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Conceptual Article

Using tasks to bring challenge in mathematics classroom

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Rich and challenging tasks can be the vehicle to bring mathematical challenge in classroom. Challenge emerges when you don't know how to solve the task at first but you can figure out, that is when the solvers are not aware of certain tools to solve the tasks and they have therefore to invent some mathematical actions to proceed. Some challenging tasks in the paper-and-pencil as well as in a digital environment will be presented. The aim is to highlight their potential (i) in engaging students to actions that make sense for them from the mathematical point of view, (ii) to support students in their experimentation and development of problem-solving strategies, (iii) to foster creative mathematical thinking, and (iv) to provoke students' curiosity as the starting point of meaning-making actions in mathematics.

Keywords: Challenging tasks; Meaning-making; Problem-solving strategies; Creative mathematical thinking

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1. Introduction: Tasks and Challenging Tasks

Although memorizing facts, mastering rules, and computational algorithms are important for mathematical learning they constitute just a part of mathematical learning since it entails much more. Conceptual understanding, investigations, experimentation, conjecturing, proving, games, and puzzles fostering mathematical knowledge are among the ingredients of mathematical learning and it seems that mathematical tasks play a central role in effectively teaching mathematics. Sullivan, Clarke, and Clarke (2013) suggest engaging students by utilizing a variety of rich and challenging tasks to allow students to better understand what mathematics is and how mathematics is developed. Walls (2005) defines mathematical tasks as 'the kinds of activity that teachers of mathematics assign or *set* their learners' (p. 751) and they take a variety of forms, length, and complexity. Sometimes they are just questions posed verbally. Others are worksheets or content of the students' textbooks. They could be open-ended questions or real-life situations that should be explored from the mathematical point of view. However, what exactly is meant by "challenging task"?

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The definitions found in the relevant research literature converge. Challenging tasks are complex and absorbing mathematical problems that meet certain criteria (Russo, 2015; Sullivan et al., 2011; 2014):

(i) They require students to process multiple pieces of mathematical information simultaneously and make connections between them and for which is more than one possible solution or solution method (Sullivan et al., 2014, p. 597).

(ii) They must involve more than one mathematical step.

(iii) They should be both engaging and perceived as challenging by most students (Russo & Hopkins, 2017, p. 290)

(iv) The solvers are not aware of procedural or algorithmic tools that are critical for solving the task and therefore they have to invent mathematical actions to solve it (Powell et al., 2009).

Smith and Stein (2011) describe the same thing as 'doing mathematics' instead of 'challenging tasks'. They argue that such tasks provide students opportunities to determine their approach, to identify and express patterns. Moreover, given that these tasks are not previously seen by the students in their textbooks, the students not only determine their own methods of solutions but also record these solutions and communicate them to others.

One key feature of challenging tasks is their *authenticity* in the sense that they are characterized by a certain degree of complexity and they are not amenable to a ready-made solution (Diezman & Watters, 2000). Students should be engaged in challenging tasks for important pedagogical, psychological and social reasons (Powell et al., 2009) even in the early years of schooling when they possess a very small fraction of formal mathematical knowledge, thus allowing students at different levels to pursue the same learning objective (Russo, 2016). Challenging tasks 'engage students in *cognitive processes* at the level of doing mathematics and engage students in high-level thinking and reasoning' (Henningsen & Stein, 1997, p. 546). More precisely, they enable students to develop a fuller understanding of the many aspects of a mathematical concept, enriching their concept image (Stillman et al., 2009). Additionally, these tasks encourage the use and development of *metacognitive skills* which is crucial for success on challenging tasks (Diezman & Watters, 2000). Solving such tasks enhances *motivation* (Lupkowski-Shoplik & Assouline, 1994) and facilitates the development of students' *autonomy* (Betts & Neihart, 1986; Applebaum & Leikin, 2014) by providing students opportunities to approach the challenges at different levels of mathematization (Stillman et al., 2009) which is necessary for equipping students with the capacity to persist with a challenging task (Russo & Hopkins, 2019). Motivation is in its peak when tasks are within students' ability to grasp and conquer but hard enough to be fun. The amount of effort students are willing to put in varies with their confidence and stamina, but all of them want at least some challenge: tasks that are too easy are boring; tasks that feel inaccessible are forbidding (Goldenberg et al., 2015).

Finally, an additional reason students should be engaged in challenging tasks is that solving them contributes to the development of creative mathematical thinking. Vale and Pimentel (2011) claim that students can be creative if they are attracted and challenged by the task. Curiosity is a critical component of creativity (Arnone, 2003) and given that "students can become unmotivated and bored very easily in ''routine'' classrooms unless they are challenged" (Holton et al., 2009, p. 208), challenging situations provide an opportunity.

The students themselves value these tasks for at least three reasons: *enjoyment*, *effort*, and *meaningful mathematics*. In their study, Russo and Hopkins (2017) in their study found that *enjoyment* was the most frequent response of the participating students. The participants derived satisfaction from the process of being challenged mathematically. Another prevalent theme that emerged from their analysis was the students' willingness to *attempt* a task they found challenging. They insisted on a task even they were initially unsure how to begin. Building the persevering habit of mind means that 'we must have enough stamina to continue even when progress is hard and enough flexibility to try alternative approaches when progress seems *too* hard' (Goldenberg et al., 2015, p. 14). By *meaningful mathematics,* the students referred to the fact that challenging tasks were more purposeful compared to mathematical tasks within their school mathematics considering the context in which the mathematics was situated as more meaningful for them (personally relevant to their real life, for example).

Implementing, however, challenging tasks in classroom implies also a series of issues concerning the role of the teacher. How to effectively introduce students to such tasks? Is there a line between making the task challenging yet accessible instead of challenging and overwhelming? How *often* challenging tasks should be used? Are there certain *techniques* to manage the challenge? Cheeseman, Clarke, Roche, and Walker (2016) advocate among others that when introducing such tasks to the students it is necessary to (i) connect the task with students' experience, (ii) communicate enthusiasm about the task including encouraging students to persist with it, (iii) hold back from telling students how to do the task, and (iv) clarify the task without explaining or demonstrating a solution method.

Teaching with challenging tasks often proceeds in three phases (Stein et al., 2008; Baxter & Williams, 2010). It begins with the '*launch phase*' of the problem to students (the teacher introduces the problem, the available tools, and the nature of the expected outcome). This is followed by the '*explore phase*' (students work on the task. At the same time the teacher offers encouragement, provides challenges, gives insight or hints as needed). The process concludes with the '*discuss and summarize phase*' (the teacher facilitates a whole group discussion providing students an opportunity to present their particular approach to solving the task).

Challenging experiences must be provided regularly to give multiple opportunities for students to access such tasks and bring them to the realization that it is an expectation of all students to be able to do so (Stillman et al. 2009). As Kadijević and Marinković (2006) claim "Only a continuous and well-planned use of challenges gives good results" (p. 33). Indeed, this regular solving of mathematical challenges indicates that it is applicable for all students no matter their learning abilities or their experiential background, thus becoming a suitable option for inclusion in the classroom. Sriraman (2006) found that when providing regularly challenging tasks in secondary classroom, students of varying mathematical abilities were consistently able to devise strategies, examine examples, and control the variability of the problem situation.

Due to the lack of knowledge to effectively use challenging tasks in the classroom, teachers are often reluctant to use them. The relevant literature offers some techniques to help teachers overcome their reluctance and cope with it. Sullivan and Clarke (1991) suggest the use of 'good questions' as they call them. They define them as having three features: they require more than recall, they are open-ended, and they promote active learning. Specific methods for constructing 'good questions' are given. However, open questions per se are not sufficient to facilitate the deeper thinking required when using challenging tasks (Herbel-Eisenmann & Breyfogle, 2005). This is why teachers need to use some questioning techniques (for example, the *funneling* and *focusing* methods. For more details see Goos, Stillman and Vale, 2007).

The relevant literature distinguishes two different ways of grouping challenging tasks. The first way is about paradoxes, counterintuitive propositions, patterns and sequences, geometry, combinatorics, and probability (Powell et al., 2009). For the second way, Applebaum and Leikin (2014) working with teachers describe five types of challenging tasks: (i) problems that require logical reasoning, (ii) nonconventional problems, (iii) inquiry-based problems, (iv) problems that require performing different ways of solutions, and (iv) problems that require a combination of different mathematical topics. The first group examines mainly the mathematical content of the tasks whereas the second is focused on the task's characteristics in alignment more or less with the definition of 'challenging tasks'.

The aim of this plenary is to present another way of grouping challenging tasks that takes place so much in the traditional environment of paper-and-pencil as well as in a digital environment (videogames, for example), on the basis of the pedagogical aim of the tasks. This group includes four types of challenging tasks: (i) tasks that engage students in mathematical meaning-making,

(ii) tasks that facilitate systematic experimentation and development of strategies, (iii) tasks that foster creative mathematical thinking, and (iv) tasks that challenge students' curiosity against a problem-solving situation.

2. Challenging Tasks That Engage Students in Mathematical Meaning-Making

This section is built upon the work of Papadopoulos (2019) who examines aspects of algebraic thinking exhibited by grade-6 (11-12-year-old) students using a rich environment called *mobiles puzzles*. They are a nice example of challenging tasks that support mathematical meaning-making in the classroom.

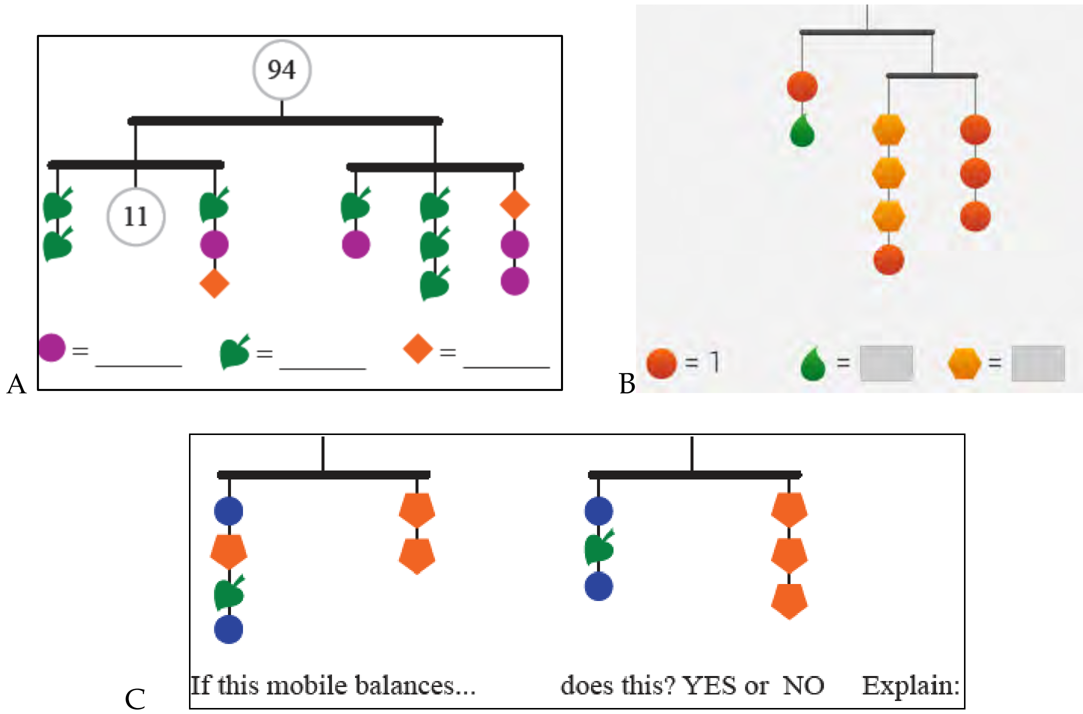

Figure 1. Mobile puzzles in balance

The core idea in these puzzles is that multiple balanced collections of objects are presented. The horizontal beams are always suspended in the middle by strings. This means that the two ends of each beam must have the same weight. It is supposed that beams and string weigh nothing. Identical shapes have the same weight. Different shapes may have the same or different weights. The total weight or the weight of some shapes might be given, and the solver is asked to determine the weight of the unknown shapes (Fig. 1, A and B). These tasks are considered as puzzles rather than as problems by the students but if one examines them carefully it is easy to see that they focus on the equality of expressions. Students just use their imagery to build the logic of balancing equations while at the same time they do not need algorithms or rules to solve them. Another kind of mobile puzzles challenges students to decide whether a mobile balances (always, sometimes, never) based on the given information (Fig. 1, C). Combining partial information to make a decision for another mobile makes these tasks more cognitively demanding.

The information included in a mobile puzzle can be presented in the form of a system of equations. So, for example, if we denote with l the leaf, c the circle, and d the diamond, the whole system A of Figure 1 might be represented by the equations below:

$$3l+c+d+11=47 \quad (\text{left branch})$$
$$2l=l+c+d \quad (\text{left branch})$$
$$4l+3c+d=47 \quad (\text{right branch})$$
$$l+c=2c+d \quad (\text{right branch})$$
$$7l+4c+2d+11=94 \quad (\text{whole mobile})$$

The mathematical meaning-making in these tasks concerns the intuitive application of the conventional formal rules for solving equations that will be later introduced to the students as the standard algebraic "moves" in the context of algebra courses. Indeed, aiming to find the unknown weights the students, informed by the structure of the mobile, induce certain rules (isolate variables, add or remove the same amount from both sides, substitute weights that are known to be equal) trying to maintain the balance. This process includes an intuitive sense of certain properties of the operations that will be later introduced formally as reflexive, symmetric, commutative, and associative properties. In the case of mobile C (Figure 1) a typical approach followed by many 6th graders was to interpret the balance situation with expressions such as 2●+🍏=3⬟. In the same system, the mobile on the left was transferred by the students in the form of 2● + 🍏 + ⬟ = 2⬟. If a pentagon is removed from both sides the system still balances and therefore a new equation is formed, 2● + 🍏 =⬟. New information now became available. After removing the pentagon the left branch (left mobile) becomes identical with the left branch of the mobile on the right. So, the left branch of the mobile on the right can be substituted by its equal (Fig. 2, left).

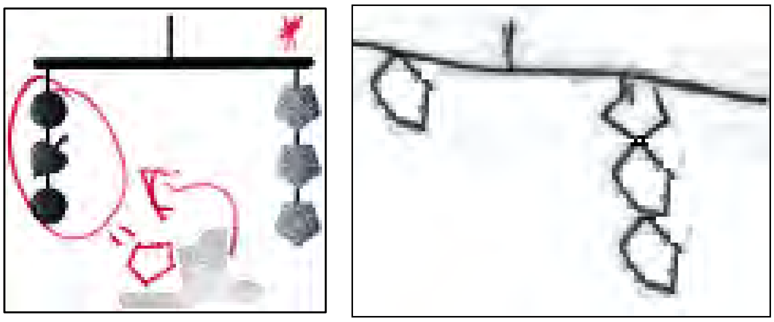

Figure 2. Intuitive application of formal rules

This action adds new information for the system. The substitution leads to another equation that might facilitate the answer to the question of whether this new mobile balances or not: 1⬟=3⬟ (Fig. 2, right). Now the answer was obvious. One pentagon can not have the same weight with three identical pentagons (unless they weigh zero).

The extent to which these tasks are considered by the students as challenging ones depends on the solvers' mathematical background. In another research study that is still in progress with pre-service teachers, a collection of mobile puzzles is used to identify their understanding of the equal sign and the existing algebraic relationships of these mobiles. So, if the solver has the adequate algebraic thinking then these tasks do not challenge them. The solvers simply turn to the use of algebraic notation to easily solve the system (Fig. 3). So, for them, these tasks are not challenging at all.

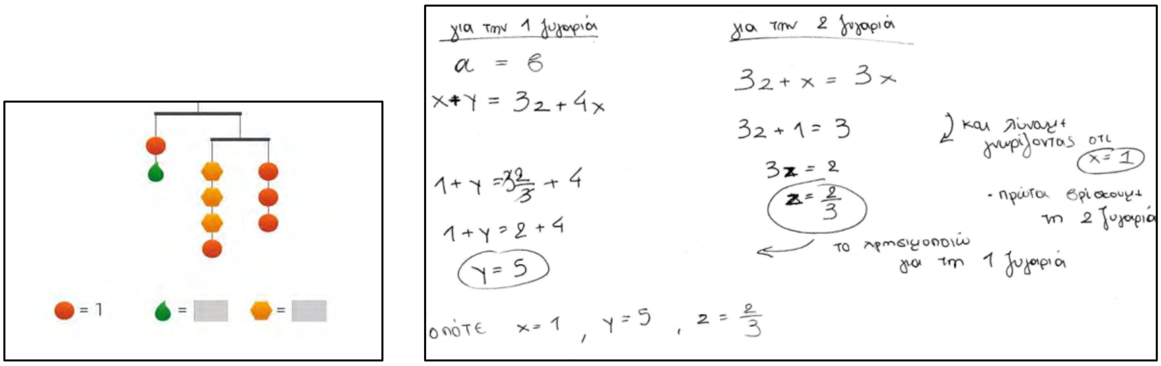

Figure 3. Algebraic solution of the mobile puzzle

However, in case the solvers miss mastery of skills necessary to algebraically solve the system, they invent smart and creative alternative ways to cope with the challenge (Fig. 4).

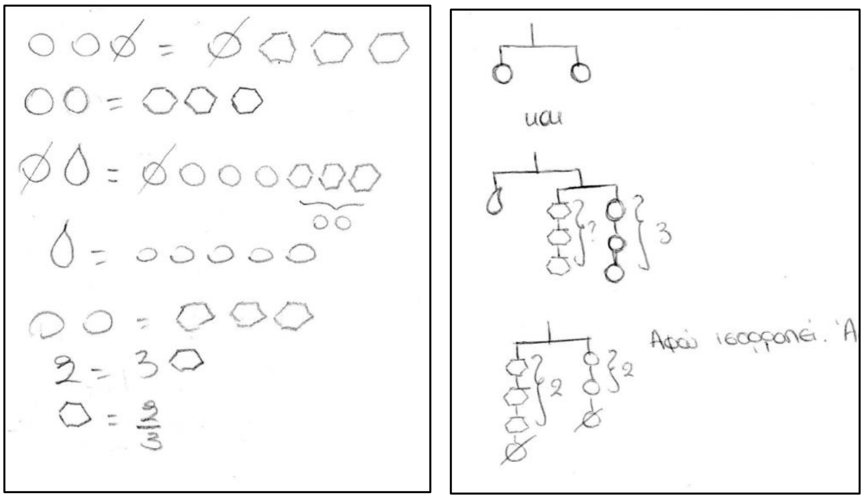

Figure 4. Alternative solutions to the mobile puzzle

If their answers examined carefully it is easy to identify all the mathematical bits of knowledge that is acknowledged as necessary to solve an equation such as subtracting the same quantity from both sides and substituting something with its equal.

In these examples the students in their effort to preserve the balance took actions that were sense-making to them (instead of following rules they do not understand).

3. Challenging Tasks That Facilitate Systematic Experimentation and Development of Strategies

This section is built upon the work of Thoma and Biza (2019) who follow four children aged 6 to 8 years old and examine their problem-solving techniques while they play the video game 'The logical journey of Zoombinis' and more specifically one of its puzzles named '*The Mudwall puzzle*' (Fig. 5).

Figure 5. The Mudwall puzzle

The Zoombinis are a race of small blue creatures living initially on a small island (called Zoombini isle). At some point, they were enslaved by their neighbors, the Bloats. So, in the game, we follow the Zoombinis as they try to get a new home facing a series of logical puzzles the solver must solve to help the Zoombinis get there. In the Mudwall puzzle, the obstacle for the Zoombinis is a tilled wall (5 x 5) and some of the tiles include a special mark (specific number of dots). The player must use a mudball launcher machine to help the Zoombinis going over the wall. The key to the solution of the puzzle is for the player to hit the marked tiles (see Figure 5). The number of dots on each tile indicates the number of Zoombinis that will go over the wall. The player must each time decide the color of the mudball (blue, red, yellow, purple and green) and the shape inscribed on it (square, triangle, star, circle, diamond). Each cell corresponds to a unique combination of color and shape. Therefore, from the mathematical point of view, the wall can be seen as a matrix that is a permutation of five shapes on one axis and a permutation of five colors on the other axis. But, this information is hidden from the player. Additionally, there is a hidden permutation of the two axes since colors and shapes -each time the game starts- can be either on the horizontal or the vertical axis. This means that there are overall 5! x 5! x 2! = 28800 possible different matrices. So, the problem that has to be solved is to find the coordinates (shape and color) of the marked tiles to help the zoombinis going over the wall. Moreover, there is a limitation on the number of mudballs. Therefore, the player must solve the puzzle within the given range of available efforts.

It is reasonable to expect that initially, the players decide to work on a trial-and-error basis. But, since this is a non-promising approach, they turn towards identifying a strategy necessary to successfully accomplish the task. The results of this study show that these very young participants were able to develop a systematic experimentation as this is described by Papadopoulos and Iatridou (2010). According to them, a systematic experimentation applies the following steps: (i) Identify the structural components of the problem, (ii) Then keep all but one unchanged and experiment with this one, changing it in various ways to identify its role in the problem's solution, and (iii) Then keep this component constant and change another one, and so on, until making clear how all these components contribute to the solution (p. 215).

Figure 6. Systematic experimentation in the Mudwall puzzle

So, one of the participants, after many efforts without success, focused on the fact that there are two elements that must be taken in mind to set up an effective strategy. The first step is to keep constant the color and change progressively the shape. This will make evident the axis that corresponds to this specific color (Fig. 6). At the same time, you get other important information. If you complete the whole row with red color mudballs of different shapes you reveal the shape that corresponds to each column. Thoma and Biza (2019) refer to it as *stepping-stone technique.*

So, something that started as a challenging game led the students to develop certain skills on problem-solving keeping at the same time the excitement of the game and the connection with the students' actual "real life". As the same researchers conclude in their paper "students are the ones discovering and adapting the technique themselves. Problem-solvers implicitly guided by a global problem, have the agency to interact and experiment in a story-driven and challenging environment and thus find the need for a more efficient solution" (pp. 2982-2983).

4. Challenging Tasks That Provoke Creative Mathematical Thinking

Our next example will be retrieved from the work of Papadopoulos, Vlachou, and Kioridou (2020) who used a learning environment called 'Staircase' (Slezáková, Hejný, & Kloboučková, 2012). The aim was to examine the notation invented by primary school students to write negative numbers (given that they have never been taught anything about negative numbers).

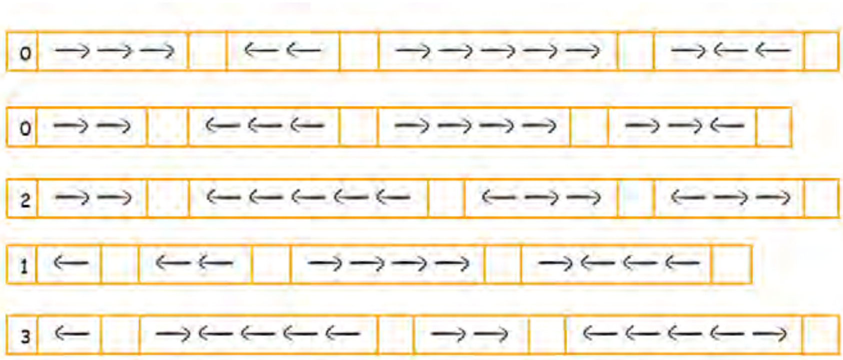

Figure 7. The Staircase examples

In this environment, an initial number as a starting point is given (*number as an address*) and all the other numbers are represented by the number of steps forward and backward (*number as an operator of change*). In their study Papadopoulos et al. (2020) asked the students to make the steps (physically or mentally) and write in the empty box the arithmetical evidence about their placement on the number line. The first example does not cause any problem since all the intermediate results are positive integers (Fig. 7). In the second example, the first operation is $0 + 2 = 2$. But after that, the students must go backward 3 steps. The operation implied here is $2 - 3$ which means that the student is now at -1. This was a challenge for the students. They did not know anything about that part of the number line. They did not know its existence. They had to invent a notation for the numbers in this part of the number line that would make sense to them.

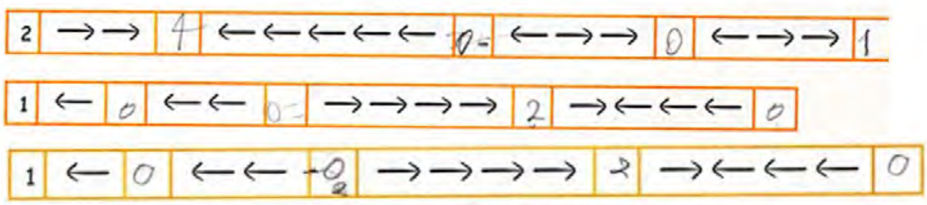

Figure 8. Students' notation about negative numbers

As can be seen in Figure 8, the challenge posed to the students led them to invent creative notations that were meaningful to them. In the first two examples, negative numbers are symbolized with zero and some dashes according to how many steps behind zero we are on the number line. More specifically, in the first example, the student was at number 4 and had to move five steps backward. This means that he would be one step behind zero. He decided to denote it by zero accompanied by one dash (**0-**). In the second example, the same student was at 0 and had to move two steps backward. This means that the new stop was two steps behind zero. He consistently used the same way of symbolizing these numbers and developed his notation system.

In this case, he used zero accompanied by two dashes (**0=**). In the last example, another student, for the same calculation (from zero two steps backward) used the notation $-\mathbf{0_2}$. This notation incorporated all the necessary information. According to the student, this means two steps (2) behind (-) zero (0). The interesting thing is that this was a functional notation since the students consistently used this for all the intermediated calculations. Perhaps one could object that this is not "creative" with the broad sense of the term. However, the way students assign meaning to every detail of their notation constitute a nice example of what the literacy calls mini-c(reativity) as it is described by Beghetto and Kaufman (2009): "Mini-c creativity pertains to the novel and personally meaningful insights inherent in learning and self-discovery" (p. 41).

5. Challenging Tasks That Provoke Curiosity against a Problem-Solving Situation

This section draws its content from a case of a secondary mathematics teacher, Dimitris, who challenged his students by redesigning a digital artifact (Kynigos, 2017). More specifically, he chose an equation problem in an interactive scales task for grade 8 students (Fig. 9). This is a classic problem. The equation that must be solved is 3x+200=x+600. The unknown and known weights are dynamically manipulable through the four sliders above the scales.

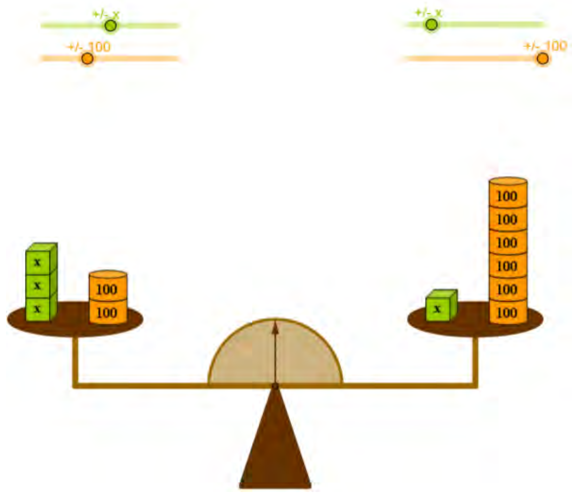

Figure 9. An interactive scales task

However, Dimitris decided to make it more interesting for his students by challenging their curiosity. In his version, the solvers imagine the weights (blue for the unknown and red for the known ones that weigh 20gr each) inside two pots and the sliders change the number of weights in each pot (Fig. 10, left).

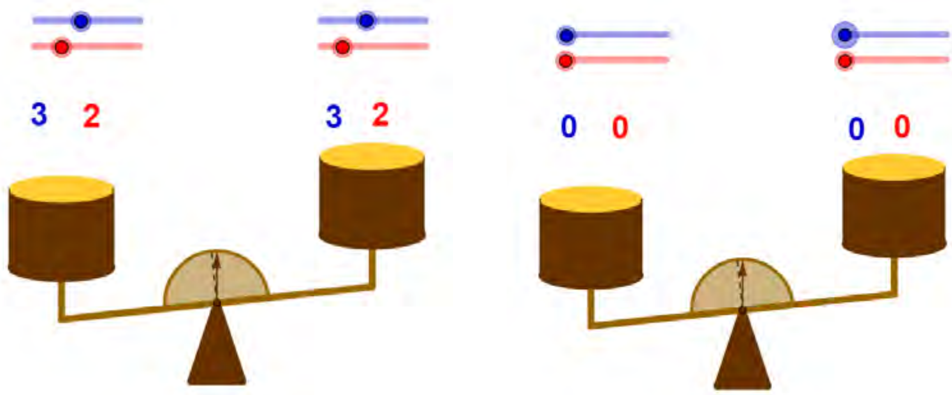

Figure 10. Modified version of the scales balance

Although it seems similar to the original one there is a significant difference since the new scale is faulty. By default, the scale includes the same number of weights on both sides (3 blue and 2 red), but the scale does not balance. The visual impression is that $3x + 2 > 3x + 2$ or equivalently $0x > 0$ (Fig. 10, right). This is surprising and triggered the interest of the students who did not abandon their effort. On the contrary, they insisted to satisfy their curiosity and find both the fault and the actual weights. So, they obtained a balance situation (Fig. 11, left) which was translated in symbolic language as $3x + 40 = 3x + 100$. This made them identify that the fault of the balance was 60gr.

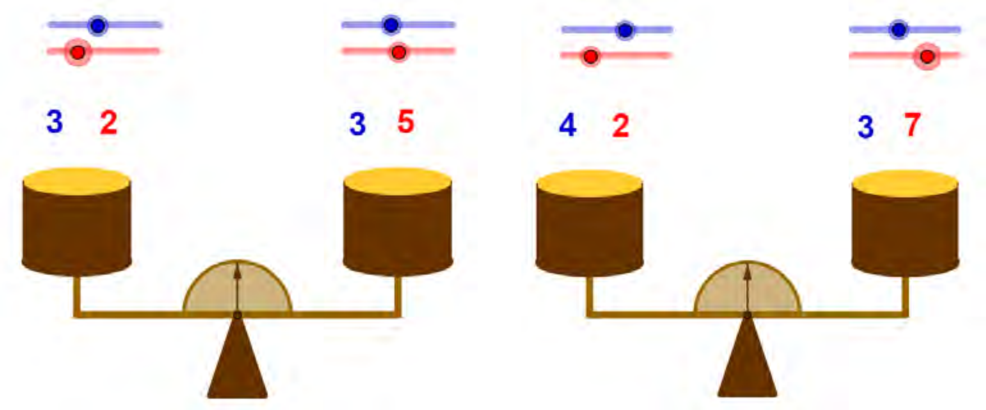

Figure 11. Finding the unknown in a faulty scale

Now they were ready to find the unknown weight. The increased the blue ones on the left by one (from 3 to 4) and then they started adding red ones on the other side until balance. They needed two red ones (Fig. 11, right). Therefore, the unknown blue entity weighs 40gr.

In an ongoing study with preservice teachers who were asked to use the same artifact, more benefits from this challenge have been identified. An initial analysis of the collected data indicates that the participants are involved in (i) Exploring (different ways of dealing with surprise and ambiguity), (ii) Explaining (their own ideas), (iii) Envisaging (predicting what the outcome might be before trying out), (iv) Exchanging (sharing different approaches), and (iv) bridg(E)ing (making links between their work with the scale and the language of the 'official' mathematics) (for more detail on the 5Es framework see Hoyles & Noss, 2016).

6. Some concluding thoughts

The whole endeavor of mathematics teaching includes a series of actions taken by the teacher. New mathematical content should be taught to all students providing at the same time plenty of opportunities for students to master this content. At the same time teacher must support those of the student who experience difficulties with mathematical understanding but also to provide supporting experiences for those who are more capable than the others. Given that normal classrooms are populated by students exhibiting a broad range of abilities the result often is an exhausted teacher. Differentiated learning has been suggested as a promising approach to overcome this difficulty. This means preparation of content in multiple levels, individualization of the content for each student, and access to a variety of learning resources. Many teachers consider this approach time consuming that absorbs all their energy. Digital technology then appeared aiming to support the individualizing effort. The fact is that many of these online mathematics learning solutions are in the spirit of 'drill-and-practice'. They should not be underestimated since they contribute to mathematical understanding and in this sense, they are necessary, but not sufficient.

Challenging tasks might lessen this problem. Sometimes researchers call them 'rich tasks' or 'low threshold, high ceiling tasks' which I find also very successful term. Their main advantage is that all students can make a start to the problem no matter if they need some kind of assistance. By being 'low threshold' these tasks allow less confident students to get some self confidence since they can have some success. By being 'high ceiling' involve students to deal with mathematics in a more advanced level. The task itself remains simple but gradually the required thinking to solve the task becomes quite complex.

Zohar and Dori (2003) explain how challenging mathematics problems in formal classrooms help *all* students to appreciate mathematics and consider them accessible and attractive: 'Instruction of higher order thinking skills is appropriate for students with high and low academic achievements alike' (p. 174).

I would like to end with a Howard Whitley Eves quote: 'A good problem should be more than a mere exercise; it should be challenging and not too easily solved by the student, and it should require some "dreaming" time' (Eves, 1990, p. 2).

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