Understanding Fractions Begins with Literacy

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Abstract

Transitioning from teaching whole number operations to teaching fraction operations can prove difficult, even for the seasoned mathematics teacher. However, with effective literacy practices, teachers can seamlessly shift their students to learning the rules for fractions. The implications can be a mastery of mathematical language used to describe fraction operations and a deeper understanding of the concepts involved with solving fraction equations. The purposes of this paper were to emphasize the importance of learning math vocabulary and offer ideas for literacy activities that integrate fraction concepts with reading and writing.

Keywords: *fractions, write-to-learn, literacy, math*

Literacy strategies can feel out of place in a math classroom because they seem irrelevant to teaching mathematical operations (Brozo & Crain, 2018), and mathematics teachers may initially resist the integration of literacy instruction into their content area (Hall, 2005; Siebert & Draper, 2008). However, a disciplinary literacy approach, in which the literacy skills used to engage in a specific content area are developed, students can learn to unpack the complex meanings in math problems (Kester Phillips, Bardsley, Bach, & Gibb-Brown, 2009). Fractions are a particularly difficult concept for students to master, but infusing disciplinary literacy into math instruction can help students develop the skills necessary to work through fraction problems.

Teaching fractions is inherently challenging because the operations are counterintuitive to what students already know about whole numbers (Ni & Zhou, 2005). Without a strong foundation in what fractions are and how they operate, as many as 50% of students will continue to struggle with fractions throughout middle and high school (NMAP, 2008). In Misquitta's (2011) meta-analysis over interventions for re-teaching fractions, he found methods such as graduated sequence, direct instruction, and strategy instruction are highly effective tools for clarifying misconceptions about fractions. He also highlights the National Mathematics Advisory Panel's (NMAP) emphasis on developing conceptual knowledge of fractions and the need for educators to develop that conceptual knowledge at the same time they are forming students' knowledge of

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fraction operations. In this article, we outline strategies for helping students advance their conceptual knowledge of fractions by utilizing two sound literacy practices, vocabulary development (Bay-Williams & Livers, 2009) and write-to-learn strategies (Burns, 2004).

Vocabulary Strategy

There are numerous new terms and concepts for students to grasp as they learn about fractions and their operations. Although these unfamiliar terms are frequently used in context during classroom instruction, and some teachers may even provide a brief definition, students struggle to comprehend the strange and unique language. It is not uncommon for students to regurgitate the definition of a few terms, but lack a true understanding of what those terms represent. For example, a student may know that the numerator is the number above the fraction bar but not understand that it identifies the number of pieces out of the whole or group. Without a deep understanding of the vocabulary, students have difficulty learning about fractions because of the strangeness of the terms. This is especially true when the unusual terms are taught using just a definition and without being integrated into the context of fractions. When students have not internalized the language of fractions and are expected to make computations using fractions they resort to simply memorizing a series of steps or arbitrarily guessing at what they are supposed to do.

Vocabulary Strategy for the Unit Fraction

A rich academic vocabulary helps students assimilate new terms into their existing knowledge base and empowers students to effectively communicate both inside and outside of the classroom. Learners who are provided with a vocabulary-rich setting experience greater learning in the classroom and increased connections to new contexts outside of the classroom (Marzano & Pickering, 2005). It is critical for students to fluently use appropriate academic vocabulary in their daily communication. To develop students' mathematical vocabulary teachers need to provide intentional and explicit instruction. This instruction goes beyond simply providing students with textbook definitions; it saturates students' experiences with frequent repetition in a variety of contexts with the intent for students to learn, retain, and rehearse the language in everyday life.

In English Language Arts, a common phrase used when teachers want students to convey an image is, "paint a picture." Numerator and denominator do not provide the student with any sort of visual. They are completely foreign words with no connection to concepts previously taught when learning number recognition, counting, and whole number operations. Unit Fraction, however, paints a picture for the math student. Simply by using these two words, the student can now see we are going to be dealing with a part of a whole, we're going to fracture a unit, or split a whole into parts. Our choice of words can make a difference in a student's understanding of abstract concepts. We recommend introducing fractions by first teaching the students the definitions of the key terms whole, unit, equal, part, and unit fraction. Conducting activities such as a pictorial word match (see Figure 1), where the students match the word or definition to a picture that represents that word, can help them learn how to use the terms in context before they start performing complex operations of fractions.

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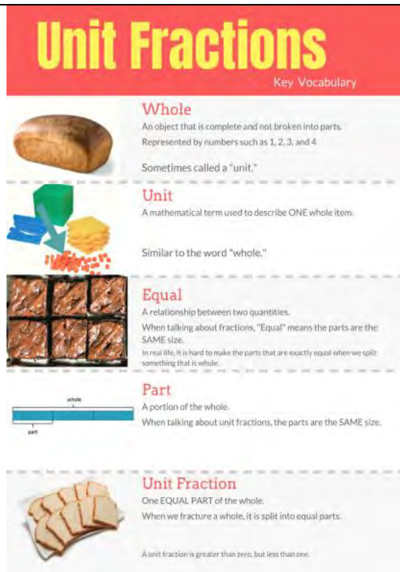

Figure 1. Unit Fractions Key Vocabulary.

Anticipating Misconceptions

In addition to strange vocabulary, students enter their math education with many misconceptions about what fractions mean. These misconceptions begin long before a child is introduced to the concept in school. Take, for example, a parent who buys a single Kit Kat Bar for her two children. She instructs the older child to split the candy bar in half so both children can have their equal share. The older child breaks the Kit Kat bar between the 3rd and 4th stick and claims the larger portion for himself while passing the smaller portion to his younger brother. Thus, one-half takes on the erroneous meaning of two parts, regardless of the comparable size. This reinforces the faulty belief that parts making up the whole unit do not have to be equal sizes. This flawed idea is further magnified by comments such as, "the bigger half" or the "smaller half" when by the definition of a fraction, the pieces must be the same size. In respect to fractions, because the whole unit is fractured into equal size pieces resulting in parts, there cannot be a bigger or smaller section. But life experiences teach that when a whole unit is separated into parts it often results in different size pieces. For example, seldom is a pizza cut into precisely equal size slices. This reinforces a spurious understanding of fractions and results in the idea that fractions are identified simply by the number of pieces it takes to make the whole unit, not considering that the parts must be equivalent.

Another misconception about fractions is caused by the inability to transfer whole number concepts to fractions. When a child enters Kindergarten and learns number recognition, they then learn how to count by whole numbers. Each item they count represents a single number and the higher the student counts, the larger the quantity. As a student begins to learn whole number operations, the rules are relatively straightforward and can easily be demonstrated with visuals and manipulatives. If I have three bears and add two bears to the group, I will have a total of five bears (Figure 2). Teachers then scaffold the student to subtract, multiply, and divide, building on prior knowledge as each new operation is taught.

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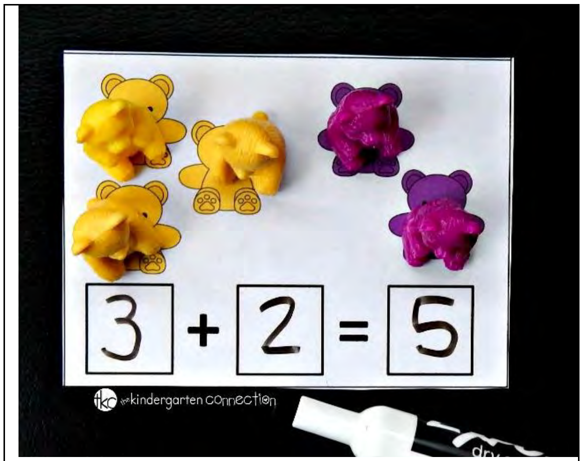

Figure 2. Whole Number Manipulatives.

Tapping into a students' prior knowledge is an efficient method for scaffolding and accelerating math concepts. However, when educators begin teaching fractions, they have to disconnect students from their prior knowledge about operations and whole numbers because fractions have their own set of rules that are completely separate from whole number operations (See Table 1). When using fractions all four basic operations are illogical from whole numbers.

Students cannot apply prior knowledge when transferring from whole number operations to fraction operations. Take, for instance, the operation of addition. When adding whole numbers, the sum is a larger quantity than either of the addends (parts). When you compute 2x3, it results in a larger product than either of the factors. Multiplication makes numbers larger, faster than addition. When multiplying a fraction, such as $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$ you get a product smaller than either of the factors. Based on previous understanding of multiplication, you would assume multiplying fractions produces a number larger than either of the factors. However, because you are taking a fraction of a fraction the product is a smaller number. This results in the operations on fractions being contradictory. In other words, fractions operate counter to prior knowledge and instincts. To be understood, fractions must be taught conceptually using hands-on physical manipulatives that are familiar to children. If math manipulatives, such as fraction tiles, fraction circles, and pattern blocks are being used, teachers must explicitly connect the concept to the students' environment.

Operation	Example	Concept	Confusion
Add Whole Number	2+2 =4	Part plus part=whole	
Add Fraction	$\frac{1}{2} + \frac{1}{4} = \frac{1}{8}$	Each part needs to have same size pieces in relation to the whole (common denominator)	Addition operation doesn't transfer Fraction pieces must first be standardized
Subtract Whole Numbers	7-3=4	Whole, take away a part = part	
Subtract Fractions	$\frac{1}{2} - \frac{1}{8}$	Each part needs to have same size pieces in relation to the whole (common denominator)	Subtraction operation doesn't transfer Fraction pieces must first be standardized
Multiply	3 x 4=12	Three groups of four	

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Multiply Fractions	⅔ x ⅗	Part of a part, or fraction of a fraction	Part of a part results in a smaller part
Divide	12/4 = 3	Fair share- fair share 12 units among 4 entities results in each entity receiving 3 units Or grouping-how many groups of 4 units are in 12 units?	
Divide Fractions	⅞ / ½	Fair share doesn't work here, however, the grouping concept does and the question is how many ½'s are in ⅞'s. To compute this, you must multiply the first fraction by the reciprocal of the second fraction	When you divide fractions the quotient is larger than both fractions and to compute you must change the operation from division to multiplication and invert the second fraction

Table 1. Confusing concepts when moving from whole number operations to fraction operations

In order to transition students from fluently moving through whole number operations to learning the concepts behind fraction operations, we propose the teacher should 1) introduce the concept of a unit fraction using vocabulary strategies; 2) provide multiple opportunities for students to work with authentic manipulatives (items that are found in everyday life, rather than created solely for classroom use); and 3) use write-to-learn strategies to help solidify the concept of unit fractions.

Introducing Manipulatives

The National Council of Teachers of Mathematics (NCTM) has touted the benefits and encouraged the use of math manipulatives to increase students' conceptual understanding of mathematics. For the purpose of this article, the researchers used Hynes' definition of manipulatives as "concrete models that incorporate mathematical concepts, appeal to several senses and can be touched and moved around by students" (Hynes, 1986, p. 11).

Learners benefit by using hands-on math manipulatives as demonstrated by increased achievement (Moch, 2002; Boggan, Harper & Whitmire, 2010; Perry & Howard, 1997, p. 27). Math manipulatives increase student learning and enhance the feeling tone of the learning environment by reducing math anxiety (Cain-Caston, 1996; Heuser, 2000). Karp and Voltz (2000, 212) encourage teachers to reflect on their instruction using manipulatives to increase student achievement.

Magnetic Apple Fractions

Once students are familiar with the key vocabulary words for unit fractions, use manipulatives to model how those words interact with each other. Magnetic Apple Fractions from Learning Resources (see Figure 3) allow students to visualize how the whole can be split into parts in three different ways, while the teacher explains how to use the words to describe what is happening to the apples (see Figure 4).

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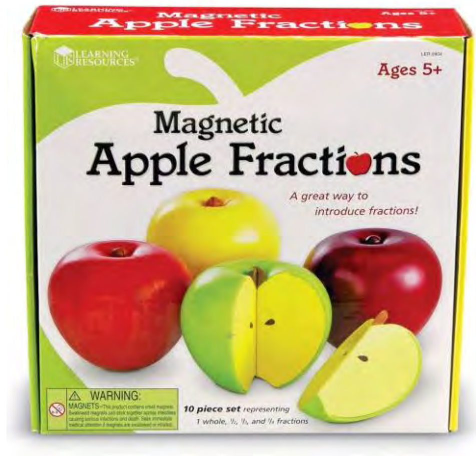

Figure 3. Magnetic Apple Fractions (available through Learning Resources)

Teacher: *(Holds up whole apple)* What term would I use to describe this apple?

Student: A whole.

Teacher: That's correct. Why would you say it's a whole and not a part?

Student: Because it's one. It's not split up.

Teacher: *(Holds up ½ apple and pulls apart the pieces.)* If this whole unit is cut into two equal size parts, how many parts does it take to make the whole?

Students: Two?

Teacher: Yes. It takes two equal size parts to make the whole unit. *(Puts the ½ size pieces back together to make the apple whole again).* If it takes two equal size parts to make the whole unit then the whole unit consists of how many parts? *(Takes the ½ size pieces apart again.)*

Students: Two.

Teacher: That's right. We would write 1 over 2 (½) to numerically represent one of the two equal size pieces. Which of our vocabulary words would we call this numerical representation? This is a unit fraction.

Let's talk about what each component of the number represents. The number on the bottom is how many EQUAL parts the whole was split into. The one on top represents how many parts I have. I also know...

- The unit fraction is ½.
- The whole is fractured into two equal size pieces.
- Two pieces make the whole unit.

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(*Repeat the same dialogue with the ⅓ and ¼ apples, gradually releasing the responsibility of describing the apple and its parts to the students.)*

Figure 4. Teacher – Student Conversation to Explain Fractions

Write-to-Learn Strategies

Write-to-learn strategies can facilitate the acquisition of students' mathematical language. As stated by the National Council of Teachers of Mathematics' Curriculum and Evaluation Standards for School Mathematics (1989) educators are encouraged to incorporate writing opportunities in the math class. By doing so, students can benefit in a variety of ways including growth in achievement (Kostos and Shin, 2010), increased conceptual comprehension (Wood, 1992), decreased math anxiety (Stix, 1994), better attitudes about math (Jurdal and Abu Zein, 1998), increased participation, the ability to communicate about mathematics (Baxter, Woodward and Olson, 2005), and improved metacognition through reflection (Pugalee, 2001). By combining vocabulary strategies with write-to-learn methodology, educators can help primary students build a strong foundation for fraction concepts and operations.

Sentence Stems

To expand on how to use fraction terms teachers can allow students to independently practice with modeling compound and sentence stems. By writing out the following sentence stems for students on a dry erase board or slips of paper, the teacher can model how to use vocabulary words and manipulatives together (see Figure 5).

The teacher can then shift the responsibility of describing unit fractions to the students by providing each student with a container of modeling compound and facilitating a discussion about sharing the modeling compound between two people. By specifying the need to fair share, teachers can reinforce the conditional statements discussed above and have students determine the unit fraction as ½. Having the students repeat the experience using four people and then eight people to fair share the modeling compound allows the students to practice filling in the blanks in the sentence stem (see Figure 5). By elaborating on student discoveries and using the term 'unit fraction' repeatedly, teachers can help solidify the concept of unit fractions before transitioning into fraction operations. Provide opportunities for the children to explain how they fair-shared, and encourage them to use their new vocabulary word 'unit fraction'.

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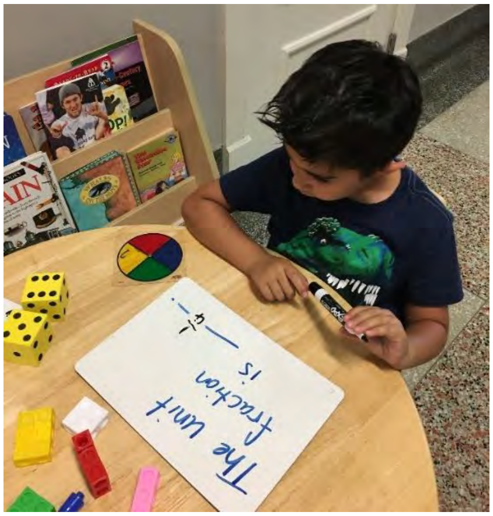

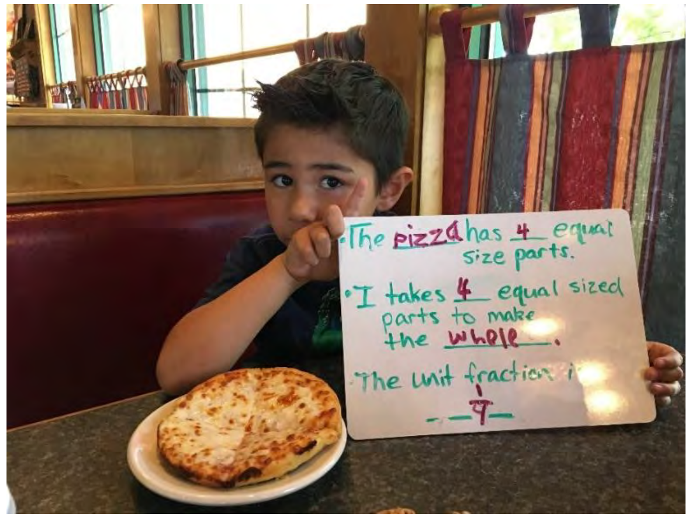

Figure 5. Sentence Stems for Unit Fractions

Quickwrites

Once students have mastered the vocabulary using sentence stems and manipulatives, they can move on to other write-to-learn strategies, such as answering quickwrites. Green, Smith, and Brown (2007) define a quickwrite as, "...a brief written response to a question or probe requiring students to explain a principle or phenomenon. Quickwrites can eliminate the frustration that frequently accompanies traditional testing methods by providing students more flexibility of response" (p. 39). By having students answer questions, such as, "How would you describe a whole?" and "What does fair share mean?" educators can discover how students are processing the concepts and vocabulary. Younger writers who are still in the early stages of spelling development can even use pictures with labels to explain their understanding of mathematical concepts (Saundry & Nicole, 2006). Students' mathematical understanding develops when they are provided with opportunities to use their mathematical language in a variety of settings, both academic and authentic.

Conclusion

Learning a new concept can be difficult when new content is in direct opposition to what we already know. In the case of fractions, teaching students to understand the vocabulary associated with fraction operations and teaching them to use that vocabulary in a variety of contexts can help educators scaffold instruction for students, so the "hows" and "whys" of performing fraction operations becomes clear. Providing a literacy-rich experience into math education by incorporating reading and writing strategies during instruction helps students cognitively process difficult math concepts and develop a deeper understanding. With the integration of literacy strategies into mathematics, teachers can improve learning outcomes regarding math standards, while at the same time practicing and improving literacy skills.

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