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Actionable 10: A Checklist to Boost Mathematics Teaching for Students With Learning Disabilities

Soyoung Park, MA[1], Diane Pedrotty Bryant, PhD[1], and Barbara Dougherty, PhD[2]

Abstract

This article presents a checklist of 10 evidence-based practices for educators to apply in mathematics instruction for students with learning disabilities. The checklist is "actionable," meaning the items on the checklist can be put into action immediately. It provides practical strategies teachers can adopt to fit their lessons regardless of their specific mathematical domain areas or student grade level. The focus of this article is translating research of evidence-based strategies into practice for mathematics instruction.

Keywords

evidence-based practices, mathematics, learning disabilities, research to practice

Over the past decade, substantial research has been conducted on mathematics instructional methods to improve the performance of students with mathematics learning disabilities (MLD; Bryant et al., 2014). Nevertheless, a large number of students struggle learning mathematics, including those who have more persistent difficulties. Researchers have developed and evaluated high-quality evidence-based interventions that include strategies and methods to support mathematics achievement (Cook & Odom, 2013).

This article provides teachers with a checklist for translating research into practice for mathematics instruction. The Actionable 10 Practices Checklist is based on findings from previous research to include the most critical aspects of interventions.

The Actionable 10 Practices Checklist

Teachers can use the Actionable 10 Practices as part of their mathematics instruction to help students with MLD better comprehend concepts and skills, which are based on the Common Core State Standards for Mathematics (CCSSM; National Governors Association Center for Best Practices & Council of Chief State School Officers [NGACBP/CCSSO], 2010). The checklist format can help teachers quickly identify how to implement evidence-based strategies in mathematics lessons. In addition, teachers can incorporate the strategies with this checklist into existing mathematics interventions for students with MLD. Explanations are included on how to incorporate the checklist, including definitions and examples (see Figure 1).

Actionable Practice 1: Explicit Instruction

Explicit instruction involves a series of instructional steps including (a) opening, (b) modeling, (c) prompted practice, (d) unprompted practice, (e) closing, (f) pacing, (g) checks for understanding, and (h) reflection (Archer & Hughes, 2010). Researchers have shown repeatedly that explicit instruction is an effective teaching strategy that leads to student success in mathematics (Archer & Hughes, 2010; Doabler et al., 2017). Explicit instruction is a way for teachers to select a learning objective, provide structured learning experiences, explain concepts and skills,

[1]The University of Texas at Austin, Austin, TX, USA
[2]University of Hawai'i at Mānoa, Honolulu, HI, USA

Corresponding Author:
Soyoung Park, College of Education, The University of Texas at Austin, 1912 Speedway, Stop D5000, Austin, TX 78712, USA.
Email: soyoungpark@utexas.edu

Actionable 10 Practices Checklist Directions: Check the box next to each practice indicating that you already implement a practice or plan to use the practice	
	Implementing Explicit Instruction. Teaching using a series of instructional steps including (a) opening, (b) modeling, (c) prompted practice, (d) unprompted practice, (e) closing, (f) pacing, (g) check for understanding, and (h) reflection.
	Promoting Contextual Teaching. Connecting real-world situations and problems to the concepts being studied.
	Building Precise Mathematical Vocabulary. Teaching accurate mathematical language, using the same terms in subsequent lessons, and expecting students to use the terms correctly.
	Conducting Error Analysis. Identifying student's errors during a task and determining detailed areas of instructional need.
	Encouraging Mathematical Discourse. Planning for student discussion as an integral part of the lesson.
	Presenting Multiple Representations. Teaching students ways to apply multiple representations for understanding concepts.
	Providing Constructive Feedback. Providing strategic and goal-directed feedback to guide students' learning.
	Reversibility. Having students solve problems presented in different ways for a given solution.
	Flexibility. Teaching students ways to apply different strategies for solving problems.
	Generalization. Teaching mathematical patterns and structures that can be applied to similar core concepts across similar problems.

Figure 1. Actionable 10 Practices Checklist.

model skills being taught, and provide scaffolded practice to achieve mastery. For example, modeling is a powerful instructional tool. Teachers begin explaining procedural steps or mathematical concepts to show students what they are supposed to do. As an example, a lesson plan provided by the National Center on Intensive Intervention shows how teachers can model to solve addition problems through demonstrating the skill and describing what is being done.

1. Present addition problem (2 + 5 =)
2. Use colored bears or other concrete manipulatives
3. Show the number 2 by counting out 2 bears of the same color
4. Place the two bears next to the number 2
5. Show the number 5 by counting out 5 bears of a different color than the 2
6. Place the five bears next to the number 5
7. Explain to students that the plus sign tells us to add
8. Explain that when we add, we put things together
9. Say, "to solve the problem, we put the 2 bears together with the 5 bears." The answer is the number of bears. Let's count together. 1, 2, 3, 4, 5, 6, 7
10. Say, "The equation to 2 + 5 = 7" (National Center on Intensive Intervention, 2016, p. 2).

Teachers can help students to build conceptual understanding and procedural fluency through the use of clear objectives, modeling, and practice (Doabler et al., 2017).

Actionable Practice 2: Contextual Teaching

Contextual teaching is the process of teaching mathematics to students in a way that connects real-world situations to the concepts being studied by integrating students' everyday experiences into the problems (Berns & Erickson, 2001). Instead of teaching mathematical concepts solely through problems in textbooks, contextual teaching engages students in problem-solving through the concrete application of problems (Williams, 2007). The National Council of Teachers of Mathematics (NCTM, 2000) suggested that "The need to understand and be able to use mathematics in everyday life and in the workplace has never been greater" (p. 4). Many researchers also have emphasized the importance of embedding real-world contexts in mathematics lessons (Williams, 2007). For example, the contextual problem could be delivered to students as a word problem in the following way:

> Last night [you] finished making [your] guest list for Snoopy's surprise birthday party. Because there will be 30 people, [you] will need to borrow some square card tables that seat one person on each side. [You] want to arrange them in a long row, end to end. What is the least number of card tables [you] need?" (Rachlin et al., 2001, p. 13)

For students with a learning disability (LD) in mathematics, providing real-life examples in mathematics lessons can help them engage in and apply the mathematics with examples from their everyday experiences.

Table 1. Conducting Error Analysis.

Problem	Example of Student Answer	Error Type
Fraction $\frac{1}{2}+\frac{3}{5}$	Student A : $\frac{4}{7}$	Adding fractions without finding the common denominator
	Student C : $\frac{5}{10}+\frac{3}{10}=\frac{8}{10}$	Numerator change errors
Place value $59 + 24$	Student D : $59 + 24 = 713$	Misunderstanding of regrouping
	Student E : $59 + 24 = 35$	Performing incorrect operation

Actionable Practice 3: Precise Mathematical Vocabulary

Building precise mathematical vocabulary refers to promoting accurate language for mathematical teaching (Hughes et al., 2016). The use of precise mathematical vocabulary plays a vital role in teaching mathematics to students because talking about the mathematics can potentially foster conceptual understanding and procedural knowledge and support communication across and within students for the CCSSM (NGACBP/CCSSO, 2010). Teachers should use precise mathematical vocabulary and expect students to use this precise vocabulary for instruction. Teachers also should consistently use the same terms in subsequent lessons. When students become adept at using the correct mathematical terminology, they can use this knowledge to better communicate with teachers and other students about the mathematics while improving their understanding.

To illustrate, in a lesson about fractions teachers should use precise mathematical language to read a fraction correctly. Teachers can write $\frac{3}{4}$ on the board and ask students to read the fraction. The student might reply, "Three over four" or "three out of four." The teacher can clarify that $\frac{3}{4}$ is read as "three-fourths" because it represents the magnitude of the number (Hughes et al., 2016). Teaching with the appropriate mathematically precise language can contribute to students' ability to discuss the mathematics in group work.

Actionable Practice 4: Error Analysis

Error analysis is a strategy that identifies student mistakes during a task and determines areas in which they need additional support through instruction (Kingsdorf & Krawec, 2014). Error analysis has been proven effective in identifying detailed information about student errors in basic computational skills and procedural and conceptual knowledge (Kingsdorf & Krawec, 2014). Researchers have confirmed that identifying error details in computation, fractions, and word problem-solving can explain students' underachievement more specifically (Ashlock, 2009). For example, a teacher can examine how students solve a fraction problem to identify where they have difficulty. If a student is asked to solve a fraction problem or a computation problem involving place value, there are several common error patterns teachers can identify (see Table 1).

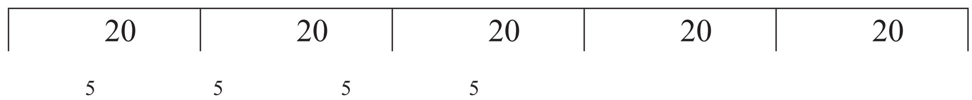
During the interaction, the teacher has the opportunity to determine the source of a student's difficulty such as identifying whether the student is experiencing difficulties with understanding the concept of the numerator and denominator. Identifying student errors can give teachers direction as to where they should focus their instruction (Cohen & Spenciner, 2010).

Actionable Practice 5: Mathematical Discourse

Encouraging mathematical discourse refers to teachers incorporating mathematical conversations into activities as students engage in mathematical reasoning (Cobb, 2006). Researchers found that students' mathematical outcomes significantly improved when teachers adopted student discussion as an integral part of the lesson (Ing et al., 2015). Students with MLD face challenges when they attempt to solve mathematical problems. Instructors can prompt them to express their thoughts by utilizing a variety of question strategies that gather information, probe thinking, make the mathematics visible and connected, and encourage reflection (NCTM, 2014). Questioning strategies for mathematics guides teachers to better understand student thinking and by intervening at the point of difficulty (Ing et al., 2015).

An example of using probing thinking in practice is, instead of simply instructing students to "discuss with your classmates the problem on the board with your classmates," teachers can use four ways to encourage mathematical discourse (Schumacher et al., 2019). The following example is a

Schematic Diagram Representation



x = total time

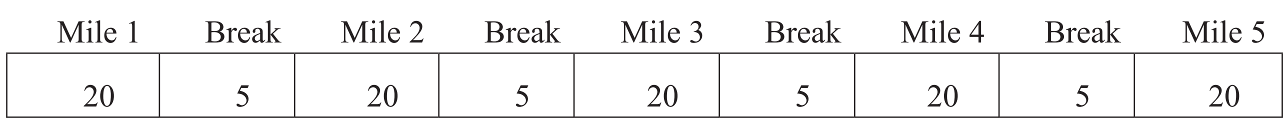
$x = (5\ \ 20) + (4\ \ 5)$

x = 120 minutes

Tabular Representation

Miles	Minutes	Break	Total Time in Minutes
1	20	5	25
2	20	5	50
3	20	5	75
4	20	5	100
5	20	End	120

Strip Diagram Representation



$4\ (20+5) + 20 = x$

Figure 2. An example of multiple representations of Jack's problem.

hypothetical scenario on how to support mathematical discourse, drawn from Schumacher et al. (2019). First, teachers encourage students to clarify an approach. For instance, they can say "Tell me more about how you answered the question." Second, during confirmation, teachers repeat the explanation that a student gave in a new way, "Have I understood correctly that X means Z?" Third, leading, the teacher should then ask the students, "How can you solve this question?" Leading allows the students to see the question as a bigger picture that requires a logical thought process. Elaboration can complete this process in which teachers have an opportunity to (a) summarize the student's thinking and (b) expand on the information that the students provided. The elaboration process can be another way of providing extended feedback (Schumacher et al., 2019).

By gathering information, probing student's thinking, making mathematics visible and connected, and encouraging reflection and justification (NCTM, 2014), teachers allow students to formulate a response and justify their answers by explaining their reasoning. Furthermore, student discussion allows teachers to evaluate student understanding (Ing et al., 2015).

Actionable Practice 6: Multiple Representations

Multiple representations are the depiction of a concept in two or more forms such as verbal, graphical, symbolic, and tabular (Nielsen & Bostic, 2018). Multiple representations are one of the eight effective mathematics teaching practices outlined in *Principles to Actions: Ensuring Mathematical*

Success for All (NCTM, 2014). The use of multiple representations allows students different ways to show a problem. Students also can switch between representations until they are able to understand the problem and proceed along a path that will lead them to a solution (NCTM, 2014). If students use multiple representations, they are more likely to connect these mathematical representations to previously learned mathematics topics and, in turn, build greater conceptual understanding (NCTM, 2014; Nielsen & Bostic, 2018).

For instructional guidance, teachers can select appropriate multiple representations for a specific mathematics problem's structure. The use of multiple representations can promote students' ability to link problem information to the multiple representations. Below is an example of introducing a problem along with various types of multiple representations:

> Jack participated in a 5-mile run this morning. Usually he runs 3 miles in 60 minutes. However, Jack pulled a muscle a few days previously, so he had to take a 5-minute break after every mile. How much time did it take Jack to complete the full 5 miles? (Adapted from Schumacher et al., 2019, p. 24).

Teachers can use symbolic, graphical, and tabular representations to represent an equation in various ways. Figure 2 illustrates the use of multiple representations for solving this problem.

Actionable Practice 7: Constructive Feedback

Constructive feedback refers to providing strategic and goal-directed feedback to guide student learning and behavior (McLeskey & Brownell, 2015). Using constructive feedback helps students understand areas that are their strengths and those that need improvement with ways to improve their performance. In comparison, ineffective feedback is where the teacher says "right" or "try again" immediately after the student gives an answer. Students with LD in mathematics need prompts and specific feedback about their work to guide them to better performance. Students are able to improve their learning by receiving consistent instructional guidance. Providing feedback motivates students to progress in their learning in the classroom setting. Teachers use feedback as a way of motivating, correcting, and communicating with students during their lessons (McLeskey & Brownell, 2015). The following example is a hypothetical scenario on how to provide constructive feedback based on the scholarship of McLeskey and Brownell (2015).

For example, "You answered the question incorrectly" would be ineffective feedback. The following would be constructive feedback:

> You correctly added 8 out of the 10 questions which is great! You solved two out of 10 questions incorrectly because you misread the addition symbol as the subtraction symbol. Next time, you could highlight the symbols and identify what they mean before attempting the questions. If you correctly identify the computational symbol, it will increase your accuracy.

Providing constructive feedback enables students to enhance their reasoning and understanding of concepts and procedures.

Actionable Practice 8: Reversibility Tasks

Reversibility tasks change the direction of student thinking. Many students think of mathematics as a series of sequential, linear steps. If the steps are followed, a correct answer is forthcoming (Dougherty et al., 2015). However, if there is a slight variation in a problem, students are often not sure how to proceed. By using reversibility questions, students develop more flexible thinking. These questions provide solutions for students to construct problems that have the given solution. Reversibility tasks enable students to develop different paths to a given solution and access the task at their own level (Dougherty et al., 2015). A teacher provides a solution and asks the students to generate multiple problems for that solution. Students can access an infinite number of solutions to the problem and perhaps gain a deeper understanding of constructing the problem (Dougherty et al., 2015).

For example, a teacher can ask the class to find expressions that simplify to 24. Teachers can use the reversibility task to support conceptual and procedural understanding across multiple mathematical areas. This is a simple activity that allows students to come up with a variety of problems based on their current level of understanding.

A teacher would say, "Write at least three expressions that can be simplified to 24."

Possible problems could include $(10 + 10 + 4)$, $(30 - 6)$, (12×2), $\left(\frac{48}{2}\right)$, and $(4[6 - 1] + 2^2)$, each demonstrating a different level of sophistication in the students' thinking. Even though some of these responses may appear to be more simplistic, a question of this type keeps students engaged in the discussion as opposed to their engagement when a factual question is asked, with only one answer. Students who struggle often wait for others to answer but when given a reversibility question, there is an expectation that all students can provide an answer. Given time for students to think independently, and then share out either in pairs or to the whole class, students with MLD have opportunities for discussion. Teachers allow more time for students with MLD to create a response as necessary since reversibility task are involved in thinking process. Through the use of reversibility questions, students gain flexibility in their thinking and can create problems that are at their level of understanding, promoting more engagement.

Actionable Practice 9: Flexibility Tasks

Flexibility tasks promote multiple solution methods and multiple strategies for exploring open-ended questions (Dougherty et al., 2015). Giving a flexibility task also provides opportunities for students to identify similarities and differences across problems (Schukajlow et al., 2019). Researchers found that when teachers presented multiple solution strategies for solving the same problem, students demonstrated improvement in procedural flexibility, conceptual knowledge, and procedural knowledge (Dougherty et al., 2015). Teachers should give students with LD in mathematics further opportunities to improve their thinking process by finding other ways to solve the same problem (Schukajlow et al., 2019).

For example, a teacher can ask students to solve a problem in multiple ways or to use what they know about one problem to solve another one by asking

1. "Solve the problem in another way."
2. "How are these solution strategies alike? How are they different?"

Mathematics teachers can help students develop their critical thinking skills instead of simply solving mathematical problems that have one solution. Giving students the option of solving a problem in a different way gives students the opportunity to expand their thinking (Achmetli et al., 2019; Schukajlow et al., 2019). More importantly, asking students how solution strategies are alike or different focuses students' attention on problem structures or characteristics that support students in solving similar problems (Dougherty et al., 2015).

Actionable Practice 10: Generalization Tasks

Generalization tasks prompt students to identify patterns within and across problems (Dougherty et al., 2015). Students with MLD have difficulties finding patterns that may help them develop concepts and solve broader classes of problems. Once students understand the core concepts, they can use this knowledge to solve various problems that use similar concepts. If students can generalize patterns, they can use those generalizations to support their solutions to similar problems. An example of a generalization question is: "What is the maximum number of digits in the sum when adding two three-digit numbers?" Students would generate examples to find a generalization about the size of the sum. This type of generalization question helps students predict what to expect for a sum when doing this addition and to determine whether a sum is reasonable. To have students see the patterns needed to arrive at the generalization, teachers should ask specific questions that focus on students' attention on big ideas that lead to more significant learning.

Using the Checklist Every Day

Teachers should reference the Actionable 10 Practices Checklist in preparation for class as a reminder to embed evidence-based strategies when designing their lessons. To be specific, teachers should consult the checklist prior to beginning their lessons. Then, they should review instructional practices that can be embedded in their lesson plan. Teachers can target particular instructional practices or cover all 10 instructional practices during the lesson. The use of a checklist format encourages teachers to consider all 10 practices throughout their lessons. Teachers can also monitor methods of enhancing their instructional practice in their classroom.

Conclusion

This article provides a list of evidence-based practices in a checklist format, which facilitates the application of these practices. It is designed to support teachers in creating a high level of active student engagement and participation. Teachers can monitor their students' progress as it relates to integrating these evidence-based strategies in the checklist, as well as set professional goals. Teachers who use the checklist could also take notes or keep a journal to assess how effective it is in the classroom and how often it is used in an everyday setting.

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ORCID iD

Soyoung Park https://orcid.org/0000-0001-6178-6818

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