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The Didactic Notion of “Mathematical Activity” in Japanese Teachers’ Professional Scholarship: A Case Study of an Open Lesson

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The Didactic Notion of “Mathematical Activity” in Japanese Teachers’ Professional Scholarship: A Case Study of an Open Lesson

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Abstract

This paper investigates how Japanese mathematics teachers produce and share didactic knowledge together. It is a case study of a post-lesson reflection meeting so-called *open lesson*. The crucial idea of this study is the dialectic between the *specific* and *generic* level of *foci* of the participants’ reflections about the observed teaching practice; namely, about applied teacher’s specific didactic technique for achieving a specific mathematical goal, and more general pedagogical issues such as realisation of the objectives of mathematics education. This dialectic is mediated by the *meso*-level notion of *mathematical activity*, described in the guidelines for Japanese national curriculum. The application of the *scale of levels of didactic co-determination*, provided by the *anthropological theory of the didactic* into the analysis shows in what way the dialectic interplay between the teachers’ comments with focus of the specific and generic levels influences the development and establishment of the Japanese teachers’ shared professional scholarship.

Keywords: Anthropological theory of the didactic, didactic praxeology, mathematical activity, paradidactic infrastructure, teacher knowledge

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La Noción Didáctica de “Actividad Matemática” en la Beca Profesional de Profesores Japoneses: Un Estudio de caso de una Lección Abierta

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Resumen

Este trabajo investiga cómo los profesores de matemáticas japoneses producen y comparten juntos el conocimiento didáctico. Es un estudio de caso de una reunión de reflexión posterior a la lección llamada lección abierta. La idea crucial de este estudio es la dialéctica entre el nivel específico y el nivel genérico de los focos de atención de las reflexiones de los participantes sobre la práctica docente observada; a saber, sobre la técnica didáctica específica del profesor aplicada para lograr un objetivo matemático específico, y cuestiones pedagógicas más generales como la realización de los objetivos de la enseñanza de las matemáticas. Esta dialéctica está mediada por la noción de nivel meso de la actividad matemática, descrita en las directrices del plan de estudios nacional japonés. La aplicación de la escala de niveles de codeterminación didáctica, proporcionada por la teoría antropológica de la didáctica en el análisis muestra de qué manera la interacción dialéctica entre los comentarios de los profesores con el enfoque de los niveles específicos y genéricos influye en el desarrollo y establecimiento de la beca profesional compartida de los profesores japoneses.

Palabras clave: Teoría antropológica de la didáctica, praxeología didáctica, actividad matemática, infraestructura paradidáctica, conocimiento del profesorado

Using Etzioni (1969)'s expression, the teaching profession is treated as a *semi-profession* in many countries. It means that teaching is not considered as a real profession, at the same level as the professions of medicine or law. One of the main reasons is the lack of explicit and justified knowledge that is clearly shared in the community of teachers, as a support to practice their profession (Chevallard, 2006). To explore the components of the knowledge required to become a “good” mathematics teacher, much international research has focused on the cultural scripts of the community of Japanese mathematics teachers (beginning with Stigler & Hiebert, 1999). Some studies focus on Japanese teachers' widely shared theory about teaching practice (e.g. Jacobs & Morita, 2002) to pursue “effective teaching” and describes the characteristic of their practice (Corey, Peterson, Lewis, & Bukarau, 2010). A considerable number of studies concern Japanese *lesson study* as one of their crucial methods for sharing and developing teacher knowledge (Stigler & Hiebert, 1999; Lewis, 2002; Winsløw, 2011; Isoda, 2015). Several recent special issues and books focus on lesson study implemented outside of Japan (Groves & Doig, 2014; Quaresma, Winsløw, Clivaz, da Ponte, Ní Shúilleabháin & Takahashi, 2018). A number of these studies emphasize the value of teachers' cooperative lesson planning and feedback receiving during the post-lesson reflection meeting.

Miyakawa and Winsløw (2013) analyze the conditions, which support the construction and distribution of knowledge in relation to Japanese mathematics teachers' didactic practice. They present a case study of an open lesson and the following post-lesson discussion and conclude that these activities enable Japanese teachers to develop and share theoretical knowledge about their teaching practice. Rasmussen (2015) investigates what impact the post-lesson reflection give to prospective teachers during the implementing of lesson study in a teacher education program in Denmark. He analyses the comments concerning the didactic practice observed by the participants during the discussions, and concludes that different institutional preferences (prospective teachers, teachers in service, teacher educators) in the post-lesson discussions are a source of new insight for the participants.

With these studies of post lesson discussions as a starting point, my study aims to investigate one specific and important case of theoretical knowledge, namely the notion of *mathematical activity*, as it appears in a reflection meeting following an open lesson. The importance of the mathematical activities is strongly emphasized in the guidelines for the Japanese national curriculum both from 2008 and from 2018, within the sections bearing on “objectives and contents”. As I will explain further in the following sections, this notion is strongly linked to teachers’ didactic techniques to organize students’ autonomous learning practice in relation to specific mathematical tasks.

Theoretical Framework

The analyses of this paper rely on several tools from the Anthropological Theory of the Didactic (Chevallard, 1999), hereafter ATD). The first tool is *praxeological modeling* (ibid.), which can in principle be used on human activity of any kind, by dissecting it in terms of *praxeologies*. A *praxeology* consists of two units; the practical block (*praxis*) and the theoretical block (*logos*). The *praxis* consists of *type of tasks* and *techniques*, which can solve the task. The *logos* “discourse about the *praxis*”, contains two levels: *technology*, which is explanatory and unifying discourse about the techniques, and *theory* which provides a unifying and justifying discourse on the technology.

A *mathematical praxeology* (MP) is evidently one in which the tasks are somehow mathematical (more precisely, are considered mathematical by the institution in which they occur). A *didactic praxeology* (DP) is one in which the tasks concern the teaching of one or more MP. It is carried out by teachers and can have more or less shared logos. These two kinds of praxeology are *co-determined*; it means that a MP developed in the classroom depends on the teacher’s DP, and the construction of the DP is depending on how the MP is described officially (in guidelines, curricula, etc). We notice that didactic theory may be both private and shared by teachers, and it is often not questioned by the community of the teachers. The didactic theory includes “a certain conception of mathematics, the rational of teaching it and the mission of schools in society” (Bosch & Gascón, 2014, p. 79).

The extent to which the praxeologies is structured depends on *paradidactic infrastructure* (Winslow, 2011), which is the second tool from

ATD applied here. This notion is related to the didactic infrastructure (Chevallard, 2009), which describes the totality of conditions for the teachers' work in the classroom, that is, the *didactic praxis*. Paradidactic infrastructure is, similarly, the totality of conditions for teachers' work *outside* the classroom; this includes their efforts to share and develop didactic knowledge, which could improve their teaching practice. Teachers' collective activities like lesson study, open lessons and practice research are called *paradidactic practices* (Miyakawa & Winsløw, 2019) – they are all essential elements in the Japanese paradidactic infrastructure. Miyakawa and Winsløw (2013) further state that the Japanese paradidactic infrastructure supports teachers' development of the knowledge about the co-determination of DP and MP.

The third tool is the notion of *scale of levels of didactic co-determination* (Bosch & Gascón, 2006). This model was originally created to help analysing how DP and MP are shaped and sometimes defigured by condition and constraints at different institutional levels (from curricular specifications relating to a mathematical technique, to generic features of the school, society and so on). In this paper, I use the scale to situate the teachers' focus during paradidactic practice of post-lesson reflection, called *paradidactic foci*, which could help to identify “unintentional regularities” (ibid., p.54) of DP in the lesson. The levels of paradidactic foci are defined following to the co-determination model as below:

- civilization (e.g., Oriental culture and ethos)
- society (e.g., Japanese national traits)
- school (e.g., Japanese lower secondary school, with its policies, goals etc.)
- pedagogy (e.g., generic teaching principles)
- discipline (here, mathematics)
- domain (e.g. algebra, geometry...)
- sector (e.g. equations, similarity...)
- theme (e.g., triangles, root...) and
- subject (e.g., one simple type of task, and corresponding technique)

In this paper, the subject and theme levels are called as the *specific-level*, the levels from the sector to discipline as the *meso-level*, and the higher levels as the *generic-level*.

Idea of the Study and Research Questions

Our empirical data come from a so-called “open lesson”, in which a number of teachers and other guests observe and discuss one particular mathematics lesson. In our case, the guests include an invited advisor from an educational university (this is quite common). All participants get a copy of the teachers’ lesson plan before the lesson starts. The lesson plan describes the flow of the whole lesson, the students’ prerequisite knowledge, the mathematical and didactic tasks of the lesson, and the teacher’s ideas for solving the teaching task (e.g. Fernandez, Cannon & Chokshi, 2003; Isoda, 2015). Thus, during an open lesson, the participants observe mainly how the teacher applies explicitly described didactic techniques to realize the mathematical praxeology described in the lesson plan, and the new MP students develop as a result.

What makes the following reflection session (*hanseikai*) significant is the *dialectic* between *specific* and more *generic* observations. Some participants comment on the realised DP of the lesson focusing on precise didactic techniques to support the students’ learning. Others have a broader focus and evaluate the realised DP and MP of the observed lesson and sometimes even more general DPs and MPs within the school mathematics framed in terms against the goal based on certain didactic theories. Miyakawa and Winsløw (2013) also described such dialectic within the reflection session:

The discussion relates the lesson to more theoretical aspects of the mathematics curriculum as such, and even to more general pedagogical and societal aims of the school. This way, the discussion provides a space—an ‘ecology’ in the sense of Chevallard (1988, p. 99)—for developing teacher knowledge that is neither narrowly limited to teaching a particular lesson nor drifting into discussions of teaching philosophies which are more or less detached from the reality of schools and teaching” (p. 204).

Then questions arise: what else could grow in this ecology, in terms of DP logos? Can one find any explicit connections between generic didactic theories and the technologies? In other words, how can *generic* didactic theories, which are directed from the general pedagogical and societal aims of the school, help to organize and validate the didactic technology that explains and informs teachers’ *specific* didactic techniques? If they are connected within the participants’ discourse, in what way, are they connected?

To sum up, the research questions of this study are as follows:

RQ1. What are the teachers' paradidactic foci? In other words, what components of didactic knowledge can appear or develop during the post-lesson reflection in an open lesson in Japan? In particular, what is the role of the notion of mathematical activity?

RQ2. How is the teachers' knowledge of didactic practice shaped during the post-lesson reflections, and how do the discussions relate to components of the different levels of didactic co-determination?

Method

In order to answer both research questions raised above, I first made a small-scale analysis of the guidelines for the Japanese national curriculum (MEXT, 2008, translated into English by CRICED, 2010). To analyse the comments based on the generic didactic theories from the teacher who conducted the open lesson, and the participants, I studied the *objectives of mathematic sin* the guidelines, since there, the fundamental aims of the mathematics education that teachers are supposed to realise are described. Considering RQ1, I studied how the guidelines defined the notion of mathematical activities, and how this notion relates to the actual mathematical contents in the guidelines. Secondly, I outlined the core episodes from the open lesson together with the analysis of the realized mathematical praxeology, and the praxis of the teacher's didactic praxeology (DP). The logos part of the DP that justifies the DP praxis is revealed by analysing the various comments of the teacher and the participants during the reflection session. There, all comments are characterized in three major patterns: 1. reflections regarding generic didactic theory, 2. reflections regarding the specific DP, and 3. reflections regarding the generic theory applied to specific techniques and technologies. In this paper, I analyse the comments of the teacher, the advisor and⁴ of the participants, which are relevant to the topic described above. As each comment of the participants is described, I have emphasized how the notion of mathematical activities is exposed in their comments. To answer RQ2, I located the comments according to the scale of levels of didactic co-determination, to reveal how the different institutional levels are related in the participants' comments, and

how the notion of mathematical activities functions connecting the participants' paradidactic foci at different levels.

The Context of the Open Lesson

The observed open lesson took place in June 2011 at a 7th grade class (age around 13) of 41 students at the Asahikawa lower secondary school in Northern Japan. This school is “attached” to Hokkaido University of Education (meaning, for instance, that it serves for preservice teachers practice). The school holds an annual one-day “research meeting” (*kenkyukai*) and invites hundreds of teachers from inside/outside of the region. Every second year, the school raises a “study theme” which is common for all disciplines in the school. For instance, the theme until the previous year was “raising students’ ability to think” and from this year, it is “raising students’ ability to express themselves with focus on *questioning*, which supports students’ use of their language”. The teachers plan and work with the lessons with this theme as focus. The annual research meeting is an important event where the teachers present the outcome of their daily efforts. The teachers in every discipline describe their achievements during the period and their texts are edited and presented in a booklet, which is distributed to all participants during the research meeting. Further, the teachers have an opportunity to improve their work by receiving reflections and advice from the participants from other schools, as well from researchers who are invited as “advisors” from other universities.

Yachimoto, who is teaching the open lesson we consider here, has worked as mathematics teacher for 16 years. The title of today’s lesson (and lesson plan) is “determination of the surface area of a cone. In the previous lesson, the students have learned how to determine the area of a sector of a circular disk by using the central angle a (namely $A = \pi r^2 = a/360$). As is usual, the reflection session was held in the classroom immediately after the 50 minutes lesson. The number of attendants to the open lesson was about 65, whereof 25 were student teachers in mathematics. The open lesson and the post-lesson reflection session were video recorded and transcribed into English. The analysis work was done based on the English transcript, and thoroughly discussed with two fellow researchers in didactics of mathematics.

Result

The Notion of Mathematical Activities

As it is mentioned in the introduction, the components of the notion of mathematical activities appears significantly within the participants' comments during the post-lesson reflection and plays a notable role to justify their argumentations. To give an insight into the phenomenon, I describe and analyse the notion as it is defined in the section "Objectives and Content of Mathematics" in the guidelines for the Japanese national curriculum (MEXT, 2008, translated by CRICED, 2010).

Normative aspect of the notion

The notion of mathematical activities figures in the Course of Study since 1998. Historically, this notion has been developed over a long period of time before gaining this official status (Isoda, 1999; Nagasaki, 2007). In the guidelines 2008, the notion appears first in the "Overall Objectives of Mathematics":

Through mathematical activities, to help students deepen their understanding of fundamental concepts, principles and rules regarding numbers, quantities, geometrical figures and so forth, to help students acquire the way of mathematical representation and processing, to develop their ability to think and represent phenomena mathematically, to help students enjoy their mathematical activities and appreciate the value of mathematics, and to foster their attitude toward to making use of the acquired mathematical understanding and ability for their thinking and judging. (MEXT, 2008, in CRICED, 2010, p. 15, emphases by the author).

The guidelines then provide further details related to every sentence marked in italics above. The sentence "*Through mathematical activities*" is described as below:

Mathematical activities are various activities related to mathematics where students engage willingly and purposefully. (...) Mathematical activities may also include engaging in trials and errors, collecting and organizing data, observing, manipulating and experimenting; however, simply listening to teachers' explanations or engaging in simple

computational exercises will not be viewed as mathematical activities. (ibid.,p. 16)

Ikeda (2008) describes two essential ideas that affected the development of the notion of mathematical activity: students' *autonomy* and *socialisation*. He emphasized the guidelines' phrasing cited above, "*where students engage willingly and purposefully...*" considering that the guidelines manifest here something *non-mathematical* as a conceptual provision of the notion of mathematical activity. Indeed, the aims described in Overall Objectives of Mathematics, such as "(Through mathematical activities) to appreciate the value of mathematics", "to foster their attitude toward to making use of the acquired mathematical understanding and ability for their thinking and judging" (p. 15), indicate strong normative prerogatives.

Mathematical activities as content and the relation to the structured problem solving

The Guidelines (MEXT, 2008, p. 16) describe three types of mathematical activities, which are particularly emphasized (numbering was added by the author for later reference):

- Type 1: activities to discover and extend properties of numbers and geometrical figures based on mathematics students have learned previously
- Type 2: activities to use mathematics in everyday life and in the society
- Type 3: activities to explain and communicate logically and with a clear rationale by using mathematical expressions

The notion of mathematical activity is included even in the *content*. In the section "Approaches to Content Organization" the contents are categorized in five domains: A. Numbers and Algebraic Expressions, B. Geometrical Figures, C. Functions, D. Making Use of Data. The guidelines (ibid., p. 77) state that to support learning in each of the content areas A to D, as well as to establish connections between them, students should be provided opportunities experience the three types of mathematical activities mentioned above (see Figure 1).

To carry out the mathematical activities, the importance of application of problem solving is emphasized: "Of course, as a principle, these mathematical activities are carried out as problem solving..." (ibid., p.32).In

fact, several scholars consider that the problem solving—especially, the *structured problem solving* (Stigler & Hiebert, 1999) approach as a most appropriate method to practicing the mathematical activities (e.g. Kunimune, 2016). I will describe the process of the structured problem-solving approach as I present detail of the open lesson in next section.

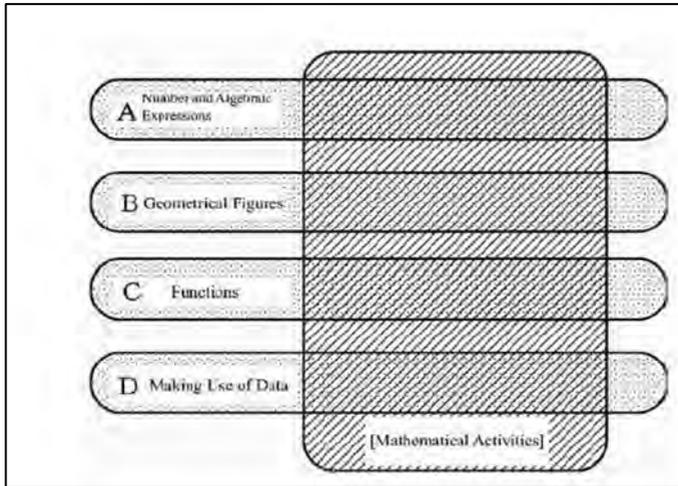


Figure 1. Engaging students in mathematical activities within the five main mathematical contents (MEXT, 2008, p. 78).

In the terminology reference book “Basic knowledge of 300 important terminologies of the teaching mathematics” (Nakahara, 2000), the notion of the mathematical activity is described as “the activities where children create mathematics autonomously” (p. 132) and categorized in three different steps: (1) problem-posing/hypothesis-setting, (2) activities of solving problem, or proving (1), (3) activities of utilization and application of (2). Shimizu (2011) considers that the major part of mathematical activities is coherent with the central property of the structured problem solving itself, and also the implementation of the objectives of mathematics education (p. 5). He states that “mathematical activities should be perceived as the trinity of the objectives of the education, the contents (of the mathematics) and the teaching methods” (ibid., p.5). This expression indicates the normative aspect of the

problem-solving approach as a method of socialization within mathematics education in Japan.

The Lesson and its Didactic Praxeology

Yachimoto shows the class a picture of two cones (see Figure 2) and poses the following initial task: *Which of the surface area of the cones is the largest?*

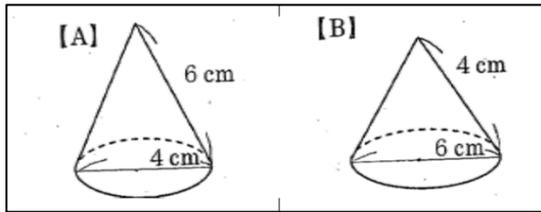


Figure 2. The picture for the task of determination of surface area

The mathematical task is: 1. to notice the unfolded view of a cone is a circular sector, 2. to find out the proportionality between the length of the arc of the circular sector and the whole circle, and 3. to find out the formula for area determination using the generatrix and the diameter [$A = \pi gd/2$], Yachimoto lets the student raise their hands to vote (teacher's didactic technique τ_1 corresponding to the didactic task: making the students engage in the initial task). Five students guess A is the largest, 14 students vote for B, and the rest of the class (about 20) vote for "equally large". The students now consider finding out the solving methods and Yachimoto observes students' work, while circulating in the classroom. This action of the teacher is called *kikan-shido*, it means "teachers' instruction at students' desk"; teacher's *scanning* of students' individual problem-solving process (Simizu, 1999). Different didactic techniques corresponding to the didactic task "making the students explore the initial task and start to find out several solution methods" are used during this *kikan-shido* moment: Yachimoto goes around between students' desks and observe what kind of solutions they are writing on their notebooks (τ_2); listens to the students asking to each other (τ_3); observe who have not and who have already found the solution methods (τ_4), and so on. Generally, these didactic techniques are frequently used by

Japanese mathematics teachers (*ibid.*) to plan the next moment—*neriage*, which I will explain later, and are recognized as “routine” techniques of *kikan-shido*. Thus, *kikan-shido* as a whole functions as a didactic technology in Japan.

Since Yachimoto notice that many students have not found out any solutions yet, he poses a question to the class “what is the problem?” and asks if they know the unfolded view of the cone. When it became clear that the majority of the class cannot imagine the unfolded view of the cone, Yachimoto takes up paper models of the two different cones and lets a student cut and open them. He put the unfolded models on the blackboard (Figure 3). The students notice that they are circular sectors (τ_5 corresponding to the didactic task “to let the students realise what information they must know to solve the initial task”).

Yachimoto lets the students to determinate the area of the circular sector A. While he circulates between the desks, he catches a student’s murmur “But we do not have the central angle of the sector...” Yachimoto remarks quite loudly (so that all students can hear) “The central angle? Must you have the central angle to determine the area?” Then he asks the class how many of them have a same problem. It shows the majority of them do. He comments: “Ok, you have a trouble not having the central angle. What can we do without the angle?” (τ_6 corresponding to the didactic task “to let the students realise that it does not work with the known technique of the mathematical praxeology (MP) and to promote finding out a new technique”). Yachimoto let a student M to write his solution of the blackboard: $6 \times 6 \times \pi \times 1/3 = 12\pi$. Then he asks the class “Is there anyone who has a problem?” Several students raise the hands and one utters: “How and where the 1/3 comes from?” Yachimoto confirms the other students have the same question (τ_4 : to illuminate the core task of the mathematical praxeology). Then he checks if they know the number 6 comes from the generatrix and asks if there is any who uses the 1/3. 8 students do. Student N explains: “The length of the arc of A is equally long with the circumference of the bottom (circle of the cone). If we compare them, we can find out the central angle of A”. Yachimoto then asks the class how the bottom of the cone looks like. It’s a circle. He then picks up a circle made by a paper and puts it on the blackboard (Figure 4).



Figure 3. Unfolded views of cones



Figure 4. The bottom circle

He repeats what student N said using the model and writes the circumference of the bottom circle and the length of the arc of A is “equally long” (τ_7 corresponding to the didactic task “to promote students’ reasoning on the key-concept by pointing out the equivalence between the length of the arc and the circumference in the bottom”). He asks again the class if they now understand where the $1/3$ comes from. They still do not. Student O now describes: “If we consider the sector as a whole circle, then we can compare to the area of the whole circle and the area of the sector” Further, student P explains that one compares the circumference of the whole circle (12π) and the length of the arc (which is equally long as the circumference of the bottom circle— 4π), then $4\pi/12\pi = 1/3$ (τ_8 corresponding to the didactic task “to give the class several different version of explanations on a certain MP technique by students”). Yachimoto writes few keywords on the blackboard: “The circumference of the whole circle”, “The length of the arc” (τ_9 : to write down key-concept of a technique of the mathematical praxeology on the blackboard, corresponding to the didactic task τ_7 aimed to) and checks if the class have grasped Student O’s explanation. These whole-class discussions are called *neriage* in Japanese (ibid.) that is the process of polishing students’ ideas and of developing an integrated mathematical idea. As it was the case of *kikan-shido*, the didactic techniques Yachimoto had applied during the *neriage*-moment are considered as routine didactic techniques for Japanese teachers. Accordingly, *neriage* as a concept can be considered as a didactic technology.

After they have found that the surface area of both cones are equally large, and it is possible to determinate the area without the central angle, Yachimoto gives the class a control task: to find the surface area of two cones with the combination of generatrix and diameter 6-8 and 8-6. The students work by pairs and one tries 6-8 combination and the other checks 8-6 (τ_{10} : to let

students explain the technique to each other, corresponding the didactic task “letting the student establish a new technique they have learned”). Finally Yachimoto presents student P’s idea to establish a formula for the determination of surface area of a cone: diameter \times generatrix $\times \pi \times \frac{1}{2}$. He let student P explain how she found out the pattern while she tried to calculate the different combinations of the generatrix and diameter. Yachimoto let the class to look at the textbook where this formula is described (τ_{11}). This didactic technique belong *tomato me*, which is another Japanese didactic technology for the *institutionalisation of* the learned knowledge. Then Yachimoto asks the students what they would associate from this formula. They answer “the formula for area of triangle” (τ_{12} corresponding to the didactic task “to promote the students explore the technology of the mathematical praxeology”). Yachimoto notes that they will work with this concept at the next lesson and with this comment, he closes the lesson.

The Reflection Session

After the students left, their desks and chairs were arranged so that all 65 participants could be seated in the same classroom during the reflection session (*hansei-kai*). Yachimoto, the chairman, the secretary and a university professor as an advisors at in front of the rest of the participating teachers.

Reflections regarding a generic didactic praxeology

In this section, I will pinpoint how the participants’ comments relate the generic educational aims to Yachimoto’s actual didactic techniques and technologies. The session starts with Yachimoto’s comments on the mathematics teachers’ work related to the theme of the math department this year—to improve students’ abilities of “to express” their thoughts autonomously and to judge properly:

Since I think it is necessary to improve our students’ ability of mathematical thinking and relate it to the lessons with problem solving, we set this goal. I think the relation between learning mathematics and engaging in mathematical activity is very important.

From the viewpoint of levels of co-determination, posing an educational “theme” as a goal to be realised within the daily work in all disciplines, is

derived from the school level. The didactic theory, which supports the goal to “improve students’ abilities of to express themselves”, is a generic educational conception of the duty of schools. Yachimoto’s comments above address that he deploys the educational theme in a specific didactic and mathematical praxeology by connecting the idea behind the lesson to the problem-solving approach. His remark on “problem solving” and “the relation between learning mathematics and engaging in mathematical activity” indicates that the problem solving is a part of the mathematical activities, which promote the realisation of the educational theme, “to express their thoughts autonomously and to judge properly”. Here, he establishes a clear conceptual link between the generic theory and the teaching practice, which has just been observed.

The construction of the conducted lesson and Yachimoto’s perception regarding mathematical activities are partly in line with Nakahara’s (2000) definition of three steps in mathematical activities described in the previous section: (1) problem-posing/hypothesis-setting, (2) activities of solving problem, or proving (1), and (3) activities of utilisation and application of (2). Concerning the third activity, we could not see how all the work of today’s lesson would be applied in the next lessons. However, students’ reactions during the open lesson showed that knowledge gained in previous lessons was invested in today’s lesson. In that way, applying the problem solving also depends on the lower levels of the co-determination like theme and subject, since all these activities (1) to (3) concern about specific didactic technologies. Afterwards, Yachimoto began to describe the concerns of his and other mathematics teachers who collaborate in this lesson study about how to design the moment of *kikan-shido*, which in didactic technology refers to the moments of teachers’ observing students’ initial work on a problem. He also explains the background of his didactic techniques used from the beginning of the lesson and to the *neriage*-moment follow the flow of his lesson plan. These descriptions of techniques clearly relate to theme and subject levels.

Participant 1 comments on the goal of today’s lesson, as it figures in the lesson plan: “students will be able to explain how to determine the surface area of a cone”. He asks:

Was the goal realized? How often did the pupils explain during the lesson? To whom did they explain?

Right after participant 1 stated this question, participant 2 criticises Yachimoto's technique to realise the more generic goal of improving students' abilities of "to express themselves":

I think the crucial attitude our students need to achieve is to know the value of mathematics, learning the logic, thinking, communication and so on. You lead the students all the time. Wasn't there a too small space to let them find out and talk without YOU telling everything? Tell me what kinds of activities were used to train their communication skills in today's lesson. It was great that student P found the formula in the end. Shouldn't you aim that your students find out things like she did, and let them reason using words and several expressions, rather than to let them follow precisely what you planned?

Then participant 2 begins to talk about the mathematical activity:

There is the text, which tells, "pupils will learn through mathematical activity" in many different parts in the national curriculum. I think there are three different types in the mathematical activity: 1 finding out (*mitsukedasu*), 2 applying (*riyousuru*: applying the methods one found out), 3 to express and communicate (*tsutaeru*: to tell how one applied it to their classmates). Tell me what kind of activities were done in today's lesson?

Both participants talk about pedagogy and discipline level issues. The comments such as "goal", "to whom they explain?" are related to the generic levels such as school and pedagogy. These comments are detached from specific didactic techniques, which aim to realize a specific mathematical praxeology. While the comments regarding "crucial attitude our students need to achieve is to know the value of mathematics, learning the logic, thinking, communication" and "mathematical activity" are related to me so-level like discipline and domain. Participant 2 gives a direct question how Yachimoto has planned managing of the linking of the praxis ("Tell me what kind of activity was used to train their communication skills...") and logos part of the didactic praxeology by drawing on the notion of mathematical activities described in the national curriculum as theory to justify or argue the use of specific didactical techniques. Then he gives a suggestion of a certain didactic techniques, which realize not only the mathematical but also the pedagogical goals, since these techniques are justified by the notion of

mathematical activities (“let them reasoning words and several expressions rather than following the teacher”).

Yachimoto replies now to the questions. For the planning of today’s lesson, he has made a small-scale research on his students’ knowledge regarding solid bodies. The students could imagine the unfolded view of cubes and cylinders, however only 42 % could correctly determine the unfolded view of a cone:

I wanted to make them realize that they can apply their previous knowledge regarding circular sectors to solid bodies. By cutting the model, they realize that possibility, of using the concept of the plane figure. Then they may see the value of mathematics.

Here, Yachimoto justifies the legitimacy of his DP. First, he describes an element of his paradidactic practice—the pre-research for the construction of the DP of his lesson. Next, he explains the didactic techniques (letting the students cut the model to make the plane figure) that is supported by a didactic technology—Japanese teachers’ general preference “to *make students realize that they can apply their previous knowledge*”. Then he relates this achievement to a generic goal at discipline level: “*Then they see the value/functionality of mathematics*”. We note that his description is well aligned with the guidelines’ description of mathematical activities of type 1 (see the previous section).

Reflections regarding a specific didactic technique

Participant 3 has observed a very specific didactic technique. He firstly mentioned Yachimoto’s technique of “questioning” to explain the expression for the area of cone A ($6 \times 6 \times \pi \times 1/3$) to the class:

It’s worth noting that the teacher did not simply engage in a dialogue with a particular student. We usually say: ‘discuss among each other’, or ‘discuss with the whole class’. However, I think it is impossible to realize, if the discussion is a ‘free talk’. The teacher must become a manager and connect different persons’ remarks. If one masters this technique, one can carry out the problem solving well.

Participant 3 evokes the *professional*, practical knowledge (didactic techniques) for managing a whole-class discussion. Then he asks Yachimoto about how to make the students relate to the idea of proportionality:

My impression is that this idea ($1/3$) is based on the concept: the area of a circular sector is proportional to the length of the arc. Without having this idea, it would never happen that the students find out the $1/3$. How did you do to make them find out that idea?

Yachimoto answers the question:

Actually, when I did a trial lesson in another class, it took 40 minutes to find out the $1/3$. So yesterday, in this class, I asked the students ‘if you know the radius of the whole circle and length of the arc (of the sector), can you find out how big the middle angle of the sector is?’ Then, they started to talk about the proportionality between the circumference of the whole circle and the length of the arc. I think they remember what we have done yesterday and applied that idea (for the determination of the area of the sector).

This dialogue concerns a specific didactic technique (for making the student get the idea for solutions to a specific mathematical task) and generic technology (applying students’ previous knowledge), related to the specific mathematical technology (proportional relation between the circumference of the whole circle and the length of the arc). Thus, this comment is formed entirely from the lower level such as theme and subject.

Relations among generic theories, specific techniques and technologies

Participant 4 remarks on Yachimoto’s technique of organization of the blackboard:

Every time I see Mr. Yachimoto’s lessons, I admire his way to organize the blackboard. Today for example, you used different colors to different matters: orange for the proportionality, blue for the circumference, green for the arc and yellow for the answers. The theme of the year is to improve students’ ability of expression. However, the issue of blackboard techniques has not been described in the booklet. If the teacher does not organize the blackboard properly, the students cannot learn about the proper expression. Can you tell me how we can help students to develop their ability of expression by effective use of blackboard?

Yachimoto replies that he carefully plans the blackboard organization.

The whole record of the blackboard will remain in the students’ notebook. During the last lesson, I was conscious that what I would

write on the blackboard would remain on their notebook. They could see it and get some hints from the note. So, I plan the use of the blackboard carefully; what topic will be written in which place, recording a student's word verbatim, and so on.

Here, participant 4's question about the specific didactic technique perfectly links to a generic didactic theory. The issue of blackboard organisation for this particular lesson is connected to establishing a shared inventory of ideas about developing students' abilities of expression. Yachimoto's response addresses that blackboard organisation techniques, such as the clear presentation of the problem (and later, solutions from students) by the teacher, are important to students' opportunities to develop their mathematical reasoning and communication, and to record main points developed in the course of the lesson.

Now is the time for the advisor, who is a professor invited from Hokkaido University of Education, giving his concluding comments.

I consider that raising students' ability of expression is about the enrichment of the use of the mathematical language. By seeing all these pictures and expressions on the blackboard (from today's lesson), we can understand exactly how the students thought. I would like you all to emphasize the value of mathematics within your lessons. As we saw in today's lesson, they can discuss and think together in pairs or groups. I observed the students eventually began to understand some issues they did not understand in the beginning, by listening their classmates' comments and writing down classmates' solutions in their notebooks. By doing these activities, their abilities to express themselves develop. That is the true training for the ability of expression.

The advisor's comments are related to educational aims and are based on generic didactic theories (related to pedagogy and school levels). As in participant 4's reflection, these generic pedagogical issues are linked to Yachimoto's didactic techniques as observed; letting the students discuss and think together in pairs or groups, and write down others' solutions in their notebooks. These activities are, according to the advisor, "*the true training for the ability of expression*". Advisor's comments above clearly relate to the mathematical activities type 3: *activities to explain and communicate logically and with a clear rationale by using mathematical expressions* (see the previous section).

Discussion and Conclusion

Here I sum up the result and the analysis of the comments to answer my two research questions. For the investigation of RQ1, the components of didactic knowledge can be characterized as follows:

- Category1: generic DP logos, which discuss how general educational aim such as “to improve students’ abilities of expressing themselves” is treated during the lesson.
- Category2: specific DP praxis, which is the discussion of precise didactic technique, such as how Yachimoto managed letting the students notice the area of a circular sector is proportional to the length of the arc.
- Category3: the combination of the discussion of generic DP logos and specific didactic technologies and techniques, such as, how Yachimoto organize the blackboard disposition to support the students’ development of their ability of expressing themselves.

The analysis shows that the notion of mathematical activities, which has been studied (by teachers) within the Japanese paradidactic practices, promotes having a shared perception of how to capture the dialectic between the generic educational aims and specific didactic technologies. This phenomenon is especially exposed within the category 1 and 3. In the comments of the participants 1 and 2, the issue of training students’ abilities of communication, was clearly connected to the mathematical activity of type 3; *activities to explain and communicate logically and with a clear rationale by using mathematical expressions*. The description of this type 3 justifies even the advisor’s comment “*By doing these activities-discussing and thinking together in pair or group, and writing down others’ solutions in their notebooks-their ability of expressing themselves are developed*”.

Also, Yachimoto’s remark such as “*It is necessary to improve our students’ ability of mathematical thinking and relate it to the lessons with problem solving; we set this goal (to improve students’ abilities of expressing their thoughts autonomously and to judge properly). I think the relation between learning mathematics and engaging in mathematical activity is very important*”; “*I wanted to make them realize that they can apply their previous knowledge (...). Then they may see the value of mathematics*” “indicates that he uses the notion of mathematical activities as overall methods for the

learning mathematics, and the *specific* didactic praxeologies for that are captured by applying the structured problem solving. In fact, the realisation of the notion of mathematical activities in the mathematical and didactic praxeologies of daily lessons is closely related especially to the structured problem solving approach, since this approach is considered as one of the most widespread didactic theories in Japan, and relates to a large set of professional notions (central in the didactic technology) such as problem posing and whole-class discussion (Simizu, 1999). As a teaching approach, the structured problem solving has capacity to carry complex mathematical praxeologies (MPs), also, the structure of its didactic praxeologies—including task construction and whole-class discussion—promotes students' autonomous work on a mathematical task (Asami-Johansson, 2015). As it described in the previous section, the Japanese teachers' and scholars' didactic focus in the use of the structured problem solving is on the cultivation, or socialization of students, as much as on the didactic techniques to enable development of students' MPs within the daily lessons.

Regarding RQ2, the participants' paradidactic foci—the components of didactic knowledge that were discussed during the post-lesson reflection—were related to different institutional levels of didactic co-determination, such as school, pedagogy, theme and subject. Given that the context was an open lesson in a regional study meeting, where the participants were from different schools, it was nevertheless expected that some comments relate to the levels beyond, or at the level of the discipline, since generic didactic theory is often in focus at such large-scale events. Also, the absence of comments related to discipline, domain and sector, is also expected phenomena, since schoolteachers have limited influence on these higher levels which are fixed by the national curriculum (*thematic confinement*, see Barbé, et al, 2005). Consequently, most of the discussion concerning the MP remains focused on the teacher's theme-specific didactic techniques, as we saw in the participant 3's remarks.

However, in the comments of category 1 (generic DP logos) and 3 (generic DP logos and specific didactic technologies), the both sides of the co-determination (generic levels—school/pedagogy, and specific levels—theme/subject) are related as these categories were extended both to the educational aims and how to actually realise the aims within the DP of the demonstrated lesson, through the notion of mathematical activities as a *mediator*. This notion basically belongs to Japanese teachers' paradidactic

practice. As we have seen in the descriptions in the guidelines, the “activity” itself has an aspect of mathematical contents, and serves to develop students’ MPs in different domains. According to the levels of co-determination, mathematical activities as described in the objectives of mathematics education pertain to generic-levels as “school” and “pedagogy”. At the same time, the mathematical activities as mathematical contents pertain to the meso-levels as “discipline”, “domain” and “sector”, since mathematical contents are settled in the national curriculum, which initiates the structure of the praxeologies of mathematics lessons in long span. Thus, this notion is directed from the generic level and meso-level in tandem. Since the main didactic technologies (problem solving, use of students’ previous knowledge, logical communication using mathematical expressions, etc.) mentioned in the post-lesson reflection, were coherent with the characteristic of the notion of mathematical activities (type 1 to 3), we can conclude that this notion functions as a shared *didactic* and *paradidactic theory* for the participants.

As the *hanseikai* exemplifies, Japanese teachers connect these generic theories explicitly to their teaching practice. Depending on the educational goal considered (here, raising students’ ability to express themselves), they design their didactic practice to realise it, and the reflection session is a main moment for evaluating the extent to which it succeeded, and for sharing alternative strategies. The dialectic discourse between generic aims and specific didactic praxeologies, described in this paper, is carried by the notion of mathematical activities, which is an important asset for the Japanese teachers. The analysis shows how this notion reflects both the objectives of mathematics education and the genuine paradidactic practice related to Japanese lessons, as it enables teacher knowledge to be developed and shared beyond a particular theme or domain. For researchers, the dialogue between the teacher and the participants provides detailed knowledge of teachers’ didactic theory blocks and how they seek to manage the co-determination of mathematical and didactic praxeologies. Studying a post-lesson reflection session exposes many aspects of Japanese teachers’ paradidactic practice, including the generic rationales underlying the planning of a lesson, which we cannot see in the lesson itself. The reflection session serves to enhance and share the theoretical block of the teacher knowledge. This and similar elements of Japanese paradidactic practice contribute to develop shared,

essential knowledge about teaching, through genuine, professional scholarship.

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