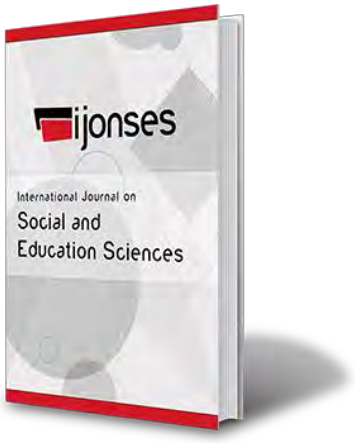

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Investigation of Proportional Reasoning Problem Solving Processes of Seventh Grade Students: A Mixed Method Research

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Abstract

This study was carried out to examine proportional reasoning problem solving processes of seventh grade students. This study was conducted with the explanatory sequential mixed method design. In this respect, firstly, quantitative data from 56 students were collected and analyzed. Then, qualitative data of the study was collected from six students selected according to their success and analyzed. The quantitative data of the study was consisted of a proportional reasoning skill test and analyzed using predictive statistics. The qualitative data of the study were collected through activity cards and the think-aloud protocol, and content analysis was applied to the collected qualitative data. The findings of the study showed that students were most successful at qualitative comparison. On the other hand, the least success was observed for quantitative comparison, and female students were found to be more successful in proportional reasoning problems.

Introduction

Mathematics is one of the ways students make sense of daily life. This situation makes it necessary to learn mathematics for primary and secondary school periods that prepare students for life. The process of learning math requires using many types of mathematical reasoning. Some of these ways of reasoning can be listed as arithmetical reasoning, proportional reasoning, and algebraic reasoning. Many studies indicated that proportional reasoning is required to facilitate the relationship between arithmetical reasoning and algebraic reasoning. Jacobson, Lobato and Orrill (2018) emphasized that proportional reasoning is a cornerstone for students' cognitive development.

Lesh, Post and Behr (1988) stated that proportional reasoning is the cornerstone of secondary school mathematics and the skeleton of mathematics in later years. All these situations reveal the importance of proportional reasoning. However, the literature showed that the studies on proportional reasoning remain very limited. In this context, this study was conducted to examine the proportional reasoning processes of seventh grade students in cognitive domain. The results of this study are expected to contribute to the design of educational environments that will be organized for proportional reasoning and revealing the worlds of thought of students for proportional reasoning.

Proportional Reasoning

Goldstein (2013, p.608) expressed reasoning as reaching new results beyond this information based on the information that the individual has. Kurtz, Gentner and Gunn (1999), on the other hand, define reasoning as a mental process in which an individual has an inference beyond his knowledge based on the information he has. When all these definitions are examined, it can be inferred that when the individual has a certain level of knowledge, the general name of the process of producing new information using his knowledge is reasoning. Mathematical reasoning is one of the reasoning types. In this study, proportional reasoning is discussed since it acts as a bridge between arithmetic and algebraic reasoning (Akkus & Duatepe-Paksu, 2006). Proportional reasoning is defined as the ability to recognize a proportion, express it with symbols and solve problems that require proportional reasoning (Cramer & Post, 1993; Cramer, Post & Currier, 1993). Turkish Ministry of National Education (MoNE, 2013, p. 5) is defined proportional reasoning as "The process of obtaining new information by using the unique tools (symbols, definitions, relations, etc.) and thinking techniques (induction, deduction, comparison, generalization, etc.) based on the information available." According to Baxter and Junker (2001), proportional reasoning involves identifying, predicting and evaluating the relationship between two concrete objects. Akkus and Duatepe-Paksu (2006) mentioned about three different types of problems related to proportional reasoning, namely "qualitative comparison", "quantitative comparison" and "finding what is not given". In the qualitative comparison of these problem types, quantitative (numerical) values are not given, and comparisons are made with qualitative expressions such as magnitude-smallness, minority-multiplicity. Such problems allow students to compare verbal expressions, as well as enable them to establish relationships between numbers by valuing them. Quantitative comparison is the types of problems that require students to relate these numerical relationships by giving them numerical relations. In other words, it is to enable students to understand numerical relations by establishing a relationship between numbers. Problems about finding the not given are proportional reasoning problem in which the student is asked to find this unknown value, leaving one not given in one of the two ratios mutually. In the process of solving proportional reasoning problem, students experience various difficulties (Jacobson et al., 2018). In addition, the current literature indicates that the proportional reasoning levels of students are low (Ayan & Isiksal-Bostan, 2018). Focusing on function might be one of the reasons that why students' proportional reasoning skills are low. While solving problems, students can ignore the other part of the problem by focusing on the prominent aspect of the problem. This may cause them to fail to solve problems (Goldstein, 2013, p. 562; Öztürk & Kaplan, 2019). Another reason may be that students ignore what is given in the question and give their first answer (hasty approach) (Demir, 2016).

Students' Cognitive Strategies in Solving Proportional Reasoning Problems

Researches emphasizes that there are some strategies that students use in the proportional reasoning process. Among these strategies, two types of strategies are very common as building additive and multiplicative relationships (Arican, 2019). The first of the strategies is to establish a multiplicative relationship. Multiplication relationships include processes such as multipliply, multiplication and duplication (Lamon, 2012). Additive relationships include mathematical processes such as addition, subtraction and difference

(Arican, 2019).

Many studies in the literature have pointed out that students establish cumulative and multiplicative relationships in the process of proportional reasoning (Arican, 2019; Ayan & Isiksal-Bostan, 2018; Duatepe, Akkus-Cikla, & Kayhan, 2005; Kahraman, Kul, & Aydogdu-Iskenderoglu, 2019; Toluk-Uçar & Bozkuş). Ayan and Isiksal-Bostan (2018) also indicated that students make comparisons in the process of solving proportional reasoning problems related to geometric shapes, while giving arbitrary values to the lengths. Lundberg and Kilhamn (2018) emphasized that students are dealing with proportional reasoning problems through trial and error. In addition, Lundberg and Kilhamn (2018) determined that students also use the strategies like direct proportion and doubling or halving during the proportional reasoning process. Langrall and Swafford (2000) stated that students use pictures, models or manipulatives to understand proportional reasoning problems and to make a qualitative comparison. Researchers emphasized that using models contributes to students' proportional reasoning.

Previous studies have pointed out that proportional reasoning skill of secondary school students differ according to gender. This differentiation may have originated from cultural differences. Because Karplus et al. (1977) stated that boys are more successful than girls in proportional reasoning in their research study conducted to examine the proportional reasoning levels of students from seven different countries. Ünsal (2009) found that proportional reasoning skills of seventh grade students were significant in favor of girls in their study with Turkish students. The TIMSS (2015) report also pointed out that girls in eighth grade Turkish students have a higher average score than boys (MoNe, 2016).

Aims of the Research

The literature pointed out that studies on proportional reasoning are carried out qualitatively and studies focus on proportional reasoning. However, supporting these findings with quantitative results remains insufficient. Especially studies on what kind of difference will be in proportional reasoning problem types are quite insufficient. This study will contribute significantly to the literature and practitioners in terms of aiming to compare according to the types of proportional reasoning problems. In addition to this, the results to be obtained in this study, which will enable to compare qualitative and quantitative findings, are expected to contribute to the literature and mathematics curriculum. In this context, this study was carried out to examine the proportional reasoning problem solving processes of seventh grade students in secondary school. The study sought answers to the following sub-problems:

1. Do the proportional reasoning problem solving skills of secondary school seventh grade students differ according to gender?
2. Is there any significant difference between the proportional reasoning successes of secondary school seventh grade students in quantitative comparison, qualitative comparison and finding the not given value problem types?
3. What kind of skills do secondary school seventh grade students demonstrate in the process of solving proportional reasoning problem in terms of understanding, process and correlation?

Method

Research Model

This study was conducted with the explanatory sequential mixed method design. In explanatory sequential mixed method design, according to Creswell (2015, p. 6) the researcher first collect quantitative data and then use qualitative data to help explain the quantitative data in more depth. Quantitative data collection and quantitative data analysis were deemed necessary in this study, since the comparison of the proportional reasoning skills of the students by gender and the comparison of the dimensions of the proportional reasoning was sought. In addition, the sub-problem for examining the proportional reasoning processes of students required the use of qualitative data collection and qualitative data analysis. Therefore, explanatory sequential mixed method design in which quantitative and qualitative researches are used consecutively is preferred.

Participants

Due to the use of explanatory sequential mixed method in the study, quantitative-qualitative sequential mixed method sampling was used in sample selection. For the quantitative dimension of the study, simple random sampling method, one of the probability-based sampling methods, was used and 56 students selected at the seventh grade formed the sampling of the quantitative part. In the selection of the students, firstly, a city located in north-east of Turkey was designated and a list of schools in the province was created. Two of the schools in this city were selected randomly and 56 volunteered students to participate in the study selected randomly from these two schools constituted the sample of the study.

Of the students participating in the quantitative part of the study, 50% are girls and 50% are boys. While 67% of the students participating in the study have low income level, 25% of them have medium income, 8% of them have high income level. When students' year-end mathematics school success scores (mathematics) are taken into consideration, it is determined that their success levels are intermediate ($\bar{x} = 2.89$). The implementation of the explanatory sequential mixed method design takes time. For this, Creswell (2015, p. 82) suggested that the number of participants should be limited. Therefore, the number of participants of the study remained limited.

Six students (3 girls, 3 boys) selected from the students participating in the quantitative part of the study constituted the qualitative part of the study. In the qualitative part of the study, criterion sampling method, one of the purposeful sample selection methods, was used. In the study, success levels were taken as criteria. The students were ranked according to their success levels, and then they were divided into three groups as high success (first 18 students), moderate success (second 19 students) and low success (last 19 students). Then, two students, one girl and one boy in each group, were randomly assigned from the endpoints. The names of the students participating in the qualitative part of the study were kept secret and pseudonyms were used to represent these students. Pseudonyms are used as Yeliz (girl) and Yakup (Boy) to represent students with high success, Oylum (girl) and Orhan (Boy) to represent students with medium success, Dilara (girl) and Davut (Boy) to represent students with low success.

Data Collection Tools

In collecting the data of the study, proportional reasoning skill test, activity card and thinking aloud protocol were applied.

Proportional reasoning skill test. In the study, the proportional reasoning skill test developed by Akkus and Duatepe-Paksu (2006) was used to determine the proportional reasoning skills of 7th grade students. When the literature is reviewed, it is determined that there is only one test prepared for Turkish secondary school students. In addition, it has been observed that this test is often used in studies with Turkish students. When the development process of the proportional reasoning test was examined, it was seen that the item discrimination index values of the test ranged between .50 and .71. Since these values were acceptable for discrimination, the test was considered sufficient in terms of discrimination. In the meant time, the reliability of the data collected during the test development process was also found to be high (α = .86). When the study was analyzed in terms of content, it was determined that the test consisted of 15 items and included questions such as "finding the value not given", "quantitative - qualitative comparison" and "inverse proportion" which are frequently included in the sub-dimensions of proportional reasoning in the literature. Because of these reasons, the proportional reasoning skill test developed by Akkus and Duatepe-Paksu (2006) was used in the study without any need to adapt or develop any proportional reasoning test. A sample question from the questions in the test to find what is not given:

> Mr. Short has a friend named Mr. Tall. When the length of the Mr. Short was measured with paper clips, it was observed that he was 6 paper clips. When the lengths of Mr. Tall and Mr. Short were measured with a button, it was found that the length of the Mr. Tall was 6 and the length of the Mr. Sort was 4 buttons. According to this, how many paper clips is the length of Mr. Tall?

An example question for qualitative comparison is: "*Nesrin and Başak are running on a running track. While Nesrin runs 8 laps in 32 minutes, Başak runs 2 laps in 10 minutes. According to this, which one is running faster?*" The sample question for quantitative comparison is the problem of comparing students' orange juice concentrate and water ratios in two jugs (Noelting, 1980). Sample question for the inverse proportion: "*The walking speeds of Nevzatcan and Nergis on a track are the same. Nevzatcan first started walking. When Nevzatcan completed 9 laps, Nergis completed 3 laps; so when Nergis completes 15 laps, how many laps will Nevzatcan complete?*"

In this study, the internal consistency coefficient was examined to calculate the reliability of the data collected. The reliability coefficient was calculated both for all sub-dimensions and for the entire test. In the calculation, the reliability coefficient was calculated as .83 for the not given questions and inverse proportion questions, .70 for the quantitative reasoning questions and .83 for the qualitative reasoning questions. The reliability coefficient of the whole test was found to be .91.

Think aloud protocol. Think aloud protocol is a method used to reveal thoughts in the mind of the individual (Öztürk, Akkan, & Kaplan, 2018). In this method, the person is asked to think everything out loud, in other

words, to say his inner voice out loud. Thus, the mental activity of the individual is expressed verbally. The researcher uses this method to determine, how he approaches the problem by observing the participant and what techniques and comments he made while solving the problem. In the think-aloud protocol, it is provided to observe whether the participant shows the desired behavior with the activity cards prepared by the researcher.

Activity Cards. Four different activity cards were used in the study. After all the activity cards were prepared, they were presented to two faculty members in a state university who have a PhD. Degree in mathematics education and their opinions were received. One of the experts stated that only the problem in the second activity card should be corrected. The expert stated that the objects in the question should be replaced with the objects that students are more likely to encounter in daily life. After making the necessary correction, the problem was put on the activity card. The first of the activity cards has been prepared in the type of finding the one not given. The problem in the activity card used for the problem of finding what is not given in the study is as follows:

> The factory, which manufactures a cardboard box, has two machines that make boxes at different speeds. The first machine can manufacture 10 boxes per minute, while the second machine can manufacture 10 boxes in 3 minutes. According to this;
>
> i. How many boxes are made in 3 minutes while both machines are running?
> ii. How many boxes does the first machine produce when the second machine makes 50 boxes?
> iii. In order to make 60 boxes within a 9-minute period by starting the second machine alone from the beginning, how many minutes after the second machine starts operating should the first machine start operating?

Finding what is not given type of questions allows students to establish numerical relationships, find proportion and reach the results by using the ones given in the question. The ratio subject enables students to establish relations between numbers and contributes to the development of skills such as comparison and correlation among students. In this context, this problem has been added to the activity card in order to examine the students' processes of solving the problems of finding the not given.

The second activity card used in the study is quantitative comparison type. In the study, the problem in the activity card used for the quantitative comparison problem is as follows:

> There are 20 cats in an animal shelter and in the store there is food for 30 days. 15 days after the start, if 10 more cats are brought to the shelter, If 20 cats were adopted 20 days after the start, is the remaining food enough more than 30 days for the remaining cats or not? Please explain?

Quantitative comparison questions enable students to establish numerical relationships, use ratio-proportion, and achieve results using what is given in the question. In the problem, this type of question was asked, as it was aimed that the students establish more than one proportional relationship, so that the thinking process could be revealed more clearly.

The third activity card used in the study is quantitative comparison type. The problem in the third activity card is

as follows:

> Preparing for the high school entrance exam, Ahmet has solved less than the number of questions he had solved yesterday. So, compared to the number of questions he solved in his work today; the number of questions Ahmet solved in his work yesterday is:
>
> a) More b) Less c) Same d) Not enough information
>
> Which options is correct? You explain the reasoning of your answer.

Qualitative comparison questions enable students to build qualitative relationships by thinking numerically or to compare directly by conducting qualitative reasoning. This type of problem is preferred because it is aimed to determine how students make comparisons (numerical reasoning or qualitative reasoning) in situations that require qualitative comparison. When qualitative comparison problems are examined, it is seen that students are usually presented with different options, so that students' different thoughts are tried to be revealed. In other words, it is aimed to reveal how students think by including distractors.

The fourth activity card used in the study is the type that contains the inverse proportion algorithm. The problem on the event card is as follows:

> If 10 pigeons are fed for 12 days with a pack of feed, how many days are 30 pigeons fed with the same amount of feed? Explain and write.

The inverse proportion algorithm is one of the frequently used question types in proportional reasoning. The inverse proportion, which forms the basis of the ratio-proportion with the direct proportion, is one of the subjects that students may experience difficulties as it requires them to think relationally. For this reason, it is aimed to study the relational thinking processes of the students by adding the inverse proportion type question.

Data Analysis

Inferential statistics were applied on the quantitative data in the study. Content analysis method was applied to qualitative data. Normality tests were carried out before starting the quantitative data analysis. For normality analysis, firstly, central tendency measurements were compared and it was determined that mean, mode and median values were close to each other. When skewness and kurtosis values were examined, it was found that all dimensions except for the quantitative comparison dimension were within the range of ∓1. In the quantitative comparison sub-dimension, it was determined that the skewness values are outside the ∓1 range, but take a value very close to 1. Indeed, when looking at the histogram chart, it was understood that the value indicates the normal distribution. Normality test results show that on Table 1.

In the study, independent t-test was conducted on whether the features that consist of proportional reasoning and its sub-dimensions differ according to the gender of the students. In order to reveal whether the proportional reasoning sub-dimensions of the students differ, the Friedman ANOVA test, which is one of the non-parametric tests, was used because it does not provide the spherical assumptions of the data for repeated measurements.

Table 1. Descriptive Statistics for Quantitative Results

Statistics	Finding the value not given and inverse proportion	Quantitative comparison	Qualitative comparison	Proportional reasoning
x	2.65	1.55	4.42	8.62
Median	2.00	1.00	2.50	7.00
Mode	.00	.00	.00	.00
Minimum	.00	.00	.00	.00
Maximum	9.00	7.00	15.00	27.00
Skewness	.792	1.173	.614	.724
Kurtosis	-.735	.352	-.987	-.396

In the analysis of the qualitative data of the study, firstly, the think-aloud protocols were transcribed and coded by the researcher(s). The encodings and explanations of these codes were prepared by the first researcher and re-coding was done by the second researcher by considering this framework. There was 83% consistence between the first and the second encoder. In the non-consistent codes, both coders made a common decision by discussing together. Despite this, non-consistent codes are not included in the study. Then, the categories were created by the researchers considering the common features of the codes. The created categories were named with the help of the literature. Thus, content analysis has been completed.

Validity and Reliability

In order to ensure validity in the study, the expressions used by students in the think-aloud protocol were given with direct transfers along with their duration. In this way, the reader was allowed to see the participants' thinking time and the connections between their answers. In this way, the reader is provided with detailed information about how the think-aloud protocol is executed. In the expressions used by the students, the transactions made on the activity card are also included. In this way, diversification of data collection tools is ensured. In order to ensure the external validity of the study, detailed information about the sample of the study is presented. Thus, it has been revealed in which cases a researcher who wants to do this study can achieve the same or very similar results. The reliability of the study is discussed in two parts as quantitative and qualitative. In order to ensure the reliability of quantitative data, reliability analyses were performed with the help of SPSS program. In order to ensure the reliability of qualitative data, reliability between encoders was examined. In addition, the method and findings of the study were presented in line with the research problem and the consistency of the study was ensured.

Findings

Differentiation of Proportional Reasoning Problem Solving Skills According to Gender Variable

An independent t-test was employed types of proportional reasoning problems as the dependent variables and gender as the grouping variable to see Table 2.

Table 2. Comparison of the Proportional Reasoning Scores of the Students by their Gender

	Gender	N	$\bar{x}$	t	Sd	p
Finding the not given value and inverse proportion	Boy	28	2.50	-.564	54	.575
	Girl	28	2.96			
Quantitative comparison	Boy	28	.64	-2.58	54	.013
	Girl	28	1.71			
Qualitative comparison	Boy	28	2.57	-2.50	54	.016
	Girl	28	5.50			
Proportional reasoning	Boy	28	5.71	-2.29	54	.026
	Girl	28	10.18			

Table 2 stated that the scores of the students did not differ significantly according to gender in finding the not given value and inverse proportion problems ($t_{54} = -0.56, p > .05, CI\ [-2.11,1.19]$). However, among other problem types, both quantitative comparison ($t_{54} = -2.58, p < .05, CI\ [-1.90,0.24]$) and qualitative comparison problems ($t_{54} = -2.50, p < .05, CI\ [-5.28, -0.58]$) it was determined that there was a significant differentiation. However, proportional reasoning total scores also differed according to gender ($t_{54} = -2.29, p < .05, CI\ [-8.38, -0.55]$). When the differences between the students' scores are examined, it is understood that the scores of female students are significantly higher than that of male students.

Comparison of Students' Achievement in Different Proportional Reasoning Problem Types

In different proportional reasoning problems, ANOVA test was applied for repeated measurements to determine whether students' scores differ. Since the maximum scores that can be obtained from each dimension will differ, ANOVA analysis was performed after the scores were converted to standard scores. In the process of converting to standard score, firstly, the maximum scores that can be obtained in each dimension were determined and then the score of each student was divided into the maximum score in the related problem type and converted into a standardized score between 0-1. As a result of the analysis, it was determined that the students' scores differ significantly ($X^2{}_{(N=57)} = 11.72, p < .05$). When the order differences are examined, it is determined that the type of problem that the students are most successful is qualitative comparison, and the type of problem that they fail the most is quantitative comparison.

Skills Displayed by the Seventh Grade Secondary School Students in the Process of Solving Proportional Reasoning Problems

According to the answers given by the students who participated in the study to the questions on the activity cards, three categories were reached as understanding, operation process and relationship. The codes obtained under each category are described in detail below.

Secondary school students' activities to understand proportional reasoning problems. In the understanding category, the activities that students carry out to understand proportional reasoning problems are included. In

this category, we reached the sub-categories of "*Tries to understand the question by reading several times*", "*Carries out the process steps by asking questions to him/herself*", "*Uses modeling*" and "*Ignores what is given in the question and gives the first answer that comes to his/her mind*".

We found that some participants performed the sub-category of "*Tries to understand the question by reading several times*." One example was Yakup's again read the question with a low voice (inside). Yakup reading several times the problem 1 to understand. Similarly, we saw that Yeliz read the question aloud again after reading the question for the first time. When asked why they read the question again, Yeliz said that she could not fully understand the question and she read again to understand better.

Some participants have asked themselves questions while performing the process steps. We called the sub-category as "*Carries out the process steps by asking questions to him/herself*". For instance, Yakup said "[*03.28] How many boxes does the first machine produce? It is asked how many boxes are produced in 15 minutes.*" In the mention expression, Yakup turned the question into a simpler form and asked himself again. Similarly, we detected Dilara while reading the question she has also made operations by asking questions to herself. Dilara's *"[00.22] I should answer this first: How many boxes are made in 3 minutes when both machines are running?*" can be given as an example to this situation.

We determined that some participants used model for solving problem. We called this sub-category as "Uses modeling". Yeliz drew a calendar model about Problem 2 (see Figure 1, Yeliz) and said that "[05.30] *I want to write the days as one-two-three.*"

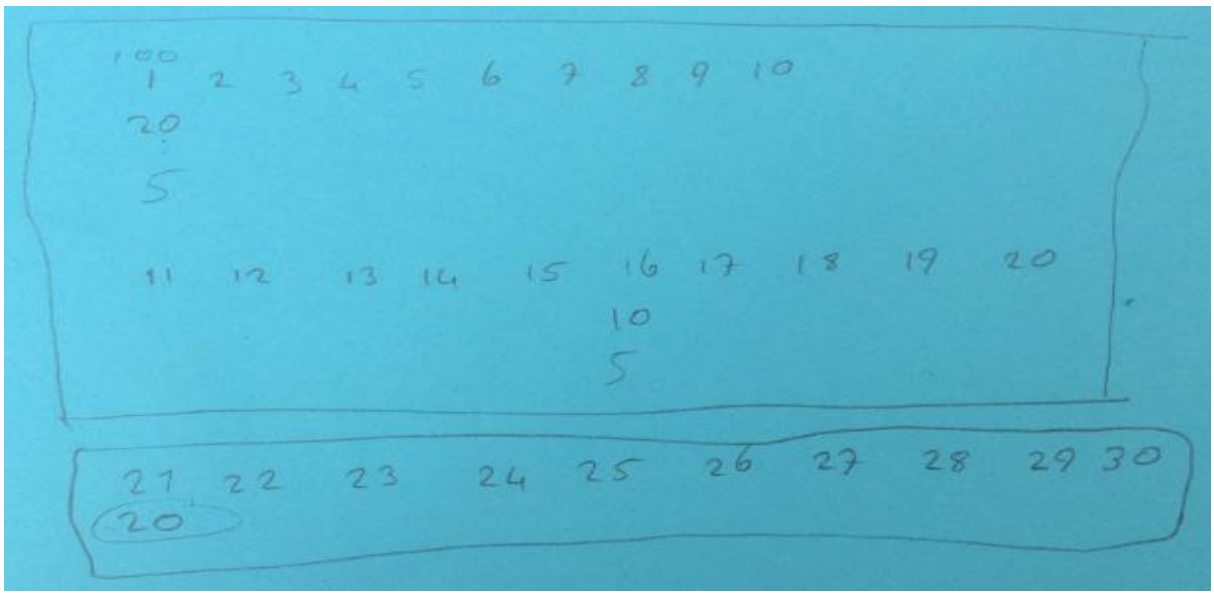

Figure 1. Image of Yeliz's Activity Card in Problem 2, which is Determined that she Used Models

We also observed that some students performed sub-category of "*Ignores what is given in the question and gives the first answer that comes to his/her mind*". For instance, Dilara said following sentences:

> "[12.16] Problem said that, yesterday he spent more time to solve fewer questions. I think the number of questions he solved today is the same as he did yesterday."

> "[12.46] In other words, Ahmet has tried to do the same again now with the number of questions he solved yesterday. I say it's the same number."
> "[14.26] Ahmet solved same questions today that he had solved yesterday. I think he wants to do the same in the same time."

Dilara's expressions show that she is not interested in what is given in the question and focuses on the answers she has in her mind.

Secondary school students' activities during the process of proportional reasoning problems. In the understanding category, the activities that students carried out to understand proportional reasoning problems are included. In this category, "*Focuses on the function (focuses on the data given in the question)*", "*Uses a trial and error strategy*", "*Calculates the ratio by making use of half*", "*Thinks qualitatively by ignoring numbers*" and "*Tries to reach the result by giving an example*" codes were reached.

We detected that some participants focused on the function. We called the sub-category is "*Focuses on the function.*" For example, Dilara had the following conversation with the researcher: "[06.20] *the machines are being stopped. After they stopped, something like this comes to my mind. It says 'manufactures for 3 minutes'. I think about 3 but on the other hand, I also think about 5.*" These representations in excerpt indicate Dilara focuses on production time of a machine that manufactures 10 boxes in 3 minutes and she thinks that the answer may be 3. Similarly, Davut stated that "[02.28] *second machine produces 50 boxes in the same amount of time. I think other machine would also produce 50 boxes*". It is understood from Davut's statements that his intuition led him to think the two machines were the same. This indicates that Davut focused on number 50.

Some participants used trial and error strategy. We called the sub-category as "*Uses trial and error strategy*". For instance, sentences used by Oylum on the fourth activity card as "[11.03] *I think one day is enough for this.*", and "[11.31] *is one or two days ok? If we write?*" indicates that she uses trial and error strategy. Similarity, Orhan performed this sub-category on first activity. Orhan said that:

> "[01.05] Because, if first machine produces 10 boxes per minute, second machine produces 3 boxes in 10 minutes, according to this how many boxes can be produced while both machines running? If it's 10 boxes for 1st and 3 boxes in 10 minutes for second, this one produces 3 and the other one produces 5?"

Dilara, who was determined that she is in "Uses trial and error strategy" repeatedly said "[06.53] *That is my guess ...*" and "[06.59] *This is my guess I can't make the operations…*". From these statements, it is understood that she made predictions about the result. Also, the expressions used by Dilara in the third activity card in the think-aloud protocol as "[11.46] *Ahmet, who is preparing for the high school entrance exam, has solved less than the number of questions he had solved yesterday. Accordingly, the number of questions that Ahmet solved in his work yesterday is the same as the number of questions he solved in his work today, I mean…*" indicate that he made operations through trial and error. Dilara made the operations on the fourth activity card in the same way. A dialogue between Dilara and the researcher was presented below:

> "[15.33] Dilara: No 13."

"[15.36] Researcher: Let me ask you if 10 pigeons are fed for 12 days or if the number of pigeons increases, will the food increase or decrease? Does the daily spend feed increase or decrease? If the number of pigeons increases, will the daily feed increase or decrease?"

"[16.01] Dilara: Decreases."

"[16.02] Researcher: You say it decreases."

"[16.03] Dilara: Yes, I think it will decrease."

"[16.04] Researcher: So how many days do 30 pigeons are fed?"

"[16.10] Dilara: I think it's 13 days, sir."

"[16.22] Researcher: Why 13? Or how did you find it?"

"[16.27] Dilara: Sorry, my teacher, again for 12 days."

The dialogues show that Dilara tried the answer and changed the answer after the questions asked by the researcher. This situation shows that Dilara uses trial and error strategy. Similarity, Davut used the strategy of trial and error, answered the researcher as follows:

"[06.24] I think it's enough."

"[06.28] Because it's 15 days at first and then 10 more comes afterwards."

"[06.36] It's 40 cats in total."

"[06.50] 10 cats are brought to the shelter and 20 cats are adopted 20 days after the start. 20 cats remain."

"[07.17] I think it is enough."

Davut's answers pointed out that he did not perform any operation and long-term thinking process which he tried to reach only by trial.

The following statements of Orhan, who are determined to exhibit the "*Calculates the ratio by making use of half*" sub-category, can be presented as evidence that they display this sub-category:

"[05.26] Because in order to make 60 boxes in 9 minutes, after how many minutes the first machine should start operating. Now the second machine makes 60 boxes in a 9-minute period, or because the first box, that is how I say, not half, but my teacher, that is, a quarter."

Orhan's statements show that he makes comparisons by half and quarter. Similarly, Davut's expression "[04.38] *It makes 60 in the beginning and this means 30*" shows that he calculated the ratio by using half.

Some participants ignored numbers and they thinks qualitatively (without quantitative operations). We called the sub-category as "*Thinks qualitatively by ignoring numbers.*" For instance, Yakup said that "[08.14] *There's enough food for 30 days. 15 days were brought, enough to be left for 15 days*." and "[09.56] *because 20 of them are not enough. Because if 20 cats have 30 days of food, 15 cats will have 15 days of food.*" These expressions showed that he performed on the second activity card the sub-category of "*Thinks qualitatively by ignoring numbers*". In the same activity card, Dilara's "[09.09] *There were already 30 foods. They were giving 20 cats, right? They give it to 20 cats, 10 cats comes and 30 foods more. I think it has decreased because they are given every day. I think so.*" sentence and "[10.06] *There were food for 20 cats for 30 days. As 10 more cats come on*

top of it, I think 30 food has decreased. Let me read it. It is not enough because there were enough food for 20 cats." sentence supports this situation. Again, Oylum, who does not use numerical data on the second activity card, said "[07.07] *I think it is enough because the cats are not adopted along with the remaining food.*"

Expressions on Yeliz's second activity card, one of the participants determined to perform the "*Tries to reach the result by giving an example*" sub-category shows that she tries to reach the result by giving an example:

> "[06.32] It is 16 if we add another 15 days. 10 more cats are coming. Twenty days after the start, there are 21, 20 more cats. Is it more than 30 days enough for the remaining cats to eat? Now I'm making it up, for example, if there is 3 tons of food."
>
> Expressions on Yeliz's 3rd activity card show that he tries to reach the result by giving an example.
>
> "[09.45] First of all, the information provided is not sufficient. Here I find the answer with information. Let's write yesterday today, it solved less than the number of questions it solved yesterday. Let's say yesterday he solved 20 questions and solved it in 30 minutes."

Again, "[09,25] *Because he says that Ahmet, who is preparing for the entrance exam for high schools today, has solved 40 less than the number of questions he had solved yesterday, that is, if he solved 50 yesterday*." expressions of Orhan in the 3rd activity card indicate that he is trying to reach the result with examples. On the same event card, Oylum's "[07.58] *So let's say 50 questions solve yesterday, but today it solves less than 50 questions, but in less time*." is an example of this code.

Association activities of middle school students in proportional reasoning problems. In the understanding category, the activities carried out by the students to understand the proportional reasoning problems are included. In this category, we reached the sub-categories of "*Uses additive relationships*", "*Establishes a multiplicative relationship*", "*Uses the direct proportion*" and "*Establishes the relationship between the difference and the double*".

The following expressions used by Yakup on the first activity card showed that he performed the sub-category of "*Uses additive relationships*". A dialogue between Yakup and the researcher was presented below:

> "[01.42] If he makes 10 boxes in a minute, 10 boxes 1 minute, 10 in the second minute and 10 in the third minute, 30 in total."
>
> "[02.42] How many boxes does the first machine produce when the second machine produces 50 boxes? The second machine produces the first machine in how many minutes before we will find it in 3 minutes. We will proceed 3 minutes by 3 minutes."
>
> "[03.06] 10 boxes are 3, 10 more boxes make 6, 10 boxes in 9 min, 10 boxes are 12, 10 boxes are 50 in 15 minutes."

When Yakup's expressions and solution in Figure 2 are examined, it is seen that he used additive relations three times in the same problem.

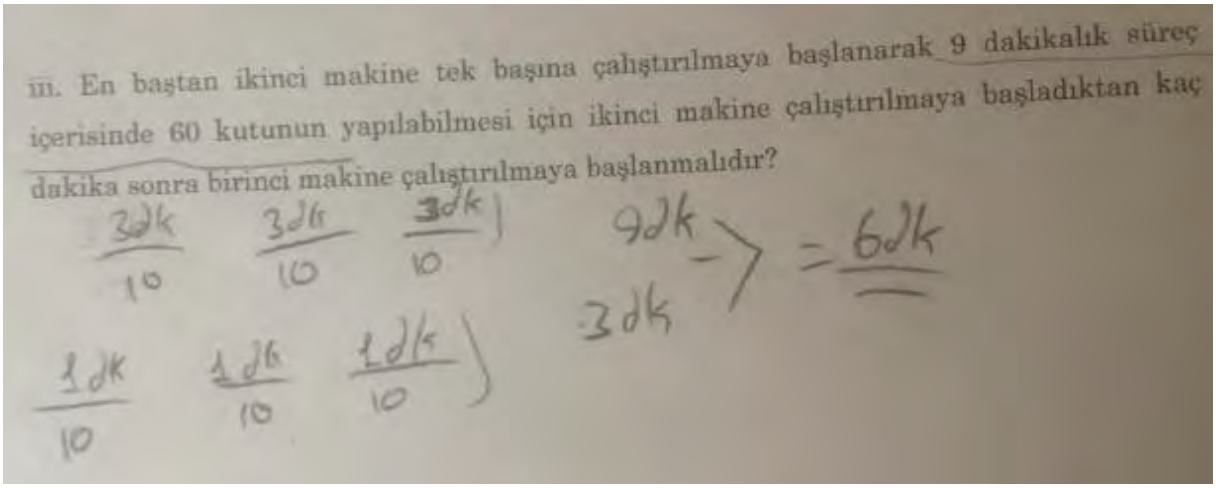

Figure 2. The Acitivity Card of Yakup's Used Additive Relationships

Similarly, Orhan used expressions in the problem on the same activity card. Orhan said that "[02.27] *If 10 boxes are in 3 minutes, 20 boxes are in 6, there are 30 boxes in 9 minutes, 40 in 12 minutes, 50 in 15 minutes. Did we collect them all 15 minutes…*" These representations in excerpt indicate Orhan focused additive relationships.

One of the participants identified as perform the "*Establishes a multiplicative relationship*" sub-category, Yeliz's "[00.28] *Since the first machine produces 10 boxes per minute, if we multiply it by 3, it makes 30 boxes in 3 minutes.*" statement on the first activity card indicates that she established a multiplicative relationship. The "[00.37] *10 boxes were produced in one machine. 10 boxes per minute. For example, 3 minutes each is 30.*" expression of Davut also shows that he established a multiplicative relationship.

The following sentence used by Yeliz, who was determined to perform the "*Uses the direct proportion*" sub-category, in the process of solving the problem in the first activity card, shows that she used the direct proportion:

> "[00.41] How many boxes does the first machine produce when the second machine makes 50 boxes? 10 boxes, then we will multiply 5 by the second machine. So if we multiply 10 by 5, 50 is 3, and if we multiply 5, it's 15. Second one was producing in 15 minutes. In 15 minutes, if it produces 10 in 1 minute, if we multiply in 15 minutes, it can produce 150 boxes."

It is understood that the "[00.28] *Okay. Now while both machines are running, it says how many boxes they make in 3 minutes. If they make 10 boxes here every minute, then they make 30 boxes in 3 minutes*." expressions used by Oylum in the same event card use the correct proportion. Three of the participants made mistakes by using the direct proportion in the problem in the fourth activity card, which contains the inverse ratio. This can be given as an example to participant Yakup's statement:

> "[16.27] 10 pigeons are fed for 12 days. In this, again, 10 pigeons are fed for 12 days. 10 pigeons are fed for 12 days. 10 more pigeons which is equal to 30 pigeons. 30 pigeons are fed 12 plus 12 makes 24, 24 plus 12 makes 36.It's 36 days."

Some participants established relationship between the difference and multiple. We called the sub-category as

"Establishes a relationship between the difference and the multiple." For instance, Oylum said that "[02.32] *It has been said that when the second machine makes 50 boxes, the first machine… Then, since it makes 50 boxes, if the first machine produces 2 minutes ahead of this, then if it is 50 then is the answer 100?*" When the expression is examined, it is understood that the participant first took the difference (3min - 1min = 2min), then considered this difference as double and multiplied the number of boxes by 2, thus reached 100.

Characterization of data obtained from qualitative data. The distribution of the categories and the codes obtained from the qualitative data according to the students is presented in Table 3 and the qualitative findings are summarized with the help of this table.

Table 3. The Distribution of Categories and Codes Obtained from Qualitative Data

Category /Codes		Yakup	Yeliz	Orhan	Oylum	Davut	Dilara
Understanding	Tries to understand the question by reading several times	■	■				
	Carries out the process steps by asking questions to him/her self	■					■
	Uses modeling		■				
	Ignores what is given in the question and gives the first answer that comes to his/her mind						■
Operation Time	Focuses on function (focuses on what is given in question)					■	■
	Uses trial and error strategy			■	■	■	■
	Calculates the ratio by making use of half			■		■	
	Thinks qualitatively by ignoring numbers	■			■	■	■
	Tries to reach the result by giving an example	■	■	■	■		
Relationship	Uses additive relationships	■	■	■			
	Establishes a multiplicative relationship	■	■			■	
	Establishes a relationship between the difference and the double				■		
	Uses the direct proportion	■	■	■	■	■	

When Table 3 is examined, it was seen that four codes for comprehension skill, four codes for processing process and four codes for relationship are formed. It is determined that the codes generally show an equal distribution according to the levels of the participants. However, it was inferred that the skill of associating is higher in students with high success.

Discussion and Conclusion

This study, which aimed to examine the proportional reasoning processes of middle school seventh grade students, did not only support the literature but would open new discussions in the related literature. The study

examined the proportional reasoning process by considering the quantitative and qualitative methods together. Thus, it revealed which types of proportional reasoning problems students were more successful in and what they thought of proportional reasoning. In addition, the processes of understanding proportional reasoning problems and their relationships in these processes are explained in more detail. Findings to compare proportional reasoning by gender indicated that the proportional reasoning skills of female students were significantly higher than that of male students.

When the findings for proportional reasoning by gender were compared, it was found that the proportional reasoning skills of female students were significantly higher than that of male students. This difference was also obtained in the types of qualitative comparison and quantitative comparison problems of proportional reasoning. However, it was determined that proportional reasoning did not differ by gender in finding the not given value and inverse proportion problem types. This finding coincides with the findings in Ünsal's (2009) study about seventh grade students' proportional reasoning skills compared to success, attitude and gender variables. Karplus et al. (1977) found that boys were more successful than girls in proportional reasoning in their survey research conducted to examine the proportional reasoning levels of students from seven different countries. The quantitative findings of the study showed that the type of problem that students were most successful in was qualitative comparison, and the type of problem that they failed most was quantitative comparison. This finding supports previous studies that have concluded that proportional reasoning skills differ according to problem types (Ayan & Isiksal-Bostan, 2018). When the qualitative results of the study are examined, the proportional reasoning skills of the students are categorized under three categories: Understanding, operation process and relationship. In the process of understanding the proportional reasoning problems, it was determined that the students were trying to understand the question by reading it several times. In the literature, many studies have shown that secondary school students read over and over again to understand the problem (Öztürk et al., 2018; Pativisan, 2006). It can be said that reading the questions several times in the process of proportional reasoning supports the literature. It was determined that the students who participated in the study carried out the process steps by asking themselves questions during the proportional reasoning process. Öztürk et al. (2018) determined that students asked themselves questions to understand the problem. Students used modeling to understand problems. Langrall and Swafford (2000) stated that students use pictures, models or manipulative to understand situations and make a qualitative comparison. Langrall and Swafford (2000) emphasized that modeling helps students practice proportionally reasoning to develop a better understanding of how the value in each ratio changes. In this context, it can be said that the students' use of models to understand proportional reasoning problems supports the literature. Some of the students participated to the study ignored what was given in the question and gave the first answer that came to their mind. This feature, defined as a hasty approach, is shown as one of the basic dimensions of problem solving approaches (Demir, 2016).

Another finding related to the proportional reasoning process was about the operation process. It was determined that some of the students participating in the study were fixated to the numbers given in the question, so they focused on these numbers instead of taking action. Goldstein (2013, p. 562) pointed out that fixating occurred during the problem-solving process and caused the student to focus on the prominent aspect of the problem, ignoring the other part. Studies have also pointed out that students are fixating in the problem

solving process (Öztürk & Kaplan, 2019). It was determined that some participants used the trial and error strategy in the operation process. This finding supports the findings of previous studies that indicated that students used the trial and error strategy in the proportional reasoning process (Lundberg & Kilhamn, 2018). Lundberg and Kilhamn (2018) emphasized that students' doing operations by using trial and error strategy in the proportional reasoning process may be due to the fact that mathematics textbooks or teachers touch upon this strategy too much. Lundberg and Kilhamn (2018) stated that in the proportional reasoning process, students can use strategies to make solutions by doubling or halving. However, the researchers emphasized that although these strategies exist in different sections in the textbooks, they are not associated with proportional reasoning. It was determined that some of the students participating in the study thought qualitatively by ignoring the numbers. This situation, known as data neglect, is mentioned in the literature. Kahraman et al. (2019) determined that students neglected data while solving problems requiring proportional reasoning. It was determined that some of the students participating in the study try to reach the result by giving arbitrary value, in other words, an example. The efforts of the students to reach the result by giving value supports the finding that Ayan and Isiksal-Bostan (2018) make arbitrary values to the lengths while making comparisons in the problem solving process of the students. The reason why students try to reach to results by giving arbitrary value may be because they do not know how to demonstrate that their knowledge is mathematically correct. Another qualitative finding is that students use additive relationships in the process of making associations. This finding supports the findings obtained in the previous studies that students use additive relationships in the proportional reasoning process (Ayan & Isiksal-Bostan, 2018; Duatepe et al., 2005; Kahraman et al., 2019; Toluk-Uçar & Bozkuş). Karplus et al. (1977) determined that female students are looking for more additive relationships in questions that require proportional reasoning. In this study, such a situation could not be determined for female students. Some of the students who participated in the study established a relationship between the difference and double, while others established a multiplicative relationship in problems involving proportional reasoning. In many studies, it has been determined that students establish a multiplicative relationship in the proportional reasoning process (Arican, 2019; Cramer & Post, 1993; Lamon, 2012; Lundberg & Kilhamn, 2018; Toluk-Uçar & Bozkuş, 2016). The reason why students establish multiplicative relations may be the use of the concept of cross multiplicative in teaching the ratio-proportion subject. Lundberg and Kilhamn (2018) emphasized that students can focus on multiplicative relationships because teachers often use the concepts of finding their doubling and halving. It was determined that the students participating in the study benefited from the direct proportion in the process of establishing a relationship. Research suggests that the direct proportion can be used as a strategy in the proportional reasoning process (Lundberg & Kilhamn, 2018). The reason why students use the direct proportion as a strategy may be because they have a good understanding of multiplicative relationships. Another reason for using the correct proportion strategy may be that they have memorized the cross multiplicative.

While some qualitative findings of the study supported the quantitative findings, some did not. For example, the quantitative findings of the study showed that students scored lower in problems requiring quantitative comparison, and were more successful in problems requiring qualitative comparison. In qualitative findings, it was determined that students think qualitatively while ignoring the numbers. On the contrary, while the quantitative findings of the study showed that girls were significantly more successful than boys in some types

of problems, the qualitative findings indicated that the codes were equally distributed to female and male students.

Practical Implications and Future Research

The results of this study showed that students have low proportional reasoning skills and make many mistakes during the proportional reasoning process. There may be several reasons for this situation. One of these reasons may be the lack of proportionality in the mathematics curriculum for reasoning. Proportional reasoning skills of students can be improved if practitioners include proportionality reasoning after they include ratio-proportion and percentage subjects in the curriculum. In addition to this, trainings to be designed for proportional reasoning can also be expected to improve proportional reasoning.

This study, which aims to examine the proportional reasoning problem solving process, was conducted under certain limitations. The first of these limitations is related to the number of participants of the study. The study was conducted with six students in order to enable detailed examination in the qualitative part of the study. For this reason, generalization could not be made. In addition, in this study, the data was collected only with activity cards and the think-aloud protocols, and these data collection tools only allow to examine students thoughts. However, thanks to the technology entering every field with digital transformation, brain imaging is performed using devices such as fMRI, PET, EEG and the mental processes of individuals are examined. In this context, future researchers can examine the proportional reasoning process mentally. For example, it is possible to determine which regions are active in the brain by examining the mental processes of the individual in terms of finding what is not given, inverse proportion, quantitative comparison and qualitative reasoning. Although this study has been conducted under certain limitations, it has made some important contributions to the current literature on proportional reasoning. Mathematics educators should be aware of the strategies students use in developing proportional reasoning and apply different strategies in the classroom. Moreover, educators should consolidate quantitative reasoning-enhancing activities or take advantage of qualitative reasoning. In addition, in the curriculum, emphasis should be made on activities that improve proportional reasoning and include proportional reasoning in teaching of the proportion subject.

Note

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