CONCEPTUAL SUBITIZING AND PRESCHOOL CLASS CHILDREN'S LEARNING OF THE PART-PART-WHOLE RELATIONS OF NUMBER

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Abstract

Few research studies within the field of mathematics education have focused on the ability to recognize quantities quickly and accurately without counting (i.e. subitizing). The aim of this research was to empirically explore the role of conceptual subitizing activities for enhancing preschool class children's learning of the part-part- whole relations of number. 24 children (aged 6-7) and two teachers participated in the intervention. Due to ethical issues, data were only collected from 18 of the children. The design was a collaborative and iterative intervention, employing a mixed-method approach. Data consisted of pre- and post-test, teacher observation notes and teacher responses to questions about the children's learning. Both the quantitative and qualitative analysis showed that conceptual subitizing activities supported children's knowledge development regarding part-part-whole relations of number. At the group level, the children´s results between pre-and post-test showed an increase of 18.1 percent units and more than half of the children showed conceptual subitizing abilities in a qualitatively more developed way after having participated in the intervention. The result indicated that conceptual subitizing activities might enhance preschool class children's understanding of the part-part-whole relations of number. The results however also elucidated that not all children improved their understanding of the part-part -whole relations of number. Future research should therefore consider individual differences when developing and carrying out interventions.
Keywords: *collaborative intervention, conceptual subitizing, part-part-whole relations of numbers, preschool class children*

Introduction

This research empirically explored the role of conceptual subitizing activities for supporting preschool class children's learning of the part-part- whole relations of number. In the Swedish educational context, preschool class is mandatory for all Swedish children from the age of six. A guarantee, regulated in the Swedish Education Act (SFS 2010:800), aims to assure that all students in need of support in their reading- writing and mathematical development will receive it early on (age 6-10 years). The purpose of the guarantee is to assure that all students will be able to meet the knowledge requirements in grade 1 and 3 in the subjects Swedish, Swedish as a second language and mathematics. This regulation of early support is in line with studies that have shown that early years mathematical skills are strongly connected to later performance (e.g. Aunola et al., 2004; Duncan et al., 2007; Morgan et al., 2009). Morgan et al., (2009) for example reported that children who experienced difficulties in mathematics in Kindergarten showed low mathematics skill growth throughout primary school. The importance of early attention regarding learning math and related skills in preschool (Aunola et al., 2004) and high-quality interventions for children at risk of academic failure (Baroody et al., 2006) have been highlighted in previous research.

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However, no guidelines are provided regarding how to teach the mathematical content appropriately to support children's early mathematical learning and knowledge development. Comprehensive research reviews on mathematical learning, have indicated that an understanding of quantities and numbers as well as an ability to operate with numbers are important aspects of early mathematical development (e.g. Dowker, 2019; Kilpatrick et al., 2001). One fundamental idea of understanding quantities and numbers includes an understanding of part-whole relations i.e. that a "quantity or a whole can consist of parts and be broken apart (decomposed) into parts, and that the parts can be combined (composed) to form the whole" (Baroody et al., 2006, p. 207). One important aspect that has been extensively reported on, regarding early mathematical development, is the ability to quickly and accurately recognize small quantities without counting, also called subitizing (e.g. Clements, 1999; Hannula-Sormunen et al., 2015; Kaufman et al., 1949). Studies have for example shown a positive significant relation between the subitizing range and mathematics performance in children (Kroesbergen et al., 2009; Yun et al., 2011). Another important aspect that has been reported on is children's counting skills (Zhang et al., 2020). Zhang et al. (2020) have for example reported that accuracy in counting sequences discriminated between average achieving children and children with mathematical difficulties. Research in the field of psychology and cognition has proven that subitizing and counting are two different systems for exact number magnitude processing (Kaufman et al., 1949; Mandler & Shebo 1982; Schleifer & Landerl, 2011). Counting is defined as a pairing of objects with the number sequence (Mandler & Shebo, 1982) whereas the term subitizing is used for defining a rapid, accurate and confident process for determining the number of objects in a set (Kaufman et al., 1949). Research studies have found a discontinuity regarding response times between enumeration of smaller (1-3) and larger (4-8) sets, suggesting that subitizing is a faster process for enumeration of small quantities than counting (Mandler & Shebo, 1982; Schleifer & Landerl, 2019). In a study, Benoit et al. (2004) investigated the role of counting abilities and subitizing abilities for young children to acquire the cardinal meaning of small-number words. The results indicated that subitizing promoted the cardinal value of the first number words, since it allowed the child to grasp the whole and the individual elements at the same time. Despite research showing that the ability to quickly and accurately recognize quantities without counting (i.e. subitizing) is an important aspect in children´s early mathematical development (Clements, 1999; Hannula-Sormunen et al., 2015; Kaufman et al., 1949), few empirical studies within the field of mathematics education have focused on this aspect (Clements et al., 2019).

Perceptual and Conceptual Subitizing

Most of the studies regarding subitizing have predominately been conducted in the field of psychology, investigating perceptual processes (e.g. Kaufman et al., 1949; Mandler & Shebo, 1982; Schleifer & Landerl, 2011; Starkey & McCandliss, 2014; Starkey,1990). Kaufman et al. (1949) coined the term subitizing for defining an innate, rapid, and accurate process for determining the number of objects in a set less than 6. Furthermore, Starkey et. al (1990) reported, based on evidence from an experiment in which 6-8-month-old babies could match drumbeats with the correct dot-card (by looking at the card), that subitizing is an innate mechanism. Research within mathematical education distinguishes between two types of subitizing processes: perceptual and conceptual subitizing. Perceptual subitizing is, similarly to Kaufman et al.'s definition (1949), referred to as an immediate, innate number recognition, i.e. recognizing the numerosity of a briefly presented collections of a small number of items without consciously using other mathematical or mental processes (Clements, 1999; Clements & Sarama, 2014; Clements et al., 2019). Conceptual subitizing is instead referred to as a more advanced ability to recognize the number of items in a larger set of items by using some kind of partitioning strategies, by mentally structuring and organizing the items in smaller identifiable parts, showing an awareness of the parts and the whole (Clements et al., 2019). Even though research in the field of educational science distinguishes between the two processes, researchers argue that this innate perceptual subitizing ability can be developed into conceptual subitizing abilities (Clements, 1999; Conderman et al., 2014). Clements and Sarama (2014) described the relations between the two processes as preschoolers recognize the number of objects in a set of three (or four) or fewer instantly (perceptual subitizing) and as they become more familiar with sets of numbers, they are able to determine the quantity in larger sets by quickly recognizing the smaller sets that make up the quantity (i.e. conceptual subitizing). Also, Conderman et al. (2014) have postulated the relationships between the two processes by the formulation that conceptual subitizing is about "identifying a whole quantity as the result of recognizing smaller quantities (recognized through perceptual subitizing) that make up the whole" (p. 20). The ability to instantly recognize quantities target the ability to spatially structure quantities which is essential for the development of insight into numerical relations (e.g. Mulligan & Mitchelmore, 2009; Van Nes & de Lange, 2007). Mulligan and Mitchelmore (2009) have for example found that the general awareness of mathematical patterns and structures of grade 1 children were related to their mathematical achievement on tasks assessing mathematical processes such as subitizing, unitizing, partitioning, multiplicative, and proportional relationships. Likewise, Van Nes and de Lange (2007) have suggested, based on an overview of research literature from mathematics education and neuroscience, that spatial structuring are beneficial when children are to develop an understanding of quantities and numbers. Similar results have been reported by van Nes and Doorman (2011) who found that the use of structuring strategies, instead of unitary counting strategies, were beneficial when children (aged 4-6) were engaged in numerical procedures such as determining, comparing and operating with quantities.

Conceptual Subitizing and the Part-Part-Whole Relations of Numbers

According to Piaget´s (1997/1952) theory of the development of number, an understanding of number requires a prior understanding of number conservations (the number of items are the same regardless of how they are arranged), class inclusion (to simultaneously consider the parts and the whole) and seriation (numbers in order). Furthermore, Piaget concluded that the additive composition of number (e.g. 8=4+4 or 3+5) is an important aspect regarding the understanding of relations between number. Additionally, Baroody (2006) has argued, in a theoretical paper, that the understanding of the big ideas of composition and decomposition, i.e. that a whole can be composed in different ways and with different parts and decomposed into its constitute parts, make a foundation for mastering quantities and the basic numbers 1-10, in flexible ways. Baroody et al. (2006) have furthermore stated, in a hypothetical learning trajectory model, saying that "composing and decomposing small, easily subitized collections may be the basis for constructing an informal concept of addition (and subtraction)" (p.193). Also, Clements (1999) has argued that conceptual subitizing activities provide a basis for understanding addition since the addends and the sums are shown simultaneously, offering the children different views of how numbers can be arranged. Furthermore, the ability to conceptually subitize has been positively linked to children's general number development such as knowledge of cardinality (Butterworth, 2005), number conservation (MacDonald, 2015; MacDonald et. al., 2015), number relationships (Jung et al., 2013; Sayers et al., 2016; Young-Loveridge, 2002) and basic arithmetic skills (Baroody, 2006; Baroody et al., 2006; Özdem & Olkun, 2019). Özdem and Olkun (2019) for example reported that the intervention group (grade 2 and 3), that were engaged in conceptual subitizing activities improved their results on a math test significantly more than the control group. In addition, the intervention group used more sophisticated calculation strategies on a follow-up test compared to the control group.

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Similar positive relations between conceptual subitizing activities and children's mathematical development have been reported by Sayers et al. (2016), MacDonald (2015) and MacDonald et al. (2015). Sayers et al. (2016) found that conceptual subitizing activities supported the children's (*N*=2, grade 1) development of foundational number sense, whereas MacDonald (2015) and MacDonald et al. (2015) found that subitizing activities (perceptual and conceptual) supported 4-5 year-old children's number conservation.

Research Problem and Research Focus

Despite research showing that subitizing is an important component in children´s early mathematics development (Clements et al., 2019; Kroesbergen et al., 2009; Yun et al., 2011), few empirical studies within the field of mathematics education have focused on this aspect. Clements et al. (2019) claimed, based on synergized research on studies across education and psychology, that subitizing is a "neglected quantifier in educational practice" (p.13). Furthermore, few intervention studies have been conducted within the age group of 6-year-old children, both in a Swedish and in a global educational context (Mononen et al., 2014; Sterner et al., 2019). The aim of this research was therefore to empirically explore the role of conceptual subitizing activities for enhancing preschool class children's learning of the part-part- whole relations of number.

The specific research questions were:

RQ 1: What knowledge regarding the part-part-whole relations of numbers did the children develop?

RQ 2: What connections can be found between the children's knowledge development and the design of the conceptual subitizing activities?

This research is based on two assumptions. The first assumption is that subitizing and counting make use of two different magnitude processing systems (Kaufman et al., 1949; Mandler & Shebo, 1982) and that subitizing is a faster process for determining quantities than counting (e.g. Schleifer & Landerl, 2011). The second assumption is that conceptual subitizing abilities are closely linked to knowledge of the part-part-whole relations of number (Baroody et al., 2006; Clements et al.,2019; Conderman et al.,2014; Piaget, 1997/1952).

Research Methodology

General Background

A case study methodology (Yin, 2011) with a mixed-methods approach (J.W. Creswell & J.D. Creswell, 2018) was used. Since the case of interest was on preschool class children's learning the study could be classified as a single case study, meaning that the case is a typical or representative case (Yin, 2011). A mixed-methods approach was chosen due to that it involves data collection and analysis that integrates both quantitative and qualitative data and it may therefore provide a more complete understanding of the problem than either quantitative or qualitative data alone can provide (Creswell & Creswell, 2018). Figure 1 gives an overview of the research design in which data collection and analysis are included. Johnson and Onwuegbuzie (2004) emphasized that the benefit of bringing quantitative and qualitative data together is to incorporate the strengths of both methodologies. In this way the study followed the suggestion in Johnson and Onwuegbuzie (2004) to incorporate the strengths of both methodologies. This was especially important in this study since it focused both on children's learning as well as on the teaching. It was therefore relevant to gain knowledge of both the impact of the intervention and on children's knowledge of the part-part-whole relations of number on the group level as well as on how the individual children responded to the intervention.

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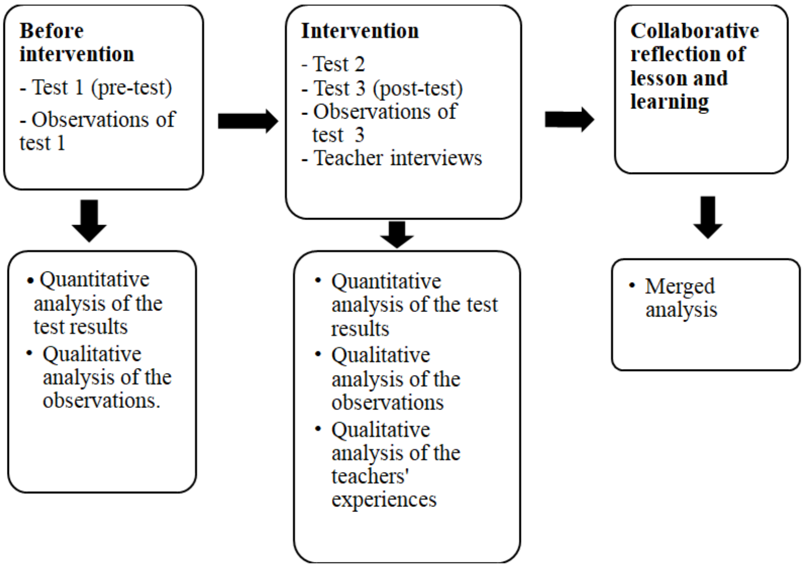
Figure 1
Overview of the Research Design



Sample

This study was conducted in January 2019 in one Swedish preschool class, which consisted of 24 children and their two teachers. Data were only collected from 18 of the children due to ethical considerations (see section Ethical Considerations). Both teachers were legitimated for teaching in preschool class and their teaching experiences were 16 and 4 years, respectively. The selection of the school can be described as a convenience sampling since the principal and the teachers were interested in participating in the intervention (Cohen et al., 2007). The school is situated in the south of Sweden in a small society with about 150 students of which 48% of the children had a foreign background (the students were born abroad or born in Sweden with both parents born abroad) (Swedish National Agency for Education, 2020). Furthermore, only 54.2 % of the "third graders" at the school (N=26) reached the knowledge requirement on a National test on numbers and their properties and the positioning system of numbers in 2018/2019, compared to the national level of 86.3%.

Procedures

The intervention was conducted in collaboration with the two teachers, and it was inspired by a lesson study model which is an iterative, collaborative process, where a lesson is planned, analyzed, and revised by researchers and teachers (Holmqvist, 2017). In table 1, an overview of the different phases of the study is presented. All 24 children participated in the intervention-lessons during the 3-week study period since the mathematical content that was taught in the intervention corresponded with the core content in the syllabus of mathematics in preschool class (Swedish National Agency for Education, 2019). The children were engaged in the activities in each lesson block (lesson block 1 and 2) for 2-3 days, in all 60-80 minutes for each block. The teachers and researcher met about 2 hours per week, altogether 8 hours. Before implementing the intervention, the children's existing knowledge of the part-part-whole relations of numbers (i.e. 5) were screened (Test 1). The test results, as well as the teachers'

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written notes from the observations of what strategies each individual child used during the test, were analyzed both quantitatively and qualitatively.

Table 1
The Different Phases of the Study

Activities and data collection	Participants	Date
Defining the learning goal and planning of the Lesson study	Teachers and researcher	6/2/19
Test 1 (version 1) and observations	Teachers	8/2/19
Collaborative analysis and planning lesson block 1	Teachers and researcher	8//2/19
Lesson block 1	Teachers	11-12/2
Test 2 (version 1) and observations	Teachers	13/2/19
Collaborative analyzing and planning of lesson block 2	Teachers and researcher	13/2/19
Lesson block 2	Teachers	14-15/2, 25/2
Test 3 (version 2) and observations	Teachers	26/2/19
Collaborative analysis	Teachers and researcher	26/2/19
Teacher responses to questions	Teachers	4/3/19

Based on children's prior knowledge of the part-part-whole relations of number five, the goal of lesson block 1 was determined. The goals and the activities used for supporting the learning goal are described in table 2. An analysis of the children's knowledge development as well as what learning opportunities were offered in lesson block 1 was then conducted. Based on the outcome, the learning goal and the activities in lesson block 2 were planned (Table 2). In many of the activities, the different numerosity was briefly presented (2 seconds) and the children were encouraged to directly determine the cardinal value without counting in ones. The different strategies used for determining the numbers of quantities in a set were discussed in the whole class setting. In order to let the children experience that smaller quantities (2-4), regardless of representation and configurations, can be instantly recognized without counting in ones, quantities of 2, 3 and 4 were used in the activities whereas the representations as well as the configurations varied in the activities. Since the focus in lesson block 2 was on identifying smaller subgroups, within the whole, only dots were used whereas the configurations varied. In lesson block 2 quantities in the range between 4 to 7 were used, since the teachers thought that most of the children would feel comfortable to handle quantities within this range.

Table 2
Activities in Lesson Block 1 and 2

Lesson block and goal	Activities
1 Goal: understand that smaller quantities can be recognized directly, without counting	Discuss similarities and differences among dice patterns Play dice games where the children were asked to directly state the number of dots (without counting); encourage the children to describe how they "knew" the number Application: subitizing dice Spread out cards with different numbers of dots and different dot patterns; ay a number (e.g., 2, 3 or 4) and let the children find all cards with the same number of dots Application: subitizing flashcards with a maximum of 4 items Show different objects (2 seconds) of 2, 3 or 4 in different patterns; encourage the children to directly state the number of smaller quantities Show different objects (2 seconds) with more than 2, 3 or 4 or less than 3 or 4; ask the children to determine if they were greater or less than 2, 3 or 4; ask the children how they knew
2 Goal: recognize smaller subgroup within a composite group, showing an understanding that quantities can be (de) composed in different ways	Use 2 or 3 different colors to "build" quantities of 4–7, such as 2 red dots and 4 blue dots when building 6; discuss different solutions for how to build 6 Use the same color for the quantities of 4–7 dots; ask the children to describe which clusters they perceive within the composite group Compose two different subgroups (with 2 different colors) and place them on 2 trays or plates; show them briefly; ask which plate had the most or least dots; ask how the children know; repeat using the same colors on the dots Application: subitizing flash cards Application: King of Math junior Organize 5 dots in 2 different ways; show them briefly; ask if they are the same number of dots in different configurations; ask how the children know; ask which grouping was easy or hard to perceive; use other quantities

Instruments

The test used for collecting data about children's knowledge development was Number Sets Test (NST) (Geary et al., 2009) which is a test that was developed by Geary and his colleagues to identify children at-risk for developing mathematical learning difficulties (MLD). In this study however, the test was not used for identifying children at risk for MLD but only for assessing children's abilities to directly recognize different non symbolic combinations that added up to 5. The original test consists of four subtests; non-symbolic object sets or Arabic numerals that add up to 5 or 9. During the test the child is supposed to decide if pairs or trios (symbolic or non-symbolic) add up to 5 or 9 respectively, within 1 minute. Due to the time limit the child must do so as fast and accurately as possible without making mistakes. The test is constructed so the children are unlikely to be able to complete all tasks within a minute. The non-symbolic stimuli (consisting of sets of dots, triangles or stars) are joined in domino-

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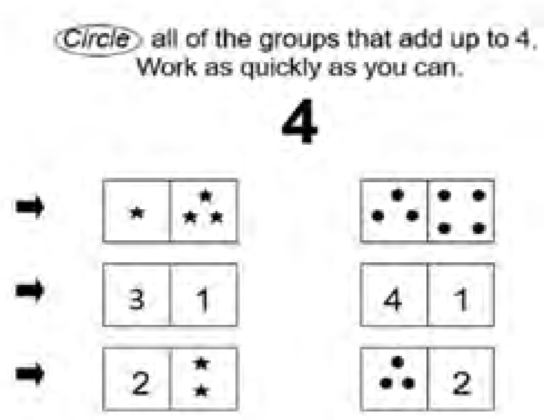
like rectangles, with varying numbers of objects and configurations. Both canonical and non-canonical patterns are used, to represent quantities between 2 and 9. In the present study only the subtest to determine if pairs and trios of non-symbolic object sets add up to 5 was used. The five first rows in the non-symbolic 5-test are pairs of objects (*N* = 30); the last two rows are trios (*N* = 6) and 18 of the combinations add up to 5. 5 was chosen since the teachers believed that the children had conceptual understanding of 5 and the quantities used in the activities were seven and fewer. The quantitative summarized scores consisted of correct answers minus incorrect answers. The children were instructed to circle all the groups that together added up to 5; they were to work as fast as possible without making mistakes. The children worked individually with the test and the time limit of 1 minute was controlled for with a digital clock. Before the test was administrated, the children practiced by circling sets that added up to 3 and 4 (Figure 2).

Figure 2
Examples of Practice Items in the NST-test



Note. From "Early Conceptual Understanding of Cardinality Predicts Superior School-Entry Number System Knowledge", by Geary, D. et al., 2018, Psychological Science, 29(2), p.195. Reprinted with permission, 2019-12-17.

Data were also collected by direct observations. The preschool class teachers observed the children in the test situation and took field notes of what strategies the children used when they did the test (i.e. determined which sets added up to 5). The approach used was according to Robson and McCartan (2018) formal observations since the observer only attended to pre-specified aspects. Some of the test situations were also video recorded by the teachers so that the researcher and teachers could view them together in the analysis and planning phases.

After the intervention, the teachers got questions via e-mail; three open-ended questions regarding the children's learning and one attitude question about to what degree the children wanted to participate in the activities. There were also some introductory questions regarding age, gender, education, and professional experiences. The impersonal nature of the e-mail questions (interviews) may help the respondent to say things they would not have done in a face-to-face situation (Robson & McCartan, 2018) which is an important aspect when the research is done in collaboration between the researcher and the respondents.

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Data Collection

Data consisted of test results (Test 1, 2 and 3), teacher observation notes (direct observations from test 1 and 3) and teacher responses on questions about the children's learning. One problem that arose regarding data collection was the difficulty in obtaining complete datasets with results for all test and observation-situations: some children stayed home due to illness or had to participate in other activities, such as mother-tongue teaching. Furthermore, some of the written observations were unclear and ambiguous and thereby not reliable to include in the analysis (see table 3). Due to missing data, the quantitative data consists of results from 12 of the children who participated in all three test situations whereas the qualitative analysis was based on written teacher observations of 9 of these 12 children (A, D, E, F, G, H, I, K and N). Table 3 provides an overview of the collected data from participants (coded with the letters A-R). The items in the test were identical in all the tests (1-3), but differently ordered; version 1 and 2 (table 1).

Table 3
An Overview of the Data Collected; a Cross (x) Indicates Complete Data whereas a Slash (/) Indicates Missing Data

Kod	Pre-test (Test 1)	Test 2	Post-test (Test 3)	Observation Pre-test (Test 1)	Observation Test 2	Observation Post-test (Test 3)
A	x	x	x	x	x	x
B	x	x	/	x	x	/
C	/	x	/	/	x	/
D	x	x	x	x	x	x
E	x	x	x	x	x	x
F	x	x	x	x	x	x
G	x	x	x	x	x	x
H	x	x	x	x	x	x
I	x	x	x	x	x	x
J	x	x	x	x	/	/
K	x	x	x	x	x	x
L	/	x	/	/	x	/
M	x	x	x	x	/	/
N	x	x	x	x	x	x
O	x	x	x	x	/	/
P	x	/	/	x	/	/
Q	x	/	x	x	/	x
R	x	/	/	x	/	/

Analysis

The scores on the test, before and after the intervention, were calculated at the group level. The knowledge development is described in percent units in table 5. The mean was also calculated to identify the average score in the test. Since the mean can be quite heavily influenced by extreme scores (Field, 2018) standard deviations were calculated as well. A qualitative

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analysis with a thematic coding approach, following the phases; familiarization, initial codes, identifying themes, constructing thematic networks and integration and interpretation (Robson & McCartan, 2018) was used to analyze the teacher's observation notes. The notes were summarized in a table. The table provided an overview of the strategies used and thus facilitated the analysis process. The notes were read repeatedly, and key words were highlighted. A deductive approach, based on the assumption that subitizing and counting make use of two different magnitude processing systems (Kaufman et al., 1949; Mandler & Shebo, 1982) were initially used in the coding process. In the next step an inductive coding analysis, where the codes and themes emerged from the data, were conducted. The codes were then sorted into five different categories, showing different strategies used by the children when determining what pairs or trios add up to 5. The categories are described in hierarchical order, reflecting qualitatively different conceptual subitizing abilities and by that knowledge of the part-part-whole relations of number, based on the assumption that conceptual subitizing abilities are closely linked to knowledge of the part-part-whole relations of numbers (Baroody et al., 2006; Clements et al.,2019; Conderman et al., 2014; Piaget, (1997/1952).

Ethical Considerations

The study was guided by ethical principles described by the Swedish Research Council (Swedish Research Council, 2017). The author obtained written informed agreement on collecting data from 18 of the children's parents meaning that no data was collected from the six children for whom an agreement was not provided. Furthermore, the participants' names were replaced with codes to avoid identification. Some of the test- situations were video recorded (see reason in section observations) however, in line with the General Data Protection Regulation (GDPR) (SFS 2018:218), the video recordings were not stored by the researcher and to avoid unnecessary exposure of the children, only their hands were recorded.

Research Results

Children's Knowledge Development

Descriptive statistics regarding children's knowledge development on a group level are shown in tables 4 and 5. Thereafter the individual child's knowledge development is shown in figure 3 and in tables 6 and 7. An increase was evident in the mean values (*M*) (Table 4) among the three tests (Test 1, Test 2 and Test 3). The standard deviation (*SD*) value also increased between tests 1 and 3), indicating that the children's understanding of the part-part-whole relations of numbers differed more after the intervention than beforehand (Table 4).

Table 4
Measures of Central Tendency with NST Scores with Tests 1, 2, and 3

	Test 1	Test 2	Test 3
M	6.9	9.8	10.2
SD	3.5	4.3	4.4

Note. The results are rounded to one decimal place.

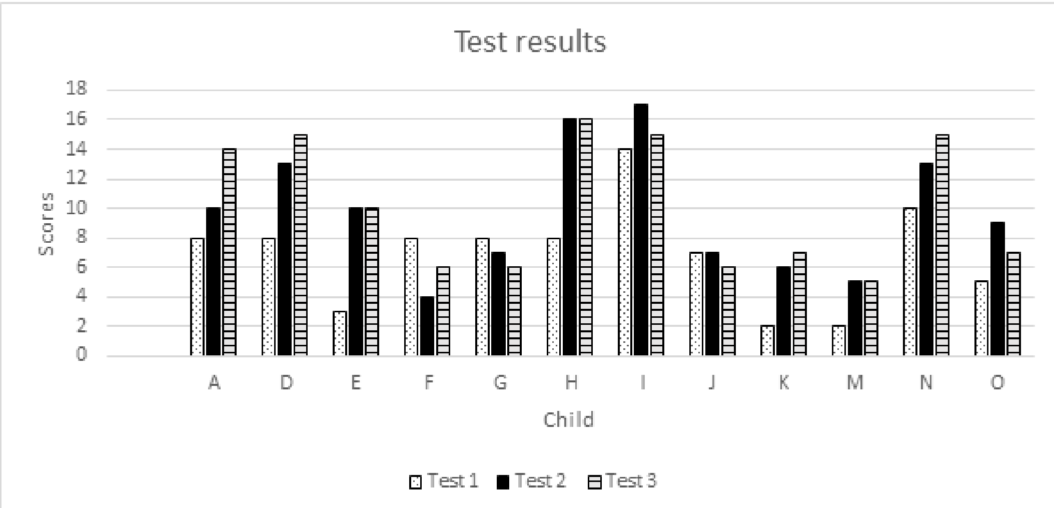
The children's knowledge development between test 1 and test 3 increased with 18.1 percent units (Table 5).

Table 5
Children's Knowledge Development (%)

	Test 1 (max 216)	Test 2 (max 216)	Test 3 (max 216)	Increase percentage points
T1–T2	83 points (38.4%)	117 points (54.2%)		15.8
T2–T3		117 points (54.2%)	122 points (56.5%)	2.3
T1–T3				18.1

Figure 3 presents the children's individual scores on test 1, 2 and 3. Most of the children (9 out of 12) had increased scores in test 2 compared with test 1. The scores between test 2 and 3 varied. Five of the children had increased scores between tests 2 and 3, whereas three children had the same scores.

Figure 3
Test Scores



The increased scores between T1 and T2 may indicate that the children used the faster subitizing magnitude process when they determined which pairs or trios added up to five. The more modest increase of scores between T2 and T3 indicated that their conceptual subitizing competencies only to some extent were further supported in lesson block 2. However, the increased scores between the pre-and post-test suggest that the children used conceptual subitizing to a greater extent for determining quantities after the intervention than they did before. The qualitatively analysis of the children's pre-knowledge resulted in five qualitatively different categories of knowledge regarding the part-part-whole relations of five (Table 6).

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Table 6
Qualitatively Different Ways of Determining what Subgroups Add up to Five

Categories	Strategies
V. Shows knowledge that 5 can be composed of different pairs and trios.	Directly circle all combinations that add up to 5 (5+0, 4+1, 3+2, 2+2+1)
VI. Shows knowledge that 5 can be composed of pairs.	Directly circle all pairs that add up to 5 (5+0, 4+1, 3+2, 2+2+1)
III. Shows initial knowledge that 5 can be composed of subgroups.	Directly circle one or two of the combinations that add up to 5 (e.g., 4+1 and 5+0)
II. Recognizes that the quantities are many more than 5, however do not show knowledge that five can be composed of smaller subgroups.	Count all the individual dots in a set, except quantities of 7, 8, or 9 dots
I. Do not show knowledge that 5 can be composed of smaller subgroups. Discern quantities as lots of ones.	Count all the individual dots, without paying attention to the whole.

The individual child's knowledge development during the intervention regarding the part-part-whole relations of five, based on qualitative analysis of observational field notes from test 1 and test 3 is shown in table 7.

Table 7
The Child's Knowledge Development Regarding the Part-Part-Whole Relations of 5

Child →	A		D		E		F		G		H		I		K		N	
Category ↓	T1	T3	T1	T3	T1	T3	T1	T3	T1	T3	T1	T3	T1	T3	T1	T3	T1	T3
V.		x										x	x	x				
IV.	x			x							x						x	x
III.			x			x										x		
II.							x	x	x	x					x			
I.					x													

Note. Description of the categories (V to I) appears in table 6.

The qualitative analysis showed that four of the children (F, G, I and N) did not develop their knowledge of part-part-whole relations of numbers during the intervention. It should however be noted that child I showed knowledge at the highest level even before the intervention (T1). The qualitative analysis further revealed neither F, nor G understood that quantities can be structured and grouped as a unit. However, they both changed their strategies. In test 1, F counted all the dots except dominos that were many more than 5. In test 2, F instead represented the dots with his/her fingers and then counted the fingers whereas F neither pointed nor used

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the fingers when determining 5 in test 3. However, the decrease in score between tests 1 and 3 may suggest that F still counted the dots separately, but instead of pointing at the individual dots, counted by nodding the head or by counting the individual dots with the eyes. G changed the strategy in a similar way: owing to the decrease in the score, this was interpreted similarly to that observed for child F. Children A, D, E, H, and K all developed their understanding. Both A and H for example circled all pairs that added up to 5 in test 1 (IV); however, in test 3, they also circled trios (V) In test 1, D recognized only the 5+0 (dice 5+0) combinations (III), whereas D distinguished all pairs that added up to 5 in test 3 (IV). E counted every dot in test 1 (I), whereas he/she circled all the 5+0 and 3+2 combinations in test 3 (III). In test 1, K did not count quantities that were many more than 5 (II). But in test 3, K circled 4+1 and 3+2 (though no 5+0 combinations) and one of the trios (III).

Connections between Teaching and Learning

The increased scores between T1and T2 suggest that the activities in lesson block 1 stimulated the children to discern that smaller quantities (2-4) can be directly recognized without using counting strategies. This strategy was beneficial when the children were to determine, as fast as possible, what pairs or trios add up to five. In lesson block 1 different sets of quantities (2, 3 and 4) were contrasted which probably made it possible for them to separate these quantities from each other. Furthermore, the use of different objects may have facilitated the children to discern that quantities of 2, 3 and 4 are the same regardless of objects or configurations. However, the low increase of scores between test 2 and 3 indicate that the activities only to a limited extent supported the children's knowledge development regarding the part-part-whole relations of number five. This may be due to that the number of objects (dots) in the activities varied a lot (between 4 to 7), which may have made it difficult for them specifically discern how five can be composed and decomposed. The teachers' answers showed that the teachers regarded the intervention beneficial for the children's knowledge development regarding part-part-whole relations of numbers—especially among children that performed on a "middle" level in mathematics. Both teachers considered activities where the children described and discussed different ways of structuring and grouping quantities the most beneficial. Both teachers answered that the children liked the activities in the lessons.

Discussion

Despite that perceptual subitizing is an innate ability (e.g. Starkey et. al., 1990), many of the children did not spontaneously subitize smaller quantities. However, due to the assumption that conceptual subitizing abilities build upon perceptual subitizing abilities (Baroody et al.,2006; Clements et al., 2019) it is an important ability to build upon when teaching mathematics in the early years. However, the increased scores between pre-and posttest, indicate that the children used the faster subitizing magnitude process (Mandler & Shebo, 1982; Schleifer & Landerl, 2019), suggesting that the conceptual subitizing activities made it possible for them to discern that quantities (2-4) can be directly recognized. It was evident that many of the children stopped counting in ones and instead subitized quantities of 1-3 objects. One interpretation of why they changed strategy when determining quantities could be that some of the children believed that they had to count all the dots, one at a time. Therefore, it seems to be important to engage the children in activities that stimulate them to directly determine smaller sets of 2-4 objects. Furthermore, the results shed light on the importance of paying attention to children's understanding of number and quantity already in preschool (Aunola et al., 2004; Baroody et al., 2006). The results of this research with preschool class children is consistent with previous research with "first graders" (Sayers et al., 2016), 4-5 year-old children ((MacDonald, 2015;

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MacDonald et al., 2015) and "second-and third graders" (Özdem & Olkun, 2019), showing that conceptual subitizing activities support children's learning of the part-part-whole relations of number (Baroody et al., 2006; Clements, 1999; Conderman et al., 2014). Conceptual subitizing activities provide the children with a ground for understanding that quantities can be (de) composed in different ways which in turn serve as a foundation for "constructing an informal concept of addition (and subtraction)" (Baroody et al., 2006, p.193). The benefits of synthesizing quantitative and qualitative analyses discussed previously (Creswell & Creswell, 2018; Johnson & Onwuegbuzie, 2004) were evident in the present study. The children's quantitative scores on a test increased at the group level—even though not all the children increased their scores. The analysis of the qualitative data revealed that more than half the children (five out of nine) developed their knowledge of the part-part-whole relations of number (i.e. 5). However, two of the children showed no knowledge that five can be composed of smaller subgroups. If the data only had been analyzed quantitative, at the group level, individual children who experienced difficulties would have been missed, which according to Morgan et al., (2009) is alarming since children who experienced difficulties in mathematics in Kindergarten often showed low mathematics skill growth throughout elementary school.

Conclusions and Implications

The aim of this study was to explore the role of conceptual subitizing activities for enhancing preschool class children's learning of the part-part- whole relations of numbers. The research questions to be answered were: what knowledge regarding the part-part-whole relations of numbers did the children develop, and connections found between the children's knowledge development and the design of the conceptual subitizing activities. The results showed that conceptual subitizing activities supported children's knowledge development regarding part-part-whole relations of number. The quantitative analysis showed that the children increased their scores between pre-and post-test and more than half of the children showed conceptual subitizing abilities in a qualitatively more developed way after having participated in the intervention. According to the teachers, activities, whereby the children were stimulated to organize and structure quantities in smaller identifiable subgroups supported the children's knowledge development regarding part-part-whole relations of numbers. There are still some questions to be answered, and by that suggestions for further research. Since the results elucidated that not all children improved their understanding of the part-part -whole relations of number, future research should consider individual differences when developing and carrying out interventions to support the individual child's development, based on his/her needs.

Limitations

The findings of this study provide knowledge of how to support 6-year-old children's number knowledge development. However, the results may not be generalized owing to the small sample and due to the lack of comparison or control groups. Further, it was not clear how representative the present sample was; hence, the risk of selection bias and sampling errors was high (Bryman, 2011). In respect of the Hawthorne effect (McCambridge et al., 2014) the children may have changed their behavior in the test due to the received attention rather than because of the teaching activities. The validity of the results is though strengthened by the qualitative and quantitative results having consistently indicated a developed knowledge of the part-part-whole relations of number (five) for most of the children. That point was also confirmed by the teachers' experiences. Owing to the small number of participants it was however possible to explore the individual child's knowledge development regarding part-

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part-whole relations of five during the intervention thoroughly. The qualitative analysis was mainly based on direct observations. More frequent, systematic use of video recordings would have allowed more detailed analyses—even beyond the intervention period. To obtain in-depth knowledge of how children perceive quantities, interviews in which the children describe their strategies in conjunction with eye-tracking data on eye-movement patterns would be beneficial.

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