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PRESERVICE MATHEMATICS TEACHERS' REPRESENTATION TRANSFORMATION COMPETENCE LEVELS IN THE PROCESS OF SOLVING LIMIT PROBLEMS[1]

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Abstract: In this study, representations used by preservice mathematics teachers in the process of solving limit problems were determined, the inter-representation transformation competence levels were investigated and the relationship between them was examined. In this context, "Limit Representation Transformation Test" with a reliability of .908 was administered to 50 preservice teachers attending to a state university in the Central of Turkey. Preservice teachers had most difficulty in solving problems that had verbal representation inputs, especially they achieved low performances in transformation from verbal to numerical representation. Although, in general, they achieved the highest performance in the problem that had numerical representation input, they also achieved very high performances in the problems that had graphical and algebraic representation inputs. Specifically, they performed very well in the problems that required transformation from an algebraic representation to a verbal representation. Moreover, significant positive correlations were found among preservice teachers' representation transformation competence levels.

Key words: external representations, inter-representation transformation, limit, multiple representations

1. Introduction

The concept of limit, which requires strong mathematical thinking skills and is among the fundamental concepts of mathematics, has been conceptualized in two ways: dynamic (informal) and static (formal) (Cornu, 1991; Tall & Vinner, 1981). Dynamic form defined by Tall and Vinner (1981) relies on the following statement:

$$x \to a \Rightarrow f(x) \to L \text{ (verbally when } x \text{ approaches } a\text{, then } f(x) \text{ approaches } L)$$

On the other hand, the static form refers to $\delta - \epsilon$ definition, which is accepted by many mathematicians, and expressed as

$$\lim_{x \to a} f(x) = L \iff \text{For } \forall \epsilon > 0 \text{ there is } \exists \delta > 0 \text{ that satisfies } \ni |x - x_0| < \delta \Rightarrow |f(x) - L| < \epsilon$$

When the studies on limit concept are examined, it is seen that students conceptualize the limit concept more in informal way (Szydlik, 2000; Tall & Vinner, 1981; Williams, 1991) and stated to have difficulty in conceptualizing it formally (Tall & Vinner, 1981), and only a limited number of students being able to develop a clear understanding of formal definition (Quesada, Einsporn & Wiggins, 2008). This situation may cause the concepts such as derivative, integral, and Taylor series that are built on the formal definition of the limit to be incomprehensible. According to Delice and Sevimli (2016), it is necessary to have a good command of mathematical language in order to make sense, use, and transfer of mathematical knowledge. The elements that make up this language are

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symbols, tables, graphics, figures, and similar representations and the use of different representations in both concept teaching and problem solving process prepares the ground for the development of high-level thinking skills in terms of knowledge and cognition (Kuzu, 2020).

In mathematics education, the concept of representation can be defined as tools that are needed/used to process mathematical realities in the mind and transfer them to another person (Delice & Sevimli, 2016). Hence, representations are forms of displaying mathematical ideas, phenomena, objects or realities that aim at editing, recording, transferring, modeling, and interpreting science or social contexts (NCTM, 2000). A mathematical object has more than one representation, and establishing relationships among these representations are a necessity for conceptual understanding (Hiebert & Carpenter, 1992). In 1989, the importance of using multiple representations was emphasized in the "Curriculum and Evaluation Standards for Schools" published by the National Council of Mathematics Teachers (NCTM) in the United States (NCTM, 1989). Goldin and Kaput (1996) defined multiple representations as a characteristic arrangement that allows the symbolization of a thing with images or concrete objects.

Janvier, Bednarz, and Belanger (1987) stated that in the most general sense, the representation concept can be classified as the internal and external representations. The internal representations are the structures that consist of mental pictures, information or images that an individual sees, formulates, and reconstructs within the framework of his or her knowledge (Goldin & Kaput, 1996). On the other hand, the external representations are observable tools that enable understanding and transfer of mathematical concepts and ideas (Goldin, 1998). Examples of the external representations include verbal, graphical, algebraic and numerical representations. Internal and external representation systems are not independent of each other but have a network of relations between them (Girard, 2002; Kendal & Stacey, 2003; Hughes-Hallett vd.2008). When the mathematics education literature are examined, it can be seen that there are many studies conducted on the concept of limit. However, these studies focused mostly on the difficulties experienced by students, the sources of these difficulties, emerging misconceptions, and the effect of different teaching methods on the learning process (e.g., Bezuidenhout, 2001; Cornu, 1991; Davis & Vinner, 1986; Sierpinska, 1987; Szydlik, 2000; Tall & Vinner, 1981; Williams, 1991). It has been emphasized that multiple representations should be used in teaching mathematics topics and concepts, and importance has been given to the representation transformation process (Delice & Sevimli, 2016; Kuzu, 2020). In the representation transformation process, if there is a transition between different systems or between different types of representations of the same system, it is called an "inter representation transformation." If there is a transition within the same system and the same kind of representations, it is called as "within representation transition" (Goldin, 1998).

In this study, the concept of limit, which is stated as difficult by the majority of students, was discussed on the basis of multiple representations approach. The representations used by preservice mathematics teachers when solving limit problems were determined according to the external representations, and the following research problems are investigated:

1. What levels of inter-representation transformation abilities do preservice mathematics teachers employ in the process of solving limit problems?
2. Are there significant relationships among preservice mathematics teachers' levels of inter-representation transformation abilities?

2. Method

In this study, when the data collection process and data analysis was considered, quantitative research approach was adopted and descriptive model was used. The participants of the study consisted of 50 preservice mathematics teachers (34 females, 16 males) who were studying in the faculty of education of a state university in the Central of Turkey during the fall semester of 2018-2019 academic year. In addition, preservice teachers have seen the limit concept in undergraduate learning processes. In the selection of the related university, simple random sampling method was used. In order to determine preservice teachers, the homogeneous sampling which is one of the purposive sampling methods was used.

2. 1. Data Collection and Analysis

As a data collection tool, the related literature was reviewed and "Limit Representation Transformation Test (LRTT)" was prepared consisting of four open-ended items was prepared in accordance with the sub-problems of the study (See Appendix). Each item in this test was prepared with only one of verbal (V), graphical (G), algebraic (A) and numerical (N) representations and supported by three different items to examine the transformation between representations. The expression of the related problem is defined as input representation and its solution is defined as output representation. For example, an item shown in VG format was prepared with verbal representation and the participants were expected to response with graphical representation. The items were examined by two mathematics teachers and three academicians who are experts in mathematics and mathematics education and the content validity of the test was provided. Prior to the implementation phase of the LRTT, the participants was informed by the researcher about a total of 16 teaching hour limit concept, two lessons per week for eight weeks. Then, this test was administered to 50 preservice mathematics teachers within 40 minutes. In order to determine the reliability of the test, the responses were examined independently by two academicians who are experts in mathematics and mathematics education, and inter-rater reliability were calculated by Krippendorff Alpha (α) statistics. Krippendorff Alpha (α) statistic is a reliability technique that includes two or more raters and can be applied to samples of any size and gives a more reliable degree of compliance since it takes into account the percentage of chance and disables it (Krippendorff, 2004). α coefficient for two raters is calculated as

$$\begin{array}{c|ccccc|l}
 & 1 & . & k & . & . & \\
\hline
1 & o_{11} & . & o_{11} & . & . & n_1 \\
. & . & & . & & & . \\
. & . & & . & & & . \\
c & o_{c1} & & o_{ck} & . & . & n_c = \sum_k o_{ck} \\
. & . & & . & & & \\
\hline
 & n_1 & . & n_k & . & . & n = \sum_c \sum_k o_{ck}
\end{array}$$

$$\alpha = 1 - \frac{D_o}{D_e} = \frac{(n-1)\sum_c o_{cc} - \sum_c n_c(n_c - 1)}{n(n-1) - \sum_c n_c(n_c - 1)}$$

and where D_o is the observed disagreement, D_e is the chance-expected disagreement (Krippendorff, 2011). When observers agree perfectly, α=1, which indicates perfect reliability. When observers agree as if chance had produced the results, α=1, which indicates the absence of reliability. If α coefficient is less than 0.67, it is weak, moderate between 0.67 and 0.80, and higher than 0.80 indicates a high level of agreement between the raters (Krippendorff, 2004). In this study, each item was examined separately, and coded as "2: concept, process and response were correct"; "1: concept was correct, process and/or response were incorrect"; "0: concept, process and/or response was incorrect". Inter-rater reliability was calculated separately for 50 participants in terms of representation type of each item. For example, for the item which is the input representation type is verbal, output representation type is graphical (VG), the calculation of the inter-rater reliability was given:

$$\begin{array}{c|ccc|c} & 0 & 1 & 2 & \\ \hline 0 & o_{00} & o_{01} & o_{02} & n_0 \\ 1 & o_{10} & o_{11} & o_{12} & n_1 \\ 2 & o_{20} & o_{21} & o_{22} & n_2 \\ \hline & n_0 & n_1 & n_2 & n = 2N \end{array} \qquad \begin{array}{c|ccc|c} & 0 & 1 & 2 & \\ \hline 0 & 28 & 4 & 0 & 32 \\ 1 & 4 & 42 & 0 & 46 \\ 2 & 0 & 0 & 22 & 22 \\ \hline & 32 & 46 & 22 & 100 \end{array}$$

$$\alpha = 1 - \frac{D_o}{D_e}$$

$$= \frac{(100-1)(28+42+22) - [32(32-1)+46(46-1)+22(22-1)]}{100(100-1) - [32(32-1)+46(46-1)+22(22-1)]} = 0.876$$

Table 1. *Inter-rater reliability values for each item*

Item	1			2			3			4		
Rep.	VG	VN	VA	GV	GN	GA	AG	AN	AS	NV	NG	NA
α_{item}	0.87	0.90	0.92	0.86	0.87	0.87	1.00	0.97	1.00	0.82	0.87	0.90

α_{test}: 0.908

In this study, the quantitative data were transferred to the SPSS 23 program and the percentage and frequency distribution of responses was examined, and the transformation competencies levels between representations was investigated with total score mean. In addition, the relationship between the total score means obtained from the input representations of preservice teachers was examined with Pearson Correlation Test.

3. Findings

In this section, representations and representation transformation process competencies used in the process of solving limit problems by preservice mathematics teachers were explained in line with the sub-problems of the study. Response percentage and frequency distributions of preservice teachers were presented in Table 2.

Table 2. *Response percentage, frequency and score distribution intended for items in the LRTT*

Item	Input Rep.	Output Rep.	2	1	0	X	Sd	X	Sd
		G	11 (%22)	24 (%48)	15 (%30)	.92	.72		
1	V	N	4 (%8)	2 (%4)	44 (%88)	.20	.57	1.38	1.41
		A	5 (%10)	3 (%6)	42 (%84)	.26	.63		
		V	21 (%42)	8 (%16)	21 (%42)	1.00	.92		
2	G	N	22 (%44)	9 (%18)	19 (%38)	1.06	.91	3.32	1.85
		A	25 (%50)	13 (%26)	12 (%24)	1.26	.82		
		G	28 (%56)	3 (%6)	19 (%38)	1.18	.96		
3	A	N	6 (%12)	31 (%62)	13 (%26)	0.86	.88	3.80	1.64
		V	44 (%88)	0 (%0)	6 (%12)	1.76	.65		
		V	40 (%80)	5 (%10)	5 (%10)	1.70	.64		
4	N	G	30 (%60)	6 (%12)	14 (%28)	1.32	.89	4.46	1.82
		A	36 (%72)	0 (%0)	14 (%28)	1.44	.90		
Total								12.96	4.99

Since the items of each input representation type were consisted of three open-ended sub-item, the lowest score for each representation type was 0 and the highest score was 6. In addition, the highest score to be obtained from LRTT was 24. According to Table 2, with a mean of 1.38, it was seen that preservice teachers had most difficulty in solving problems that had verbal representation inputs. The problem with the lowest rate of correct response of preservice teachers was found to be transformation from verbal representation to numerical representation (VN) with 8%. When the solution examples given to the question of the VN representation type are examined, it was seen that two preservice teachers gave the same partially correct response in the same way. In this item, for the values x of variable close to 5, finding which number the function $f(x)$ approaches, preservice teachers applied an approach by considering each t point instead of values close to 5 points and they found $f(x)$ funciton as 12, 16, 24, 32, 40, 48 for x values such 1, 2, 3, 4, 5, 6 values, respectively. If the function $f(x)$ had not been defined at $x = 5$, it would be difficult to estimate the value of this function with a table based on each t moment. In other words, ϵ being a positive number, because of the definition of the limit, $x = 5$ is an accumulation point. Hence, there are infinitely many elements belonging to the input domain of the function in the ϵ neighboring of this function. Therefore, instead of approaching $x = 5$ point by considering each t moments, approaching to this point with the elements in the neighborhood of ϵ will lead us to obtaining the limit of this function. In this item, it was seen that 44 preservice teachers gave the incorrect response. When the responses obtained were examined, although the tables showing the limit result correctly, it was determined that preservice teachers had deficiencies in the concept and process as a result of the interviews. For example, 14 preservice teachers expressed that they wrote 40 directly to the table, knowing that the result was 40, and did not know how to approach it. 11 preservice teachers stated that they focused on "Two hours later..." in item text, therefore they approached x value to 2 points. In addition, it was seen that 7 preservice teachers have unrelated responses and 12 preservice teachers did not give any responses.

The correct response rate of preservice teachers was found to be the highest (88%) in the item which was transformed from algebraic representation to verbal representation (AV). When the solution examples given to AV representation type problem were examined, it was not seen that any preservice teachers gave partially correct response. Only six preservice teachers respond incorrectly. Two preservice teachers who gave the wrong answer used a statement such as “Since the function is not defined in 1 points, we cannot talk about its limit”. Four preservice teachers did not give any responses. When the mean scores were examined, it was seen that the items whose input representation type was numerical could easily transform to other representation types. In general, not only preservice teachers achieved the highest performance in the transformation of the items which numerical input representation, but they also achieved very high performances in the transformation of the items that were graphical and algebraic input representation. The relationship between the total scores obtained from the items by preservice teachers was presented in Table 3 according to the representation types.

Table 3. *Relationship between input-output representation types*

r	VG	VN	VA	GV	GN	GA	AG	AN	AV	NV	NG	NA
VG	1	.089	.314*	.183	.100	.206	.080	.020	-.041	-.052	.104	.117
VN		1	.531	.039	.329*	.103	.156	.024	.131	.055	.112	.063
VA			1	.244	.078	.336*	.089	.044	-.043	-.105	.103	.259
GV				1	.121	.186	.115	.145	.336*	.102	.247	.340*
GN					1	.357*	.080	.384*	.297*	.273	.177	.239
GA						1	.145	.155	.117	.110	.162	.687*
AG							1	.114	.457*	.351*	.884*	.352*
AN								1	.324*	.203	.085	.300*
AV									1	.788*	.483*	.318*
NV										1	.383*	.195
NG											1	.378*
NA												1

$^*p<.05$

The relationship is very weak if r <.20; is weak if .20 < r <.40; moderate if .40 < r <.60; high if .60 < r <.80; very high if r > .80 (Evans, 1996). When Table 3 was examined, there is a significant weak relationship in the positive direction between VG-VA, VN-GN, VA-GA, GV-CV, GV-NA, GN-GA, GN-AN, GN-AV, AG-NA, AN-AV, AN-NA, AV-NA, NV-NG, NG-NA. Moreover, the positive relationship was moderate between AG-AV, AV-NG; high between GA-NA, AV-NV; very high between AG-NV, AG-NG. The relationship between input representation types was given in Table4.

Table 4. *Relationship between input representation types*

r	Verbal	Graphical	Algebraic	Numerical
Verbal	1	.352*	.104	.153
Graphical		1	.365*	.520*
Algebraic			1	.796*
Numerical				1

*p<.05

When Table 4 is examined, there was a statistically weakly significant relationship between the total score means obtained by the verbal items and the total score mean obtained from the items that are representative of the input (p = .012; r = .352). There was a weakly positive relationship between the total score means obtained from the items that are graphs of input representation and the items that are input representation algebra (p = .009; r = .365). There was a positively moderately significant relationship between the total score means obtained from the items that were graphical of input representation and the items that was numerical of input representation (p = .000; r = .520). On the other hand, there was a high positive correlation between the total scores obtained from the items that were algebraic input representation and the items that were numerical representation (p = .000; r = .796).

3. Discussion, Conclusion and Implications

In this study, representations used by preservice mathematics teachers in the process of solving limit problems were determined, and it was observed that they had most difficulty in making transformations in the items with the verbal input representation type. In the limit problems provided by verbal representations, it was determined they had difficulty in transformations to numerical and algebraic representations (VN and VA). The fact that there are difficulties in interpreting verbal data (Kuzu, 2020), numerical and algebraic representations are the least used representation types in problem solving (Delice & Sevimli, 2010a; Polat & Sahiner, 2007) and that preservice teachers are not being successful in such problems (Kendal & Stacey, 2003) can be the reason of why the limit problems given with verbal representations could not solved with numerical and algebraic representations (VN and VA).

In the numeric items, although preservice mathematics teachers easily transformed numerical representations into other types of representations, they also performed well in transformation of items with input types graphical and algebraic representations. In particular, although the number of they who could make transformations from verbal representations to numerical and algebraic representations (VN and VA) was not that high, there were quite high number of they who could make transformations from numerical and algebraic representations to verbal representations (NV and AV). Considering that it is more comfortable to express a problem verbally (Yaman, 2010), it can be explained why the number of preservice teachers who can do NV and CV is high.

In thus study, it was obtained relatively low performances on answering items with graphical representations using verbal representations (GV). This situation may occur due to preservice mathematics teachers' poor performances in developing problems for visual representations (Isik, Isik, & Kar, 2011), or the interpretation of this type of problems requiring more advanced cognitive performances (Dreyfus & Eisenberg, 1991). Preservice teachers performed relatively higher performances in the transformation from the graphical representations to numerical representations (GN). Delice and Sevimli (2010b) also observed their high performances from graphical representations to numerical representations. This may be due to the fact that, the values given to the function for each *x* value in the items can be easily seen in the graph. Moreover, when the items with the algebra output representation types (VA, GA and NA) were examined, it has been found that the highest performance was obtained from the items that had numerical inputs (NA). In the items presented by a table, while preservice teachers could make more easily transformations to the algebraic representations (Yesildere-Imre, Akkoc, & Bastürk-Sahin, 2017), they reported to have difficulty transforming the graphical representations to the algebraic representations (Delice & Sevimli, 2010b).

In this study, the analysis showed that there was a weak statistically significant relationship between the mean total scores obtained from items with verbal representations and items with graphical representations (V-G). Kwon (2002) reported that providing verbal expression of a given graph requires interpretation competence, and this interpretation competence is closely related with the learners' skills in using graphs. The fact that both types of representations require reading, understanding, and interpretation skills might have caused the existence of such positive relationship between them. On the other hand, there was a moderate statistically significant relationship between the mean total scores obtained from items with graphical representations and items with numerical representations (G-N). In addition, there was a weak statistically significant relationship between the mean total scores obtained from items with algebraic representations and items with graphical representations (A-G), and there was a strong statistically significant relationship between items with algebraic representations and items with numerical representations (A-N). Some studies (e.g., Ostebee & Zorn, 1997; Sevimli, 2009) reported that using numerical and graphical representations in problem solving can increase preservice teachers' concept knowledge. When teaching concepts of mathematics, there is dominance of using algebraic representations. This dominance of using algebraic representations may be an indicator of meaningful relationships between the mean total scores obtained from items with algebraic, graphical, and numerical representations.

In this study, preservice mathematics teachers were asked to solve given limit problems with different types of representations, and it was observed that preservice mathematics teachers had difficulty during the representation transformation processes. Hence, in order to improve preservice mathematics teachers' conceptual understanding levels and cognitive process skills multiple representations can be used when teaching concepts and solving problems, and these representations can be related to each other. A variety of traditional and technological materials that include multiple representations can be used in the teaching processes, and in the teaching process, the course contents can be enriched by including real-world problems that enable the representation transformation process.

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Appendix

Limit Representation Transformation Test

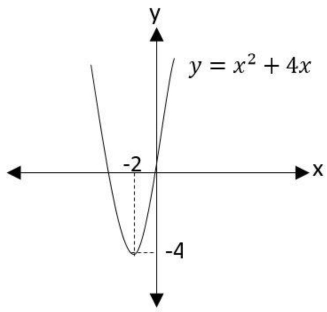
Item 1: Ahmet starts running at a constant speed of 12 km/h. Two hours later, he's taking one hour break. After the break, he goes back to the point where he started by running at a constant speed of 8 km/h. According to this, how many kilometers has Ahmet starts to complete as he approaches the 5th hour of the race.

a) Present with Distance-Time graph

b) Explain which value it approaches using the table $x \mid f(x)$

c) Calculate the result using $\lim_{x \to a} f(x)$

Item 2: It is desired to calculate the limit at 2 points of the function given in the graph. According to this;



a) Explain what the limit of the function at 2 points means.

b) Explain which value it approaches using the table $x \mid f(x)$

c) Calculate the result using $\lim_{x \to a} f(x)$

Item 3: $f(x)$ function is given as

$$f(x) = \begin{cases} 2x & ; \ x < 1 \\ x + 1 & ; \ x > 1 \end{cases}$$

Then, for $\lim_{x \to 1} f(x)$,

a) Present with graph

b) Explain which value it approaches using the table $x \mid f(x)$

c) Express verbally.

Item 4: For a limit problem, the following table is presented and as a result of some values given to the variable *x*, the values taken by the *f* (*x*) function are shown. Accordingly, what does the table presents to us,

x	$f(x)$
1,9	3,80
1,91	3,82
1,95	3,90
1,99	3,98
1,999	3,998
2	4
2,001	4,002
2,01	4,02
2,05	4,10
2,08	4,16
2,1	4,20

a) Express verbally.

b) Present with graph

c) Calculate the result using $\lim_{x \to a} f(x)$