (e.g., Florida's Benchmarks for Excellent Student Thinking [B.E.S.T.] Standards for Mathematics; Nebraska's College and Career Ready Standards for Mathematics) affect not only K-12 instruction, but also university teacher preparation programs charged with producing the next generation of teachers responsible for attaining and sustaining Common Core's vision of developing mathematically competent citizens. This report adds to emerging research into teachers' conceptions of the mathematical practices by addressing the following research question: How do the mathematical practices pre-service teachers identified as being relevant during problem solving compare with those practices identified by mathematics teacher educators?

Methods

Participants

Fifteen pre-service teachers enrolled in a middle grades mathematics methods course (seeking 4th-9th grade licensure) at a mid-size Midwestern university were asked to solve mathematics problems related to the six 6th-

8th grade Common Core content domains (i.e., Ratios and Proportional Relationships, The Number System, Expressions and Equations, Geometry, Statistics and Probability, and Functions). Furthermore, it was requested that PSTs solve the problems in a manner aligned with how they envisioned students might engage with the problems and a manner PSTs believed exhibited engagement in one or more of the mathematical practices in their written work.

The identical set of problems were given to six mathematics teacher educators to generate a sample of the mathematical practices that professionals in the field identified as being associated with attempts to solve these same problems. MTEs received instructions identical to those given to PSTs. The MTE sample included one mathematics teacher education doctoral candidate and five practicing mathematics education professors with varying areas of research and teaching focus and diverse years of experience.

Problems and Data

Problems were chosen from several standards-based sources, including the Connected Mathematics Project (CMP) (Michigan State University, n.d.). Data comprised the mathematical practices PSTs and MTEs indicated students would engage in and exhibit in their written work as they attempted to solve the five problems. At the time these problems were assigned, PSTs' main experiences with the mathematical practices, in relation to the mathematics methods course, involved supporting their conceptions of what each practice ―looks like" during verbal classroom interactions and how they could manage instruction (as teachers) to focus on specific mathematical practices. Such support included watching and discussing video of classroom instruction correlated to the mathematical practices from the Charles A. Dana Center's (2017) Inside Mathematics website. At the time MTEs were given the problems, each was teaching one or more mathematics methods course—for pre-service teachers seeking licensure in grades pre-K-3 (children ages 2-9 years), grades 4-9 (children ages 9-15 years), or grades 7-12 (children ages 12-18 years)—that incorporated discussions about and examples of the mathematical practices as part of their curriculum.

Data Analysis

Analysis consisted of summary statistics that focused on the frequencies with which particular mathematical practices were indicated by PSTs and MTEs across and within problems or by a particular PST or MTE. In addition, the percentage of PSTs and MTEs that indicated specific practice standards for each problem were examined and compared. Finally, where appropriate, chi-square tests were performed to test for deviations of observed frequencies from frequencies expected by chance.

Results

Results will be divided into three sub-sections: (1) descriptions of the mathematical practices PSTs identified as being associated with attempts to solve each problem, (2) descriptions of the practices MTEs identified as associated with attempts to solve each problem, and (3) comparisons between the mathematical practices PSTs and MTEs identified for each problem.

Pre-Service Teachers' Identified Mathematical Practices

The mathematical practices each PST indicated as being engaged in and exhibited for each problem are displayed in Table 2. For example, Connie identified MP5 (Use appropriate tools strategically) and MP6 (Attend to precision) as being associated with Problem #2 (see Figure 1). A random name generator was used to designate PSTs.

Table 2. Mathematical Practices Identified by Problem and by PST or Group of PSTs

PST or PST Group	Problem #1	Problem #2	Problem #3	Problem #4	Problem #5
Amber			1, 4, 5	8	1, 3
Austin		7, 8		2	4
Bertie				1, 6	4
Beverly				1, 5, 6	1, 4, 5, 6
Carlene					2, 4
Connie	2, 8	5, 6	1, 4	6, 8	1, 4, 5
Doretha	1, 4, 6	6, 7, 8	1, 4, 6	1, 3, 4, 5	1, 2, 3, 4, 5, 6
Evan	1, 4, 6	4, 7, 8	1, 4, 5	2, 3	1, 2, 7
Flora	4	6		7	1, 2
Luther		1, 3, 4, 5, 8		1, 2, 3, 5	1, 3, 4, 5, 6
Marquita	1, 5			1, 4, 6, 8	1, 3, 6
Pamela					
Rene	1, 4, 5, 6		1, 4, 6	1, 3, 7	1, 4
Stuart	1, 4, 5	4, 6, 7, 8		1, 4, 5, 7	1, 4, 5, 6
Tyrone		6			
Group #1 (Austin, Bertie, Carlene, Stuart)	1, 4, 5, 6, 8				
Group #2 (Evan, Marquita, Rene)	1, 4, 5, 6, 8		1, 4, 5, 6		
Group #3 (Amber, Beverly, Doretha, Luther)	1, 3, 4, 5, 6		1, 3, 4, 5, 6		
Group #4 (Connie, Flora, Pamela, Tyrone)	2, 8	1, 7, 8			
Unanimous MP Indication	none	none	1, 4	none	none

As Table 2 illustrates, not every PST identified practices for every problem. PSTs asserted that such absences did not indicate their belief that no practices were involved. Rather, PSTs indicated such instances indicated they had not had time to settle on specific mathematical practice identifications or had inadvertently failed to write down which practice standards the given problem involved. After identifying practices for Problems #1-3 individually, PSTs were asked to re-examine one or two of the same problems in groups of three to four. As illustrated in Table 2, the number of practices identified during these group discussions frequently increased over what PSTs had selected individually.

As displayed in Table 2, there was a reasonable degree of variability among the combinations and number of practices identified. For example, for Problem #2 (Figure 1), which was given to PSTs as part of the Number System Problem Set, Tyrone identified only MP6, whereas Luther identified MP1, MP3, MP4, MP5, and MP8.

(2) Use the number sentence 78 × 12 = 936 to find each product below. Explain your thinking.

a) 7.8 × 1.2
b) 7.8 × 0.12
c) 7.8 × 0.012
d) 0.78 × 1.2
e) 0.078 × 1.2
f) 0.78 × 0.12

Figure 1. Number System Domain (adapted from Lappan, et al., 2009a)

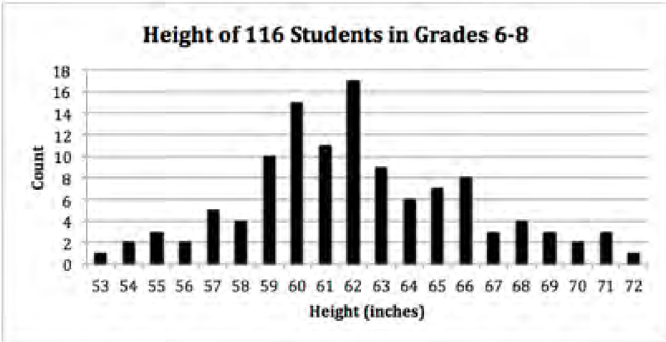
For Problem #2, each mathematical practice was indicated, on average, by 3.0 of the 9 PSTs and groups of PSTs that completed the problem. That is, each practice was identified, on average, by 33.3% of PSTs or PST groups. The standard deviation with which each individual practice was indicated for this problem was 2.14. According to McHugh (2013), the chi-square test assumes “data were obtained through random selection. However, it is not uncommon to find inferential statistics used when data are from convenience samples rather than random samples” (p. 144).

Chi-square also assumes all expected values “should be 5 or more in at least 80% of the cells, and no cell should have an expected [value] of less than one” (McHugh, 2013, p. 144)—a restriction cited by several authors (e.g., Gingrich, 2004; Moore, McCabe, & Craig, 2009). Since only nine PSTs or PST groups identified mathematical practices for Problem #2, the expected value for each mathematical practice was 9×0.5=4.5 (since any PST or PST group either identifies a particular mathematical practice or not, the probability is 0.5). Therefore, since every expected value was less than five, we were unable to perform a chi-square test to determine how likely it was that the observed distribution (of PSTs’ identified practice standards) was due to chance.

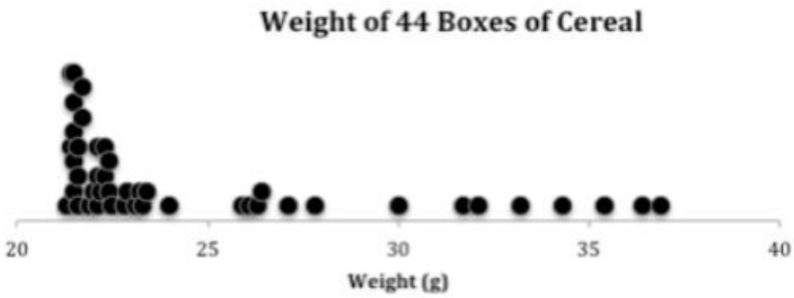
The problem with the least variability in frequency with which individual practices were identified was Problem #4 (Figure 2), which was given to PSTs as part of the Statistics and Probability Problem Set.

(4) Look at the mean and median for each of the data distributions below. Describe how the shape of each distribution is influencing the location of the mean and the median. Explain your reasoning.

a) Mean = 62 inches, Median = 62 inches,
Minimum and Maximum Values = 53 and 72 inches



b) Mean = 24.8 grams, Median = 22.4 grams,
Minimum and Maximum Values = 21.3 and 36.9 grams



c) Mean = 135 minutes, Median = 138 minutes,
Minimum and Maximum Values = 88 and 201 minutes

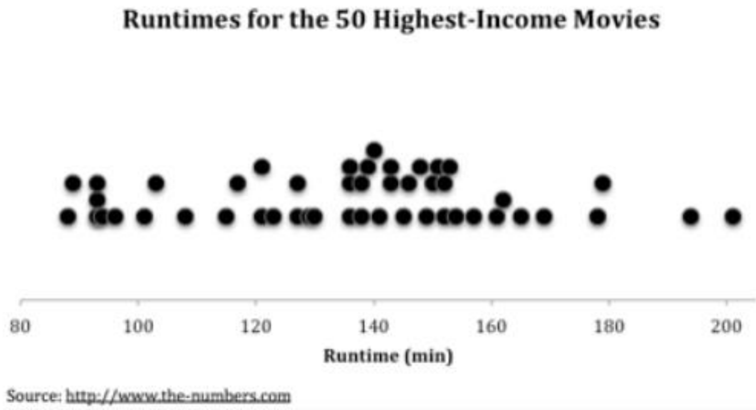


Figure 2. Statistics and Probability Domain

For Problem #4, each mathematical practice was indicated, on average, by 3.88 of the 12 PSTs that completed the problem. That is, each practice was identified, on average, by 32.3% of PSTs. The standard deviation with which each individual practice was indicated for this problem was 1.36. Figure 3 displays the percent of PSTs that indicated each of the eight mathematical practices for Problem #4. As illustrated in Figure 3, MP1 (Make sense of problems and preserve in solving them) was identified by 26.0% more PSTs than average.

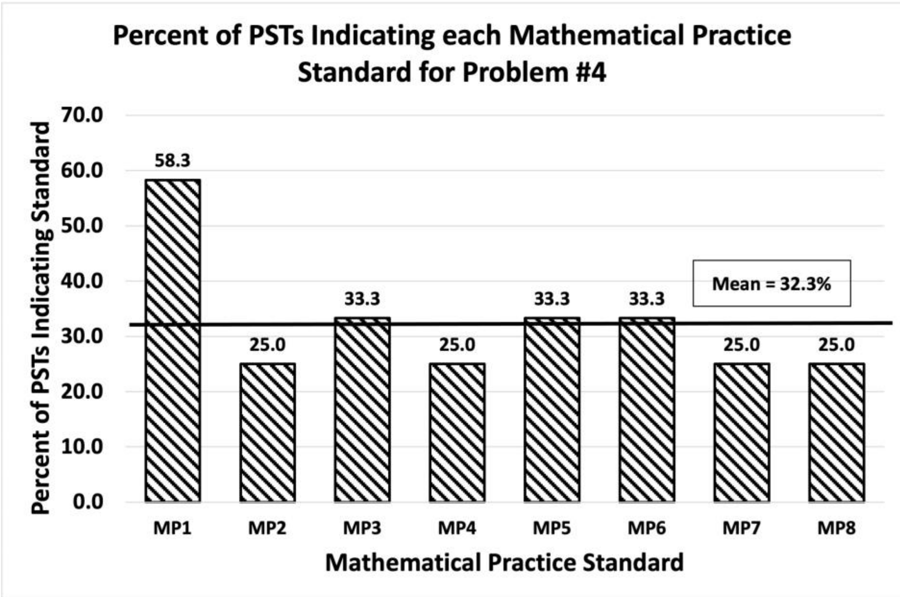


Figure 3. Percent of PSTs indicating each practice for Problem #4

Since the number of individual PSTs was 12, we were able to perform a chi-square test with both the number of expected PSTs identifying each individual practice and the number of expected PSTs not identifying each practice given as six. Results indicate that the number of PSTs identifying each practice for Problem #4 was not significantly different than that expected by chance, X^2 (7, N = 12) = 8.167, p = 0.3181.

The problem with the most variability in the frequency with which practices were identified by PSTs was Problem #5 (Figure 4), which was given to PSTs as part of the Geometry Problem Set.

(5) A drink can is a cylinder with radius 3 cm and height 12 cm. Ms. Doyle's classroom is 6 m wide, 8 m long, and 3 m high. Estimate the number of drink cans that would fit inside Ms. Doyle's classroom. Explain your estimate.

Figure 4. Geometry Domain (adapted from Lappan, et al., 2009c)

For Problem #5, each mathematical practice was identified, on average, by 4.75 of the 13 PSTs that completed the problem. That is, each practice was indicated, on average, by 36.5% of PSTs. The standard deviation with which each individual practice was indicated was 3.45. Figure 5 displays the percent of PSTs that indicated each of the eight mathematical practices on Problem #5. As illustrated in Figure 5, MP1 and MP4 (Model with mathematics) were identified by 40.4% and 32.7% more PSTs than average, respectively. Conversely, MP7 (Look for and make use of structure) was identified by 28.8% less PSTs than average and MP8 (Look for and express regularity in repeated reasoning) was indicated as not being involved at all; that is, identified by 36.5% less PSTs than average.

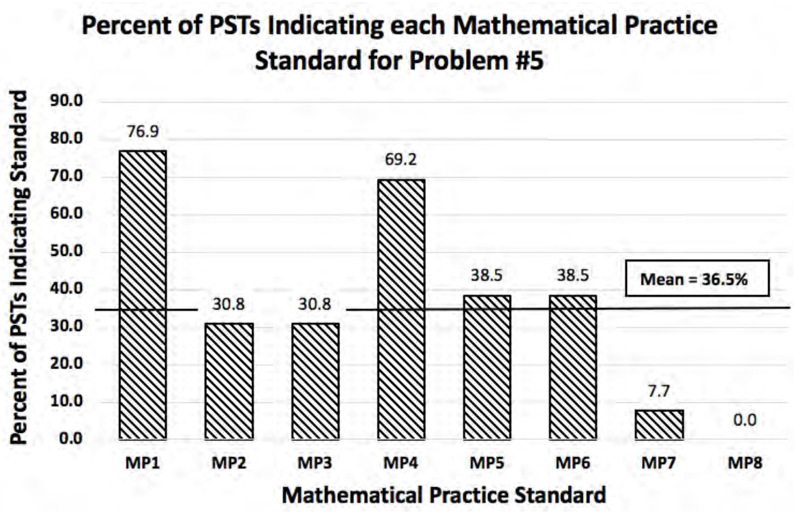


Figure 5. Percent of PSTs indicating each practice for Problem #5

Since the number of individual PSTs was 13 we were able to perform a chi-square test, with both the number of expected PSTs identifying each individual practice and the number of expected PSTs not identifying each practice given as 6.5. Results indicated that the number of PSTs identifying each practice for Problem #5 was significantly different than that expected by chance, X^2 (7, N = 13) = 16.615, $p < 0.05$.

The only problem with unanimous practice identification was Problem #3 (Figure 6), in which MP1 and MP4 were identified by all PSTs and groups of PSTs that completed the problem. Problem #3 was also the problem that was completed by the least number of individuals and groups of PSTs (n = 7), a fact that could have contributed to this result. For Problem #3, each mathematical practice was identified, on average, by 2.88 of the seven PSTs and groups of PSTs that completed the problem. That is, each practice was indicated, on average, by 41.1% of PSTs or PST groups. The standard deviation with which each individual practice was indicated was 3.04.

(3) Jumel and Ashley have two of the most popular phones on the market, a Droid and an iPhone. Jumel's monthly cell phone plan is shown below, where c stands for the cost in dollars, and t stands for the number of texts sent each month.

- Jumel: $c = 60 + 0.05t$
- Ashley's plan costs $.35 per text, in addition to a monthly fee of $45.

a) Whose plan, Jumel's or Ashley's, costs less if each of them sends 30 texts in a month? Explain how you determined your answer.
b) How much will Ashley's plan cost for the same number of texts as when Jumel's costs $75.00?
c) Explain in writing how you know if there is a number of texts for which both plans cost the same amount.

Figure 6. Expressions and Equations Domain (adapted from New York City Department of Education, 2012)

As illustrated in Figure 7, MP1 and MP4 were each identified by 58.9% more PSTs than average. Conversely, MP3 (Construct viable arguments and critique the reasoning of others) was identified by 26.8% less PSTs than average. Finally, MP2 (Reason abstractly and quantitatively), MP7, and MP8 were each not identified by any PST; that is, each were identified by 41.1% less PSTs than average.

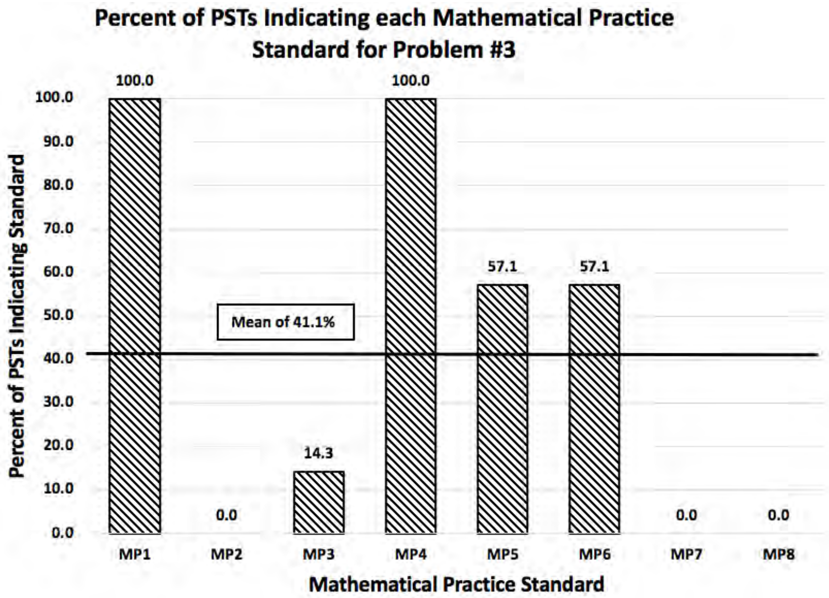


Figure 7. Percent of PSTs indicating each practice for Problem #3

Since only seven PSTs or PST groups identified mathematical practices for Problem #2, the expected value for each mathematical practice was $7 \times 0.5 = 3.5$. Therefore, since every expected value was less than five, as with Problem #2, we were unable to perform a chi-square test to determine how likely it was that the observed distribution (of PSTs' identified practice standards) was due to chance. Although the results presented here might be expected, considering the idiosyncratic nature of interpretations of the mathematical practices and the interaction and overlap amongst practices (PARCC, 2017, p.13), our intent was to gather data with which to develop a baseline for PSTs' interpretations of the practices. Such a baseline would then serve to guide future engagements with these and other teachers.

Mathematics Teacher Educators' Identified Mathematical Practices

As indicated earlier, our MTE sample was given the same five problems and directions as our PST sample. Table 3 displays the practices each MTE indicated as being engaged in and exhibited for each problem. MTEs were randomly assigned the Greek letter names Alpha through Zeta. Zeta failed to identify practices for Problems #1 and #5 but indicated that this was merely an oversight and not meant to indicate that no practices were involved.

As illustrated in Table 3, there was a relative degree of variability among the combinations and number of practices identified. For example, for Problem #5 (Figure 4) Alpha indicated only MP1, whereas Beta indicated practices MP1, MP2, MP3, MP4, and MP6. For MTEs, problems with unanimous mathematical practice indications included: Problem #2 (MP7 indicated by all MTEs), Problem #3 (MP2 and MP3), and Problems #4 and #5 (MP1). Since there were only six MTEs in our sample, we were unable to perform a chi-square test to determine how likely it was the observed distribution (of MTEs' identified practice standards) was due to chance on any of the problems.

Table 3. Mathematical Practices Identified by Problem and by MTE

MTE	Problem #1	Problem #2	Problem #3	Problem #4	Problem #5
Alpha	1, 5, 8	1, 7, 8	1, 2, 3	1, 2, 6	1
Beta	1, 2, 3, 4, 8	3, 7, 8	1, 2, 3, 4, 5	1, 3, 6	1, 2, 3, 4, 6
Gamma	2, 4, 5, 6	2, 3, 6, 7	2, 3, 4, 6	1, 3, 4	1, 2, 3, 4
Delta	1, 4, 5, 7	1, 2, 3, 6, 7, 8	1, 2, 3, 4, 5, 7, 8	1, 2, 3, 7, 8	1, 2, 3, 5, 6
Epsilon	1, 2, 4	1, 2, 7	1, 2, 3, 4	1, 2, 3	1, 2, 5, 6, 8
Zeta		1, 3, 6, 7, 8	1, 2, 3, 4, 7	1, 2, 3, 5	
Unanimous MP Indication	none	7	2, 3	1	1

As with their PST counterparts, MTEs' results might be expected. Aside from idiosyncratic mathematical practice interpretations and the connections among practices, MTEs' knowledge of and experiences with both the NCTM Process Standards (e.g., Communication) and NCR Strands of Mathematical Proficiency (e.g., Adaptive Reasoning) might have impacted their choices. For example, NCTM's Connection Process Standard correlates with mathematical practices MP4, MP6, MP7, and MP8 (NCTM, 2010).

Problem #3 (Figure 6), which derives from New York City Department of Education's (2012) WeTeachNYC online resource library and professional learning space, was the only problem where the source itself indicated which mathematical practices they envisioned the problem involving. Documents accompanying Problem #3 indicate the problem engages students in practice standards MP1, MP2, MP3, MP4, MP6, and MP7—a combination that no MTE or PST in our study selected. This result, along with the variability among MTEs' identified practices for all five problems (Table 3), suggests there may not be definitive answers as to which practices should be identified for any given problem. As such, perhaps the most that can be asserted are which mathematical practices are most relevant in attempts to make sense of and work toward a solution to a problem.

In characterizing the types of high-quality assessment tasks required to span the range of performance goals set forth in the Common Core State Standards for Mathematics (CCSSM), Daro and Burkhardt (2012) assert, ―Designing rich assessment tasks that allow students *to show what they know, understand, and can do* across the range of practices and content set out in CCSSM is among the most challenging areas of educational design" [authors' italics] (p. 21). As such, ―It is obviously desirable that…different people with some expertise in mathematics education will classify a given task in much the same way" (Daro & Burkhardt, 2012, p. 24). The results displayed in Table 3 fail to represent problems with associated mathematical practices that have been classified ―in much the same way."

For Problem #5 (Figure 4), each mathematical practice was identified, on average, by 2.71 of the 5 MTEs that completed the problem. That is, each practice was indicated, on average, by 54.3% of the MTEs. The standard deviation with which each individual practice was indicated was 1.60. Figure 8 displays the percent of MTEs that indicated each of the eight mathematical practices for Problem #5.

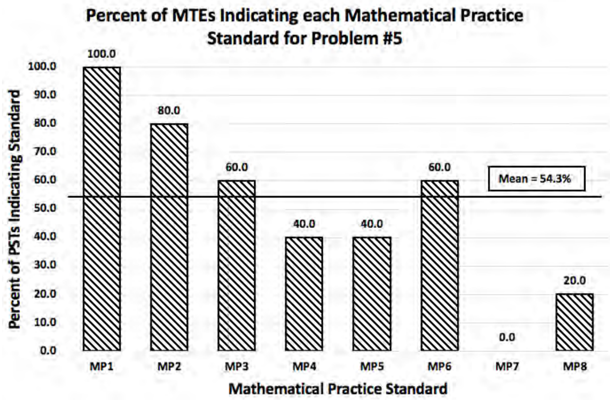


Figure 8. Percent of MTEs indicating each practice for Problem #5

As illustrated in Figure 8, MP1 and MP2 were identified by 45.7% and 25.7% more MTEs than average, respectively. Conversely, MP8 was identified by 34.3% fewer MTEs than average and MP7 was indicated as not being involved at all; that is, identified by 54.3% fewer MTEs than average.

Comparisons of PSTs' and MTEs' Identified Mathematical Practices

Comparisons of the practices identified by PSTs and MTEs across the five problems helps identify the types of problems that could best serve PSTs' developing conceptions of the Standards for Mathematical Practice (NGA Center & CCSSO, 2010b). For example, for Problem #1 (Figure 9), each mathematical practice was identified, on average, by 4.375 of the 11 individual or groups of PSTs that completed the problem. That is, each practice was indicated, on average, by 39.8% of the individual or groups of PSTs. The standard deviation with which each individual practice was indicated was 3.11. Since the number of individual and groups of PSTs was 11 we were able to perform a chi-square test, with both the number of expected PSTs identifying each individual practice and the number of expected PSTs not identifying each practice given as 5.5. Results indicated that the number of PSTs (or groups of PSTs) identifying each practice for Problem #1 was significantly different than that expected by chance, X^2 (7, N = 11) = 14.182, $p < 0.05$.

(1) Students in Eric's gym class must cover a distance of 1,600 meters by running or walking. Most students run part of the way and walk part of the way. Eric can run at an average speed of 200 meters per minute and walk at an average speed of 80 meters per minute.

a) Suppose that Eric runs for 4 minutes and walks for 8 minutes. How close is he to the 1,600-meter goal?
b) Write an equation for the distance (d) Eric will cover if he runs for x minutes and walks for y minutes.
c) Find three combinations of running and walking times for which Eric would cover 1,600 meters.
d) Plot the ordered pairs for part (c) on a graph. Use the graph to estimate four other combinations of running and waling times which Eric could cover 1,600 meters.

Figure 9. Ratios, Proportional Relationships, and Functions Domains (adapted from Lappan, et al., 2009d)

This same problem was identified, on average, by 2.375 of the five MTEs that completed the problem, with a standard deviation of 1.30. Figure 10 displays the percent of PSTs and MTEs that indicated each of the eight mathematical practices on Problem #1.

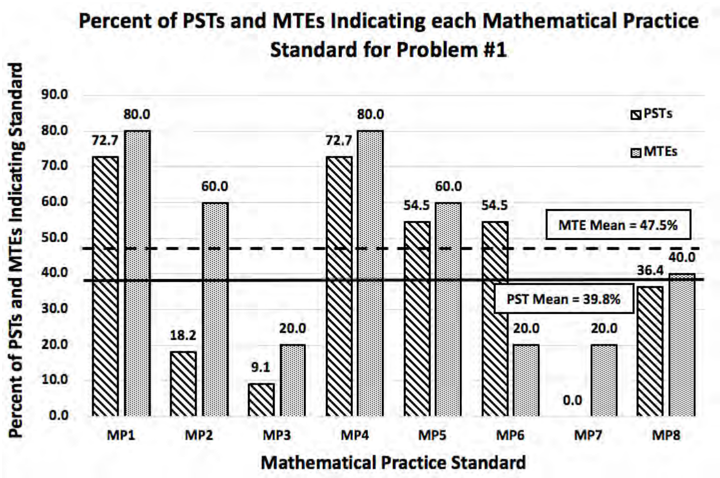


Figure 10. Percent of PSTs and MTEs indicating each practice for Problem #1

As illustrated in Figure 10, MP6 was identified by 34.5% more PSTs than MTEs. Conversely, MP2, MP3, and MP7 were identified by 41.8%, 10.9%, and 20.0% fewer PSTs than MTEs, respectively. As such, activities involving problems similar to Problem #1 could support PSTs' developing images of mathematical practices MP2, MP3, MP6, and MP7. For each problem, the difference in the percent of participants identifying a practice standard, PST% – MTE%, was calculated for each practice. Those practices with differences that we deemed substantial (|PST% – MTE%| ≥ 25%) are displayed in Table 4.

Table 4. Differences in Percent of Mathematical Practices Identified by Problem

Problem Number	Content Domain	Mathematical Practice	Percent Difference (PST% – MTE%)
Problem #1	Ratios and Proportional Relationships	MP2	-41.8
		MP6	+34.5
Problem #2	The Number System	MP1	-44.4
		MP2	-50.0
		MP3	-55.6
		MP4	+33.3
		MP7	-44.4
Problem #3	Expressions and Equations	MP2	-100.0
		MP3	-85.7
		MP6	+40.5
		MP7	-33.3
Problem #4	Statistics and Probability	MP1	-41.7
		MP2	-41.7
		MP3	-50.0
Problem #5	Geometry	MP2	-49.2
		MP3	-29.2
		MP4	+29.2

As indicated in Table 4, Problems #2 and 3, involving distinct content domains and contexts, could serve as particularly productive problems to engage PSTs in due to the large number of practices that were identified by substantially different numbers of PSTs than MTEs. In addition, such comparisons support the identification of those Common Core content domains (i.e., The Number System and Expressions and Equations in this study) PSTs might have the most difficulty conceptualizing the role specific mathematical practices play. Comparisons of the practices identified by PSTs and MTEs across the five problems also provides a glimpse into which practices might be most problematic for pre-service mathematics teachers to operationalize (Table 5).

Table 5. Differences in Percent of Practices Identified by Mathematical Practice

Mathematical Practice	Problem Number	Content Domain	Percent Difference (PST% – MTE%) < 0	Percent Difference (PST% – MTE%) > 0
MP1	Problem #2	The Number System	-44.4	
	Problem #4	Statistics and Probability	-41.7	
MP2	Problem #1	Ratios and Proportional Relationships	-41.8	
	Problem #2	The Number System	-50.0	
	Problem #3	Expressions and Equations	-100.0	
	Problem #4	Statistics and Probability	-41.7	
	Problem #5	Geometry	-49.2	
MP3	Problem #2	The Number System	-55.6	
	Problem #3	Expressions and Equations	-85.7	
	Problem #4	Statistics and Probability	-50.0	
	Problem #5	Geometry	-29.2	
MP4	Problem #2	The Number System		+33.3
	Problem #5	Geometry		+29.2
MP6	Problem #1	Ratios and Proportional Relationships		+34.5
	Problem #3	Expressions and Equations		+40.5
MP7	Problem #2	The Number System	-44.4	
	Problem #3	Expressions and Equations	-33.3	

As illustrated in Table 5, MP1 (on two problems), MP2 (on five problems), MP3 (on four problems), and MP7 (on two problems) were each indicated substantially less frequently by PSTs than MTEs. Conversely, MP4 and MP6 (on two problems each) were each indicated considerably more frequently by PSTs than MTEs. These results indicate PSTs' under-identified practice standards MP1, MP2, MP3, and MP7 and over-identified of MP4 and MP6, suggesting particular difficulties in PSTs' attempts to operationalize these four mathematical habits of mind. Such results further suggest a need to engage participating PSTs in additional problems, activities, and discussions regarding these six particular mathematical practices (see Table 1).

Conclusion

In this report, we identified the mathematical practices that small samples of PSTs and MTEs identified as being relevant in their attempts to solve five domain-specific mathematics problems. In addition, through comparisons of PSTs' and MTEs' selections, we identified the mathematical practices with the potential to be most problematic for participating PSTs to operationalize and sample problems that might best support PSTs' development of more practical conceptions of mathematical habits of mind. The results presented in this report suggest the problems chosen in the study were such that it was difficult to isolate individual, pairs, or groups of mathematical practices that students might exhibit as they engage in solving the problem. Daro and Burkhardt (2012) indicate that it is easier to classify a problem or task ―if the factors used [to classify the problem] are close to directly observable rather than, for example, depending on inference from a deep theoretical model" (p. 24). This suggests the results illustrated in Tables 2 and 3 might be attributed more to the inferences each MTE and PST made as to what constitutes engagement in individual, pairs, or groups of mathematical practices than the problems themselves. It should be noted that attempting to determine which contributed more to the results in this study, the problems themselves, the inferences participants attributed to the mathematical practices, some combination of each, or alternative notions not mentioned here, is not possible with the current data. Such conclusions will need to be explored through future research.

Although results from this report lack generalizability due to the small sample size and unequal numbers within each group, they do suggest the benefits of engaging larger samples of teachers (both pre- and in-service), mathematics teacher educators, and mathematicians in similar activities. Specifically, such research could serve teacher education and professional development in identifying and refining the list of those mathematical practices with the potential to be most problematic for teachers to operationalize, both within and across specific content domains. In addition, such research could help generate a more defined set of domain specific problems and tasks with the greatest potential to support teachers in developing practical conceptions of each practice and for specific mathematical practice combinations. Finally, results from this report fail to make explicit what participating teachers took as evidence that any given practice was being engaged in. Future research must explore this line of inquiry to determine how best to support teachers in developing conceptions that enable them to design and manage instruction that engages students in activities that develop students' understandings, skills, and habits of mind.

Though our focus was on the Common Core's Standards for Mathematical Practice NGA Center & CCSSO, 2010b) in the U.S, similar data could be collected and analyzed to explore and compare pre-service teachers', in-service teachers', and teacher educators' conceptions of general or content-specific processes, proficiencies, or competencies, including the Next Generation Science Standards' (NRC, 2013) Scientific and Engineering Practices (e.g., developing and using models, engaging in argument from evidence), PISA's (OECD, 2019) Reading framework processes (e.g., represent literal meaning; reflect on content and form), the Common Core's (NGA Center & CCSSO, 2010a) English Language Arts Standards' capacities of a literate individual (e.g., demonstrate independence, value evidence), the Danish Mathematical competencies (Niss & Højgaard, 2019) (e.g., engaging in mathematical inquiry, assessing and producing justification of mathematical claims), or other content-specific or general habits of mind (e.g., Costa & Kallick, 2008). Therefore, the process of having pre-service or early career teachers (i.e., novices in the field) and teacher educators (i.e., experts in the field) categorize the same problems, tasks, and activities, and comparing these categorizations could be done with any content-specific habits of mind (e.g., Science, Social Studies, and English Language Arts).

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