

The Investigation of Middle School Student Learning Difficulties and Concept Misunderstandings in Multipliers and Factorization

Harun Dogrucan, Danyal Soybas, and Sevim Sevgi

Abstract

The aim of this study was to determine learning difficulties and misunderstanding in multipliers and factors of middle school students in Kayseri, Turkey. One hundred and seven students from 6th grade and 48 students from 8th grade were selected randomly from three middle schools for the study. A questionnaire, which was developed by the first researcher, including two open-ended questions, eleven multiple choice questions, and one true-false question was prepared for each group. Students solved each question and explained how they solved them. Also, it was requested verbally to fill in the solutions. Besides these questions, it was requested from the students to answer how they use the subjects of multiplication and factors in their daily lives and how these subjects affected their attitudes and treatments towards mathematics. According to the evaluations of this research, some suggestions were made for students' learning difficulties and concept misunderstanding about multipliers and factors.

Keywords: multipliers, factorization, concept, misconception, learning difficulties.

Harun Dogrucan is a mathematics teacher at a public school in Kayseri, Turkey. He works at the Ministry of National Education. He can be reached at hnrnstf8414@gmail.com

Danyal Soybas is a professor in the Department of Mathematics and Science Education at Erciyes University, Turkey. He can be reached at danyal@erciyes.edu.tr

Sevim Sevgi is an assistant professor in the Department of Mathematics and Science Education at Erciyes University, Turkey. She can be reached at sevimsevgi@erciyes.edu.tr

Mathematics has a very wide range and includes interdisciplinary study; so mathematics in one discipline is closely related to many other disciplines (Baki, 2004). Learning mathematics helps students to get to know themselves, while at the same time preserving their life happenings. In addition, mathematics prepares them for the future by providing readiness to life, directs human life and helps people to adapt to the society in which they live (Bryk & Treisman, 2010). There is at least one target in each system that is intended to be achieved. From this point of view, the goals of mathematical instructional systems are the behaviors that are desired and planned to be gained by the students (Sönmez, 1998). Today, mathematics teaching is given regularly in schools, as well as in the natural environment of human beings. Also, teaching mathematics is a tool that makes our life easier. As a result of planned and tailored mathematics teaching activities, students can get to know life and be predictive about life.

There are many definitions of mathematics. It is much more useful to try to understand these definitions and not to see them as a kind of explanation (Baykul, 2011). It can be thought that the most important factor that separates mathematics from other courses is abstract structure. The concept is generally defined as a form of social and environmental factors that people imagine as objects and perceptions. Being able to make sense of objects and events in the mind can vary from person to person. Mathematical concept is defined as to categorize similar objects, people or situations (Senemoglu, 2005). While mathematic concepts come to life in our minds, it is also important that we interpret the actual situations in our minds. Interpreting these situations can also be related to our life. By nature, students are intertwined with social life. As a result of these experiences, interpreting events creates a reaction against them. Sometimes the situation in which students understand life and their life may not overlap. These situations may cause some difficulties in students' minds. The differences between the real situation and the situation played by the students in the teaching of mathematics can cause misconceptions. One of the biggest

problems that we encounter in teaching mathematics is misconceptions. Because misconceptions are situations that we accept correctly, but they do not exactly match reality. There are individual differences between students and these differences can lead them to learn in different ways. The use of appropriate methods and techniques in the learning of concepts can also reduce misconceptions that may occur in the student (Ozkan, 2017). Misconceptions may arise from the experiences and experiences of students in the past (Altınyüzük, 2008). Misconceptions are an important obstacle to real learning. Since misconceptions are accepted by the students, they do not try to correct them. More precisely, they are not aware that this information that they know is wrong. Misconceptions may be due to certain reasons. We can list some of these reasons as follows:

- Inability of the student to fully control the subject and to ensure the integrity of the meaning,
- The teacher is found to be antipathic by the students,
- The student's life causes him to perceive the subject differently,
- Not trying to fully understand the subject and not motivating the subject, and the
- Mathematics teacher also has misconceptions.

Students acquire many mathematical skills, and these skills are the product of learning (Erden, 2008). Mathematical learning can also be unconscious in some cases. To be able to realize mathematical learning because of events that come across during human life. The mathematical learning process is to create an activity or change a situation by reacting against a situation we encounter (Alkan & Kurt, 2007). Mathematical learning can also take place after this reaction.

Some Methods and Techniques Used in Mathematics Teaching

Many methods can be used for more permanent and effective learning in education. The purpose of these learning methods is the whole of the mental plans and methods used to realize

learning (Duman, 2009). Individual differences reflect and reveal one's own characteristics (Bacanli, 2012). In mathematics teaching, students can have a way of telling according to their understanding.

Method of instruction is a way that can be used when starting the course, summarizing long topics to large audiences or transferring information directly on any subject. It is generally transmitted through presentations (Demirel & Şahinel, 2006). Questioning method is based on the student's answers to the teacher's questions (Kuçukahmet, 1998). In this method, the teacher is the active leader of the class. The teacher should have a good command of the subject, know what (s)he is going to ask and should not ask questions that are beyond the purpose of the subject, disseminate the subject, nor bother the students. Teaching by invention is a method in learning through invention; the teacher is not in the center and the teacher does not give the concepts and principles directly to the student. Instead, the teacher helps students to create a beautiful learning environment for the student to access information (Baykul, 2009). Teaching by presentation is very superior to the method of plain expression. The student is active in this method. The teacher gives the student information systematically, and the student reaches a conclusion by establishing a relationship between what (s)he has learned and what (s)he will learn. In this method, the teacher has responsibilities. At the same time, the teacher must also have a superior rhetoric power here and should not bore students while presenting information. As a result, techniques and methods are used in teaching; mainly that the teachers are leaders of the class and the subject and give themselves to this task.

Elvan (2012) used a semi-experimental design model with pretest-posttest control group in a study with 57 students in total. He stated that the use of worksheets is beneficial, increases the participation of students in the lesson and is effective in reducing misconceptions. Başün (2016) carried out his study in a school in Samsun with a total of 42 students with two groups of

21 each. Results of the study demonstrate that there was a significant mean difference between the experimental and the control group students about the multipliers and factorization, and the subject of multipliers and factorization in the group taught by the game yielded more successful results and had higher averages than the group described by the appropriate teaching.

Furthermore, Damli (2011) utilized a pretest-posttest semi-experimental research model with experimental-control group in the research. He concluded that web-based interactive teaching is effective in eliminating misconceptions.

Research Problem

Mathematics is a science that helps people to think analytically in daily life and find practical solutions to the problems they face. For this reason, it is a science, which needs to be conveyed to students in a basic and regular way throughout their educational life. One of the biggest problems that teachers face in teaching mathematics is misconceptions. Multipliers and factorization are explained to students at the elementary level and related to other subjects, as in all other learning areas in mathematics. In this research, multipliers and factorization as well as misconceptions about this topic and learning difficulties were investigated and some solutions were proposed.

Aim of Study

Today, deficiencies in mathematics textbooks, the physical inadequacy of the classrooms, and reasons stemming from the teachers affect mathematics achievement in a negative way. In addition, the fact that the subjects in mathematics are related to each other and that the intelligibility of each subject affects other subjects also increases the importance of this course. In today's national education system, middle school is the time in which the foundation of mathematics is laid, and students begin to gain their mathematical skills. Multipliers and Factorization is one of the important issues that can be considered as the basis of many subjects

in mathematics and should be well understood. For this reason, Multipliers and Factorization should be explained thoroughly, and no misconceptions should be given. In this research, students' misconceptions about Multipliers and Factorization and learning difficulties caused by this were determined and the measures and solutions that can be taken against them were defined.

Research Questions

Multipliers and factorization are taught in both the sixth and eighth grade in middle school.

Objectives of this subject can be listed as follows:

- Determine the multipliers of natural numbers.
- Determine the factorization with natural numbers.
- Determine the common divisors of two natural numbers and their common multiples; solve related problems.
- Calculate the largest common divisor and the smallest common multiplier of two natural numbers; solve related problems (MoNE, 2013).

Limitations of this study are:

- The research was carried out in three middle schools in Kayseri, Turkey.
- The research was carried out in the spring semester of the 2017-2018 academic year.
- The research was carried out with 48 middle school students in 8th grade and 107 middle school students in 6th grade for a total 155 middle school students.

Hypotheses of this study are:

- Middle school students at the sixth grade have learning difficulties and misconceptions about Multipliers and Factorization.
- Middle school students at the eighth grade have learning difficulties and misconceptions about Multipliers and Factorization.

Assumptions of the study are:

- The students were not affected by each other during the administration of questions,
- Students were not affected by their mathematics teachers during the administration of questions,
- All students were taught in accordance with the middle school mathematics curriculum,
- All students willingly responded to the questions,
- All students answered the questions voluntarily,
- Physical conditions for the students were made suitable during the research.

Method

This is a descriptive study from general screening models since it is aimed to determine learning difficulties and misconceptions about multiples and factorization. In the descriptive method, the aim is to define and make a quantitative description of a situation or event that happened in the past or present (Fraenkel, Wallen, & Hyun, 2011).

Subject Sampling

When conducting research, all of the elements that are required to generalize the results can be called universal (Karasar, 2009). The population of this research consisted of sixth and eighth grade students in a middle school. The sample consisted of 48 eighth grade students and 107 sixth grade students in the three middle schools in the Melikgazi of Kayseri, Turkey. The students participating in the research were administered a test including multiple choice, true, false, open-ended questions and their views on multipliers and factorization.

Instruments

While conducting this research, tests were used to collect data and the opinions of mathematics teachers were taken into consideration. In addition, questions were prepared with high validity and reliability. In this research, a test was prepared at the 6th and 8th grade level and

these tests were prepared in a way that the students could answer in one hour. At the same time, all the situations in which the students would be negatively affected were considered and the effects of these situations were eliminated. In addition, before the test was applied to the students, general information about the research was given and the importance and purpose of the research was explained to the students and suggestions were made to give confidentiality of answers. These explanations made us think that the students would give realistic answers.

Analysis of Data

In this research, the Microsoft Office Excel program was used to analyze the test given to students. Frequencies, percentages, and standard deviations of the questions in the tests were calculated. By considering these data, all questions were interpreted one by one and misconceptions about multipliers and factorization were determined.

Results

Sixth Grade Questions and Answers

The research questions and the analysis of the answers given by the 6th grade students are as follows. The first question is given in Figure 1.

1) Enter all the multipliers of the numbers given below.	
I. 6	_____→
II. 12	_____→
III. 15	_____→
IV. 20	_____→
Explanation:	

Figure 1. Question 1 of the Test

Table 1. The analysis of the answers given by 6th grade students to the first question.

Options	0	1	2	3	4	5	6	Right Answer	SD
	Points	Point	Points	Points	Points	Points	Points	%	
1a	3	3	4	10	87	-	-	81.31	36.79
1b	2	3	3	2	3	12	82	76.64	29.63
1c	2	5	2	13	85	-	-	79.44	35.84
1d	2	4	4	6	12	10	69	64.49	23.95

The comprehensibility of the students' multiplier issue was tested with the first question. In the first question, the prime factors of the number six were asked, and the students were expected to answer 1, 2, 3, 6. The number of students who found all four factors was 87, the number of students who answered three of factors correctly was 10, the number of students who found two factors was four, the number of students who found one factor was three, while the number of students who could not find any factor was three. The proportion of students who answered all the factors correctly corresponds to 81.3%. In 1b, students were asked to supply multipliers of 12 (the numbers 1, 2, 3, 4, 6, 12), and 82 of them answered all the factors correctly. Twelve students gave five correct answers. There were three students with four correct factors, two students with three correct factors, three students with two correct factors, three students with one correct factor, two students who could not find any factor. The proportion of students who answered all correctly was 76.6%. In the question of multipliers of 15 in 1c, students should have answered 1, 3, 5, 15. Eighty-five students found all the factors. Thirteen students answered three factors, two students answered two factors, five students answered one factor correctly, while two students could not find any factor. The ratio of students who

answered all correctly was 79.4%. In 1d, students were asked to find the multipliers of the number 20 (1, 2, 4, 5, 10, 20). The number of students who answered all correctly was 69. There were 10 students with five factors, 12 students with four factors, six students with three factors, four students with two factors, and four students with one factor. The number of students who could not find a multiplier was two. The proportion of students who answered all correctly was 64.5%. When the results were examined for the first question, it was observed that the number of students who answered all the answers correctly was less than the other questions.

Question 2 for the sixth graders is in Figure 2 and the answer distribution of this question is in Table 2.

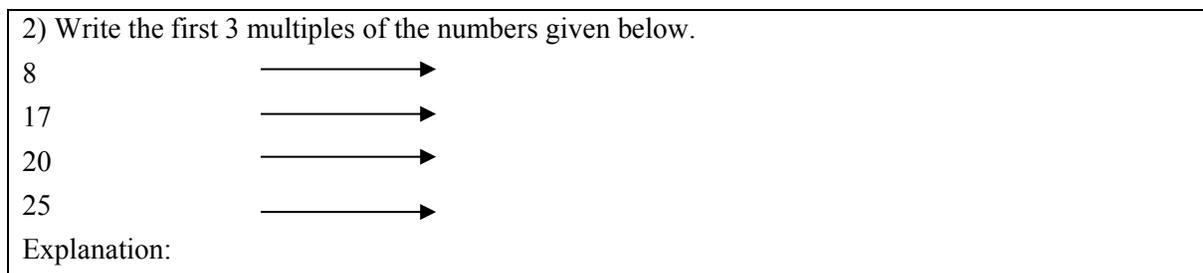


Figure 2. Question 2 of the test

Table 1. The analysis of the answers given by the sixth grade students to the second question

Options	0 Points	1 Point	2 Points	3 Points	Right Answer Percentage	SD
2a	5	13	7	82	76.64	36.99
2b	5	16	11	75	70.09	32.48
2c	8	11	8	80	74.77	35.53
2d	5	14	11	77	71.96	33.71

In the second question, students were given some numbers and asked to find the first three multiples of these numbers. In 2a, while the number of students who found the first three

levels was 82, seven students with two factors, 13 students with one factor, five students did not answer the question at all. 76.6 percent of students answered all questions correctly. This is quite a high rate for students. In 2b, 75 students answered all factors, 11 students answered two factors, 16 students answered one factor, and five students did not answer the question. The percentage of students who answered all the factors correctly was 70.1%. In this question, it was observed that some students understood the logic of the question but made a transaction error while multiplying the number 17. This shows that students should know the multiplication at the level of multipliers and factorization and that mathematics is an additive course and the lack of a subject affects other topics. In 2c, the first three factors of the number 20 were asked, and the number of students who answered all correctly was 80. The number of students with two factors was eight, the number of students with one factor was 11, and the number of students with no factor answer was 8. 74.8% of the students answered the first three factors correctly. In 2d, the number of students who answered the first three factors of the number 25 was 77, the number of students who responded with two factors was 11, the number of students who answered one factor correctly was 14, and five students left the question blank. The percentage of students who found all was 72%. In general, when the first two questions were examined, it was observed that the middle school students' comprehension of multiples and multiples of numbers is close.

The questions 3 to 10 are given in Table 3. The answer distribution to these questions is in Table 4.

Table 3. Questions 3 to 10

3) What is the smallest multiple of 12 that is greater than 50? A)50 B)51 C)59 D)50 Solution and Explanation:
4) What is the maximum multiple of 15 that is less than 60? A)45 B)59 C)60 D)75 Solution and Explanation:
5) Which of the following numbers is the common multiplier of 15 and 20? A)2 B)3 C)4 D)5 Solution and Explanation:
6) Which of the following numbers is the common multiplier of all numbers? A)1 B)2 C)3 D)4 Solution and Explanation:
7) Which of the following numbers is not a multiple of 50? A)2 B)3 C)5 D)10 Solution and Explanation:
8) Which of the following is not a multiple of 13? A)26 B)39 C)52 D)64 Solution and Explanation:
9) How many times does the number 7 go into between 80 and 90? A)1 B)2 C)3 D)4 Solution and Explanation:
10) {1, 2, 3, 4, 6, 12} What is the number given above? A)12 B)15 C)18 D)24 Solution and Explanation:

In the third question, the students were asked for the multiples of the number, while four of them marked A, five of them marked B, and four of marked C, while 94 marked the correct answer option D and no student left the question blank. When we convert the data to percentages, 3.7% of the students participating in the study marked A, 4.7% marked B, 3.7% marked C, while 87.9% answered the question correctly. As this question was one of the introductory questions of multipliers and factorization, it was generally well understood and the correct answer rate was high. It is believed that the students who marked 51 were approaching with the first number logic exceeding 50 and responded incorrectly because they did not read the question carefully. The fourth question, was asked to be solved with a logic like the third question. The correct answer, option A, was chosen by 88 students, 6 of them marked B, 5 of

them marked C, 6 students marked D, and 2 students left the question blank. When we convert the data to percentages, 82.2% of the students participating in the study marked A, 5.6% marked B, 4.7% marked C, and 5.6% marked D. When the answers of the students are examined, it is observed that the correct answers to the third question and this question are close to each other, and a large part of the students answered the question correctly. It is thought that the students who marked the number 59 in the fourth question were wrong because of the expression "less than 60". In the fifth question, students were asked to find common divisors of numbers. When the answers given by the students were examined, it was seen that none of the students marked A and C options. Two students marked B, 104 students marked D, which is the correct answer, and 1 student did not answer the question. When the data is converted to a percentage, 1.9% answered B, 97.2% answered D, while 0.9% did not answer the question. It is thought that most of the students gave the correct answer to the fifth question because it was well understood, and it is an easy question. In the sixth question, the students were asked whether they know the rule that one divides all numbers. When the answers were analyzed, it was seen that 99 students marked option A, 5 students marked option B, one student marked option C, and two students marked option D. When we convert the data to percentages, it was observed that option A was chosen by 92.5%, option B was chosen by 4.7%, option C was chosen by 0.9%, and option D was chosen by 1.9%. It is thought that the number of correct answers to this question is because the fact that one divides all of the numbers is well understood. In the seventh question, the students were asked to find the factors of the number and the answers given by the students were as follows. Six students chose option A, 97 students chose option B, three students chose option D, and one student did not answer the question. This is to say that 5.6% chose option A, 90.7% chose option B, 2.8% chose option D, and 0.9% of students did not give any answer. In the eighth question, the multipliers objective of the sixth grade was tested and when the answers

given by the students were examined, it was seen that only one student marked option A, three students marked option B, four students marked option C, and 99 students marked option D. When we convert the number of options marked to percentage, students chose option A by 0.9%, option B by 2.8%, option C by 3.7%, and option D by 92.5%. The fact that the rate of correct answers to this question is high is thought to stem from the problem. In the ninth question, students were asked about the multiple of the number and this number was limited to a strict range. When the answers given by the students to the question were examined, it was observed that 86 students marked option A, five students marked option B, 10 students marked C option, three students marked D option, while three students did not answer the question. When we convert the data to percentages, 80.4% chose option A, 4.7% chose option B, 9.3% chose option C, and 2.8% chose option D, while 2.8% of students did not give any answer to the question. In the tenth question, the multipliers of the number were given to the students, and the number with these multipliers was asked. When the answers given by the students to the question were examined, it was observed that 94 students marked A, three students marked C, six students marked D, while four students did not answer the question. When we turn the data into a percentage, it is seen that 87.9% of students chose A, 2.8% chose option C, and 5.6% of students chose D, while 3.7% of students did not answer any questions. The reason why the D option was the most frequent wrong answer is that students think that the elements of this set belong to 24, and they do not realize what the number must be in order to be 24.

Table 4. The analysis of the answers given for the 3rd to 10th questions by the 6th grade students

Question	Option A	Option B	Option C	Option D	Empty	Right Answer %	SD
3	4	5	4	94*	0	87.85	40.63
4	88*	6	5	6	2	82.24	37.27
5	0	2	0	104*	1	97.20	46.18
6	99*	5	1	2	0	92.52	43.42
7	6	97*	0	3	1	90.65	42.32
8	1	3	4	99*	0	92.52	43.41
9	86*	5	10	3	3	80.37	36.23
10	94*	0	3	6	4	87.85	40.64

Question 11 is given in Figure 3. Answers to this question 11 is in Table 5.

- 11) Write R at the beginning of each statement for the correct statements and W for the incorrect statements.
- () The number 2 is a multiplier of the number 10.
- () The number 5 is a divisor of the number 15.
- () Each number is its own multiplier.
- () The number 1 is a divisor for each number.
- () 2 is the divisor of all even numbers.
- () 3 is the divisor of all odd numbers.

Figure 3. Question 11

In the eleventh question, the students were expected to answer the right and wrong questions about Multipliers and Factorization. When the answers given by the students are

Students were asked to find the multipliers of 2,500. When the students' responses to the question were examined, it was observed that five students marked A, three students marked B, 90 students marked C, five students marked D, and four students did not answer the question. When we turn the data into percentages, it is seen that 4.7% chose option A, 2.8% chose option B, 84.1% chose option C, 4.7% chose option D, while 3.7% of the students did not give any answer to the question. In the thirteenth question, the question of the multipliers of the number was asked of the students. When the students' responses to the question were examined, it was observed that six students marked A, four students marked B, 95 students marked C, and two students did not answer the question. When we turn the data into percentages, it is seen that students marked A by 5.6%, B by 3.7%, and C by 88.8%, while 1.9% of the students did not answer any questions. In the fourteenth question, the question of multipliers of the number was asked to the students as a problem. When the answers given by the students to the question are examined, it is seen that one student marked A, 98 students marked B, two students marked C, and four students marked D. It was observed that two students did not answer the question. When we convert this data to percentages, students choose A as 0.9%, B as 91.6%, C as 1.9%, and D as 3.7%. 1.9% of the students did not give any answer to the question.

The analysis of the answers given by 6th grade students to questions 12 to 14 are given at Table 6.

Table 6. The analysis of the answers given by 6th grade students to questions 12 to 14

Question	Option A	Option B	Option C	Option D	Empty	Right Answer %	SD
12	5	3	90*	5	4	84.11	38.36
13	6	4	95*	0	2	88.79	41.20
14	1	98*	2	4	2	91.59	42.83

When the questions were considered, it was thought that multipliers and multiples are generally understood at the 6th grade level. The high percentage of respondents who answered correctly in most of the questions is the biggest indicator of this. A good understanding of this issue is thought to affect other issues. This is because mathematics is a course of interrelated subjects and forms the basis of many topics in multipliers and multiples.

At the end of all questions, the students were asked to interpret this topic in order to measure whether the logic of this subject was understood, and they were asked a question, "After you have been told by your teacher about Multipliers and Factorization, you can relate this subject to what you have in your lives and how do you think this subject affects your attitude and behavior towards mathematics lesson?" An example of the responses received from the students is as follows in Figure 5.

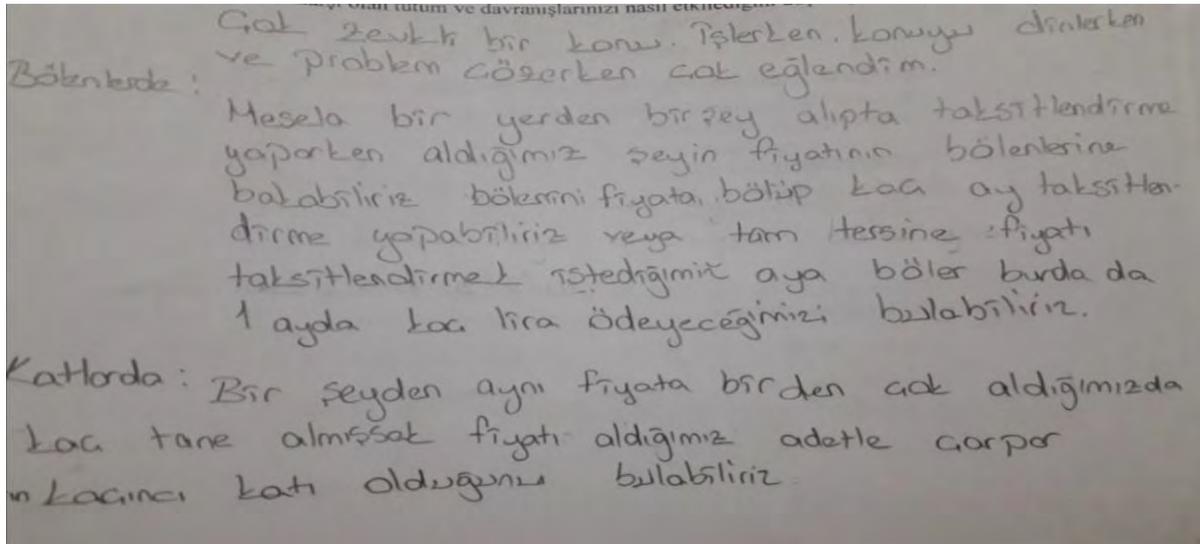


Figure 5. The example response received by the 6th grade middle school students on the multipliers and factorization

Eighth Grade Questions and Answers

The questions asked to the 8th grade students and their analysis are as follows. The first question is given in Figure 6. Eighth grade middle school students' answers to the first question are given in Table 7. In the first question, tested the comprehensibility of the Multipliers of Numbers subject which was explained to the students. Although the numbers in the four options are different from each other, they are questions to be solved with the same solution strategy. Since the number 23 in 1a is the prime number, it is only the factor of 1 and the number itself. The number of students who found these two multipliers was 35 students. The number of students who found only one of them was four, and the number of students who could not find any factor was nine. Furthermore, 72.9% of students wrote 2 multipliers. In 1b, when the answers given by the students to the factors of the number 16 were examined, the number of students who found all the factors of 1, 2, 4, 8, 16 were 34 students, four students wrote four factors, and two students found five factors. Eight students could not find any of the multipliers. The proportion of those who answered all correctly was 70.8%. When students' responses to the

factors of 25 were examined in 1c, 35 students found all the factors of 1, 5 and 25. Two students found two factors, and one student found one factor. Ten of the students could not find any factor. The percentage of students who found it all was 72.9%. In 1d, when the answers given by students were examined, 24 students found all the factors of 1, 2, 3, 4, 6, and 12. Six students found five, eight students four, and one student found one. There were nine students who did not answer any questions. This means that 50% of students found all of the multipliers correctly. When the answers given to these four questions are examined, it is understood from the high percentages that the subject is generally understood.

1) Enter all the multipliers of the numbers given below.	
I. 23	_____→
II. 16	_____→
III. 25	_____→
IV. 12	_____→
Explanation:	

Figure 6. Question 1

Table 7. The analysis of the answers given by the 8th grade students to the first question.

Options	0 Point	1 Point	2 Points	3 Points	4 Points	5 Points	6 Points	Right Answer %	SD
I.	9	4	35	-	-	-	-	72.92	16.64
II.	8	2	0	0	4	34	-	70.83	13.08
III.	10	1	2	35	-	-	-	72.92	15.85
IV.	9	1	0	0	8	6	24	50.00	8.45

The second question, which was administered to eighth grade middle school students is given in Figure 7.

2) Write the first 4 times of the numbers given below.	
13	—————▶
15	—————▶
17	—————▶
20	—————▶
Explanation:	

Figure 7. Second question of test

The answers of the eighth grade middle school students to the second question is in Table 8. In the second question, the comprehensibility of the multiples was tested. In 2a, the number of students who answered the first four multipliers of the number 13 correctly was 21, the number of students who answered the three correctly was three, the number of students who answered two correctly was three, and the number of students who answered one correctly was 13. The number of students who did not answer the question was eight. In this question, the percentage of students who answered all correctly was 43.8%. In 2b, the number of students who answered the first four times of the number 15 correctly was 20, the number of students who answered three correctly was four, the number of students who answered two correctly was three, the number of students who answered one correctly was nine, and the number of students who could not answer the question was 12. In this question, the percentage of students who answered all correctly was found to be 41.7%. When we look at 2c, the number of students who answered the four correctly was 18, the number of students who answered three correctly was six, the number of students who answered two correctly was three, the number of students who answered one correctly was eight, and the number of students who did not answer the question was 13. In this question, the percentage of students who answered all correctly was 37.5%. It is thought that the reason for the low number of correct answers to this question is because the number 17 is less common and less familiar than the other numbers. When we look at 2d, the number of students who answered the first four times of 20 correctly was 21, the number of students who answered three correctly was

four, the number of students who knew two correctly was two, the number of students who answered one correctly was nine, and the number of students who did not answer the question was 12. In this question, the ratio of students who answered all correctly was 43.8%.

Table 8. The analysis of the answers given by the 8th grade students to the second question.

Options	0 Point	1 Point	2 Points	3 Points	4 Points	5 Points	Right Answer %	SD
2a	8		13	3	3	21	43.75	7.60
2b	12		9	3	4	20	41.67	6.88
2c	13		8	3	6	18	37.50	5.94
2d	12		9	2	4	21	43.75	7.50

In general, when the first two questions were examined, it was observed that the middle school students' comprehension of multiples and multiples of numbers is close. Questions 3 to 10 are given in Table 9. The analysis of the answers by 8th grade students given to questions 3 to 10 were given at Table 10.

Table 9. Questions 3 to 10

3) What is the smallest multiple of number 13 greater than 50? A) 50 B) 51 C) 52 D) 53 Solution and explanation:
4) What is the largest multiple of 17 below 60? A)45 B) 50 C) 51 D) 52 Solution and explanation:
5) Which of the following numbers is the common multiplier of 18 and 28? A) 2 B) 3 C) 4 D) 5 Solution and explanation:
6) Which of the following numbers is the common multiplier of all numbers? A)1 B) 2 C) 3 D) 4 Solution and explanation:
7) Which number is not a multiple of 100? A) 2 B) 3 C) 5 D) 10 Solution and explanation:
8) 102:a is an integer. What is a? A) 2 B) 4 C) 8 D) 12 Solution and explanation:
9) How many common multipliers do the numbers 16 and 20 have? A) 1 B) 2 C) 3 D) 4 Solution and explanation:

In the third question, the situation of determining the multiples was measured. In this question, two students selected A, six students selected B, 32 students selected C, three students selected D, and five students preferred not to answer the question. When we convert the data to percentage, option A was chosen by 4.2%, option B by 12.5%, the correct answer was C with 66.7%, and option D was 6.3%, while the percentage of students who did not respond was 10.4%. The fact that the correct markers were 66.7% indicated that the problem was generally well understood. It was thought that the most marked option B, after the correct answer, was chosen because students marked the “the number greater than 50 is 51” without thinking carefully about the question. In the fourth question, the situation of determining the multiples of the students was determined. In this question, two of the students marked A, five of them B, 34 of them C, five of them chose D, and two students chose not to answer the question. When we convert the data to percentage, option A corresponds to 4.2%, option B to 10.4%, option C to

70.8%, and option D to 10.4%, while the percentage of students who did not respond is 4.2%. Another feature that draws attention to this question is that the problem was similar to the previous question and the ratio of those who gave the correct answer was close, which might give clues about the reliability of the research. In the fifth question, students were asked to determine the multipliers of the numbers and find the common ones. In this question, 41 students marked A, two students B, four students C, and one student D selected. When we turn the data into a percentage, it is seen that the correct answer was 85.4%, B was 4.2%, C was 8.3%, and D was 2.1%. The fact that 85.4% of the answers were correct shows that the problem was generally understood. In the sixth question, students demonstrated their skills to find divisors of numbers. In this question, 41 students marked A, four students marked B, two students marked C, and one student selected D. When we convert the data to percentages, students marked the correct answer option A at 85.4%, option B at 8.3%, option C at 4.2%, and D at 2.1%. The fact that 85.4% of the answers were correct shows that the problem was generally understood. In the seventh question, students were directed to find divisors of numbers. In this question, one student marked A, 38 students marked B, two students marked C, and seven students selected D. When we convert the data to percentages, option A was 2.1% (correct answer), option B was 79.2%, option C was 4.2% and option D was 14.6%. The fact that 79.2% answered correctly indicates that the problem was generally understood. In the eighth question, the students were asked to find the divisors of the numbers. In this question, 26 students marked A, two students B, one student C, 10 students selected D, and nine students preferred not to answer the question. When we turn the data into a percentages, it showed that the students marked the correct answer option A at the rate of 54.2%, option B at 4.2%, option C at 2.1% and option D at 20.8%, while the percentage of students who did not answer was 18.8%. Although the question is not a difficult question, the way the question is asked indirectly may mislead the students. In the ninth question, it was aimed

to find the divisors of the students and to grasp the common ones. In this question, six students marked A, 21 students marked B, 17 students selected C, and three students was selected D, and one student did not choose to answer the question. When we convert the data to percentages, option A was 12.5%, option B was 43.8%, the correct answer was 35.4%, option D was 6.3%, while the percentage of students who did not answer was 2.1%. Although the students who gave wrong answers from the students did not give much explanation, when the papers were examined, it is thought that the most wrong answer was 2. Because both numbers were double, this might have caused an error for the students, or the students may have found the numbers two and four and forgot the number one, which was generally overlooked.

Table 10. The analysis of the answers by 8th grade students given to questions 3 to 9

Question	Option A	Option B	Option C	Option D	Empty	Right Answer %	SD
3	2	6	32*	3	5	66.67	12.62
4	2	5	34*	5	2	70.83	13.72
5	41*	2	4	1	0	85.42	17.62
6	41*	4	2	1	0	85.42	17.62
7	1	38*	2	7		0	79.17
8	26*	2	1	10	9	54.17	10.01
9	6	21	17*	3	1	43.75	8.88

10) Write R at the beginning of each statement for the correct statements and W for the incorrect statements.

() The number 3 is the multiplier of the number 33.

() The number 7 is the divisor of the number 49.

() Each number is its multiplier.

() The number 1 is the divisor of each number.

() 2 is the divisor of all even numbers.

() 3 is the divisor of all odd numbers.

Figure 6. Question 10

Table 11. The analysis of the 8th grade students' answers to the 10th question

	0 Point	1 Point	2 Points	3 Points	4 Points	5 Points	6 Points	Right Answer %	SD
10	0	0	0	4	6	20	18	41.67	8.63

Question 10 is given in Figure 6. The analysis of the 8th grade students' answers to the 10th question were given at Table 11. In the tenth question, six questions were asked to the students about the factors and multiples and they were asked to write R (right) next to the questions they thought were correct and W (wrong) next to the questions they thought were wrong. The number of students who answered all questions correctly was 18, the number of students who made five correctly was 20, the number of students who made four correct was six, and four of the students who made three correct. In this question, the ratio of students who answered all questions correctly was 37.5%.

<p>11) Ahmet paid 2000 TL for a television. He wants to pay in equal installments. How many TL can Ahmet pay?</p> <p>A)150 B)200 C)300 D)350</p> <p>Solution and Explanation:</p>
<p>12) How many students can be in a class that can be divided into groups of 2, 3 and 5?</p> <p>A)15 B)20 C)25 D)30</p> <p>Solution and Explanation:</p>
<p>13) Ali has 27 walnuts. Ali cannot separate the walnuts into groups?</p> <p>A)1 B)3 C)9 D)18</p> <p>Solution and Explanation:</p>
<p>14) It is desired to put 15, 20 and 25-liter milk into bottles of equal volume without mixing them together. How many liters can the bottle be used for this?</p> <p>A)2 B)3 C)4 D)5</p> <p>Solution and Explanation:</p>

Figure 7. Questions 11 to 14

Questions 11 to 14 is given in Figure 7. The analysis of the 8th grade students' answers to the 11-14th questions are given at Table 12. The eleventh question aimed to get students to divide the divisors and to grasp the common ones. In this question, five students selected A, 37 students selected B, two students selected C, one student selected D, and three students preferred not to answer the question. When we turn the data into percentages, 10.4% represent option A, 77.1% represent option B, 77.1% represent option C, 4.2% represent option D, and the percentage of students who did not respond was 6.3%. In this problem, the divisors of the number 2000 were tried to be asked in the form of a problem and the fact that many of the students gave the correct answer indicated that this objective was understood. In the twelfth question, two students marked A, three students marked B, one student marked C, 42 students selected D, and all of the students answered the question. When we convert the data to percentages, students answered option A at

4.2%, option B at 6.3%, option C at 4.2%, and option D at 87.5%. In this question, the students were asked the common multipliers question and most of them answered the question correctly.

The thirteenth question was aimed at getting students to divide the divisors and to grasp the common ones. In this question, two students marked A, five students marked B, five students marked C, 36 students selected D, and all students answered the question. When we converted the data to a percentage, students answered option A at 4.2%, option B at 10.4%, option C at 10.4%, and option D at 75%. On the answer sheets of the students, the least wrong answer was 1, "The number 1 is the multiplier of each number." This was thought to be due to the question being well understood.

In the fourteenth question, the students were directed to find the divisors of the numbers and to grasp the common ones. In this question, three students marked A, one student marked B, six students marked C, 36 students selected D, and two students preferred not to answer the question. When we convert the data to percentages, students answered option A at 6.3%, option B at 2.1%, option C at 12.5%, and option D at 75%. The percentage of students who did not respond corresponded to 4.2%. This question was asked to find the common divisors of the numbers. Again, most of the students answered the question correctly. It is thought that the correct answer rate was high, because the numbers were small, and these numbers were due to the familiarity of the students.

Table 12. The analysis of the answers given by 8th grade students to questions 11 to 14

Question	Option A	Option B	Option C	Option D	Empty	Right Answer %	SD
11	5	37*	2	1	3	77.08	15.39
12	2	3	1	42*	0	87.5	18.15
13	2	5	5	36*	0	75	14.91
14	3	1	6	36*	2	75	14.88

When the data were analyzed, it was found that both the 6th and 8th grade students generally grasped the factor of multipliers and factorization well. At the end of all questions, the students were asked a question such as, "What are the factors you can relate to this subject in our lives after multiplier and factorization were told by your teacher and how do you think that this affects your attitude and behavior towards mathematics lesson?" A representative sample response is below:

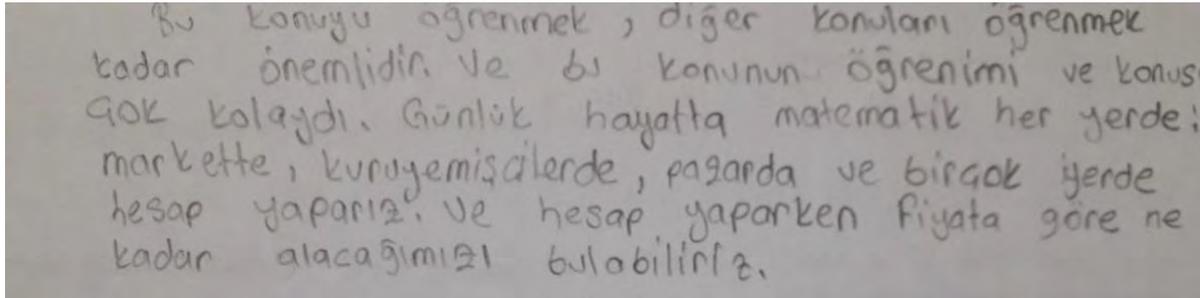


Figure 8. The example response received by the 8th grade middle school students on the multipliers and factorization

Discussion

One hundred fifty-five middle school students participated in this study, which was conducted on learning difficulties and misconceptions of middle school students on the topic of "Multipliers and Factorization". The findings are listed below:

- Students had more difficulty with indirectly asked questions. Student achievement was higher when asked directly about objectives. This is thought to be because students read at a low level and therefore have difficulty in understanding what they read.
- It was seen that the percentage of student achievement was lower for the questions in which distractions were placed.
- When the data were analyzed, it was shown that 6th grade students generally got higher scores than 8th grade students, and that 8th grade students had to have more accumulation of skill to answer questions at their level.
- Similar results were obtained in questions of the same style and logic. This shows that the reliability of the test is high.
- In the study, it was comprehended that the number 1 is a multiplier and that the number itself is both a multiplier and a multiple.

When students were asked an interpretation question about where the multipliers and base were used today, they often said that they were used in markets, shopping, or sharing something. This shows that mathematics is in life. Uysal (2013) stated that mathematics is a necessity, a part of life, and that it appears in many parts of our lives and accelerates social and scientific development.

Suggestions

From the study, several suggestions can be made. These suggestions are listed below:

- When explaining mathematics topics, students should first be asked what they think about that subject and their thoughts should be understood. To do this, a question-answer or pre-test can be applied. This may be one of the most effective methods to understand misconceptions of students. If pretesting is to be carried out, the teacher should identify the questions (s)he will ask well and be predictive of the misconceptions that may occur.

Otherwise, trying to describe new concepts on misconceptions makes the concepts more complex and difficult to understand, which can cause students to deepen their misconceptions.

- Visual materials should be used when explaining multipliers, multiples and factorization. Mathematics courses are already composed of abstract concepts in general. In this case, the subject may not be fully understood, which leads to misconceptions. In short, abstract concepts should be made concrete with visual materials. For example, when asked for the multipliers of 16, teachers can have 16 items (beans, chickpeas, coins, pens, etc.) that they can use in the classroom, and use the visual aids to help make it easier for students to understand. If there is an equal number of objects in each group, then the total number cannot be divided by the number of objects into the group.
- The places where students have learning difficulties and misconceptions that may occur should be envisioned while preparing textbooks, and a separate chapter should be prepared for the books outlining possible misconceptions. For example, when explaining multipliers and multiples, it is also necessary to specify that a multiplier is not multiplying the number, and that the number itself is a multiple when the multiples of the number are taken.
- As in many cases, most of the solutions lie with the teachers, in that the teacher must be competent in the subject and know the students well. In this context, physical conditions of the classes, class facilities, and technological facilities should be appropriate.
- Teachers should give examples from daily life as much as possible. Examples not from experiences will remain in the student's mind, which may lead to further deepening of existing misconceptions or new misconceptions. Giving examples by the teacher in accordance with the conditions of the location in which (s)he is working may make it

easier for the student to understand the subject.

- One of the most common pedagogical errors is always giving the same numbers as examples. This confuses the student when a different number or a negative number is given and alienates the student from the question. For example, although the multiples of the numbers encountered in daily life are easy to find, multiples of the numbers such as 17 and 23 have been given incorrectly by some students, which is thought to be due to the fact that the students are not familiar with the numbers.
- When asking questions, teachers should ask questions that make students think rather than memorize the subject. The teacher should constantly try to get to the bottom of the student's thinking and provide the student with a comfortable environment to ensure that (s)he expresses all aspects of the student's thinking. This provides an understanding of the misconceptions that exist in the student and the teacher can continue to eliminate misconceptions in the student.
- If grades are given, the gain should be explained before the grade. For example, instead of directly saying "1 is the multiplier of all numbers.", the student should be able to see that 1 is a multiplier in each number, which provides the basis for the student to better understand the subject and not misunderstand the concept.

At the end of the study, it was seen that the questions that the students did incorrectly were caused by lack of knowledge or because they had difficulty in understanding. For this reason, the duty of teachers and parents is to ensure that students read plenty of books.

Acknowledgements

This article is from the study of "İlköğretim öğrencilerinin çarpanlar ve katlar konusundaki öğrenme güçlükleri ve kavram yanlışlarının incelenmesi- The investigation of the secondary school students into learning difficulties and concept misunderstanding about

multiplication and factor”. This study was supported by Erciyes University Scientific Research Projects Unit, project number SYL-2018-7976.

References

- Alkan, C. & Kurt, M. (2007). *Özel öğretim yöntemleri: disiplinlerin öğretim teknolojisi [Teaching methods: instructional technologies for disciplines]*. (3rd Edition). Ankara: Anı Publishing.
- Altınyüzük, C. (2008). *İlköğretim sekizinci sınıf öğrencilerinin fen bilgisi dersi kimya konularındaki kavram yanlışları ilköğretim sekizinci sınıf fen bilgisi dersi kimya konularındaki kavram yanlışları [8th graders misconceptions in science lesson chemistry concepts]*. Unpublished master's thesis, Social Sciences Institute, İnönü University, Malatya.
- Bacanli, H. (2012). *Eğitim psikolojisi [Educational psychology]* (18th Edition). Ankara: Pegem Academy Publishing.
- Baki, A. (2004). Problem solving experiences of student mathematics teachers through Cabri: A case study. *Teaching Mathematics and Its Applications*, 23(4), 172-180.
- Baykul, Y. (2009). *İlköğretimde matematik öğretimi (6-8. sınıflar) [Teaching mathematics at middle grades (6th to 8th Grades)]*. (1st Edition). Ankara: Pegem Academy Publishing.
- Baykul, Y. (2011). *İlköğretimde matematik öğretimi (1-5. sınıflar) [Teaching mathematics at elementary grades (1st to 5th Grades)]*. (10th Edition). Ankara: Pegem Academy Publishing.
- Başun, A. R. (2016). *Oyunla öğretimin çarpanlar ve katlar alt öğrenme alanında başarı ve kalıcılığa etkisi [The effect of teaching with games on achievement and permanence in the subject of sub-learning of multipliers and multiples]*. Unpublished master's thesis, Ondokuz Mayıs University, Samsun.
- Bryk, A. S. & Treisman, U. (2010). Make math a gateway, not a gatekeeper. *Chronicle of Higher Education*, 56(32), 19-20.

- Demirel, Ö. & Şahinel, M. G. (2006). *Türkçe ve sınıf öğretmenleri için Türkçe öğretimi* [Teaching Turkish literature for Turkish literature teachers and elementary teachers. (7th Edition). Ankara: Pegem A Publishing.
- Damlı, V. (2011). *Kavramsal değişim yaklaşımına dayalı web tabanlı etkileşimli öğretimin üniversite öğrencilerinin ısı ve sıcaklık konusundaki kavram yanlışlarını gidermeye etkisi* [The effect of web based interactive instruction based on conceptual change approach for overcoming university students? misconceptions about heat and temperature]. Unpublished master's thesis, Gazi University, Ankara.
- Dogrucan, H. (2019). *İlköğretim öğrencilerinin çarpanlar ve katlar konusundaki öğrenme güçlükleri ve kavram yanlışlarının incelenmesi* [The investigation of the secondary school students into learning difficulties and concept misunderstanding about multiplication and factor]. Unpublished master's thesis, Institute of Educational Sciences, Erciyes University, Kayseri.
- Duman, B. (2009). *Neden beyin temelli öğrenme? [Why brain based learning?]* (2nd Edition). Ankara: Pegem Academy Publishing.
- Elvan, Ö. (2012). *Sosyal bilgiler öğretiminde çalışma yaprakları kullanılmasının kavram yanlışlarını gidermeye etkisi* [The effect of the usage of worksheets for resolving misconceptions in teaching social studies]. Unpublished master's thesis, Ahi Evran University, Kırşehir.
- Erden, M. (2008). *Eğitim bilimlerine giriş [Introduction to Educational Sciences]*. (2nd Edition). Ankara: Arkadaş Publishing.
- Fraenkel, J. R., Wallen, N. E., & Hyun, H. H. (2011). *How to design and evaluate research in education*. New York: McGraw-Hill Humanities/Social Sciences/Languages.

- Karasar, N. (2009). *Bilimsel araştırma yöntemi: kavramlar, ilkeler, teknikler [Scientific research methods: concepts, principles, techniques]* (19th Edition). Ankara, Nobel Publishing.
- Kucukahmet, L. (1998). *Öğretim ilke ve yöntemleri [Teaching principles and methods]* (9th Edition). İstanbul, Alkım Publishing.
- Millî Eğitim Bakanlığı [Ministry of National Education – MoNE] (2013). *Ortaokul Matematik Dersi (5, 6, 7 ve 8. sınıflar) Öğretim Programı [Middle school mathematics curriculum (5th, 6th, 7th, and 8th grades]*, Ankara
- Ozkan, F. (2017). *7.sınıf sindirim sistemi konusunda iki aşamalı test geliştirilerek kavram yanlışlarının tespit edilmesi [Determination of misconceptions by developing the two-stage multiple-choice test for 7th grade digestive system]*. Unpublished master's thesis, Institute of Educational Sciences, Erciyes University, Kayseri.
- Senemoglu, N. (2005). *Gelişim öğrenme ve öğretim: Kuramdan uygulamaya [Developmental learning and teaching: Theory to practice]* (12th Edition). Ankara: Gazi Bookstore.
- Sönmez, V. (1998). *Sosyal bilgiler öğretimi ve öğretmen kılavuzu [Teaching social sciences and teachers' manual]*. Ankara: Anı Publishing.
- Uysal, Y. (2013). *İlköğretim 6. sınıf matematik derslerinde geometrik cisimler konusunun dinamik matematik yazılımı ile öğretiminin öğrenci başarısına ve matematik dersine yönelik tutumlarına olan etkisinin belirlenmesi [The effect of learning the 6th grade primary geometric objects mathematics lesson with dynamic mathematics software onto the achievement and attitude of the students]*. Unpublished master's thesis, Institute of Educational Sciences, Gazi University, Ankara.