



Mathematics Teachers' Inclusion of Modelling and Problem Posing in Their Mathematics Lessons: An Exploratory Questionnaire

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Abstract:

In this paper the results of a questionnaire about mathematics teachers' inclusion in their teaching of mathematical modelling and problem posing are presented. To make teachers able to choose a coherent teaching method for a mathematical topic, teachers' knowledge of valuable methods for the teaching of mathematics should be fostered. In this sense, studying teachers' practice of modelling and problem posing is crucial in order to know if and which kind of support and professional development they need on such educational strategies. The questionnaire was administered to Italian in-service mathematics teachers of primary and secondary school. Findings indicate that despite teachers implement regularly some aspects of mathematical modelling in their lessons, they ask more materials to support their preparation and practice. Problem posing, instead, might be more integrated in the classroom work, and consequently in teachers' professional development courses.

Keywords: mathematical problem posing, mathematical modelling, exploratory study, questionnaire

INTRODUCTION

In May 2018, The Council of European Union recommended for some key competencies for lifelong learning (2018/C189/01). One of these competencies concerns *mathematical competence*, seen as the ability to develop and apply mathematical thinking and insight in order to solve a range of problems in everyday situations, with an emphasis on the process and activity, as well as knowledge. Mathematical competence involves the ability to use mathematical modes of thought and presentation, such as formulas, models, etc... In this direction, teachers need to know mathematics in ways useful for making mathematical sense of students' work and choosing powerful ways of representing the subject so that it is understandable to students (Ball et al., 2008).

According to the Realistic Mathematics Education perspective, an improvement in students' critical thinking and reasoning should be fostered with activities based on realistic and rich contexts (Gravemeijer & Doorman, 1999). Realistic refers to problem situations that students can image and that are, at a certain stage, meaningful for them. Therefore, problems can come from the real world, but also from a fantasy world or from the formal world of mathematics, as long as they are experientially real in students' mind (Van den Heuvel-Panhuizen & Drijvers, 2014). As a natural consequence these contexts must also be rich (Freudenthal, 1991), in the sense that they promote a structuring process as a means of organizing phenomena, physical and mathematical, and even mathematics as a whole, i.e., contexts that give more opportunities in the mathematization process. In this direction a fundamental educational tool is given by cultural artefacts (Bonotto, 2013) that, thanks to their complexity and

richness in mathematical meaning, live in both the world of symbols and the real one. Artefacts can create a new dialectic between school mathematics and extra school experiences, by bringing students' everyday-life experiences and informal reasoning into play. In particular, classroom activities based on the use of suitable artefacts and interactive teaching methods could foster a mindful approach towards realistic mathematical modelling and problem posing attitude (Bonotto, 2013).

Modelling and problem posing represent typical ways of work and thought of mathematicians at every level. Moreover, modelling and problem posing can be seen as valuable educational strategies that, starting from and working with rich and realistic contexts, enhance students' reasoning and critical thinking and give sense to their mathematical activity, promoting also essential skills necessary for involvement in a democratic society (Hansen & Hana, 2015). A modelling perspective, in fact, provides basic arguments for including authentic situations in the mathematics classroom (Maass, Doorman, Jonker, & Wijers, 2019) and represents a critical tool to understand the reality or society in general. Moreover, allowing students to write their own mathematical problems may help them to make connections between mathematics in the classroom and their real life (Kopparla et al., 2018). In this direction, there is the necessity to start working with teachers, in order to understand which kind of support they need in developing and implementing classrooms activities that could bring the gap between mathematical competences in- and out-of-school and that enhance students' reasoning and critical thinking. Since modelling and problem posing are valuable educational strategies to achieve such results, this study started investigating teachers' practice in modelling and problem posing. In the specific, the aim of the research consisted in exploring if mathematics teachers of every school level include in their lessons some aspects of modelling and/or problem posing.

THEORETICAL BACKGROUND

Several studies have shown that mathematics has assumed a stereotypical nature (Verschaffel et al., 1997), whose main consequence was an increasing gap between mathematics and reality (Gravemeijer, 1997) that did not favour mathematical modelling (Blum & Niss, 1991). This situation is still substantially unchanged after almost twenty years (Bonotto, 2013). Indeed, despite the importance of mathematical modelling, in everyday mathematics teaching practice there is still relatively few modelling (Blum, 2015). One of the main reasons for this gap between the educational debate and the classroom practice is that teaching modelling is high demanding (DeLange, 1987; Freudenthal, 1973; Ikeda, 2007; Pollak, 1979). Mathematical modelling is quite difficult for teachers because teaching become more open and less predictable and specific competencies for mathematical modelling are requested (Borromeo Ferri & Blum, 2010).

Moreover, the majority of the posed problems in the classrooms focused on memorization and procedural understanding, rather than on mathematical reasoning and conceptual understanding (Stein et al., 2000). Not enough attention is paid to the role of teachers in problem posing activities. Teachers' knowledge of problem posing, instead, must be considered and developed to improve students' success in their mathematical performances (Lee, Capraro, & Capraro, 2018). Indeed, enhancing teachers' pedagogical content knowledge is essential to improve the teaching and learning of mathematics (Appova & Taylor, 2019). Just knowing a subject well may not be sufficient for teaching. Teachers, instead, need to know mathematics in ways useful for, among other things, making mathematical sense of students' work and choosing powerful ways of representing the subject so that it is understandable to students (Ball et al., 2008).

To overcome these problems, the teaching of mathematics might be seen as a human activity of *guided reinvention* (Freudenthal, 1991) that starts from rich and realistic contexts. Students are active participants in the learning process in a balance between students' freedom of invention and the power of teachers' guidance. Moreover, rich and realistic contexts, i.e., problem situations that are meaningful for both mathematics and students, offer students opportunities to attach meaning to the mathematical constructs they develop while solving problems. Modelling and problem posing are powerful

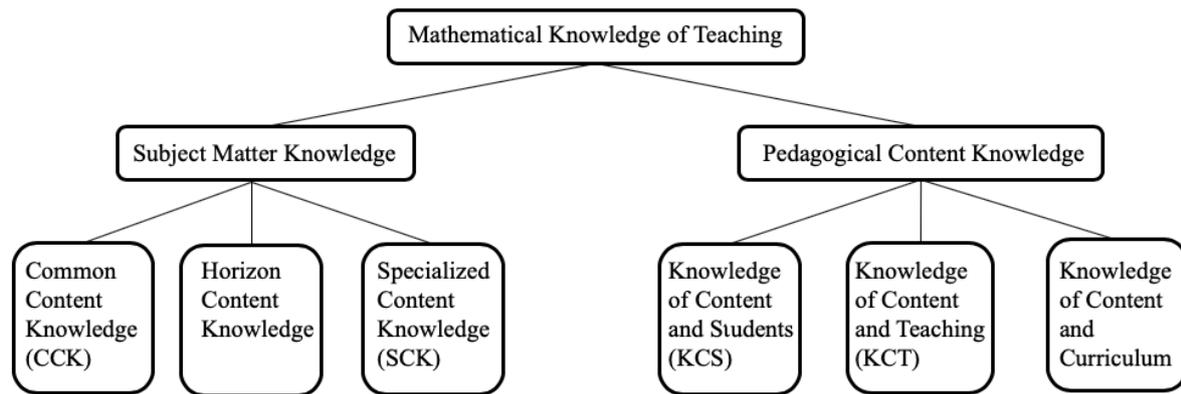


Figure 1. Domains of Mathematical Knowledge for Teaching (Ball et al., 2008)

educational strategies to improve the teaching of mathematics in a guided reinvention approach. In addition, an introduction to modelling and problem posing can be seen as a mean of recognizing the potential of mathematics as a critical tool to interpret and understand the reality, the community students live in, or society in general. Teaching students to interpret critically the world they live in and to understand its codes and messages should not be excluded, but become an important goal for education (Bonotto, 2013).

Teachers' Knowledge

Knowing a subject for teaching requires more than knowing its facts and concepts (Schulman, 1986). Based on the previous consideration, Schulman introduced a special domain of teachers' knowledge called *Pedagogical Content Knowledge* (PCK), that concerns "the most useful ways of representing and formulating the subject that make it comprehensible to students" (Schulman, 1986). In the same work, Schulman proposed some major categories of teacher knowledge grouped in the two main areas of *Subject Matter Knowledge* (SMK) and *Pedagogical Content Knowledge* (PCK). The most important point is that in Schulman's perspective, there is not a radical division between knowledge of content and knowledge of pedagogy. Instead, a combination between knowledge of content and pedagogy is at the core of the knowledge needed for teaching. In the domain of mathematics education, Ball et al. (2008), proposed a refinement of Schulman's categories, identifying and defining empirical subdomains of PCK (**Figure 1**).

Among the various components of **Figure 1**, there is a need to improve the domain *Knowledge of Content and Teaching* (KCT). KCT deals with the combination of knowing about mathematics and about teaching. In particular, teachers have to:

- know which contexts to start with and which examples to use to take students deeper into the content;
- evaluate the instructional advantages and disadvantages of representations used to teach a specific idea;
- identify what different methods and procedures could didactically afford better insights into a topic than others.

As a consequence, to make teachers able to choose a coherent teaching method, teachers' knowledge of some valuable methods for the teaching of mathematics should be fostered. In this sense, teachers' inclusion in their teaching of some aspects of modelling and problem posing is investigated in this paper. Indeed, investigating teachers' practice of modelling and problem posing could represent a crucial point in order to know if teachers need support and professional development on such educational strategies. In this way, i.e. increasing teachers' knowledge and abilities in specific

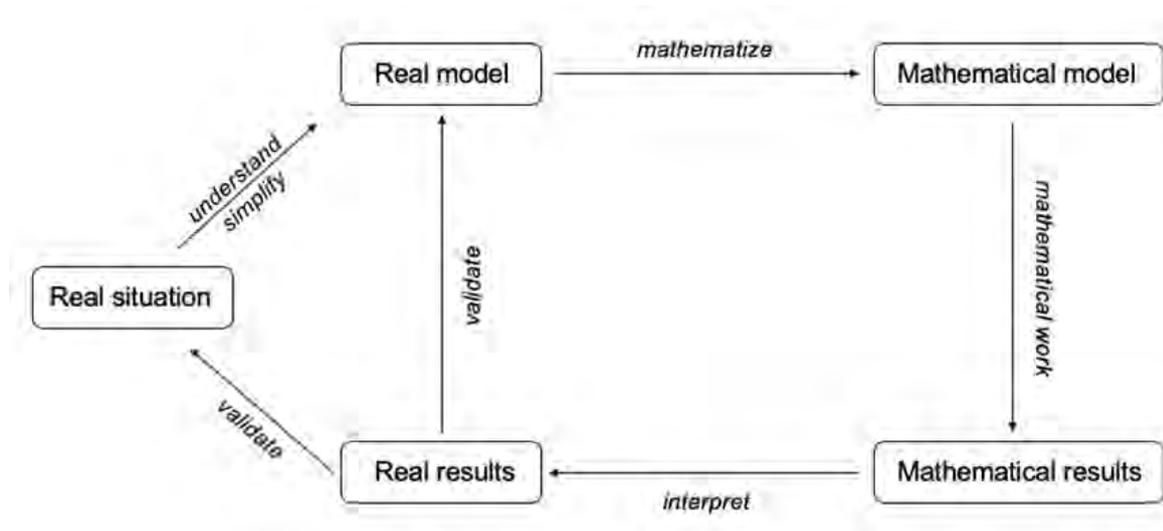


Figure 2. Modelling cycle from Kaiser and Stender (2013)

educational methodologies and strategies for the teaching of mathematics, teachers' KCT component of PCK should be improved.

Mathematical Modelling

How and why to include mathematical modelling in mathematics education has been the focus of many research studies until the half of the twentieth century. Indeed, researchers worldwide pointed at the educational value of engaging students with modelling activity (Shahbari & Daher, 2016), especially if mathematics education aims to promote responsible citizenship (Kaiser, 2017). Modelling is seen as a creative process of making sense of the real world to describe, control, optimize aspects of a situation, interpret results, and make modifications to the model if it is not adequate for the situation.

In the last decades empirical research on mathematical modelling has improved considerably, in terms of both quality and quantity. Several studies introduced different teaching approaches that have been analyzed by quantitative and/or qualitative methods (Schukajlow, Kaiser, & Stillman, 2018). Kaiser and Sriraman (2006) developed a framework for the description of the various approaches to mathematical modelling, according to their aims, types of mathematical modelling examples, epistemological background. Those approaches to mathematical modelling resulted in different characterization of the modelling process, emphasizing either the solution of the original problem or the development of mathematical concepts or ideas. Corresponding to the different perspectives on mathematical modelling, various modelling cycles developed with different emphases (see for an overview Borromeo Ferri (2006)). However, in all the approaches, the idealized process of mathematical modelling is described as a cyclic process to solve real problems using mathematics comprising different steps or phases (Kaiser, 2017).

In this paper, mathematical modelling refers to the modelling cycle given by Kaiser and Stender (2013), **Figure 2**.

Mathematical Problem Posing

An educational methodology that directly cooperates with modelling is problem posing (Bonotto, 2013; English, 1998; Xie & Masingila, 2017), representing an effective practice for real life problems (Arikan & Unal, 2014). Indeed, there is some consensus that the process of getting from a problem outside of mathematics to its mathematical formulation in mathematical modelling begins with the formulation of questions (Pollak, 2003), favouring abilities such as data collection, distinction between relevant and incoherent data, formulation of hypothesis and conjectures and advanced problem solving strategies

(Cai et al., 2015). These activities are typical of the modelling process and can help students to prepare to cope with natural situations they will have to face out of school (Bonotto, 2013). In agreement with English (1998), there is a need to capitalize on the informal activities situated in children's daily lives and get children in the habit of recognizing mathematical situations wherever they might be.

Different definitions of problem posing can be found in literature (e.g., Silver & Cai, 1996). In this study problem posing is considered as the process by which, on the basis of mathematical experience, students construct personal interpretations of concrete situations and formulate them as meaningful mathematical problems (Stoyanova & Ellerton, 1996).

These concrete situations considered as starting points for the practice of problem posing could be divided in three categories (Stoyanova & Ellerton, 1996):

- *free situations*, where students are asked to pose problems without restrictions;
- *semi-structured situations*, where students are provided with an open situation and are invited to explore its structure and to complete it using their personal previous mathematical experience;
- *structured situations*, where students pose problems reformulating or varying given problems.

In order to provide students with such meaningful contexts, a precious contribution is given by *cultural artefacts* (Bonotto, 2013). Thanks to its complexity and richness in mathematical meaning, an artefact lives in both the world of symbols and the real one, creating a sort of hybrid space that connects mathematics and everyday contexts. A re-mathematization process is thereby favoured, wherein students are invited to unpack from artefacts the mathematics that has been hidden in them, in contrast with the de-mathematization process in which the need to understand mathematics that becomes embodied in artefacts disappears (Gellert & Jablonka, 2007). As a consequence, movement from common use situations to mathematical structures and vice-versa is allowed, in agreement with the process of modelling.

Problem posing is a characterizing component of the mathematical activity at every level (English, 1998). For this reason, reports such as the National Council of Teachers of Mathematics (2000) have called for an increased emphasis on problem-posing activities in mathematics classrooms. Given the importance of problem posing activities in school, researchers started to investigate various aspects of problem posing processes, such as: the close relation between problem posing and problem solving (Peng, Cao, & Yu, 2020; Silver, 1994); problem posing integration in instruction; problem posing positive influence in students' ability in solving word problems; problem posing as a way to have a deeper understanding of mathematical concepts. Despite the interest in integrating problem posing into classroom practice, its knowledge remains relatively limited, representing a rich area for research (Cai et al., 2015).

RESEARCH DESIGN

Research Questions

This study is part of an ongoing project to improve mathematics in-service teachers' competencies in terms of modelling and problem posing. Teachers' education is crucial in the implementation of both modelling (Blum, 2015) and problem posing in mathematics classrooms (Osan & Pelczer, 2015). This paper started investigating teachers' practice in mathematical modelling and problem posing. Accordingly, the research questions are:

1. How do mathematics teachers include in their lessons some aspects of mathematical modelling?
2. How do mathematics teachers include in their lessons some aspects of problem posing?

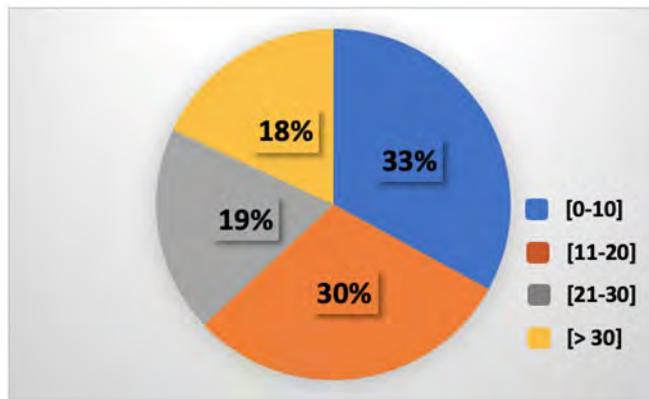


Figure 3. Distribution of the sample respect to years of teaching

In the specific, concerning the first research question, only two aspects of the modelling cycle were considered: (i) using real contexts as starting situations for mathematical activities and (ii) presenting (and working with) mathematical applications.

Methodology

In Schukajlow, Kaiser, and Stillman (2018) the authors asked for: monitoring the development of pedagogical content knowledge of in-service teachers; using a quantitative approach for the analysis of the research questions; developing a questionnaire for quantitative analysis. In order to support this perspective and in agreement with the research questions, a questionnaire for in-service mathematics teachers of primary and secondary school was developed (**Appendix**). The questionnaire was anonymous and made by closed and open questions and Likert-scales. In the first part of the questionnaire teachers were asked to give some personal details, concerning their higher degree of instruction, years of teaching and teaching level. The second part was dedicated to the investigation of teachers' educational practices. Regarding the first research question about mathematical modelling, only two aspects of the modelling process were considered, respectively the use of real contexts as starting situations for mathematics lessons and the work with mathematics applications. As a consequence, in the questionnaire two items in a five-Likert scale were inserted: the first dealing with the use of starting real situations for mathematics lessons and the second with mathematics applications. Regarding the second research question about problem posing, it was split in two questions: a closed question in which teachers were asked if they include or not problem posing in their school practice, and an open question where teachers that actually implement problem posing activities could report an example. In the questionnaire other questions were inserted, in order to analyze some relations between the use of specific educational strategies and/or tools and the implementation of problem posing activities, and to explore what teachers believed indispensable to improve their teaching. In the specific: one question in seven items of a five-Likert scale about the performance frequency of some educational strategies (e.g.: individual work, group work, laboratories, ...); one question in ten items of a five-Likert scale about the performance frequency of some educational tools (e.g.: textbooks, notes, software, artefacts, ...). The questionnaire ended with an open question in which teachers could express some suggestions they believed indispensable to improve the teaching of mathematics.

The sample comprised one-hundred and seven primary school and seventy-two secondary school teachers from the North of Italy. The method of sampling was randomly stratified (Cohen, Manion, & Morrison, 2011): the teachers' population was divided into the two groups of primary and secondary teachers, and in each of them teachers who participated to the questionnaire had been randomly chosen. The 66% of the sample had a master's degree and the 34% a high school diploma (in Italy before 2001 the master's degree was not mandatory to teach into Primary School). In **Figure 3**, the distribution of the sample respect to years of teaching is reported.

Table 1. Averages of the answers to the first research question

Item	Primary teachers	Secondary teachers	Total
First (<i>real contexts</i>)	4.3	3.7	4
Second (<i>applications</i>)	3.8	3.8	3.8
Total	4.1	3.8	3.9

In Italy, the Mathematics Curriculum is based on a National Curriculum (MIUR, 2012). In this curriculum some references to modelling and problem posing are present. Mathematical knowledge is seen as a precious tool to interpret everyday life. In particular, students should be engaged in activities of mathematization, that could help them in translate real situations in mathematical terms and work on it. Despite this accent on the mathematization process, no attention is given to all the modelling process, of which the mathematization constitute only a partial part. Concerning problem-posing, the authors of the curriculum describe mathematics as a context to pose meaningful problems, explore and recognize relations that occurs in nature and human creations. However, no compulsory training is offered for pre-service or in-service mathematics teachers in both the educational strategies of modelling and problem-posing, despite some sporadic and isolated example. Regarding this study, no one of the teachers that participated to the research had ever took part to a professional course on mathematical modelling or problem posing. The questionnaire was administrated directly by the author. The approach for the data analysis was mixed quantitative and qualitative. The answers to the open questions had been closed and grouped in categories and families. Univariate and bivariate analysis had been performed.

RESULTS AND DISCUSSION

In this section data from the questionnaires are reported and analysed. Recall that the first research question dealt with teachers' inclusion in their school practice of some aspects of mathematical modelling and the second one with teachers' knowledge of problem posing and its implementation. In addition, answers to the last question of the questionnaire, concerning teachers' suggestions on how to improve their teaching, will be presented and analysed in relation of the purpose of the study.

The results show that modelling is actually inserted by teachers in their school practice, in terms of real contexts as starting situations for mathematical activities and mathematical applications. However, teachers ask for more materials based on realistic situations in order to implement more meaningful modelling activities. Problem posing, instead, is quite absent from today's Italian school context.

To answer to the first research question, a question about the implementation of modelling activities was inserted in the questionnaire. Only two aspects of the modelling process were considered: (i) using real contexts as starting situations for mathematical activities and (ii) presenting and working with mathematical applications. This question was split in two items of a five-Likert scale. The first item dealt with real contexts as starting points for the introduction of a new mathematical topic, while the second item dealt with mathematical applications. In the specific, teachers were asked the frequency by which they implement such activities in their classrooms: 1 = never; 2 = rarely; 3 = sometimes; 4 = often; 5 = always. Considering both the first and the second items, the total average was 3.9 (**Table 1**). Successively, in order to examine the presence of differences between primary and secondary school, data were divided between primary and secondary school teachers. The findings indicate that primary school teachers use real contexts as starting situations for modelling activities more (4.3) than secondary school teachers (3.7).

The second research question was studied through one closed and one open question. The closed question asked teachers if they included or not problem posing during their classroom activities. Teachers' distributions highlights how almost a half of the sample did not implement problem posing in their lessons, indeed only the 39.6% of teachers reported that they actually include problem posing in their teaching, while the 49.2% of teachers instead did not include problem posing in their mathematics lessons (the 11.2% did not answer). To have a deeper understanding of teachers'

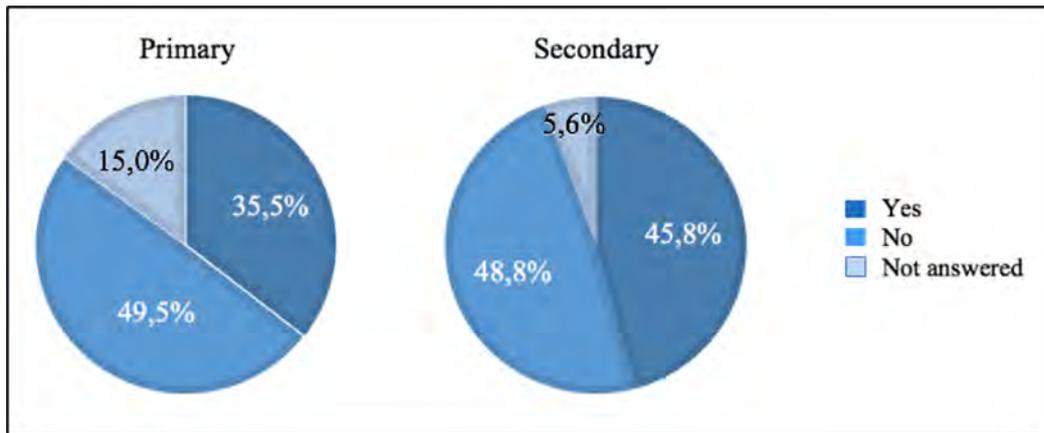


Figure 4. Teachers' distribution concerning their inclusion of problem posing in mathematics lessons divided between primary and secondary school teachers

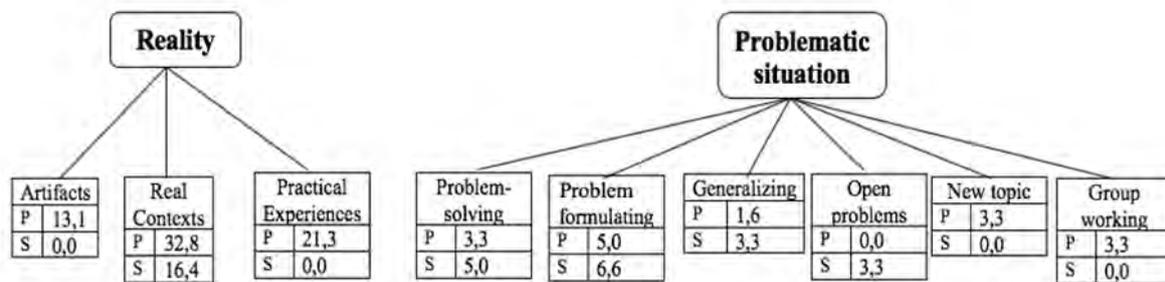


Figure 5. Families and categories of problem posing activities

implementation of problem posing activities respect to their school level, percentages respect to teachers' school level were calculated (Figure 4). In Figure 4 is clearly proved that problem posing is not common in school practice. Indeed, respectively the 49.5% of primary school teachers' and the 48.8% of secondary school teachers did not include problem posing activities in their mathematics teaching. The main difference between the two school levels is that in secondary school the 45.8% of teachers implement problem posing activities, while at primary school only the 35.5%.

To investigate teachers' problem posing practices, teachers who positively answered to the closed question, i.e., who included problem posing in their school practice, were asked to answer to an open question, in which they had to present one (or more) significant situation they used as starting point for problem posing activities. The analysis of teachers' answers consisted in an open coding (Cohen, Manion, & Morrison, 2011), translating the questions responses into specific categories. First, to each response was associated a code derived from the data and created by the author. Then, the generated codes were grouped into categories. For each category was calculated its distribution among teachers. From the coding of the answers to this open question, nine categories were identified and grouped in two families (Figure 5). The percentages of the distributions were divided in primary (P) and secondary (S) teachers. Each teacher could report more options, so the total is higher than 100%.

In Table 2 there are some examples of teachers' answers relative to the categories of Figure 5.

As for the first research question, for each category the distributions divided between primary and secondary school teachers were calculated. This analysis indicates that problem posing contexts expressed by teachers could be divided in two families: *reality* and *problematic-situations*, with distributions respectively of 83.6% and 34.7%. Note that the total is higher than 100% because teachers could express more than one situation. Moreover, the most suggested category, which is linked to the first family, was *real contexts* (49.2%). This fact remarks the importance in the choice of meaningful contexts for the implementation of problem posing activities. Such contexts are realistic and rich

Table 2. Examples of teachers' answers concerning significant situations used to implement problem posing activities

Category	Teachers' answers
Artifacts	Labels, figurines, namely artefacts
Real contexts	Real life situations such as comparing the ratio between tuna and oil in different packs
Practical experiences	Practical and concrete situations from students' experience, such as identifying the most convenient telephone company
Problem solving	Invent problems to be solved by themselves and their peers
Problem formulating	Extension of a problem
Generalizing	Generalizing the solution related to a specific problem
Open problems	Starting from open problems
New topic	Situations useful to introduce new topics
Group working	Working in groups

Table 3. Use of artefacts and implementation of problem-posing activities

Implementation of problem-posing activities		Frequency in the use of artefacts		
		Less than sometimes	Sometimes	More than sometimes
problem-posing activities	Yes	22.5	36.6	39.4
	No	46.5	31.8	19.3

contexts in the perspective of RME, and that are at the basis of the modelling process. As a consequence, contexts play a crucial role in bridging modelling and problem posing to promote students' reasoning, critical thinking and give meaning to their mathematical activity. Data split between primary and secondary school teachers allow to have an evidence in the fact that some starting situations are used only at primary school (artefacts, practical experiences, new topic, group working), while others only at the secondary school (open problems). Instead, teachers of every level should have the possibility to face with several different contexts or tools and learn to choose which is the most appropriate in relation of the classroom and the learning process.

To have a deeper insight in the relations between the implementation of problem posing and the use of artefacts, that in previous studies were proved to be able to foster a mindful approach towards a problem posing attitude (Bonotto, 2013), a bivariate analysis between the implementation of problem posing and the use of artefacts was performed. Recall that ten items concerning the use of some educational tools in a five-Likert scale (1 = never; 2 = rarely; 3 = sometimes; 4 = often; 5 = always) were inserted in the questionnaire. In the specific, teachers had to state the frequency they adopt the following educational tools during their lessons: textbooks, notes, interactive board, software, calculator, math games, audio and video supports, artefacts, library, others. Findings indicate a significant correlation between the use of artefacts and the implementation of problem posing activities ($p < 0.01$; $\phi^2 = 0.13$), as shown in **Table 3** (the total is less than 100% because the distributions of teachers who did not answer are not reported).

Moreover, to obtain some information on the relations between problem posing and problem solving, teachers were asked if they implement or not problem solving activities during their teaching. The double distribution is given in **Table 4** ($p < 0.001$; $\phi^2 = 0.44$). Considering the row percentages of **Table 4**, almost every teacher who implemented problem posing activities implemented also problem solving ones (95.8%). The vice-versa is not true, in fact if we consider teachers who implemented problem solving, the 49.3% adopts also problem posing and the 46.5% did not.

Table 4. Implementation of problem solving and problem posing activities

Implementation of problem solving activities					
Implementation of problem posing activities		Yes	No	Not answered	Total
	Yes	68	3	0	71
	No	64	24	0	88
	Not answered	6	14	6	20
	Total	138	41	6	179

Table 5. Categories for teaching improvement

Category	Distribution (%)
Laboratory	32.1
Mathematics and Reality	21.5
Teacher training	14.4
Students' motivation	19.1
Classroom equipment	12.0
Research in Education	10.7
More hours of Mathematics	12.0
Activities on Problem-Solving	13.1

Table 6. Examples of teachers' answers to improve their teaching

Category	Teachers' answers
Laboratory	<ul style="list-style-type: none"> Show the importance and use of mathematics with laboratorial activities. Materials based on laboratorial experiences.
Mathematics and Reality	<ul style="list-style-type: none"> Activities based on realistic situations and daily life experiences. Starting from concrete contexts and apply mathematics to real situations.
Teacher training	<ul style="list-style-type: none"> Incrementation of training courses for both pre-service and in-service teachers. Provide more practical professional development courses for teachers.
Students' motivation	<ul style="list-style-type: none"> Enhance students' involvement, stimulate discovery and playfulness. Students' motivation must be improved.
School equipment	<ul style="list-style-type: none"> Improve school facilities by equipping them with laboratories or teaching tools. Adequate classrooms (tools, software, etc.) available in all school levels.
Research in Education	<ul style="list-style-type: none"> More cooperation between schools and universities. Teachers' need valid materials based on valuable teaching strategies.
More hours of Mathematics	<ul style="list-style-type: none"> More hours in the week schedule for mathematics are needed to have a deeper insight in various topics. Renovate programs or increase the time to have more time to deepen the topics.
Activities on Problem-Solving	<ul style="list-style-type: none"> Pay more attention in the solution process of a problem and less time in calculations. Work regularly on problem-solving activities.

In conclusion, the last question of the questionnaire is reported, in which teachers were asked to express some suggestions they believed indispensable to improve their teaching of mathematics: "in conclusion, I ask you one (or more) suggestion you believe indispensable to improve your teaching of mathematics". The 71.0% of the sample answered to this question. From the coding of teachers' answers, four families (*educational strategies*, 103.4%; *math topics*, 33.3%; *school organization*, 26.1%; *teacher training*, 26.0%) and thirty-one categories were identified. Note that, since each teacher could express more options, the total percentage is higher than 100%. In **Table 5** data whose distribution was higher than 10% are reported.

In **Table 6** there are some examples of teachers' answers relative to the categories of **Table 5** (only the categories with distributions higher than 10% are reported).

From the analysis of teachers' answers, it is clear that teachers need more opportunities to be engaged in modelling activities. Indeed, they ask more effective experiences and practical materials based on realistic and laboratorial activities. Consequently, there is the necessity of an improvement in teachers' trainings, offering them occasions to be involved in modelling activities. It is remarked that also teachers recognized the importance in the choice of concrete and stimulating contexts for students. In addition to the examples reported in **Table 6**, other teachers' answers were directly linked to problem posing:

- *Learn to problematize from concrete situations.*
- *Attitude to pose problems and observe.*

The request of paying more attention in problem posing and problem solving situations, by a regular implementation of these educational strategies, proves that students' reasoning must be increased. Activities based on modelling and problem posing, starting from meaningful contexts given for example by suitable artefacts, should represent a valuable occasion to achieve such results.

CONCLUSIONS

This paper discussed the results of a questionnaire concerning teachers' inclusion in their lessons of some aspects of mathematical modelling and problem posing are discussed.

Concerning modelling, the results showed that teachers regularly included some aspects of the modelling process in their classroom activities, in terms of using real contexts as starting situations for mathematics lessons and showing real applications of mathematics. Despite this disposition, teachers expressed a need in both materials and preparation to implement activities based on realistic contexts. This need is in line with Blum (2015), in which it is underlined the high demanding features of implementing modelling at school, and that teachers' professional development in modelling competencies (Borromeo Ferri, 2018) is indispensable. Two directions for future research seem to be important: (i) improving teachers' professional development courses, offering teachers occasions to face with modelling activities based on rich and realistic contexts, and (ii) developing prototypes of practices and textbooks based on realistic problematic contexts available for teachers of every school level. In this way teachers should have at their disposal prototypes of modelling activities that can be adapted and implemented in their classrooms.

The analysis of the second research question indicates that problem posing is not regularly implemented at school. In fact, less than a half of the participants (39.6%) adopted it during its school practice. To overcome this lack, problem posing should become an integral part of pre-service and in-service teacher training courses, in order to give teachers opportunities to increase their knowledge, before, and their practice on problem posing. Such improvement in teachers' knowledge could help teachers to recognise intersection points between different methodologies and strategies and to adopt coherent teaching methods. Also, in our study both the relations between problem posing and modelling and problem posing and problem solving had been confirmed. Indeed, looking at the categories of **Figure 5**, the most frequent is *real contexts*, which highlights the cooperation between modelling and problem posing. This cooperation is natural in the choice of meaningful contexts for the implementation of both these educational strategies in order to enhance students' reasoning and critical thinking. A possible choice for that contexts is given by artefacts, whose precious contribution in problem posing activities emerged from the bivariate analysis, in agreement with Bonotto (2013). Moreover, the positive and strong relation between problem posing and problem solving was confirmed, in line with previous research (Bonotto, 2013; Kilpatrick, 1987; Peng, Cao, & Yu, 2020; Silver, 1994; Silver et al., 1996).

In conclusion, teachers' opinions on how to improve their teaching of mathematics were analysed. The most suggested family dealt with *educational strategies*. In the specific, linked with this family there were some categories linked to modelling and problem posing: *laboratory, math&reality, problem solving, group work, practical experiences*. The most suggested category was *laboratory*. This means that teachers realize that a change in the way of doing mathematics is necessary. However, standard mathematics and "lab-mathematics" might not be distinguished by teachers, but activities based on modelling should become integrated in the daily mathematics activities. Nevertheless, teachers pointed some difficulties and limitations they encounter, such as limited time available during class periods, in line with Lee, Capraro, and Capraro (2018). Although also institutional changes are required, it is believed that researcher could help teachers to overcome, or at least manage, such difficulties increasing teachers' knowledge and practice in suitable educational strategies for the teaching of mathematics. Therefore, an improvement

in teachers' KCT could support not only in improving their knowledge and competencies in specific educational strategies, but also in the choice of the most appropriate methodologies, strategies, contexts or tools in relation to the mathematical topic, the classroom and the learning process.

Limits and Future Work

In the present study only some aspects of modelling were considered, the ones of using real contexts as starting situations for mathematical activities and working with mathematical applications. As a consequence, a deeper understanding of teachers' effective practice of the entire modelling cycle and about their knowledge of other aspects of modelling did not emerge. Moreover, findings are based only on what teachers self-reported. As a consequence, we only have a first overview about this issue, that was actually the aim of this study. For the future a deeper investigation in teachers' practices linked to both modelling and problem posing will be performed through a series of interviews and classroom observations.

To conclude, from the findings presented in this research, in order to enhance teachers' competencies in mathematical modelling and knowledge of problem posing and to provide them with materials available in their school practice, an improvement in pre-service teacher training and in-service teachers professional development are needed, in particular: (i) changing the type of activities with more realistic problem situations; (ii) improving teachers' KCT through the knowledge of some teaching strategies, such as problem posing, that could be adequately chose by teachers for the teaching of specific mathematical topics; (iii) connecting mathematics and classroom teaching creating prototypes of practices based on modelling and problem posing available for teachers. In this direction, a fundamental factor is given by teachers' motivation for professional development (Blume et al., 2010; Porsh, & Whannel, 2019). Consequently, in order to improve teachers' professional development, further research is needed also in exploring potential motives of mathematics teachers to take part in a professional development course (Lunne, Schnell, & Biehler, 2020).

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APPENDIX

Questionnaire: Educational Practices in the Teaching of Mathematics

The following questionnaire is part of a project at the University of Padova, Italy. The questionnaire is made by three sections and it deals with the educational practices of Mathematics teachers. 20 minutes are needed to complete it.

We remind you that the questionnaire is anonymous, and the data collected will be used only for this research in respect of the privacy.

Thank you for your collaboration.

Anagrafic section

1. Year of birth:
2. Nationality:
3. Gender: M F I don't want to say
4. Higher Graduation:
5. School level in which you are teaching:
 - Primary
 - Secondary
6. How many years are you been teaching in this level?
7. In which town are you teaching?

Educational practices

8. During your teaching activity, you adopt the following strategies:

	<i>Never</i>	<i>Rarely</i>	<i>Sometimes</i>	<i>Often</i>	<i>Always</i>
Lectures	1	2	3	4	5
Individual work	1	2	3	4	5
Group work	1	2	3	4	5
Guided lessons	1	2	3	4	5
Support activity	1	2	3	4	5
Laboratories	1	2	3	4	5
Other.....	1	2	3	4	5

9. During your teaching activity, you adopt the following tools:

	<i>Never</i>	<i>Rarely</i>	<i>Sometimes</i>	<i>Often</i>	<i>Always</i>
Textbooks	1	2	3	4	5
Notes	1	2	3	4	5
Interactive board	1	2	3	4	5
Software	1	2	3	4	5
Calculator	1	2	3	4	5
Math games	1	2	3	4	5
Audio and video tools	1	2	3	4	5
Artefacts	1	2	3	4	5
Library	1	2	3	4	5
Other.....	1	2	3	4	5

10. Based on your teaching experience, you perform the following activities:

	<i>Never</i>	<i>Rarely</i>	<i>Sometimes</i>	<i>Often</i>	<i>Always</i>
I use starting real contexts for mathematical lessons	1	2	3	4	5
I show and work with some applications of mathematics	1	2	3	4	5

11. Do you include *problem-solving* activities during your teaching? Yes No

If yes, describe a significant example.

12. Do you implement *problem-posing* activities during your teaching? Yes No

If yes, describe a significant example.

Teaching difficulties

13. Based on your experience, express the level of difficulty you have found teaching the following topics:

	<i>No one</i>	<i>Just a few</i>	<i>Enough</i>	<i>Many</i>	<i>A lot</i>
Arithmetic and Algebra	1	2	3	4	5
Euclidean geometry	1	2	3	4	5
Analitic geometry	1	2	3	4	5
Functions	1	2	3	4	5
Probability and Statistics	1	2	3	4	5
Logic	1	2	3	4	5

14. In conclusion I ask you one (or more) suggestions you believe indispensable to improve your teaching of mathematics.